1) $f(x) = \dfrac{\sqrt{2+\cos(x^3)}}{|x^2-1| \cdot \log(x+\sqrt{2+x})}$ D = insieme di definizione di f

$$D: \begin{cases} 2+\cos(x^3) \geq 0 & (\forall x \in \mathbb{R}) \\ |x^2-1| \neq 0 \\ 2+x \geq 0 \\ x+\sqrt{2+x} > 0 & (\cdot) \\ x+\sqrt{2+x} \neq 1 & (\cdot\cdot) \end{cases} \qquad \begin{cases} x \neq \pm 1 \\ x \geq -2 \\ x > -1 \\ x \neq \frac{3-\sqrt{13}}{2} \end{cases} \Rightarrow \boxed{D = (-1,+\infty) \setminus \left\{\frac{3-\sqrt{13}}{2}, 1\right\}}$$

$$(\cdot) \quad x+\sqrt{2+x} > 0 \iff \sqrt{2+x} > -x \iff \begin{cases} 2+x \geq 0 \\ -x \geq 0 \\ 2+x > x^2 \end{cases} \vee \begin{cases} 2+x \geq 0 \\ -x < 0 \end{cases} \iff x > -1$$

$$(\cdot\cdot) \quad x+\sqrt{2+x} = 1 \iff \sqrt{2+x} = 1-x \iff \begin{cases} 2+x \geq 0 \\ 1-x \geq 0 \\ 2+x = 1-2x+x^2 \end{cases} \iff \begin{cases} -2 \leq x \leq 1 \\ x^2-3x-1 = 0 \end{cases}$$

$$\iff x = \frac{3-\sqrt{13}}{2}$$

2) $$\lim_{n\to+\infty} \frac{2^n-\sqrt{3^n+4^n}}{n^3\cdot\sin\left(\frac{2}{n}\right)-n} = L$$

- $$2^n-\sqrt{3^n+4^n} = 2^n-\sqrt{4^n\left(1+\frac{3^n}{4^n}\right)} = 2^n\left(1-\sqrt{1+\frac{3^n}{4^n}}\right) \sim 2^n\cdot\left(-\frac{1}{2}\cdot\frac{3^n}{4^n}\right) = -\frac{3^n}{2^{n+1}}$$

- $$n^3\cdot\sin\left(\frac{2}{n}\right) \sim n^3\cdot\frac{2}{n} = 2n^2, \qquad n^3\cdot\sin\left(\frac{2}{n}\right)-n \sim n^3\cdot\sin\left(\frac{2}{n}\right) \sim 2n^2$$

$$\Rightarrow L = \lim_{n\to+\infty} \frac{-\frac{3^n}{2^{n+1}}}{2n^2} = \lim_{n\to+\infty} -\frac{\left(\frac{3}{2}\right)^n}{4n^2} = \boxed{-\infty}$$ per gerarchia degli ∞

3) $\displaystyle\sum_{n=2}^{+\infty} \underbrace{\frac{\sqrt[3]{n^3+2n} - n}{\log(n+\sqrt{n}) - \log(n+\sqrt[3]{n})}}_{a_n}$ serie a termini positivi

- $\sqrt[3]{n^3+2n} - n = n\left(\sqrt[3]{1+\frac{2}{n^2}} - 1\right) \sim n \cdot \frac{1}{3} \cdot \frac{2}{n^2} = \frac{2}{3n}$

- $\log(n+\sqrt{n}) - \log(n+\sqrt[3]{n}) = \log\left(\frac{n+\sqrt{n}}{n+\sqrt[3]{n}}\right) = \log\left(1 + \frac{\sqrt{n}-\sqrt[3]{n}}{n+\sqrt[3]{n}}\right) \sim \frac{\sqrt{n}-\sqrt[3]{n}}{n+\sqrt[3]{n}} \sim \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$

$\Rightarrow$ $a_n \sim \frac{2}{3n} \cdot \sqrt{n} = \frac{2}{3\sqrt{n}}$ $\Rightarrow$ la serie $\boxed{\text{diverge a } +\infty}$ per confronto asintotico

4) $f(x) = \log\left(\dfrac{x^2+1}{x+2}\right)$

i) • $D = (-2, +\infty)$

• $f(x) = 0 \iff x^2+1 = x+2 \iff x = \dfrac{1 \pm \sqrt{5}}{2}$

$f(x) > 0 \iff \begin{cases} x^2+1 > x+2 \\ x+2 > 0 \end{cases} \iff x \in \left(-2, \dfrac{1-\sqrt{5}}{2}\right) \cup \left(\dfrac{1+\sqrt{5}}{2}, +\infty\right)$

$f(x) < 0 \iff x \in \left(\dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right)$

• $\lim\limits_{x \to -2} f(x) = \lim\limits_{y \to +\infty} \log(y) = +\infty$, $\quad \lim\limits_{x \to +\infty} f(x) = \lim\limits_{t \to +\infty} \log(t) = +\infty$

$x = -2$ è asintoto verticale, non esistono asintoti a $+\infty$

$$\left(\begin{array}{l} f(x) = \log\left(\dfrac{x^{\cancel{2}}\left(1+\frac{1}{x^2}\right)}{\cancel{x}\left(1+\frac{2}{x}\right)}\right) = \log(x) + \log\left(\dfrac{1+\frac{1}{x^2}}{1+\frac{2}{x}}\right) \sim \log(x) \quad \text{per } x \to +\infty, \quad \text{quindi} \\ \lim\limits_{x \to +\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to +\infty} \dfrac{\log(x)}{x} = 0, \quad \text{ma } \lim\limits_{x \to +\infty} \left[f(x) - 0 \cdot x\right] = \lim\limits_{x \to +\infty} f(x) \notin \mathbb{R} \end{array} \right)$$

ii) $f'(x) = \dfrac{2x}{x^2+1} - \dfrac{1}{x+2} = \dfrac{2x^2+4x-x^2-1}{(x^2+1)(x+2)} = \dfrac{x^2+4x-1}{(x^2+1)(x+2)}$ $\quad \forall x \in D$

f derivabile in tutto D

$f'(x) > 0 \iff \begin{cases} x^2+4x-1>0 \\ x>-2 \end{cases} \iff x > -2+\sqrt{5}$

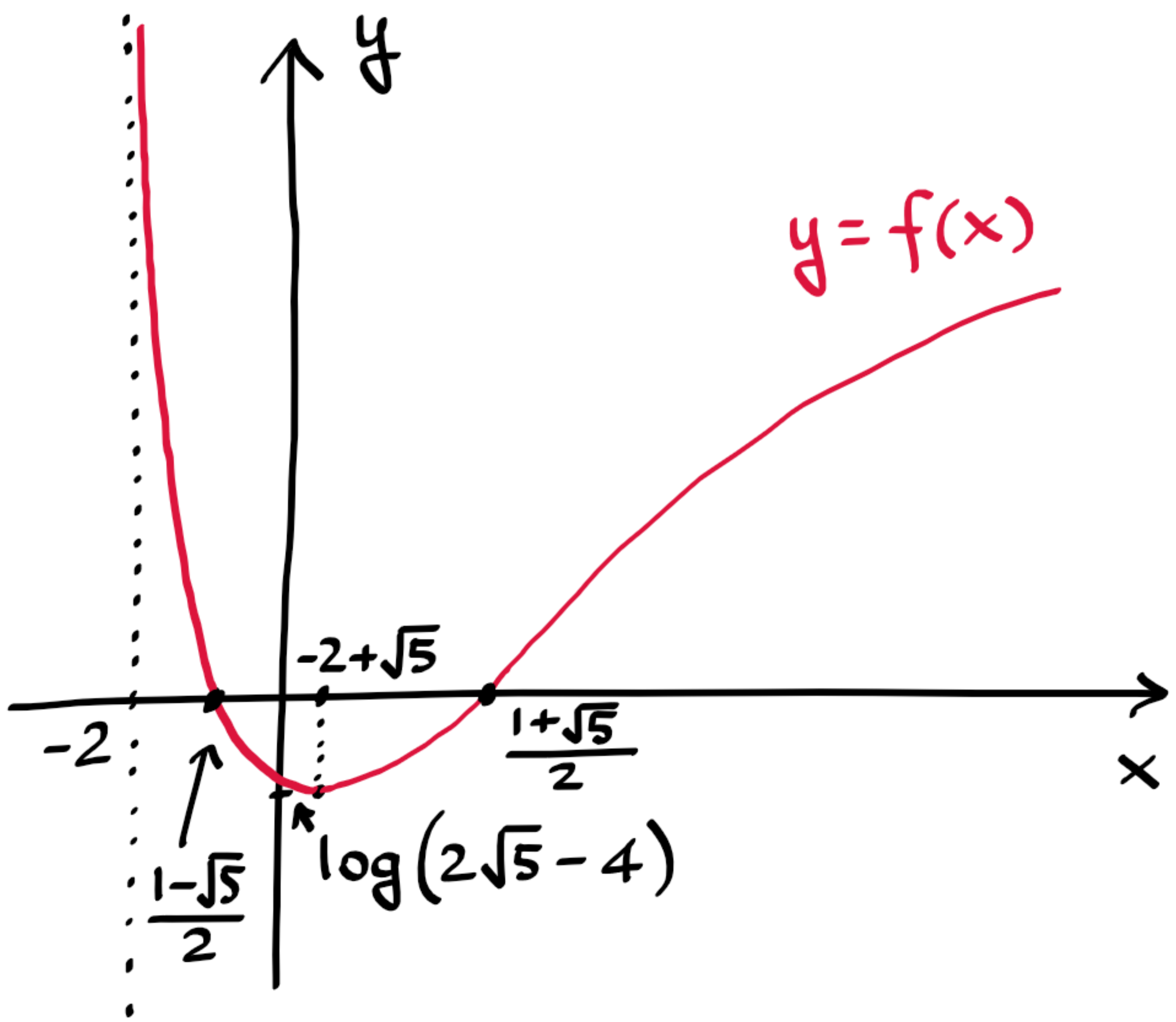
$f'(x) < 0 \iff -2 < x < -2+\sqrt{5}$

f strettam. decrescente in $(-2, -2+\sqrt{5}]$,

f strettam. crescente in $[-2+\sqrt{5}, +\infty)$

$x = -2+\sqrt{5}$ è punto di minimo (assoluto)

$$f(-2+\sqrt{5}) = \log\left(\frac{4+5-4\sqrt{5}+1}{\sqrt{5}}\right) = \log\left(\frac{10-4\sqrt{5}}{\sqrt{5}}\right) = \log(2\sqrt{5}-4) \quad (<0)$$



5) $$\lim_{x\to 0} \frac{\log(1+2x+2x^2) - \sin(2x)}{x^3}$$

- $$\log(1+2x+2x^2) = 2x + 2x^2 - \frac{(2x+2x^2)^2}{2} + \frac{(2x+2x^2)^3}{3} + o((2x+2x^2)^3)$$

$$= 2x + \cancel{2x^2} - \frac{\cancel{4x^2} + 8x^3 + \underbrace{4x^4}_{=o(x^3)}}{2} + \frac{8x^3 + \underbrace{24x^4 + 24x^5 + 8x^6}_{=o(x^3)}}{3} + o(x^3)$$

$$= 2x - 4x^3 + \frac{8}{3}x^3 + o(x^3)$$

$$= 2x - \frac{4}{3}x^3 + o(x^3) \qquad \text{per } x \to 0$$

- $$\sin(2x) = 2x - \frac{(2x)^3}{6} + o(x^3) = 2x - \frac{4}{3}x^3 + o(x^3) \qquad \text{per } x\to 0$$

$$\Rightarrow \log(1+2x+2x^2) - \sin(2x) = \cancel{2x} - \cancel{\frac{4}{3}}x^3 - \cancel{2x} + \cancel{\frac{4}{3}}x^3 + o(x^3) = o(x^3) \qquad \text{per } x \to 0$$

$$\Rightarrow \lim_{x\to 0} \frac{\log(1+2x+2x^2) - \sin(2x)}{x^3} = \lim_{x\to 0} \frac{o(x^3)}{x^3} = 0$$

6) $$\int_1^2 \frac{x^3+1}{x^3-2x^2+3x}\,dx$$

$$x^3+1 = x^3-2x^2+3x+2x^2-3x+1 \Rightarrow \frac{x^3+1}{x^3-2x^2+3x} = 1 + \frac{2x^2-3x+1}{x^3-2x^2+3x}$$

$$x^3-2x^2+3x = x\,(\underbrace{x^2-2x+3}_{\text{irriducibile}})$$

irriducibile, $\Delta = (-2)^2 - 4\cdot 3 = 4-12 = -8$

$$\frac{2x^2-3x+1}{x^3-2x^2+3x} = \frac{A}{x} + \frac{B(2x-2)+C}{x^2-2x+3} = \frac{Ax^2-2Ax+3A+2Bx^2-2Bx+Cx}{x^3-2x^2+3x}$$

$$\begin{cases} A+2B = 2 \\ -2A-2B+C = -3 \\ 3A = 1 \end{cases} \qquad \begin{cases} A = \frac{1}{3} \\ B = 1-\frac{A}{2} = \frac{5}{6} \\ C = -3+2A+2B = -3+\frac{2}{3}+\frac{5}{3} = -\frac{2}{3} \end{cases}$$

$$\Rightarrow \frac{x^3+1}{x^3-2x^2+3x} = 1 + \frac{1}{3x} + \frac{5}{6}\cdot\frac{2x-2}{x^2-2x+3} - \frac{2}{3}\cdot\frac{1}{x^2-2x+3}$$

$$\int \frac{x^3+1}{x^3-2x^2+3x}\,dx = \int \left(1 + \frac{1}{3x} + \frac{5}{6}\cdot\frac{2x-2}{x^2-2x+3} - \frac{2}{3}\cdot\frac{1}{x^2-2x+3} \right) dx$$

$$= x + \frac{1}{3}\log|x| + \frac{5}{6}\log\left(x^2-2x+3\right) - \frac{2}{3}\cdot\frac{2}{\sqrt{8}}\,\mathrm{arctg}\left(\frac{2x-2}{\sqrt{8}}\right) + C$$

$$= \underbrace{x + \frac{1}{3}\log|x| + \frac{5}{6}\log\left(x^2-2x+3\right) - \frac{1}{3\sqrt{2}}\,\mathrm{arctg}\left(\frac{x-1}{\sqrt{2}}\right)}_{F(x)} + C$$

$$\int_1^2 \frac{x^3+1}{x^3-2x^2+3x}\,dx = F(2) - F(1) = 1 + \frac{1}{3}\log 2 + \frac{5}{6}\log\left(\frac{3}{2}\right) - \frac{1}{3\sqrt{2}}\,\mathrm{arctg}\left(\frac{1}{\sqrt{2}}\right)$$