

Computational Statistics II

Assignment 3: a simple EM

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Aim of this assignment is to write in a simple case, the code for performing an EM algorithm on a mixture of bi-variate Gaussians with $K = 3$ components of the mixture, and to apply it for performing a simple clustering.

1. Describe the EM algorithm applied to a mixture of Gaussians. Specify the properties of the method (without proving them), and underline the differences with the k-means algorithm (max. 1 page).
2. Simulate a data set of sample size $n = 100$ from a mixture of gaussians with 3 components in the two following scenarios:

$$\text{A } \mu_1 = (0, 0)', \mu_2 = (4, 3)', \mu_3 = (3, -1)', \\ \pi_1 = \pi_2 = \pi_3,$$

$$\Sigma_1 = \Sigma_2 = \Sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{B } \mu_1 = (0, 0)', \mu_2 = (4, 3)', \mu_3 = (3, -1)', \\ \pi_1 = \pi_2 = \pi_3,$$

$$\Sigma_1 = \begin{pmatrix} 1 & -0.9 \\ -0.9 & 1.5 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 1 & -0.4 \\ -0.4 & 0.45 \end{pmatrix}, \quad \Sigma_3 = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 0.9 \end{pmatrix}$$

3. In both scenarios, initialize the parameters of the EM using $\pi_1 = \pi_2 = \pi_3 = 1/3$, $\Sigma_k = I$, and choose some random values for the three means. Perform an E-step and an M-step. Visualize the results of this first iteration.
4. Apply the EM to the two data sets (scenarios A and B), iterating through E-step and M-step. Use the convergence of the log likelihood as stopping criterion.
5. Use a threshold on the estimated responsibilities to assign with EM each data point to a cluster. Compare the results with what was obtained from the k -means in terms of rate of misclassified points in both scenarios.
6. **Bonus.** Use a MC simulation based on 100 runs to estimate the rate of misclassified points in the two scenarios.