Computational Statistics II
Assignment 2: Comparison of Bootstrap confidence intervals

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Aim of this assignment is to evaluate and compare the performances of the Bootsterap confidence intervals on simulated data sets. The objective is to estimate the mean of the population. You will be asked to simulate 1D samples of size $n = 10$ and $n = 100$, and to compare the obtained performances.

1. Write down a theoretical description of the Bootstrap confidence intervals in the case of estimating a generic the parameter of one population. Describe briefly all studied confidence intervals specifying their differences (max 1 page).

2. Simulate data from: Normal distribution; Chi-Squared distribution; Cauchy distribution. Choose the parameters of Normal and Chi-Squared distributions so that the mean is zero and the variance 1. For the Cauchy distribution, choose location = 0 and scale = 1. Explore the cases $n = 10$ and $n = 100$. Compute the Boostrap estimates of the mean of the population and its standard error.

3. Compute the classical χ^2-distribution confidence intervals for the mean of the popolation and the Bootstrap intervals: asymptotic normal, Bootstrap-t, percentile, and BCa.

4. Evaluate the coverage probabilities of all confidence intervals in the cases $n = 10$ and $n = 100$, and for the three distributions. Compare the performances of the intervals. Is it what is expected from theory?

5. **Bonus.** Compute the BCA confidence intervals of previous point 4, and its coverage probability, and compare its performances with the previous case.