Es calcolo dei limiti di funzioni

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es. Calcolare

$\lim_{x\to 0} \frac{2x+1}{\sqrt{x}} = \frac{1}{0^+} = +\infty$

$f(x) = \frac{2x+1}{\sqrt{x}}$ è definita in $(0;+\infty)$ $\left[\frac{2x}{\sqrt{x}} = 2\sqrt{x}\right]$

es

$\lim_{x\to 0} \frac{2x+1}{\sqrt[3]{x}} = \pm\infty$ il limite non esiste

$x \to 0^+$ $f(x) \to +\infty$

$x \to 0^-$ $f(x) \to -\infty$

es

$\lim_{x\to 0^+} \frac{x+x^3}{2x^2-\sqrt{x}}$

Scrivo $x+x^3 = x(1+x^2)$

$2x^2-\sqrt{x} = \sqrt{x}\left(\frac{2x^2}{\sqrt{x}}-1\right) = \sqrt{x}\,(2x^{\frac{3}{2}}-1)$

$\lim_{x\to 0^+} \frac{x(1+x^2)}{\sqrt{x}(2x^{\frac{3}{2}}-1)} = \lim_{x\to 0^+} \frac{-x}{\sqrt{x}} = \lim_{x\to 0^+} (-\sqrt{x}) = 0^-$

Def

Se $D \subseteq \mathbb{R}$, $x_0 \in \mathbb{R}$ punto di accumulazione per D, $f, g : D \to \mathbb{R}$ con $g \neq 0$ allora si dice

$f(x) \sim g(x)$ per $x \to x_0$ se $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 1$

$x+x^3 \sim x$ per $x \to 0$

$2x^2-\sqrt{x} \sim -\sqrt{x}$ per $x \to 0$

$\frac{x+x^3}{2x^2-\sqrt{x}} \sim \frac{x}{-\sqrt{x}} = -\sqrt{x}$ per $x \to 0$

es

$\lim_{x\to +\infty} \frac{x^2+x+3}{x+1} = +\infty$ $x^2+x+3 \sim x^2$, $x+1 \sim x$ per $x \to +\infty$

es

$\lim_{x\to 1^-} \frac{2-x^2}{x-1} = \frac{1}{0^-} = -\infty$

$\lim_{x\to 1^+} \frac{2-x^2}{x-1} = \frac{1}{0^+} = +\infty$

es

$\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \left[\frac{0}{0}\right]$ F.I

$x^3-1 = (x-1)(x^2+x+1)$

$x^2-1 = (x+1)(x-1)$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} = \frac{3}{2}$

es

$\lim_{x\to 2} \frac{1}{\sqrt{x^2-4}} = \frac{1}{0^+} = +\infty$

es

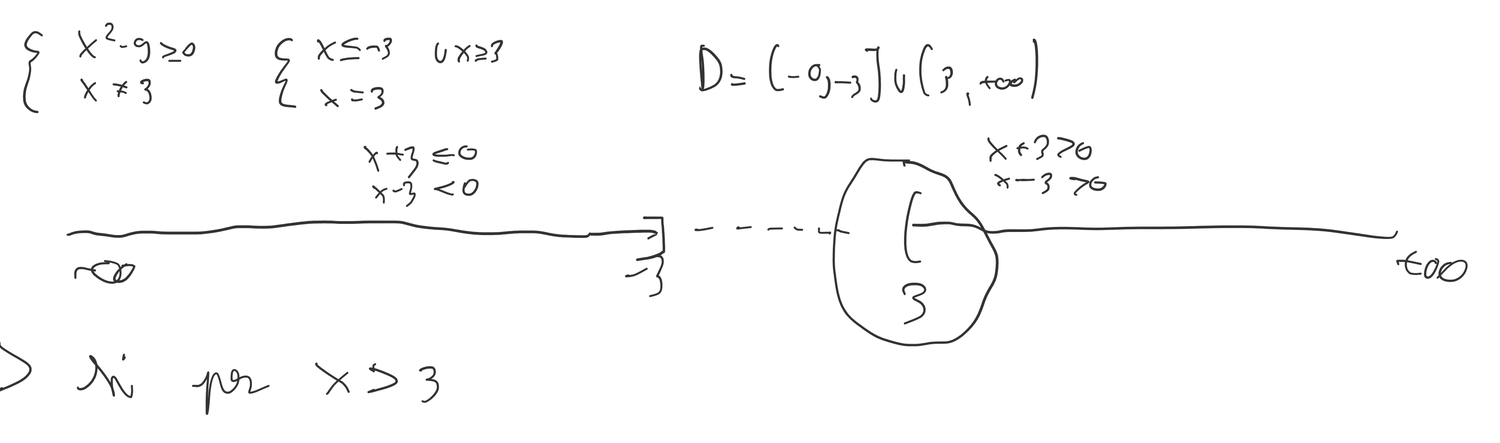
$\lim_{x\to 3} \frac{\sqrt{x^2-9}}{3-x} = \left[\frac{0}{0}\right]$ F.I

$\sqrt{x^2-9} = \sqrt{(x-3)(x+3)}$

$\frac{\sqrt{x^2-9}}{3-x} = \frac{\sqrt{(x-3)(x+3)}}{3-x} = \frac{\sqrt{x-3}\cdot\sqrt{x+3}}{3-x}$

$f(x) = \frac{\sqrt{x^2-9}}{3-x}$ definita in

$D: \begin{cases} x^2-9 \geq 0 \\ x \neq 3 \end{cases}$ $\begin{cases} x \leq -3 \cup x \geq 3 \\ x = 3 \end{cases}$ $D = (-\infty, -3] \cup (3, +\infty)$



si per $x > 3$

per $x > 3$ vale $\frac{\sqrt{x^2-9}}{3-x} = \frac{\sqrt{x-3}\cdot\sqrt{x+3}}{3-x}$

$\frac{\sqrt{x-3}\cdot\sqrt{x+3}}{-(x-3)} = \frac{-\sqrt{x+3}}{\sqrt{x-3}} = -\infty$

LIMITI NOTEVOLI

- $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$
- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim_{x\to 0} \frac{e^x-1}{x} = 1$
- $\lim_{x\to 0} \frac{(1+x)^a-1}{x} = a$
- $\lim_{x\to 0} \frac{\tan x}{x} = 1$ $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

Gerarchia degli infiniti

se $b > 1$ e $c > 0$

$\lim_{x\to +\infty} \frac{\log_b x}{x^c} = 0^+$

$\lim_{x\to +\infty} \frac{x^c}{b^x} = 0^+$

$\lim_{x\to 0^+} x^c \cdot \log_b x = 0^-$

es

$\lim_{x\to 0} \frac{\sin x}{\sqrt{x}}$

$\frac{\sin x}{\sqrt{x}} = \frac{\sin x}{x}\cdot\frac{x}{\sqrt{x}} = 1\cdot\sqrt{x} \sim \sqrt{x}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{x}{\sqrt{x}} = 1 \cdot 0 = 0^+$

es

$\lim_{x\to 0} \frac{1-\cos(3x)}{\log(1+x^2)}$

$1-\cos(x) \sim \frac{x^2}{2}$

$1-\cos(3x) \sim \frac{9x^2}{2}$

$\log(1+x^2) \sim x^2$

$\lim_{x\to 0} \frac{\frac{9}{2}x^2}{x^2} = \frac{9}{2}$

$f(x) = 3x$ $g(y) = \frac{1-\cos y}{y^2}$

$g(f(x)) = \frac{1-\cos 3x}{9x^2} = \frac{1}{2}$

$f(x) \to 0$ per $x \to 0$

$g(y) \to \frac{1}{2}$ per $y \to 0$

es

$\lim_{x\to 1} \frac{\log(x)}{3^x-3} = \left[\frac{0}{0}\right]$ F.I

$\log(x) = \log(1+(x-1)) \sim x-1$ per $x \to 1$

$3^x-3 = 3(3^{x-1}-1) \sim 3\cdot\log 3\cdot(x-1)$ $\left[3^t-1 = (e^{\log 3})^t - 1 = e^{\log 3\cdot t}-1 \sim \log 3\cdot t\right.$

$\lim_{x\to 1} \frac{x-1}{3(3^{x-1}-1)} =$ $t = x-1$ $\frac{t}{3(3^t-1)}$

$f(x) = x-1$ $g(y) = \frac{y}{3(3^y-1)}$

$g(f(x)) = \frac{\log x}{3^x-3}$ $f(x) \to 0$

per $y \to 0$

$\left.\begin{array}{l}\log(1+y) \sim y \\ 3^y-1 \sim \log(3)\cdot y\end{array}\right\}$ $g(y) = \frac{y}{3\log 3\cdot t} = \frac{1}{3\cdot\log 3}$

es

$f(x) = \frac{(x-1)\,e^{\frac{1}{x}}}{\sqrt{x^2-3x+2}}$

Determinare l'insieme di definizione D di f e limiti di f in tutti gli estremi degli intervalli di cui è composto D

D.

$x^2-3x+2 > 0$ $x = \frac{3\pm\sqrt{9-8}}{2} = \frac{3\pm1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$x \neq 0$

$\begin{cases} x < 1 \cup x > 2 \\ x \neq 0 \end{cases}$

$D = (-\infty, 0) \cup (0, 1) \cup (2, +\infty)$

$\lim_{x\to -\infty} \frac{(x-1)\cdot e^{\frac{1}{x}}}{\sqrt{x^2-3x+2}} = \frac{x}{-x} = -1$

$\lim_{x\to 0^-} \frac{(x-1)\cdot e^{\frac{1}{x}}}{\sqrt{x^2-3x+2}} = \frac{+\infty}{\sqrt{2}} = +\infty$

$\lim_{x\to 0^+} \frac{(x-1)\cdot e^{\frac{1}{x}}}{\sqrt{x^2-3x+2}} = \frac{-\infty}{\sqrt{2}} = -\infty$

$\lim_{x\to 1^-} f(x) =$

$\lim_{x\to +\infty} f(x) = 1$

$\lim_{x\to 1^-} \frac{(x-1)\overbrace{e^{\frac{1}{x}}}^{\to e}}{\sqrt{1-x}\,\underbrace{\sqrt{2-x}}_{\to 1}} = \frac{x-1}{\sqrt{1-x}} = -\lim_{x\to 1^-} \frac{1-x}{\sqrt{1-x}} = -\lim \sqrt{1-x} = 0^-$

$\lim_{x\to 2^+} f(x) = \frac{\sqrt{e}}{0^+} = +\infty$