Funzioni

giovedì 5 ottobre 2023 09:34

La funzione si dice **monotona** o strettamente monotona se si verifica una di queste condizioni

Si dice che f è:

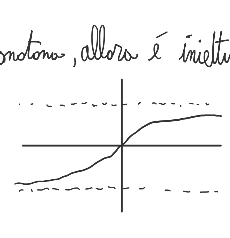
- Strettamente crescente in A se $f(x) > f(y)$
- Non decrescente in A se $f(x) \geq f(y)$
- Strettamente decrescente se $f(x) < f(y)$
- Non crescente se $f(x) \leq f(y)$

$f: D \to \mathbb{R}$, $D \subseteq \mathbb{R}$, $A \subseteq D$ $A \neq 0$ $\forall x, y \in A$

Proposizione: Se $f: D \to \mathbb{R}$, $D \subseteq \mathbb{R}$ è strettamente monotona, allora è iniettiva ed esiste f^{-1}

es $f: \mathbb{R} \to \mathbb{R}$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

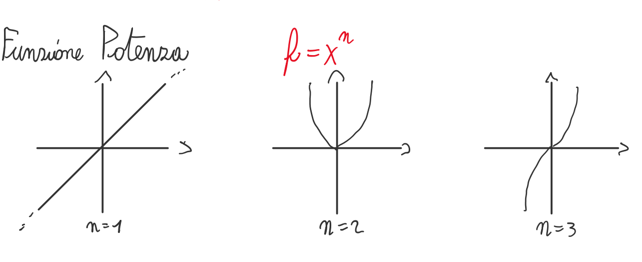
$f = \arcsin x$



Le Funzioni elementari

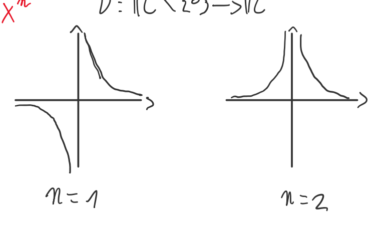
1) Funzione Potenza

$f = x^n$



$n=1$ $n=2$ $n=3$ $D: \mathbb{R} \to \mathbb{R}$

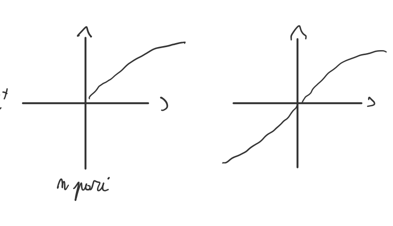
$f = x^{-n} = \frac{1}{x^n}$ $D: \mathbb{R} \setminus \{0\} \to \mathbb{R}$



$n=1$ $n=2$

Per gli altri n cambia il modo in cui la funzione si avvicina agli assi

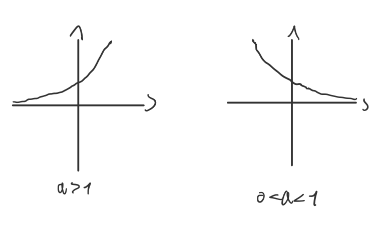
$f = \sqrt[n]{x}$



$D: \mathbb{R}^+ \to \mathbb{R}^+$ n pari $D: \mathbb{R} \to \mathbb{R}$

2) Funzioni Esponenziali: Se $a \in \mathbb{R}$, $a > 0$, $a \neq 1$ $f: \mathbb{R}^+ \to \mathbb{R}$

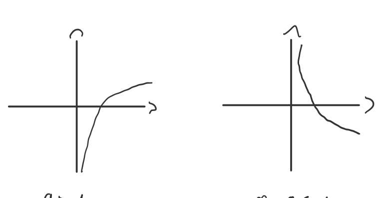
$f = x^a$



$a > 1$ $0 < a < 1$

3) Le funzioni Logaritmiche: Se $a \in \mathbb{R}$, $a > 0$, $a \neq 1$ $f: \mathbb{R}^+ \to \mathbb{R}$

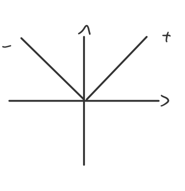
$f = \log_a x$



$a > 1$ $0 < a < 1$

4) Le funzioni Valore Assoluto

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$



Proposizione

$|x| \geq 0$

$|x| = 0$ se $x = 0$

$|x \cdot y| = |x| \cdot |y|$

$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ se $y \neq 0$

$|-x| = |x|$

Proposizione disuguaglianza triangolare: $\forall x, y \in \mathbb{R}$ $|x+y| \leq |x| + |y|$

es: Considero $f: \mathbb{R} \to \mathbb{R}$ definita da $f(x) = |x+1| - 2x \cdot |3-x|$

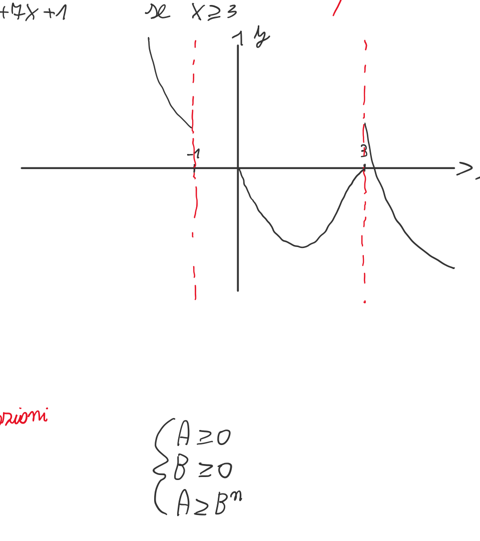
Rappresentiamo il grafico di f

$$|x+1| = \begin{cases} x+1 & \text{se } x+1 \geq 0 \to x \geq -1 \\ -x-1 & \text{se } x+1 < 0 \to x < -1 \end{cases}$$

$$|3-x| = \begin{cases} 3-x & \text{se } 3-x \geq 0 \to x \leq 3 \\ x-3 & \text{se } 3-x < 0 \to x > 3 \end{cases}$$

$$f(x) = \begin{cases} -x-1-2(3x-x^2) & \text{se } x < -1 \\ x+1-2x(3-x) & \text{se } -1 \leq x < 3 \\ x+1-2x(x-3) & \text{se } x \geq 3 \end{cases}$$

$$f(x) = \left.\begin{cases} 2x^2-7x-1 & \text{se } x < -1 \\ 2x^2-5x+1 & \text{se } -1 \leq x < 3 \\ -2x^2+7x+1 & \text{se } x \geq 3 \end{cases}\right\} \to \text{Funzione definita a tratti}$$



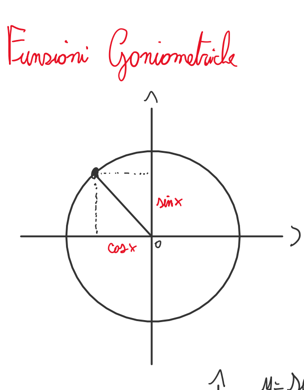
Risolvere Disequazioni

1) $\sqrt[n]{A} = B$ $\begin{cases} A \geq 0 \\ B \geq 0 \\ A \geq B^n \end{cases}$

2) $\sqrt[n]{A} \geq B$ $\begin{cases} A \geq 0 \\ B \geq 0 \\ A \geq B^n \end{cases} \vee \begin{cases} A \geq 0 \\ B < 0 \end{cases}$

3) $\sqrt{A} \geq B$ $\begin{cases} A \geq 0 \\ B \geq 0 \\ A \leq B^n \end{cases}$

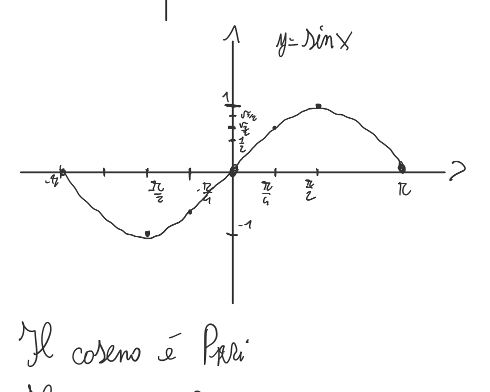
Funzioni Goniometriche

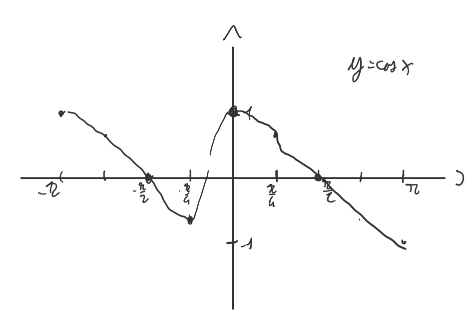


$x^2 + y^2 = 1$

$\alpha \in [0, 2\pi)$

$\alpha^\circ : \alpha' = 180 : \pi$





Il coseno è Pari

Il seno è Dispari

Proprietà

$\cos(2\pi + x) = \cos x$

$\sin(x + 2\pi) = \sin x$

$\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

$\cos^2 x + \sin^2 x = 1$

Valori notevoli

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$tg x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\nexists$	0	$\nexists$

Proposizione

$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

$\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$

$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$

$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

$tg(x+y) = \frac{tg x + tg y}{1 - tg x \cdot tg y}$

$tg(x-y) = \frac{tg x - tg y}{1 + tg x \cdot tg y}$

$\sin(2x) = 2 \sin x \cdot \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$ $\begin{cases} 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$

$tg\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos x}{1+\cos x}}$ $\begin{cases} \frac{\sin x}{1+\cos x} \\ \frac{1-\cos x}{\sin x} \end{cases}$

Proposizione

$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$

$\cos x + \cos y$

$\cos x - \cos y$

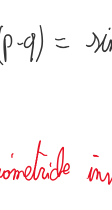
$x = \frac{x+y}{2} + \frac{x-y}{2} = p + q$ $p = \frac{x+y}{2}$

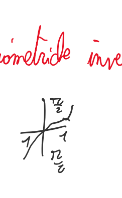
$y = \frac{x+y}{2} - \frac{x-y}{2} = p - q$ $q = \frac{x-y}{2}$

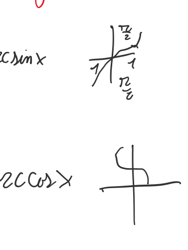
$\sin x = \sin(p+q) = \sin p \cdot \cos q + \cos p \cdot \sin q$

$\sin y = \sin(p-q) = \sin p \cdot \cos q - \cos p \cdot \sin q$

Funzioni Goniometriche inverse

$y = \arcsin x$  $\arcsin [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \arccos x$  $\arccos [-1, 1] \to [0, \pi]$

$\operatorname{arctg} x$  $\operatorname{arctg} \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$