

Computational Statistics II

Assignment 2: Bootstrap confidence intervals

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Aim of this assignment is to evaluate and compare the performances of the Bootstrap confidence intervals for estimating the variance on simulated data sets. You will be asked to simulate 1D samples of size $n = 10$ and $n = 100$, and to compare the obtained performances.

1. Write down a theoretical description of the Bootstrap confidence intervals in the case of estimating the parameter of one population. Describe briefly all studied confidence intervals specifying their differences (max 2 pages).
2. Simulate data from: Normal distribution; Uniform distribution; Cauchy distribution. Choose the parameters of Normal and Uniform distributions so that the mean is zero and the variance 1. For the Cauchy distribution, choose location = 0 and scale = 1. Explore the cases $n = 10$ and $n = 100$.
3. Compute the classical t-distribution confidence intervals and the Bootstrap intervals: asymptotic normal, Bootstrap-t, percentile, and BCa for estimating the variance. Use the unbiased variance estimator.
4. Evaluate the coverage probabilities of all confidence intervals in the cases $n = 10$ and $n = 100$, and for the three distributions. Compare the performances of the intervals. Is it what is expected from theory?
5. **Bonus.** Compute the BCA confidence intervals of previous point 4, and its coverage probability.