Exercise 1 - Analytical exercises

- $f(x) = b^T a$

  $f_i = b_i a_i$

  $\frac{\partial f_i}{\partial a_k} = b_k \quad \forall k = 0, \dots, m-1$

  $\frac{\partial f}{\partial a} = b$

- $f(x) = a^T A a = \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} a_i a_{ij} a_j$

  $f_i = \sum_{j=0}^{m-1} a_i a_{ij} a_j$

  $\frac{\partial f_i}{\partial a_k} = \sum_{j=0}^{m-1} a_{kj} a_j + \sum_{i=0}^{m-1} a_i a_{ik} \quad \forall k = 0, \dots, m-1$

  $\frac{\partial f}{\partial a} = a^T A + a^T A^T = a^T (A + A^T)$

- $f(x) = (x - As)^T (x - As)$

  $f_i = (x_i - \sum_{j=0}^{n-1} a_{ij} s_j)^2$

  $\frac{\partial f_i}{\partial s_k} = 2 (x_i - \sum_{j=0}^{n-1} a_{ij} s_j) \cdot (-a_{ik})$

  $\frac{\partial f}{\partial s} = -2 (x - As)^T A$