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Preliminary Definitions

Definition. *A matrix, $A \in \mathbb{R}^{m\times n}$ is positive semidefinite if:*

$$\forall x \in \mathbb{R}^n,\, x^T A x \geq 0$$

Definition. *A function f is affine if*

$$f(v) = Av + b$$

for some matrix A and some vector b.

Definition. *For some minimisation problem with objective function $f : \mathbb{R}^n \to \mathbb{R}$ and inequality constraints $g_i : \mathbb{R}^n \to \mathbb{R}$ and equality constraints $h_j : \mathbb{R}^n \to \mathbb{R}$ that are continuously differentiable at the optimal solution $x^\star$ the KKT conditions are (providing some constraints hold): $\exists\ \mu_i$ and λ_i such that*

$$\begin{aligned}
&\nabla f(x^\star) + \sum_{i=1}^{m} \mu_i \nabla g_i(x^\star) + \sum_{j=1}^{l} \lambda_j \nabla h_j(x^\star) = 0 \quad \textit{(stationarity)}\\
&g_i(x^\star) \leq 0 \quad \textit{(Primal Feasibility)}\\
&h_j(x^\star) = 0 \quad \textit{(Primal Feasibility)}\\
&\mu_i \geq 0 \quad \textit{(Dual Feasibility)}\\
&\mu_i g_i(x^\star) = 0 \quad \textit{(Complementary Slackness)}
\end{aligned}$$

Definition. *A point $x \in \mathbb{R}^n$ is a critical point of some $f : \mathbb{R}^{n\times m} \to \mathbb{R}$ if:*

$$\nabla f(x) = \boldsymbol{0}$$

Theorem. *Fermat's Theorem (critical points): If $x \in \mathbb{R}^n$ is a local minima of some function $f : \mathbb{R}^n \to \mathbb{R}$ then:*

$$\nabla f(x) = \boldsymbol{0}$$

Definition. *The ball of size $R > 0$ around a point $x \in \mathbb{R}^n$, written $B(x, R)$, is the set of all points $y \in \mathbb{R}^n$ such that:*

$$\|y - x\| < R$$

Definition. *An point, $x \in \mathbb{R}^n$ of some set $C \subseteq \mathbb{R}^n$ is an interior point if there exists an $R > 0$ such that:*

$$B(x, R) \subseteq C$$

Theorem. *2^{nd} Order Taylor's Approximation: If $f : U \to \mathbb{R}$ is a twice differentiable function over an open set $U \subseteq \mathbb{R}^n$ and $x \in U$ select some $r > 0$ such that $B(x, r) \subseteq U$. Then for any $y \in B(x, r)$*

$$f(y) = f(x) + \nabla f(x)^T (y - x) + \frac{1}{2}(y - x)^2 \nabla^2 f(x)(y - x) + o(\|y - x\|^2)$$

Definition. *A linear program is an optimisation of the form:*

$$\begin{aligned}
\underset{x}{\textit{maximise}}\ & \boldsymbol{c}^T \boldsymbol{x}\\
\textit{such that}\ & \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}\\
\textit{and}\ & \boldsymbol{x} \geq \boldsymbol{0}
\end{aligned}$$

$A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ *and* $x \in rnum^n$