Portfolio Management: Assignment 1

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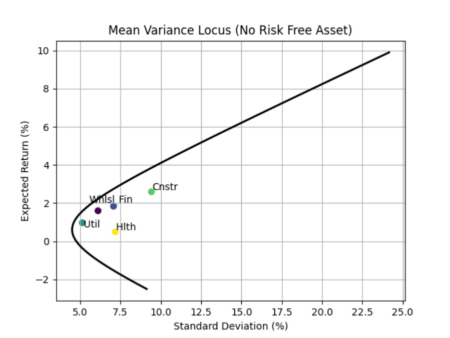
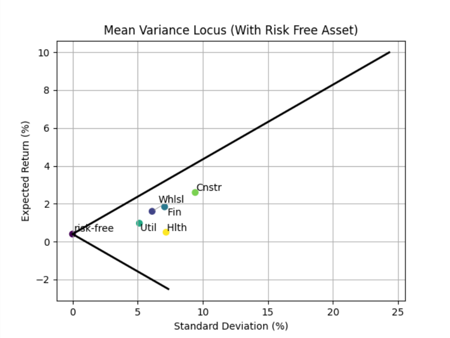
HEC Montréal

Part A: Mean-Variance Optimization

Mean-Variance Locus

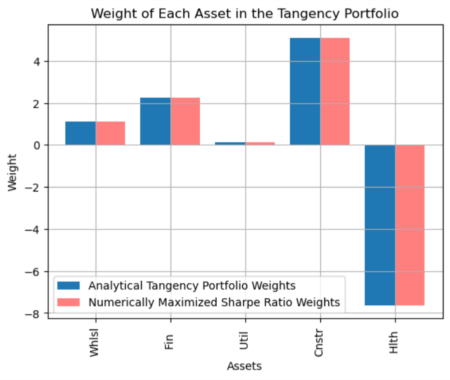
Industries selected arbitrarily for analysis: Wholesale, Financial, Utilities, Construction, and Health

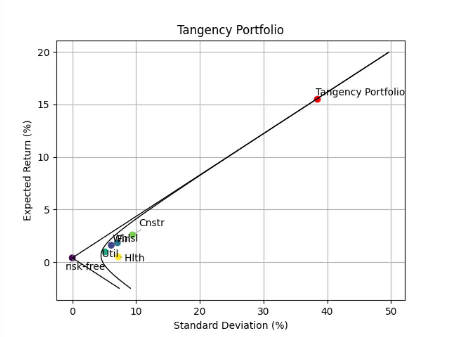




Tangency Portfolio

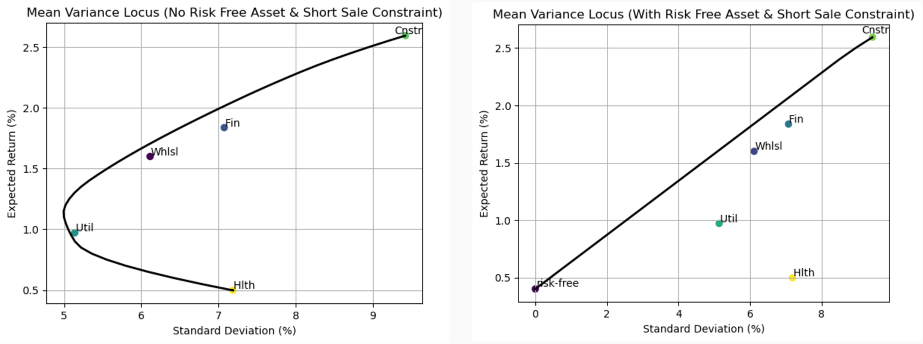
- Solve for the tangency portfolio analytically, verify numerically
- SR = 0.394, but with massive long/short positions
    - e.g. Short almost 800% of the portfolio in the Health Industry and taking a long position of almost 450% in the Construction Industry





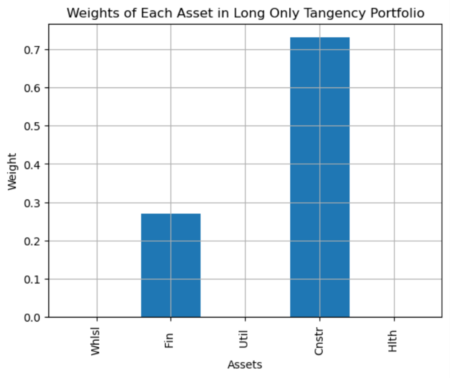
Short-Sale Constraints

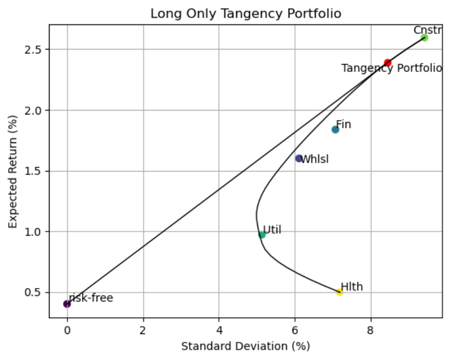
- Under the short sale constraint, returns are bounded from below by the risk-free asset and highest returning asset from above



Short-Sale Constraints

- Tangency portfolio under the short-sale constraint has $\hat{\mu} = 2.39\%$, $\hat{\sigma} = 8.45\%$ and thus $SR = \frac{2.39\%}{8.45\%} = 0.236$

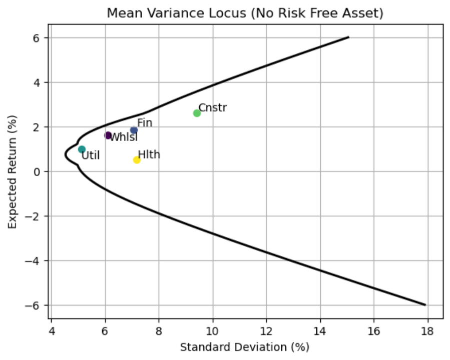


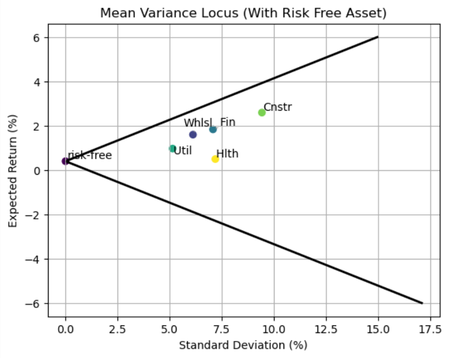


Part B: Uncertainty and Portfolio Selection under Constraints

Asset Constraints

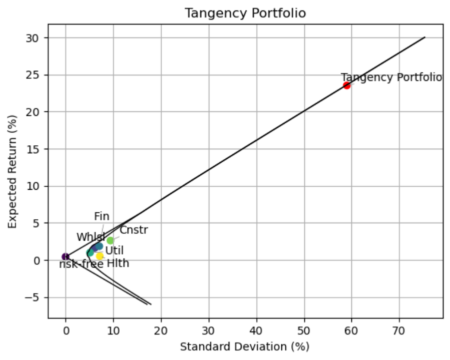
- Reproducing part A with the constraint on selecting a maximum of 3 out 5 assets:

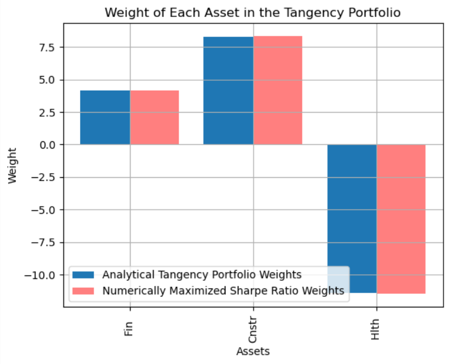




Asset Constraints

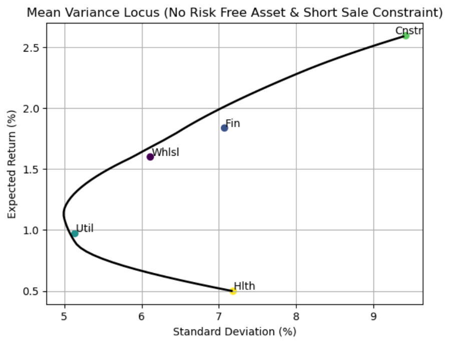
- Tangency portfolio:
  $\hat{\mu} = 23.53\%$ & $\hat{\sigma} = 58.87\% \implies SR = 0.39$
- Industries included: Fin (414%), Cnstr (833%), Hlth (-1148%)

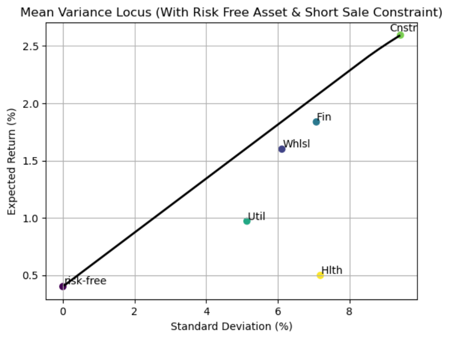




Asset Constraints & Short Sale Constraints

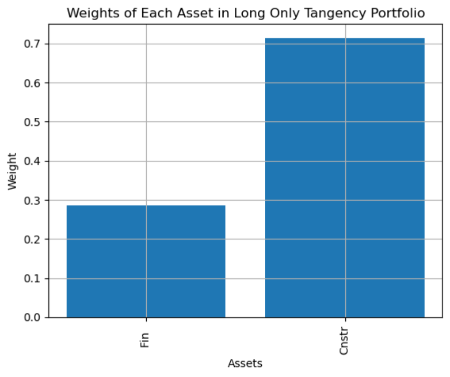
- Reintroducing the short sale constraint:

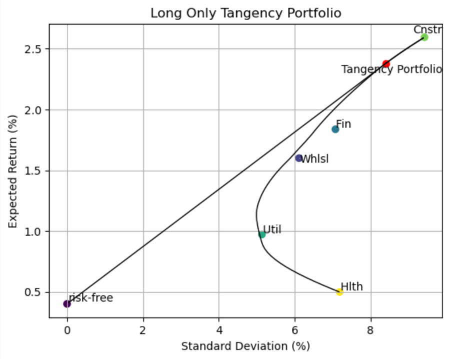




Asset Constraints & Short Sale Constraints

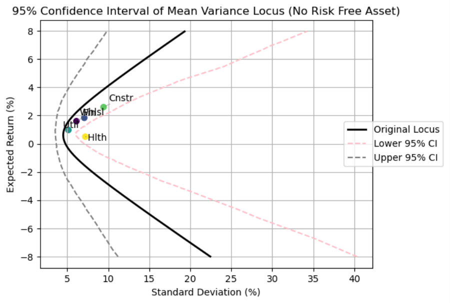
- Tangency portfolio:
  $\hat{\mu} = 2.39\%, \ \ \hat{\sigma} = 8.45\% \implies SR = 0.235$
- Industries included: Fin (28.6%), Cnstr (71.4%)

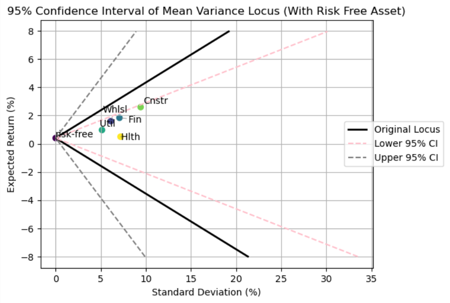




Bootstrapping the Mean-Variance Portfolio

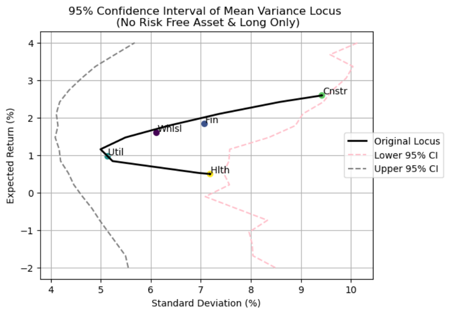
- Bootstrap procedure helps visualize the uncertainty around the estimation of the mean-variance locus
- We generate 1000 bootstrapped samples original 5 industries, and form the 95% confidence interval for each targeted return.

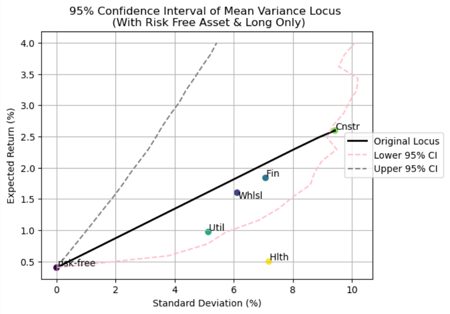




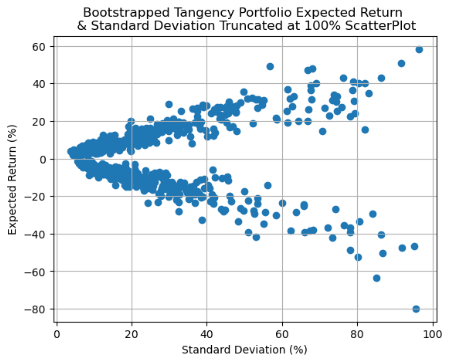
Bootstrapping the Mean-Variance Portfolio

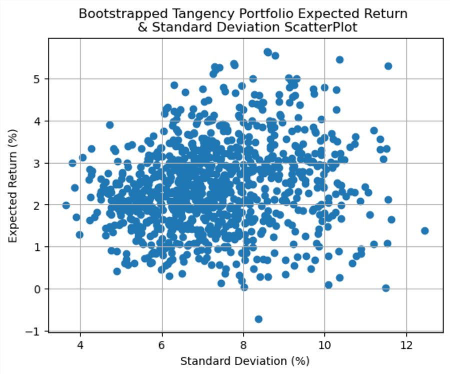
- Applying the same procedure to the long-only M-V locus:





Bootstrapping the Mean-Variance Portfolio





48C5 and Max Sharpe Ratio

- Selecting 5 assets from all 48 and optimizing for Sharpe ratio
  - `scipy` is too sensitive to initial parameters and only finds local optima, brute force approach is too slow
  - We use `gurobipy` and minimize variance for a range of possible returns and consider largest Sharpe Ratio among these to be the global maxima.

$$\text{Sharpe Ratio} = \frac{R_{portfolio} - R_f}{\sigma_p}$$

48C5 and Max Sharpe Ratio

$$
\begin{aligned}
\min_{x} \quad & x^\top \Sigma x \\
\textbf{subject to} \quad & \textstyle\sum_{i=1}^{n} x_i = 1, \\
& x_i \leq \ell \, b_i, \quad \forall\, i = 1, \ldots, n, \\
& \textstyle\sum_{i=1}^{n} b_i \leq 5, \\
& \textstyle\sum_{i=1}^{n} x_i \, \mu_i = \text{target\_return}, \\
& x_i \geq 0, \quad \forall\, i = 1, \ldots, n, \\
& b_i \in \{0, 1\}, \quad \forall\, i = 1, \ldots, n.
\end{aligned}
$$

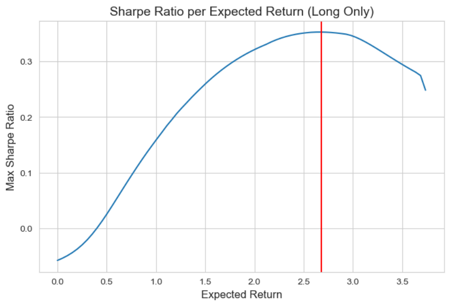
- x: Asset weights
- Σ: Covariance matrix
- ℓ: Max allocation
- b_i: Asset selection
- μ_i: Expected return
- target_return: Desired return

48C5 and Max Sharpe Ratio

Long only results:

- Agric: 3.9%
- Guns: 13.4%
- Coal: 22.1%
- Chips: 60.6%

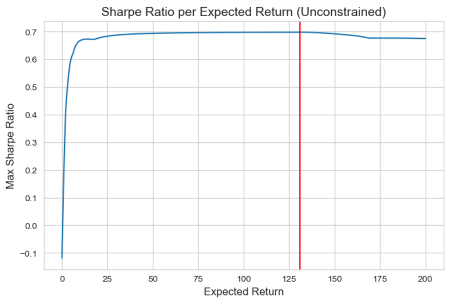
- $\hat{\mu} = 2.68\%$
- $\hat{\sigma} = 6.46\%$
- $SR = 0.35$

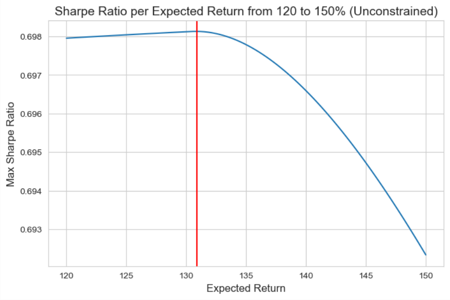


48C5 and Max Sharpe Ratio

With short selling and unbounded weights:

- Flat peak
- Sharpe ratios above theoretical maximum
- Crazy weights (e.g. Chems: -5000%!)
- $\hat{\mu} = 130.91\%$, $\hat{\sigma} = 186.94\%$, $SR = 0.70$



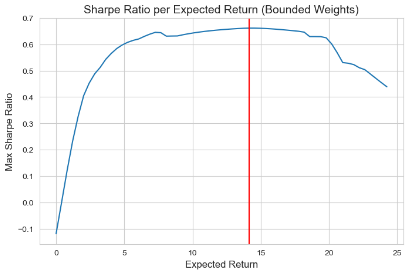


48C5 and Max Sharpe Ratio

With short selling with bounded weights:

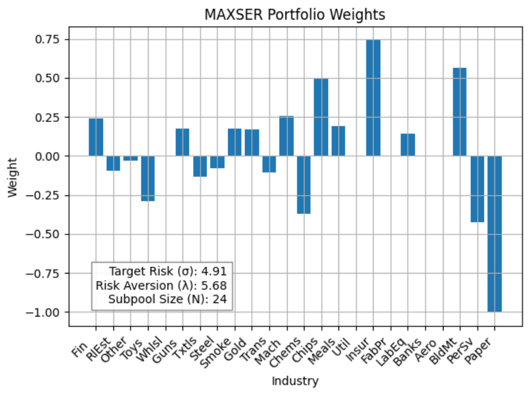
- Bounded weights between $\pm 300\%$
- Significantly improves the process with only a slight hit to the Sharpe ratio
- $\hat{\mu}$: 14.141%
- $\hat{\sigma}$: 20.783%
- SR : 0.66

- Toys: -173.75%
- Chems: -300.00%
- Constr: 241.95%
- Coal: 108.89%
- Chips: 222.91%



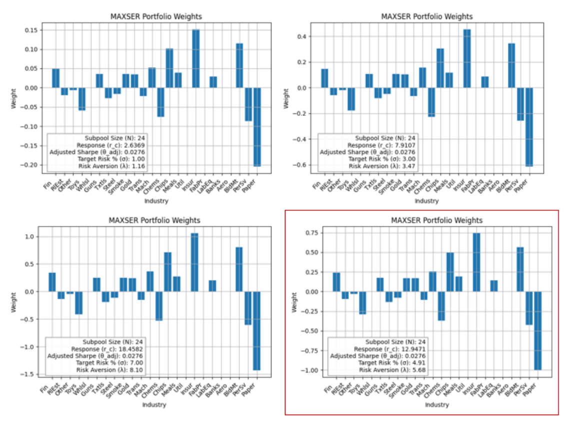
MAXSER

- Base case scenario:
  - $N = 24$ subpool size
  - $\sigma_{tgt} = 4.91$ target risk level (naive risk from the equally weighted 48-asset portfolio)
  - $SR = 0.6278$



MAXSER

- Portfolio weights for $\sigma_{tgt} = \{1, 3, 4.91, 7\}$:
  - Proportional fluctuation in weights
  - Total amount invested increases
  - Sharpe Ratio increases



MAXSER

- Portfolio weights for $N = \{12, 24, 36, 48\}$:
  - Total amount invested decreases as N becomes large
  - Likewise, the Sharpe Ratio decreases