POLITECNICO
MILANO 1863

HW2 report

Traffic Theory

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1. P_1

a. **Analyze the Limiting Distribution starting from different initial state distribution.**

We start by analyzing the limit of the transition probability matrix P1 to check if there is a convergence.

We start from 4 different initial state distributions:

```
P1_01 = [0.1 0.3 0.4 0.2];
P1_02 = [0.8 0.2 0 0];
P1_03 = [0.75 0.1 0.1 0.05];
P1_04 = [0.4 0.4 0.2 0];
```

But it doesn't converge to a proper value.

b. **Provide the Steady State Distribution, if exists.**

Since the matrix converges to different points, it doesn't exist.

c. **Analyze the limit: $\lim_{n\to\infty} P^n$.**

0.5385	0.4615	0	0
0.5385	0.4615	0	0
0	0	0.5000	0.5000
0	0	0.5000	0.5000

d. **Is the Stationary Distribution unique? Motivate.**

No, it's not because they converge to different values.

- **If it is not unique, list all possible (or provide a function to find them) Stationary Distribution and verify numerically they are indeed stationary.**

For process P1 we have two classes (1-2 and 3-4) with distributions, respectively, [0.5385, 0.4615] and [0.5 0.5]. The distributions are the linear combination of the padded distribution of these two classes.

2. P_2

a. Analyze the Limiting Distribution starting from different initial state distribution. Analyze the limit: $\lim_{n\to\infty} P^n$.

We start by analyzing the limit of the transition probability matrix P2 to check if there is a convergence.

We start from 4 different initial state distributions:

```
P01 = [0.1 0.3 0.4 0.2 0];
P02 = [0 0.7 0.1 0.2 0];
P03 = [0 1 0 0 0];
P04 = [0.4 0.4 0 0.2 0];
```

And they all converge to the vector [0 0 1 0 0]:

0	0	1.0000	0	0
0	0	1.0000	0	0
0	0	1	0	0
0	0	1.0000	0	0
0	0	1	0	0

As we can see, the limit of the transition probability matrix P2 exists which means we have a limiting distribution.

b. Provide the Steady State Distribution, if exists.

For every initial state probability distribution evolves to the same state probability distribution. Since the limiting distribution converges, we have a Steady State Distribution.

c. Is the Stationary Distribution unique? Motivate.

The stationary distribution is a probability distribution that remains unchanged by the transition probabilities and it's unique for irreducible and aperiodic Markov chain. When dealing with the stationary distribution, we can analyze a non-irreducible process by breaking it down into closed communicating classes, treating each as an irreducible process. Each individual closed communicating class within the process has one stationary distributions. In this context, we can examine two processes, P1 and P2. For P1, which comprises two classes (1-2 and 3-4), the respective stationary distributions are [0.5385, 0.4615] and [0.5, 0.5]. On the other hand, P2 consists of three classes (1-2, 3, and 4-5), each with its own stationary distribution. 1-2 and 3-5 are 0, so 3 it's the unique stationary distribution.

- **If it is unique, compute the Stationary Distribution by solving GBE in MATLAB**

Calculating it, the result is [0, 0, 1, 0, 0].

```
A = (P2 - eye(size(P2)))';
b = zeros(1,length(P2))';

An = [A(1:end-1,:); ones(1,length(P2))];
bn = [b(1:end-1); 1];

pi_transp = An \ bn;
pi = pi_transp';
```