1) $$\frac{\log(1+x^3)}{2x-\sqrt{4x+3}} \geq 0$$

$$\text{C.E.:} \begin{cases} 1+x^3>0 \\ 4x+3\geq 0 \\ 2x-\sqrt{4x+3}\neq 0 \end{cases} \quad \begin{cases} x>-1 \\ x\geq -\frac{3}{4} \\ x\neq \frac{3}{2} \end{cases} \quad \Rightarrow \quad \text{C.E.: } x\geq -\frac{3}{4} \text{ con } x\neq \frac{3}{2}$$

$$\left(2x-\sqrt{4x+3}=0 \iff 2x=\sqrt{4x+3} \iff \begin{cases} 2x\geq 0 \\ 4x+3\geq 0 \\ 4x^2=4x+3 \end{cases} \iff x=\frac{3}{2} \right)$$

Studio del segno:

- $\log(1+x^3)>0 \iff 1+x^3>1 \iff x>0$
- $2x-\sqrt{4x+3}>0 \iff 2x>\sqrt{4x+3} \iff x>\frac{3}{2}$

	$-\frac{3}{4}$		0		$3/2$	
N	$-$	$-$	0	$+$	$+$	$+$
D	$-$	$-$	$-$	$-$	0	$+$
N/D	$+$	$+$	0	$-$	$\nexists$	$+$

Soluzione: $\boxed{-\frac{3}{4}\leq x\leq 0 \quad \vee \quad x>\frac{3}{2}}$

2)

$$\lim_{n\to+\infty}\sqrt{4^n+n}\left(\frac{1}{2}+\frac{1}{n}\right)^n = \lim_{n\to+\infty}\underbrace{\cancel{\sqrt{4^n}}}_{=2^n}\cdot\sqrt{1+\frac{n}{4^n}}\cdot\underbrace{\cancel{\left(\frac{1}{2}\right)^n}}_{=\frac{1}{2^n}}\cdot\left(1+\frac{2}{n}\right)^n$$

$$= \lim_{n\to+\infty}\sqrt{1+\frac{n}{4^n}}\cdot\lim_{n\to+\infty}\left(1+\frac{2}{n}\right)^n = 1\cdot e^2 = \boxed{e^2}$$

3) $$\underbrace{\sum_{n=3}^{+\infty} \frac{n-\sqrt{n^2-3n}}{\sqrt{n}\cdot\log(1+n^2)+n}}_{a_n}$$ serie a termini positivi

- $$n-\sqrt{n^2-3n} = \frac{n^2-(n^2-3n)}{n+\sqrt{n^2-3n}} = \frac{3n}{n+\sqrt{n^2-3n}} \longrightarrow \frac{3}{2} \quad \text{per } n \longrightarrow +\infty$$

- $$\log(1+n^2) = \log(n^2(1+n^{-2})) = 2\log n + \log(1+n^{-2}) \sim 2\log n$$

$\Rightarrow$ $\sqrt{n}\cdot\log(1+n^2) \sim \sqrt{n}\cdot 2\log n = o(n)$ per gerarchia degli ∞

$\Rightarrow$ $\sqrt{n}\cdot\log(1+n^2)+n = n+o(n) \sim n$

$\Rightarrow$ $a_n \sim \frac{3}{2n}$ e la serie diverge a $+\infty$ per confronto asintotico

4) $f(x) = \sqrt[3]{x^2(x+1)}$

i) $D = \mathbb{R}$, $\quad f(x) = 0 \iff x = 0 \lor x = -1$

$f(x) > 0 \iff x \in (-1,0) \cup (0,+\infty)$

$f(x) < 0 \iff x \in (-\infty,-1)$

$$\lim_{x\to-\infty} f(x) = -\infty, \quad \lim_{x\to+\infty} f(x) = +\infty \ ;$$

$$\lim_{x\to\pm\infty} \frac{f(x)}{x} = 1, \quad \lim_{x\to\pm\infty} \left[f(x) - x\right] = \lim_{x\to\pm\infty} \left[\sqrt[3]{x^3+x^2} - x\right]$$

$$= \lim_{x\to\pm\infty} x\left(\sqrt[3]{1+\frac{1}{x}} - 1\right) = \lim_{x\to\pm\infty} x \cdot \frac{1}{3x} = \frac{1}{3}$$

$\Rightarrow \ y = x + \frac{1}{3}$ è asintoto obliquo a $+\infty$ e a $-\infty$.

ii) $f'(x) = \dfrac{3x^2+2x}{3\left(x^2(x+1)\right)^{2/3}} = \dfrac{3x+2}{3x^{1/3}(x+1)^{2/3}}$ per ogni $x \in \mathbb{R} \setminus \{-1, 0\}$

f continua in $x=-1$, $\lim_{x \to -1} f'(x) = \dfrac{-1}{3 \cdot 0^- \cdot 0^+} = +\infty \Rightarrow$ $x=-1$ è punto a tangente verticale

f continua in $x=0$, $\lim_{x \to 0^+} f'(x) = +\infty$, $\lim_{x \to 0^-} f'(x) = -\infty \Rightarrow$ $x=0$ è cuspide

$f'(x) = 0 \iff x = -\frac{2}{3}$

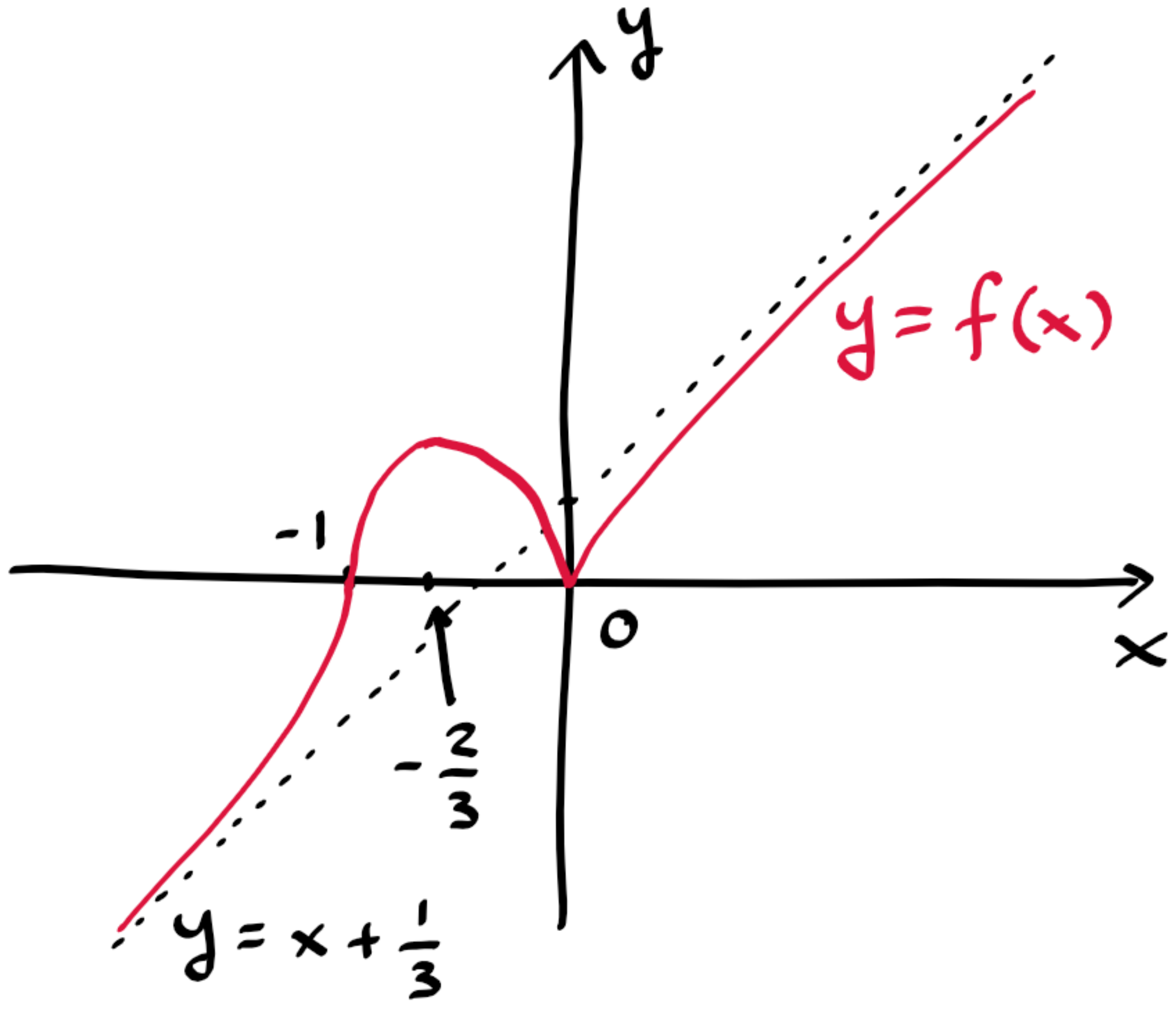
$f'(x) > 0 \iff x \in (-\infty, -1) \cup (-1, -\frac{2}{3}) \cup (0, +\infty)$

$f'(x) < 0 \iff x \in (-\frac{2}{3}, 0)$

f strettam. crescente in $(-\infty, -\frac{2}{3}]$ e in $[0, +\infty)$

f " decrescente in $[-\frac{2}{3}, 0)$

$x = -\frac{2}{3}$ punto di max locale $\left(f(-\frac{2}{3}) = \frac{\sqrt[3]{4}}{3}\right)$



5) $$\lim_{x\to 0} \frac{2\sin(x-x^2)-\log(1+2x)}{\arctan x - x\cdot\cos x} = \lim_{x\to 0}\frac{-3x^3}{\frac{x^3}{6}} = \frac{-3}{\left(\frac{1}{6}\right)} = -18$$

- $$\sin(x-x^2) = x - x^2 - \frac{x^3}{6} + o(x^3)$$

- $$\log(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 + o(x^3)$$

$$\Rightarrow \quad 2\sin(x-x^2) - \log(1+2x) = \cancel{2x} - \cancel{2x^2} - \frac{x^3}{3} - \cancel{2x} + \cancel{2x^2} - \frac{8}{3}x^3 + o(x^3) \sim -3x^3$$

- $$\arctan x - x\cdot\cos x = x - \frac{x^3}{3} + o(x^3) - x\left(1 - \frac{x^2}{2} + o(x^2)\right) = \cancel{x} - \frac{x^3}{3} - \cancel{x} + \frac{x^3}{2} + o(x^3) \sim \frac{x^3}{6}$$

6) $$\int_0^1 \frac{x^3-1}{x^3+1}\,dx = \int_0^1 1\,dx - 2\int_0^1 \frac{1}{x^3+1}\,dx = 1 - 2\int_0^1 \frac{1}{x^3+1}\,dx = I$$

$$\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{B(2x-1)+C}{x^2-x+1} = \frac{Ax^2 - Ax + A + 2Bx^2 + Bx - B + Cx + C}{x^3+1}$$

$$\begin{cases} A + 2B = 0 \\ -A + B + C = 0 \\ A - B + C = 1 \end{cases} \qquad \begin{cases} A = -2B \\ 3B + C = 0 \\ -3B + C = 1 \end{cases} \qquad \begin{cases} A = 1/3 \\ B = -1/6 \\ C = 1/2 \end{cases}$$

$$\int \frac{1}{x^3+1}\,dx = \frac{1}{3}\int \frac{1}{x+1}\,dx - \frac{1}{6}\int \frac{2x-1}{x^2-x+1}\,dx + \frac{1}{2}\int \frac{1}{x^2-x+1}\,dx$$

$$= \frac{1}{3}\log|x+1| - \frac{1}{6}\log(x^2-x+1) + \frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + C$$

$$\Rightarrow \int_0^1 \frac{1}{x^3+1}\,dx = \frac{1}{3}\log 2 + \frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \boxed{I = 1 - \frac{2}{3}\log 2 - \frac{4}{\sqrt{3}}\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right)}$$