

Es lim

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es

Calcolare

$$\lim_{n \rightarrow \infty} \sqrt[n]{2} = 1 \quad \sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\frac{1}{\frac{1}{3}}} = 2^0 = 1$$

In generale per ogni numero  $a \in (0, \infty)$   $\rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a} = 1$

es

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{3n^2+2}{n^2-1}} = 1$$

$$\sqrt[n]{\frac{3n^2+2}{n^2-1}} = \left( \frac{3n^2+2}{n^2-1} \right)^{\frac{1}{n}} = (a_n)^{\frac{1}{n}} \rightarrow 1^0 = 1$$

$$\text{con } a_n = \frac{3n^2+2}{n^2-1} \quad l_{\infty} = \frac{1}{n}$$

$$\text{Se } \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^2(3+\frac{2}{n^2})}{n^2(1-\frac{1}{n^2})} = \frac{3}{1} = 3$$

$$\lim_{n \rightarrow \infty} l_{\infty} = 0$$

es

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = (+\infty)^0 = ?$$

$$\sqrt[n]{n} = (n^{\frac{1}{n}}) = (\infty)^0$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = n^{\frac{1}{n}} = e^{\frac{\log(n)}{n}} = e^{\frac{1}{n} \cdot \log n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \cdot \log n$$

$$\lim_{n \rightarrow \infty} \frac{\log n}{n} = 0 \quad \text{per la gerarchia}$$

$$\sqrt[n]{n} > n^{\frac{1}{n}} = e^{\frac{\log n}{n}} \rightarrow e^0 = 1$$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{n} = 1$$

es Il numero di  $a \in \mathbb{R}$  significa che

$$\lim_{n \rightarrow \infty} \sqrt[n]{n^a} = 1$$

$$(\text{in particolare } \sqrt[n]{n^2} \rightarrow 1 \quad \sqrt[n]{\frac{1}{n}} \rightarrow 1)$$

$$\sqrt[n]{n^a} = (n^a)^{\frac{1}{n}} = (n)^{\frac{a}{n}} = e^{\frac{a}{n} \cdot \log n} = e^0 = 1$$

$$\lim_{n \rightarrow \infty} \frac{a}{n} \cdot \log n = a \cdot \lim_{n \rightarrow \infty} \frac{\log n}{n} = 0$$

es

$$\lim_{n \rightarrow \infty} \left( \frac{n+2}{n+1} \right)^{\frac{1}{\log n}} = 1^0 = 1$$

es

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^{2n} = 1^\infty = ?$$

$$\text{Siccome } \left( 1 + \frac{1}{n} \right)^n \rightarrow e$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^{2n} = e^2$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^{n^2}$$

$$\lim_{n \rightarrow \infty} \left( \left( 1 + \frac{1}{n} \right)^n \right)^n = e^n$$

$$\text{considero } a_n = \left( 1 + \frac{1}{n} \right)^n \quad a_n = e$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^{n^2} = e^e = +\infty$$

Proposizione Se  $\{a_n\}$  è successione di numeri reali tale che  $\lim_{n \rightarrow \infty} a_n = +\infty$  o  $\lim_{n \rightarrow \infty} a_n = -\infty$

allora  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{a_n} \right)^{\frac{1}{a_n}} = e$

$$\text{Siccome } \left( 1 + \frac{1}{a_n} \right)^{\frac{1}{a_n}} = e^{\frac{1}{a_n}}$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{a_n} \right)^{\frac{1}{a_n}} = \left[ \left( 1 + \frac{1}{a_n} \right)^{a_n} \right]^{\frac{1}{a_n}} = a_n \rightarrow e$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{a_n} \right)^{\frac{1}{a_n}} = e^{\lim_{n \rightarrow \infty} \frac{1}{a_n}} = e^{-1}$$

es

$$\lim_{n \rightarrow \infty} \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+2}}$$

$$\text{Dalla proposizione } \lim_{n \rightarrow \infty} \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{-\sqrt{n+1}} = e$$

(convergono alla stessa proposizione per  $\sum n = -\frac{1}{\sqrt{n+1}}$ )

$$\text{Siccome allora } \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+2}} = \left[ \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{-\sqrt{n+1}} \right]^{-\frac{\sqrt{n+2}}{\sqrt{n+1}}} = a_n \rightarrow e$$

$$a_n = \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{-\sqrt{n+1}} \rightarrow e$$

$$a_n = -\frac{\sqrt{n+2}}{\sqrt{n+1}} \rightarrow -1$$

$$\lim_{n \rightarrow \infty} \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+2}} = \frac{1}{e}$$

$$\lim_{n \rightarrow \infty} \left( \frac{n+1}{n-2} \right)^n$$

$$\lim_{n \rightarrow \infty} \frac{n+1}{n-2} \rightarrow 1$$

$$\text{Siccome nella forma } \frac{n+1}{n-2} = 1 + \frac{3}{n-2} \text{ per un'operazione successiva } \frac{3}{n-2} \rightarrow 0$$

$$\text{In particolare } a_n = \frac{n+1}{n-2} - 1 = \frac{n+1 \cdot (n-2)}{n-2} = \frac{3}{n-2}$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{3}{n-2} \right)^n = e^3$$

$$\text{Siccome } \left( 1 + \frac{3}{n-2} \right)^n = \left[ \left( 1 + \frac{3}{n-2} \right)^{\frac{n-2}{3}} \right]^{\frac{3n}{n-2}} = a_n \rightarrow e$$

$$a_n \rightarrow e \quad \lim_{n \rightarrow \infty} e^3 = e^3$$