Esercizi su equivalenza asintotica tra successioni - 1

In ognuno dei casi seguenti determinare, se esistono, dei parametri $a \in \mathbb{R} \setminus \{0\}$ e $b \in \mathbb{R}$ tali che

$$a_n \sim an^b \qquad \text{per } n \to +\infty \,.$$

(In altre parole, si chiede di determinare $b \in \mathbb{R}$ tale che esista *finito* e *diverso da 0* il limite $a = \lim_{n\to+\infty} \frac{a_n}{n^b}$)

1. $a_n = \dfrac{3n^2 - n^5 + 1}{n^6 - (n+2)^3}$

2. $a_n = \dfrac{n^4 + \log(5n^7)}{1 - 3n^2}$

3. $a_n = \dfrac{1}{n^3} - \dfrac{1}{\sqrt{n}}$

4. $a_n = \dfrac{n+1}{n^2} - \dfrac{1}{n+1}$

5. $a_n = \dfrac{2 - \sqrt[n]{n}}{\sqrt{n+2} - n}$

6. $a_n = \dfrac{\sqrt[3]{n+1}}{n - \sin(n)}$

7. $a_n = \sqrt{n+1} - \sqrt{n}$

8. $a_n = \sqrt{n+1} - \sqrt{n+2}$

9. $a_n = \sqrt{n^2 + n} - n$

10. $a_n = \sqrt{n^2 + \sqrt{n}} - n$

11. $a_n = \sqrt{n^2 + n^{3/2}} - n$

12. $a_n = \sqrt{n^3 + n^2} - \sqrt{n^3 + 1}$

13. $a_n = \sqrt{n+2} - \sqrt[4]{n^2 + 1}$

14. $a_n = \log(n+3) - \log(n+2)$

15. $a_n = \log\left(\dfrac{n+1}{\sqrt{n^2 - 3}}\right)$

16. $a_n = \sqrt[n]{2} - 1$

17. $a_n = \sin\left(\dfrac{1}{n^2} + \dfrac{1}{n}\right)$

18. $a_n = 1 - \sqrt{1 + \tan\left(\dfrac{\sqrt{n+1}}{n}\right)}$

19. $a_n = \sqrt{1 - \cos\left(\dfrac{3}{n+1}\right)}$

20. $a_n = \sqrt[3]{\cos\left(\dfrac{5}{n^2}\right) - 1}$

21. $a_n = \log\left(\cos\left(\dfrac{1}{n}\right)\right)$

22. $a_n = \log\left(\dfrac{1 + \sqrt{n}}{1 + \sqrt{n+1}}\right)$

23. $a_n = \log\left(\dfrac{2 + \sqrt{n}}{1 + \sqrt{n+1}}\right)$

24. $a_n = \dfrac{e^{1/n} - 1}{\tan(\sqrt{n+2} - \sqrt{n+1})}$

25. $a_n = \dfrac{1 - \cos\left(\dfrac{2}{\sqrt{n}}\right)}{\sin\left(\dfrac{n+2}{n^3+1}\right)}$

26. $a_n = (\sqrt{n^4 + 2n^3} - n^2) \log\left(\dfrac{n+2}{n-1}\right)$