A Multiple-Model Adaptive Observer implementation using Neural Networks for Oncological Hyperthermia

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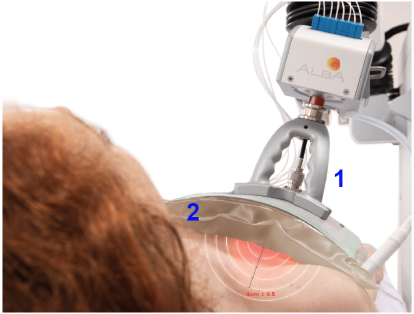
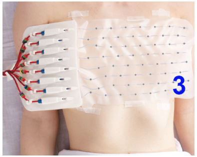
May 17, 2024

Superficial hyperthermia in a nutshell

Heating tumors in the 41-43 °C range is a powerful radio- and chemo-sensitizer [1]

1. heat is delivered by a Microwave Radiating Antenna
2. Water Bolus is used to prevent skin burns
3. superficial temperature measurements are obtained through a Matrix of Thermocouples





Mechanism

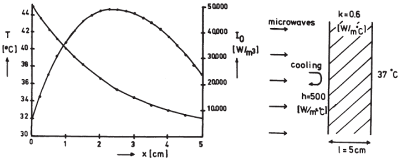


Figure 5: Temperature distribution in an infinite uniform halfspace with properties: $h = 500$ [W/m²/°C], T(water) = 30 [°C], T(tis) = 37 [°C], $d = 3$ [cm], $k = 0.6$ [W/m²/°C] and $l = 5$ [cm].

Figure: Lagendijk, 1987 [2]

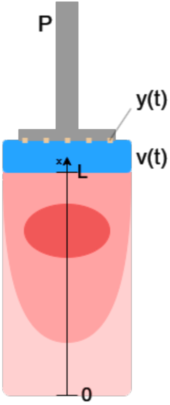
Clinical treatment success hinges upon:

- achieving and maintaining high target temperatures at appropriate timings and durations;
- ensuring reproducibility across successive treatments.

Pennes' Bio-Heat Equation [3]

1D case, homogeneous medium

$$\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - W C_b (T - T_a) + P$$



$$\begin{cases} T(x,0) = T_0(x) & x \in [0, L] \\ T(0,t) = T_a & t \geq 0 \\ \frac{\partial T}{\partial x}\Big|_{x=L}(t) = v(t) & t \geq 0 \end{cases}$$

W is the perfusion rate, unknown and variable in a known range for a given tissue type.

Physics-informed neural networks (PINNs [5])

Embed a PDE into the loss via automatic differentiation [4]

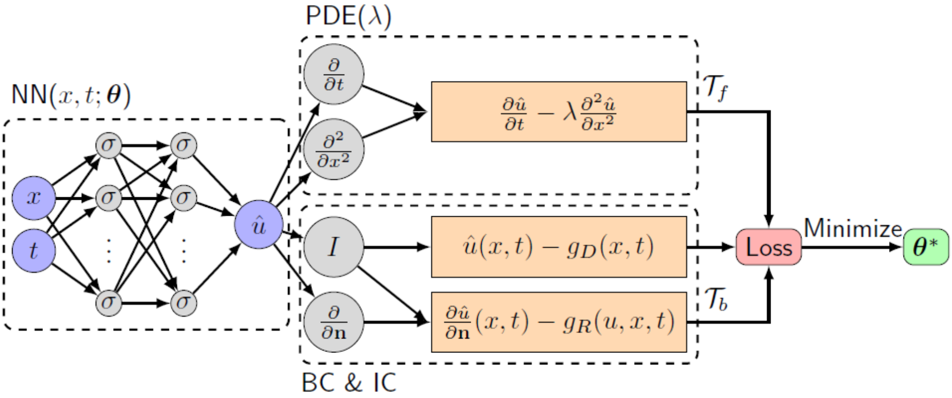


Figure: Lu et al, 2021

State of the Art and Future Perspectives

Towards non-invasive feedback control of superficial Hyperthermia

Today: feedback information about the treatment is only achievable using invasive temperature probes $\rightarrow$ discomfort for the patient, pointwise information.

Objective: retrieve temperature distribution at the target in real-time employing superficial measurements, patient models, and treatment settings.

Conventional numerical methods are not suitable for real-time application because of long computation time. A non-invasive, AI-oriented tool may be able to predict in a fast and accurate way the temperature distribution at the target.

The Problem

Perfusion plays a crucial role in Hyperthermia treatments:

PRO: stabilization term $\rightarrow$ faster achievement of stationary state

CON: heat removal due to mass transport of blood $\rightarrow$ cold tracks, inhomogeneous heating

How to retrieve the temperature distribution inside the domain, knowing boundary measurements and in the presence of unknown patient properties?

Adaptive Estimation of Pennes' Bio-Heat Equation I [6]

Under the assumption that:

- the input $v(t)$ is accessible and the output $y(t)$ is available at any $t \geq 0$;
- W is constant and $0 \leq W_{\text{min}} \leq W \leq W_{\text{max}}$.

With some guess $\tilde{W} \in [W_{min}, W_{max}]$, the observer can be designed as follows:

$$\rho C \frac{\partial \hat{T}}{\partial t} = k \frac{\partial^2 \hat{T}}{\partial x^2} - \tilde{W} C_b(\hat{T} - T_a) + P \tag{1}$$

$$\begin{cases} \hat{T}(x,0) = \hat{T}_0(x) & x \in [0, L] \\ \hat{T}(0,t) = T_a & t \geq 0 \\ \left.\frac{\partial \hat{T}}{\partial x}\right|_{x=L}(t) = v(t) + \alpha(y(t) - \hat{T}(1,t)) & t \geq 0 \end{cases}$$

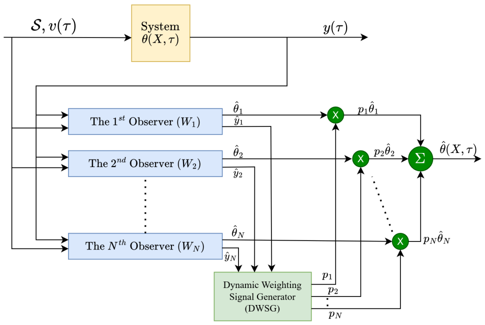
Being $\alpha > 0$ the output injection gain, and $\hat{T}_0$ subject to compatibility conditions.
The error $e_T(x,t) = T(x,t) - \hat{T}(x,t)$ is ultimately bounded and the convergence towards the attractive set is exponential.

Multiple-model observer and overall estimator

Weighted average of each observer's prediction. Weights $p_i(t)$ are obtained from:

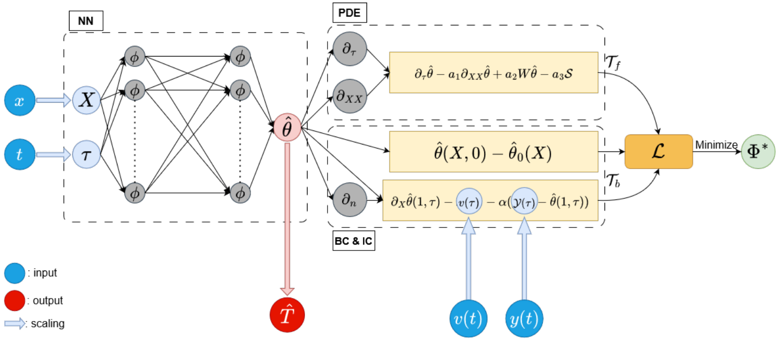
$$\dot{p}_i(t) = -\lambda \left(1 - \frac{e^{-\mu_i(t)}}{\sum_{\ell=1}^{N} p_\ell(t) e^{-\mu_\ell(t)}} \right) p_i(t)$$

with $\mu_i(t)$ the absolute output error and $\lambda > 0$ the adaptive gain.



Adaptive Estimation of Pennes' Bio-Heat Equation II [7]

Computational issues arising in the integration of multiple PDEs and ODE are handled by a PINNs surrogate of the multiple-model observer.

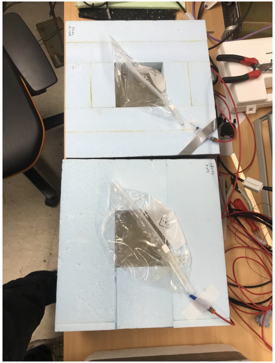


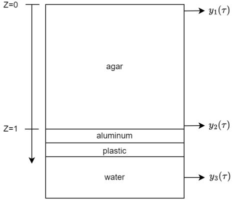
PRO: scale to multiple input seamlessly and probe continuously the system

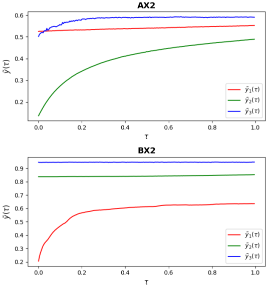
CON: accuracy depends on hyperparameters choice and on regularization techniques

Experiments at AMC Hospital

Agar phantom

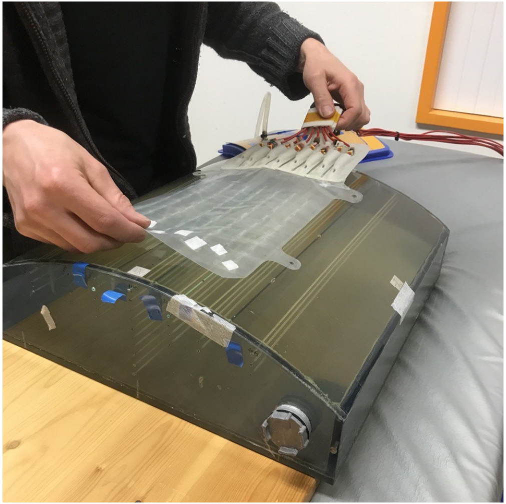


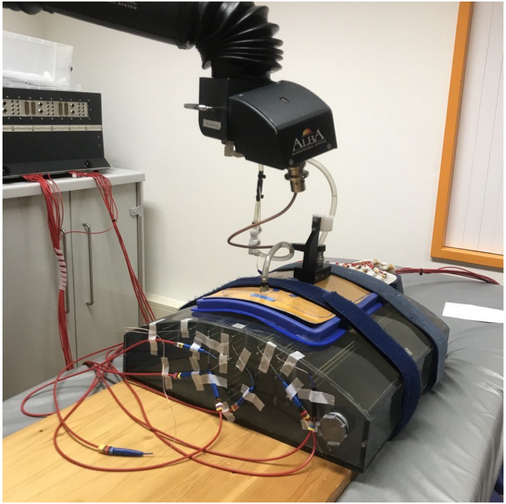




Experiments at AMC Hospital

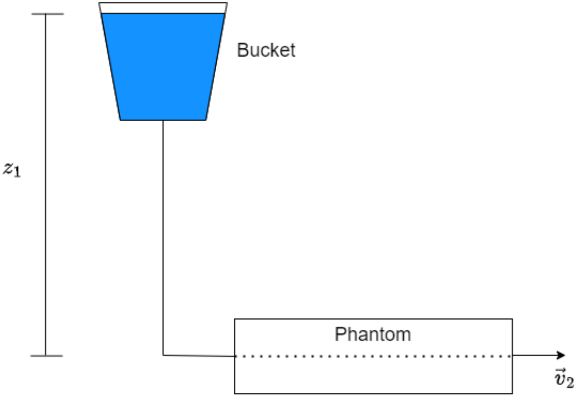
Wallpaper paste-filled phantom





Experiments at AMC Hospital

Perfused wallpaper paste-filled phantom



Project

Sometimes PINNs fail to train[8]. Several techniques to improve accuracy can be adopted [9].

- Implement Multi Scale Fourier Feature Embedding[10] with PyTorch. You can use both DeepXDE (with PyTorch as backend) or a built-from-scratch model. You can take a hint from here.

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