

# Es lim

venerdì 20 ottobre 2023 12:48

es

Calcolare

$$\lim_{n \rightarrow +\infty} \sqrt[n]{2} = 1 \quad \sqrt[n]{2} = 2^{\frac{1}{n}} = 2^{\frac{1}{\infty}} = 2^0 = 1$$

In generale per ogni numero  $a \in (0, +\infty) \rightarrow \lim_{n \rightarrow +\infty} \sqrt[n]{a} = 1$

es

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{3n^2+2}{n^2-1}} = 1$$

$$\sqrt[n]{\frac{3n^2+2}{n^2-1}} = \left( \frac{3n^2+2}{n^2-1} \right)^{\frac{1}{n}} = (a_n)^{b_n} \rightarrow 3^0 = 1$$

$$\text{con } a_n = \frac{3n^2+2}{n^2-1} \quad b_n = \frac{1}{n}$$

$$\text{Si ha } \lim_{n \rightarrow +\infty} a_n = \lim_{n \rightarrow +\infty} \frac{n^2(3+\frac{2}{n^2})}{n^2(1-\frac{1}{n^2})} = \frac{3}{1} = 3$$

$$\lim_{n \rightarrow +\infty} b_n = 0$$

es

$$\lim_{n \rightarrow +\infty} \sqrt[n]{n} = (+\infty)^0 = F.1 \quad ?$$

$$\sqrt[n]{n} = (n)^{\frac{1}{n}} = (+\infty)^0$$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{n}$$

$$\text{Scrivendo } \sqrt[n]{n} = n^{\frac{1}{n}} = e^{\frac{\log(n)}{n}} = e^{\frac{1}{n} \cdot \log n}$$

$$\lim_{n \rightarrow +\infty} \frac{1}{n} \cdot \log n$$

$$\lim_{n \rightarrow +\infty} \frac{\log n}{n} = 0 \quad \text{per la regola di L'Hôpital}$$

$$\sqrt[n]{n} = n^{\frac{1}{n}} = e^{\frac{\log n}{n}} \rightarrow e^0 = 1$$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{n} = 1$$

es Al variare di  $a \in \mathbb{R}$  verificare che

$$\lim_{n \rightarrow +\infty} \sqrt[n]{n^a} = 1$$

$$(\text{in particolare } \sqrt[n]{n^2} \rightarrow 1 \quad \sqrt[n]{\frac{1}{n}} \rightarrow 1)$$

$$\sqrt[n]{n^a} = (n^a)^{\frac{1}{n}} = (n)^{\frac{a}{n}} = e^{\frac{a \cdot \log n}{n}} = e^0 = 1$$

$$\lim_{n \rightarrow +\infty} \frac{a}{n} \cdot \log n = a \cdot \lim_{n \rightarrow +\infty} \frac{\log n}{n} = 0$$

es

Calcolare

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{n}{n^2+2}}$$

$$\text{Scrivendo } \frac{n}{n^2+2} = \frac{n}{n^2(n+\frac{2}{n})} = \frac{1}{n} \cdot \frac{1}{1+\frac{2}{n}}$$

$$\text{quindi } \sqrt[n]{\frac{n}{n^2+2}} = \sqrt[n]{\frac{1}{n} \cdot \frac{1}{1+\frac{2}{n}}} = \frac{1}{\sqrt[n]{n}} \cdot \frac{1}{\sqrt[n]{1+\frac{2}{n}}}$$

$$\text{So che } \sqrt[n]{n} \rightarrow 1$$

$$\sqrt[n]{1+\frac{2}{n}} = \left(1+\frac{2}{n}\right)^{\frac{1}{n}} \rightarrow 1^0 = 1$$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{n}{n^2+2}} = 1$$

es

$$\lim_{n \rightarrow +\infty} \left( \frac{n+2}{n+1} \right)^{\frac{1}{\log n}} = 1^0 = 1$$

es

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{2n} = 1^\infty = F.1$$

$$\text{Scrivendo } \left( 1 + \frac{1}{n} \right)^{2n} = \left[ \left( 1 + \frac{1}{n} \right)^n \right]^2$$

$$\text{Siccome } \left( 1 + \frac{1}{n} \right)^n \rightarrow e$$

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{2n} = e^2$$

es

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{n^2}$$

$$\lim_{n \rightarrow +\infty} \left[ \left( 1 + \frac{1}{n} \right)^n \right]^n$$

$$\text{Considero } a_n = \left( 1 + \frac{1}{n} \right)^n \quad a_n \rightarrow e \quad b_n = n$$

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{n^2} = e^n = +\infty$$

es

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{\sqrt{n}} = 1$$

$$\left[ \left( 1 + \frac{1}{n} \right)^n \right]^{\frac{1}{\sqrt{n}}} \rightarrow \quad a_n = \left( 1 + \frac{1}{n} \right)^n \rightarrow e \quad b_n = \frac{1}{\sqrt{n}} \rightarrow 0$$

$$e^0 = 1$$

es

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{n \cdot \sqrt{n}} = \lim_{n \rightarrow +\infty} \left[ \left( 1 + \frac{1}{n} \right)^n \right]^{1 \cdot \frac{1}{\sqrt{n}}} \quad a_n \rightarrow e \quad b_n = 1 \cdot \frac{1}{\sqrt{n}} \rightarrow 0$$

$$e^1 = e$$

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{n} \right)^{n \cdot \sqrt{n}} = e$$

Proposizione Se  $\{\epsilon_n\}$  è successione di numeri reali tale che  $\lim_{n \rightarrow +\infty} \epsilon_n = +\infty$  o  $\lim_{n \rightarrow +\infty} \epsilon_n = -\infty$

$$\text{allora } \lim_{n \rightarrow +\infty} \left( 1 + \frac{1}{\epsilon_n} \right)^{\epsilon_n} = e$$

per successione a segno

Proposizione Se  $\{\epsilon_n\}$  è successione di numeri reali tali che  $\epsilon_n \neq 0$  finito e  $\lim_{n \rightarrow +\infty} \epsilon_n = 0$

$$\text{allora } \lim_{n \rightarrow +\infty} \left( 1 + \epsilon_n \right)^{\frac{1}{\epsilon_n}} = e$$

es

$$\lim_{n \rightarrow +\infty} \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+1}}$$

$$\text{Dalla proposizione } \lim_{n \rightarrow +\infty} \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+1}} = e$$

$$(\text{consequenza della proposizione per } \epsilon_n = -\frac{1}{\sqrt{n+1}})$$

$$\text{Scrivendo } \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+1}} = \left[ \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{-\sqrt{n+1}} \right]^{-\frac{\sqrt{n+1}}{\sqrt{n+1}}} = a_n^{b_n}$$

$$a_n = \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{-\sqrt{n+1}} \rightarrow e$$

$$b_n = -\frac{\sqrt{n+1}}{\sqrt{n+1}} \rightarrow -1$$

$$\lim_{n \rightarrow +\infty} \left( 1 - \frac{1}{\sqrt{n+1}} \right)^{\sqrt{n+1}} = \frac{1}{e}$$

es

$$\lim_{n \rightarrow +\infty} \left( \frac{n+1}{n-2} \right)^n$$

$$\text{Scrivendo } \frac{n+1}{n-2} \rightarrow 1$$

$$\text{ma scrivendo nella forma } \frac{n+1}{n-2} = 1 + \epsilon_n \quad \text{per una opportuna successione } \epsilon_n \rightarrow 0$$

$$\text{In pratica } \epsilon_n = \frac{n+1}{n-2} - 1 = \frac{n+1-(n-2)}{n-2} = \frac{3}{n-2}$$

$$\lim_{n \rightarrow +\infty} \left( 1 + \frac{3}{n-2} \right)^n = e^3$$

$$\text{Scrivendo } \left( 1 + \frac{3}{n-2} \right)^n = \left[ \left( 1 + \frac{3}{n-2} \right)^{\frac{n-2}{n-2}} \right]^{\frac{3n}{n-2}} = a_n^{b_n}$$

$$a_n \rightarrow e \quad b_n \rightarrow 3$$