Numerical Methods For Finance: Groups 2 and 4

This assignment gives you from 0 to 5 points. The deadline is April 8th 2023 at 11.59 p.m.. The result of the assignment is valid for the academic year 2022/2023.

Using the Option prices in the file excel `Dataset.xlsx` (sheet of your group number) solve the following points:

1. Clean your dataset removing the options that do not satisfy the no-arbitrage bounds. Determine the percentage of the options in the cleaned dataset.

2. Use the remaining options to obtain the volatility smile.

3. Use the interpolated Volatility smile, to price the strategy composed of the following positions:

    - 1 Long position in a Call with Strike price $1.05 * S_{t_0}$ and same time to maturity of the options in the dataset.

    - 1 Long position in a Put with Strike price $0.95 * S_{t_0}$ and same time to maturity of the options in the dataset

    Determine the probability of a negative profit (loss) for this strategy under the Martingale Measure $\mathbb{Q}$.

4. Using your dataset calibrate the $B\&S$ model by minimizing the Relative Mean Squared Error. Use the calibrated σ, compute the monte carlo price for a contingent claim (using B&S) with time to maturity 30 days and final payoff defined as:

$$\max\left\{\left(S_{t_N} - \prod_{j=0}^{N} S_{t_j}\right)^{\frac{1}{N+1}} ; 0\right\}$$

    where $t_0 = 0$ and $t_j = t_0 + j\Delta$ (Δ is 5 days). Determine Upper and Lower Bounds at 95% level. Analyse the behaviour of the MC price as the number of simulations increases.

To download the dataset from the excel file **you must use the function `read_excel` of the R package `readxl`. PLEASE DO NOT USE THE USER FREINDLY TAB IN R STUDIO**. Assume Continuously compound regime for the Interest rate on yearly basis. **You are not allowed to use `fOption`**.

Prepare a file.zip (`Group_numb.zip`, for example, Group1 uses `Group_1.zip`) containing the following files:

- A report (`report_Group_numb.pdf`) that describes the results of your analysis and reports details about the used methodology.

- The R files containing all functions and all command lines for reproducing all results in the report. I recommend to write two separate files: the file `functions.R` where you define all functions used for the analysis and the file `main.R` where you reports the command lines for reproducing the analysis. Comment properly your functions and codes using the symbol #.

- The dataset.

Verify that, after unzipping the file.zip, the R files reproduce perfectly your analysis.