Convex Optimisation

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Convex Optimisation?

- What is it?
  - Finding the maxima / minima of convex functions over convex sets with respect to convex or affine constraints
- Why do we care?
  - Convex functions display many theoretical properties that are suited to optimisation
  - Solving convex optimisation allows you to place a lower bound on non-convex functions
  - Many real world optimisations are convex

Convexity

A set $C \subseteq \mathbb{R}$ is convex if $\forall\, x, y \in C, \forall\, \theta \in [0, 1]$

$$\theta x + (1 - \theta) y \in C$$

i.e. all points on the line segment between x and y lie in C

Convex Functions

A function $f : C \subseteq \mathbb{R}^n \to \mathbb{R}^m$ is convex on some convex set C if $\forall\, x,\ y \in C, \forall\, \lambda \in [0, 1]$

$$f(\lambda x + (1 - \lambda)y) \leq f(\lambda x) + f((1 - \lambda)y)$$

picture of convexity or further explanation

Minima and Maxima

$f : S \subseteq \mathbb{R}^n \to \mathbb{R}$ has a global minimum (maximum resp.) $x^\star$ if:

$$\forall x \in S,\ f(x^\star) \leq f(x),\ \ (f(x^\star) \geq f(x))$$

f has a local minimum (maxmum resp.) around $x^\star$ if $\exists\, R \in \mathbb{R}$ such that

$$\forall x\ \in B(x^\star, R),\ f(x^\star) \leq f(x),$$

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