① convergenza della serie

$$\sum_{m=1}^{+\infty} \frac{\sin(mx)}{m^2 + x^2}$$

$$\lim_{m \to +\infty} \frac{\sin(mx)}{m^2 + x^2} = 0 \quad \text{può' convergere}$$

conv totale

$$\left| \frac{\sin(mx)}{m^2 + x^2} \right| \leq \frac{1}{m^2 + x^2} \leq \boxed{\frac{1}{m^2}}$$

termine generale serie armonica generalizzata
convergente

② $F(x,y) = \dfrac{(x+y)^4}{4} + \dfrac{(x-y)^4}{4} - x^2$

$F_x = (x+y)^3 + (x-y)^3 - 2x$

$F_y = (x+y)^3 - (x-y)^3$

$$\begin{cases} 2x^3 - 2x = 0 \Rightarrow 2x(x^2-1)=0 \\ (x+y)^3 = (x-y)^3 \Rightarrow x+y = x-y \Rightarrow y=0 \end{cases}$$

$(0,0) \qquad (1,0) \quad (-1,0)$

$F_{xx} = 3(x+y)^2 + 3(x-y)^2 - 2$

$F_{yy} = 3(x+y)^2 + 3(x-y)^2$

$F_{xy} = F_{yx} = 3(x+y)^2 - 3(x-y)^2$

$HF(0,0) = \begin{vmatrix} -2 & 0 \\ 0 & 0 \end{vmatrix} = 0$

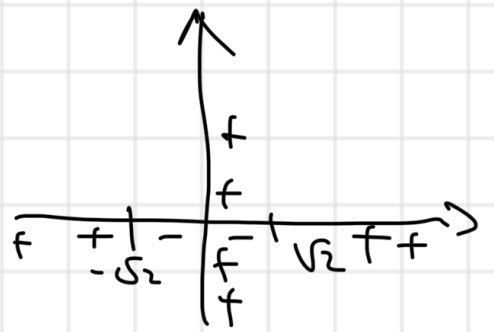$HF(\pm 1, 0) = \begin{vmatrix} 4 & 0 \\ 0 & 6 \end{vmatrix} = 24 \qquad$ Sono minimi

$F_{xx} > 0$

$F(x,y) - F(0,0) = \dfrac{(x+y)^4}{4} + \dfrac{(x-y)^4}{4} - x^2 = F(x,y)$

$F(0,y) = \dfrac{y^4}{4} + \dfrac{y^4}{4} > 0 \ \forall \ y$

$F(x,0) = \dfrac{x^4}{2} - x^2 = x^2\left(\dfrac{x^2}{2} - 1\right) > 0$

$$x < -\sqrt{2} \lor x > \sqrt{2}$$

no max no min in $(0,0)$

③ $\omega(x,y) = \dfrac{2x-y}{x^2+y^2}\,dx + \dfrac{2y+x}{x^2+y^2}\,dy$

e calcolare $\int_\gamma \omega\ dar$ γ e: $\{x^2+y^2=1\}$.

$D = \mathbb{R}^2 \setminus \{(0,0)\}$

è chiusa?

$$\frac{\partial a}{\partial y} = \frac{-x^2-y^2-2y(2x-y)}{(x^2+y^2)^2} = \frac{-x^2+y^2-4xy}{(x^2+y^2)^2}$$

$$\frac{\partial b}{\partial x} = \frac{x^2+y^2-2x(2y+x)}{(x^2+y^2)^2} = \frac{-x^2+y^2-4xy}{(x^2+y^2)^2}$$

è chiusa

$$\int \frac{2x-y}{x^2+y^2}\,dx = \int \frac{2x}{x^2+y^2}\,dx - \int \frac{y}{x^2+y^2}\,dx$$

$$= \log(x^2+y^2) - \int \frac{\not{y}}{y^2\left(1+\left(\frac{x}{y}\right)^2\right)}\,dx$$

$$= \log(x^2+y^2) - \arctan\left(\frac{x}{y}\right) + g(y)$$

$$F_y = \frac{2y}{x^2+y^2} + \frac{x}{y^2\left(1+\frac{x^2}{y^2}\right)} + g'(y) = \frac{2y+x}{x^2+y^2}$$

$$\frac{2y}{x^2+y^2} + \frac{x}{x^2+y}\,\varphi' = \frac{2y+x}{x^2+y^2} \implies \varphi'(y) = \cos t$$

$$F(x,y) = \log(x^2+y^2) - \operatorname{arctg}\left(\frac{x}{y}\right) + \cos t$$

$$\text{Dom} f = \{y \neq 0\} \neq \text{Dom } \omega = \mathbb{R}^2 \setminus \{(0,0)\}$$

$$\implies \int_{C(0,1)} \omega = 2\pi \quad \text{(anche così era ovvio non fosse esatta)}. \; \circledast$$

$$\gamma: \begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases} \qquad \theta \in [0, 2\pi]$$

$$\int_0^{2\pi} \frac{2\cos\theta - \sin\theta}{1}(-\sin\theta) + \frac{2\sin\theta + \cos\theta}{1}(\cos\theta)$$

$$\int_0^{2\pi} -2\sin\theta\cos\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$$

$$\int_0^{2\pi} 1\, d\theta = 2\pi$$

$$\implies \text{l'integrale su una curva chiusa}$$

$$\text{non è } 0 \implies \text{non può essere esatta.}$$

④ $y'' - 2y' + y = e^x \operatorname{sen} x$

$y'' - 2y' + y = 0$

$\lambda^2 - 2\lambda + 1 = 0 \implies (\lambda - 1)^2 = 0 \implies \lambda_{1,2} = 1 \text{ doppie}$

$y_1 = e^x \qquad y_2 = xe^x$

$\lambda + i\mu = i \qquad \text{non è sol}$

$y_p(x) = e^x (A \operatorname{sen} x + B \cos x)$

$y'_p = e^x (A \operatorname{sen} x + B \cos x) + e^x (A \cos x - B \operatorname{sen} x)$

$\qquad = e^x ((A - B) \operatorname{sen} x + (A + B) \cos x)$

$y''_p = e^x ((A - B) \operatorname{sen} x + (A + B) \cos x) +$

$\qquad e^x ((A - B) \cos x - (A + B)) \operatorname{sen} x$

$e^x [-2B \operatorname{sen} x + 2A \cos x] - 2e^x [(A - B) \operatorname{sen} x +$

$+ (A + B) \cos x] + e^x [A \operatorname{sen} x + B \cos x]$

$\qquad = e^x \operatorname{sen} x$

$(-2B - 2A + 2B + A) \operatorname{sen} x + (2A - 2A - 2B + B)$

$\cos x = \operatorname{sen} x$

$-A \operatorname{sen} x - B \cos x = \operatorname{sen} x \implies \begin{array}{l} A = -1 \\ B = 0 \end{array}$

$y_p = -e^x \operatorname{sen} x$

$y(x) = c_1 e^x + c_2 xe^x - e^x \operatorname{sen} x$

⑤ $\iiint_E f \, dx \, dy \, dz$

$f(x,y,z) = [4 - x^2 + 2x - 1 - y^2] z$

$E = \{ (x-1)^2 + y^2 \leq 4, \ y \geq 0 \quad 0 \leq z \leq 2 \}$

$\iint_D \left[\int_0^2 (4 - x^2 + 2x - 1 - y^2) \right) z \, dz \right]$

$\iint_D 2 (3 - x^2 + 2x \qquad - y^2) \right] dx \, dy$

$\begin{cases} x = 1 + \rho \cos\theta \\ y = \rho \sin\theta \end{cases} \qquad \begin{array}{l} \rho \in [0, 2] \\ \theta \in [0, \pi] \end{array}$

$\int_0^2 d\rho \int_0^\pi 2\rho \{ 3 - 1 - \rho^2 \cos^2\theta - 2\rho\cos\theta +$

$\qquad 2\rho\cos\theta - \rho^2 \sin^2\theta + 2 \} \, d\theta$

$\int_0^2 (8\rho - 2\rho^3) \pi = \pi \left(4\rho^2 - \frac{\rho^4}{2} \right)_0^2$

$\qquad\qquad = \pi (16 - 8) = 8\pi$

metodo variazione delle costanti es. 5

$$y_p = C_1(x)e^x + C_2(x)xe^x$$

$$\begin{cases} C_1'(x)e^x + C_2'(x)xe^x = 0 \\ C_1'(x)e^x + C_2'(x)(e^x + xe^x) = e^x \text{sen}x \end{cases}$$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = \begin{array}{l} e^{2x}(1+x) - xe^{2x} \\ = e^{2x} \end{array}$$

$$C_1' = \frac{\begin{vmatrix} 0 & xe^x \\ e^x \text{sen}x & e^x(1+x) \end{vmatrix}}{e^{2x}} = \frac{-xe^{2x}\text{sen}x}{e^{2x}}$$

$$C_2' = \frac{\begin{vmatrix} e^x & 0 \\ e^x & e^x \text{sen}x \end{vmatrix}}{e^{2x}} = \frac{e^{2x}\text{sen}x}{e^{2x}} = \text{sen}x$$

$$C_1(x) = \int -x\text{sen}x \, dx = x\cos x - \int \cos x$$
$$= x\cos x - \text{sen}x$$

$$C_2(x) = \int \text{sen}x = -\cos x$$

$$y_p(x) = (x\cos x - \text{sen}x)e^x - xe^x\cos x$$

$$y_p(x) = -e^x \operatorname{sen} x$$

$$\Rightarrow y(x) = c_1 e^x + c_2 x e^x - e^x \operatorname{sen} x.$$

(i due metodi sono equivalenti).