Sviluppi di Taylor-MacLaurin per funzioni elementari

Per ogni $n \in \mathbb{N}$ valgono per $x \to 0$ le seguenti relazioni:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots + \frac{x^n}{n!} + o(x^n)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \frac{x^n}{n} + o(x^n)$$

$$-\log(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots + \frac{x^n}{n} + o(x^n)$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \cdots + \binom{a}{n} x^n + o(x^n) \qquad (a \in \mathbb{R})$$

dove, per $a \in \mathbb{R}$ e $n \in \mathbb{N}$, $\binom{a}{n}$ è il coefficiente binomiale generalizzato così definito:

$$\binom{a}{n} = \frac{a(a-1)\cdots(a-n+1)}{n!}$$

In particolare per $a = -1$, $a = \frac{1}{2}$, $a = \frac{1}{3}$ si ottengono

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^n)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + o(x^4)$$

$$\sqrt[3]{1+x} = 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5}{81}x^3 - \frac{10}{243}x^4 + o(x^4)$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3}{40}x^5 + o(x^6)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^6)$$