3

Project specification – Part II

The aim of this second part is to design and develop a fuzzy inference system to fix the deficiencies of Eq. (1.5), explained in Section 1.5.3.

3.1 CIE L*C*h color space

The CIE L*C*h color space is a perceptually uniform color space that represents colors by means of *lightness* (L^*), *chroma* or *saturation* (C^*), and *hue angle* or simply *hue* (h or h_{ab}). The L*C*h color space has the same diagram as the L*a*b* space, but uses cylindrical coordinates, as shown in Fig. 3.1. Given a

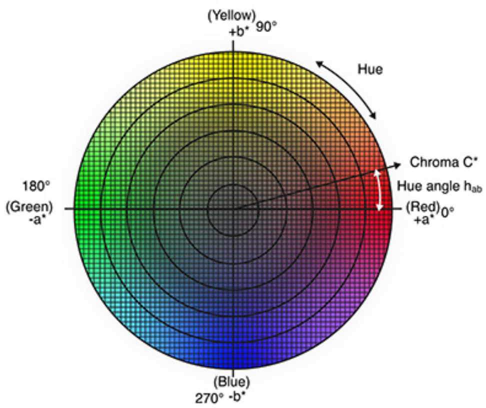

Figure 3.1: A slice of the L*a*b* color space at L*=50.

color (L^*, a^*, b^*), the C^* and h coordinates can be obtained according to the following equations:

$$C^* = \sqrt{(a^*)^2 + (b^*)^2} \tag{3.1}$$

$$h = \begin{cases} \arctan\left(\frac{b^*}{a^*}\right) & \text{if } \arctan(b^*/a^*) \geq 0 \\ \arctan\left(\frac{b^*}{a^*}\right) + 360^\circ & \text{otherwise.} \end{cases} \tag{3.2}$$

The value C^* of chroma is the distance from the lightness axis. The higher C^*, the higher the color saturation and purity. Chroma is 0 along the L* axis, and its maximum value C^*_{max} depends on the value of L*. For example, if $L^* = 50$ then C^*_{max} is approximately equal[1] to 127.

Values of L^* that are lower/higher than 50 lead to lower values of C^*_{max}. Chroma is also expressed as a percentage. Looking at Fig. 3.1, the color at the center of the circle has $C^* = 0$, whereas the colors on the outermost circumference (i.e., the surface of the L*a*b* sphere) are those with 100% saturation/chroma.

The hue angle is expressed in degrees, and starts from the +a* axis, where $h = 0$. Given a slice of the L*a*b* space, the hue angle varies in $[0°, 360°)$ counterclockwise. The hues at 0°, 90°, 180° and 270° lie on the four axes that correspond to the L*a*b* primaries red, yellow, green, and blue, respectively.

A CIE L*a*b* color can easily be converted into the corresponding CIE L*C*h coordinates by using either the `lab2lch()` conversion function of the *optprop* toolbox, or by means of Eqs. (3.1) and (3.2). The reverse transformation can be done with function `lch2lab()`[2].

3.2 Structure of the fuzzy system

The system to develop is a Mamdani-type fuzzy inference system, shown in Fig. 3.2. The system takes four inputs and has one output.

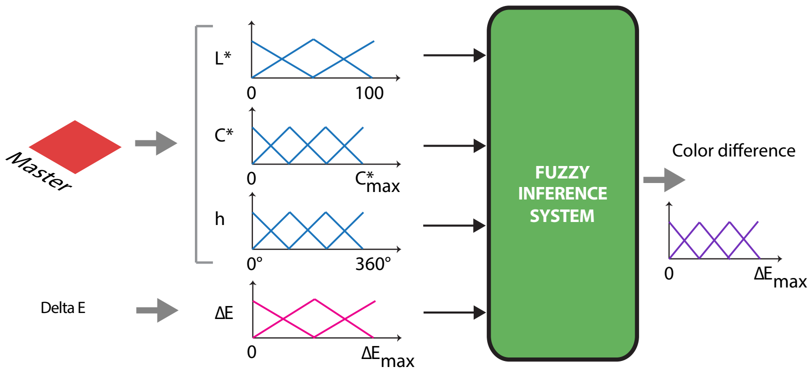

Figure 3.2: The fuzzy inference system.

The first three inputs stem from an appropriate linguistic modeling of the (L^*, C^*, h) coordinates, which indicate a specific region of the color space. The fourth input stems from the linguistic modeling of $\Delta E^\star_{ab}$, from now on simply referred to as ΔE. When designing the system, consider the values of ΔE that

[1] More precisely, when varying the hue at $L^* = 50$, you have that $C^*_{max} \in [127, 128]$ because $a^*, b^* \in [-128, 127]$. There is thus a little bit of asymmetry at any L^*, and then C^*_{max} slightly depends on h. However, you can consider 127 as the maximum value of C^* for all the hues at $L^* = 50$, and can suppose that the maximum value of C^* is the same for all hues at a given L^*.

[2] Before using these conversion functions, as well as all the other functions of the toolbox, read the help documentation carefully.

range from 0 to a maximum value ΔE_{max}. A reasonable maximum value may be $\Delta E_{max} = 10$.

The linguistic modeling of the inputs has to be performed by using appropriate sets of membership functions. Pay particular attention when designing the linguistic modeling of C^*, because the maximum value of this color coordinate changes depending on the value of L^*, as explained in the previous section.

The output of the system is a linguistic variable that expresses the perceptual difference, based on the input value of ΔE, between two colors that come from the region identified by L^*, C^* and h. The linguistic modeling of the output has to be performed by means of appropriate membership functions.

3.3 How to use the fuzzy inference system to improve ΔE

The output of the fuzzy inference system can be used to train the neural network described in Part I. This makes it possible to obtain an improved version of ΔE, whose behavior is the same whatever the region of the L*a*b* space where the difference between two colors is measured. As a result, using this improved measure, a given amount of change applied to two different colors is perceived the same way by a human observer, no matter the regions of the L*a*b* space the two colors come from.

In order to achieve this result, the output of the fuzzy inference system described in this section has to be first defuzzified and then used to replace the value of ΔE calculated using Eq. (1.5), where needed.