NBER WORKING PAPER SERIES

ON THE INTERACTION OF MEMORY AND PROCRASTINATION: IMPLICATIONS FOR REMINDERS

Keith M Marzilli Ericson

Working Paper 20381
http://www.nber.org/papers/w20381

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2014

I thank Stefano DellaVigna, David Laibson, Ted O'Donoghue, Matthew Rabin, and seminar participants at the Univeristy of California-Berkeley, Harvard University, and ESA-North America for helpful comments. I thank the Williams College Tyng Committee and the National Science Foundation for financial support. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

On the Interaction of Memory and Procrastination: Implications for Reminders
Keith M Marzilli Ericson
NBER Working Paper No. 20381
August 2014
JEL No. D03,D81,D86,D9

ABSTRACT

I examine the interaction between present-bias and limited memory. Individuals in the model must choose when and whether to complete a task, but may forget or procrastinate. Present-bias expands the effect of memory: it induces delay and limits take-up of reminders. Cheap reminder technology can bound the cost of limited memory for time-consistent individuals but not for present-biased individuals, who procrastinate on setting up reminders. Moreover, while improving memory increases welfare for time-consistent individuals, it may harm present-biased individuals because limited memory can function as a commitment device. Thus, present-biased individuals may be better off with reminders that are unanticipated. Finally, I show how to optimally time the delivery of reminders to present-biased individuals.

Keith M Marzilli Ericson
Boston University School of Management
595 Commonwealth Avenue
Boston, MA 02215
and NBER
kericson@bu.edu

1 Introduction

People forget to follow through on plans they have made, despite the availability of ubiquitous and inexpensive memory technologies, such as calendars, task lists, and reminder services. Forgetting can have substantial impacts on productivity via missed deadline. Forgetting to take medication can even be life threatening, and even though reminder systems can be effective, adherence to medication remains low (McDonald, Garg, and Haynes 2002). It is a puzzle that while individuals can set up their own reminders, small reminder interventions can have an economically significant impact on behavior in a variety of domains, including savings (Karlan, McConnell, Mullainathan and Zinman (2010) and loan repayments (Cadena and Schoar 2011).[1]

Understanding and optimally designing reminders is valuable in many different contexts. Project managers and educators must determine when to remind their employees or students about upcoming deadlines. Firms selling products with recurring purchases (e.g. health insurance) or offering retail promotions must determine when to remind customers about an upcoming decision deadline. Adherence to medical treatment could be improved through better use of reminders, and even small tasks of living (home and car maintenance, portfolio rebalancing, etc.) could benefit from improved reminder design.

The design of reminders must take into account not just limited memory, but how memory interacts with other psychological phenomena to produce behavior. Economists examining how psychological phenomena affect behavior often consider a bias or deviation in isolation, a logical first step. Yet psychological phenomena may interact in complex ways, offsetting or magnifying each other. This paper examines the interaction between two major components of decision making– present-bias and limited memory– and their implications for reminders and deadlines. It shows that this interaction can have counter-intuitive effects. For instance, while reminders increase the probability an individual remembers about a task, they also can increase the probability an individual procrastinates, and in the end, leave her worse off.

Behaviors that involve costs in the present but benefit in the future are prevalent: e.g. looking for a job, enrolling in a retirement plan, investing in a health behavior. The way in which individuals attempt to complete these tasks depends heavily on whether they are time-consistent or present-biased (O'Donoghue and Rabin 1999a, 1999b, 2001). Present-biased individuals have a declining rate of time preference, overweighting the present relative to the future and leading to preference-reversals (Laibson 1997).[2] As a result of present-

[1] See also appointment show-ups (Guy et al. 2012), appointment sign-ups (Altmann and Traxler 2012), rebate claims (Letzler and Tasoff 2013), and donations (Damgaard and Gravert 2014)

[2] Both laboratory and field evidence indicate that individuals tend to make relatively impatient choices for

bias, individuals procrastinate[3] in doing tasks. For instance, an individual may repeatedly plan to act tomorrow rather than today, but when tomorrow arrives, she changes her plan. Sophisticated individuals who recognize their present-bias will wish to bind their future behavior via commitment devices. Yet evidence also suggests that individuals are at least partially naive about their present bias, and fail to predict their future behavior (Ariely and Wertenbroch 2002, DellaVigna and Malmendier 2006, Acland and Levy 2013).

When action is delayed, memory is relevant. A growing literature on memory shows that individuals not only have limited prospective memory[4]– remembering to take an action– but are also overconfident about their memory ability.[5] For instance, Ericson (2011) elicits incentivized forecasts of subjects' subjective probability of remembering to claim a delayed payment, and finds that while their choices imply at least a 75% probability of claim, they only claim the payment about half the time. Similarly, Letzler and Tasoff (2013) find that individuals are overoptimistic about claiming rebates at a delay, claiming about 30% of the time despite believing they would claim about 80% of the time. Reminders need to take account that individuals may have biased beliefs about their memory.

This paper shows that present-bias and memory interact in ways that have substantial effects on welfare and behavior.[6] I examine individuals who are choosing whether and when to complete a valuable task that entails immediate costs and delayed benefits. I extend the framework of O'Donoghue and Rabin (1999a, 1999b, 2001) to analyze how individuals with both imperfect memory behavior and present-bias behave. I then use that framework to consider the effect of improving memory by providing reminders or changing the deadline length.

Present-bias expands the range of problems in which limited memory might have an effect on behavior and magnifies memory's economic consequences in three ways. First, because present-bias induces delay, individuals have more of an opportunity to forget. Second,

decisions that involve immediate costs or benefits, but relatively more patient choices when choosing for the future. Discount rates are high at short horizons, but lower discount rates at long horizons, found not only for money (Frederick, Loewenstein and O'Donoghue 2002) but also for direct consumption experiences (e.g. McClure et al. 2007 for juice, Read and van Leeuwen 1998 for food). Augenblick et al. (2013) show that choices for real consumption display a larger present-bias than choices for money. Present-biased preferences have been used to explain patterns of consumption (Laibson 1997, Angeletos et al. 2001) and default-taking or status quo bias (e.g. Madrian and Shea 2001, Carroll et al. 2009).

[3] I use "procrastination" to refer to any additional delay in action induced by present-bias.

[4] Prospective memory is memory for action, and is distinct from retrospective memory (recalling information about the past). See McDaniel and Einstein (2007) for a review.

[5] This literature is closely connected to that on the salience of a decision (Gilbert and Graff Zivin 2013) or limited attention (Reis 2006, Taubinsky 2013).

[6] In related work, Holman and Zaidi (2010) model the economic consequences of limited memory for time-consistent individuals. They show that in the presence of limited memory, the probability of task completion may decrease in the length of time allocated to it, and that overconfidence in prospective memory can explain the existence of free trials, automatic renewal offers and rebates.

present-biased individuals can procrastinate on setting up a memory technology (e.g. lists, calendars, and smartphones), as doing so entails immediate costs for future benefits. Consistent with these theoretical predictions, Karlan et al. (2010) find that randomly assigned reminders increase total savings and have a larger effect on present-biased individuals. Finally, I show that sophisticated present-biased individuals may deliberately reject even free reminders, using limited memory as a commitment mechanism to act.

Biases that seem small may combine to yield larger effects than a single bias alone. Naiveté about present-bias and overconfidence regarding memory ability both lead individuals to inefficiently delay completing a task, as they do not recognize that failing to act today may lead to procrastinating or forgetting next period. For a time consistent individual, small amounts of overconfidence regarding memory have no first-order welfare effect, since she was close to her optimum. Yet for even small amounts of present bias, small amounts of overconfidence regarding memory entail a first-order welfare loss, as the individual has a pre-existing bias.[7] The combination of these two biases can place a wedge between action and welfare (as the individual herself would judge it), and is an example of a "psychological factor leading to inaction" in the framework of Ericson (2014).

Present-bias affects the cost of limited memory and the design of reminder systems. The cost of reminder systems bound the cost of limited memory to time-consistent individuals with correct memory beliefs, though overconfidence can lead to large welfare losses. However, if reminder systems themselves are costly enough to setup, naifs will not use them, even if they ultimately forget a valuable task; sophisticates will only use them with some probability less than 1. The timing of providing one-shot reminders (e.g. an email reminding individuals about an upcoming deadline) is affected by present-bias as well. The optimal time to provide reminders to present-biased individuals can be much later than for time consistent individuals: even once reminded about a task that had been forgotten, present-biased individuals will procrastinate on completing the task and may forget again.

This paper is organized as follows. Section 2 lays out the task completion model, and Section 3 describes behavior and the effect of memory in the simple case in which the cost of completing the task is constant over time. Section 4 considers the more general case of stochastic task completion costs. Section 5 considers an individual's decision to invest in reminder systems. Section 6 shows how to choose when to provide a one-shot reminder, along with numerical simulations showing the difference between present-biased and time consistent individuals. Finally, Section 7 concludes.

[7] For instance, Bernheim, Fradkin and Popov (2011) argues that present-bias alone cannot calibrationally explain default-taking in retirement savings plans due to the large stakes involved; however, it is possible present-bias and memory interacting could produce such behavior.

2 Model: Completing a Task

2.1 Setup

Consider an individual choosing when to do a particular task that involves costs today (effort) but gives benefits in the future. This is a general set up that captures many economically relevant tasks: for instance, choosing to enrolling in a health insurance or retirement plan, working on a research paper, or starting an exercise program. O'Donoghue and Rabin (1999a, 1999b, 2001) use a similar framework to analyze the behavior of present-biased individuals. I adapt this framework to allow for the possibility of imperfect memory, nesting the case of both perfect memory and no present-bias.

Each period, the individual decides whether to do a task that can be done only once. She can only determine her behavior this period, and cannot commit to future actions.[8] If the individual does the task in period t, she pays cost c_t that period and receives benefit y in the next period.[9] The cost of action is allowed to be stochastic, with draws each period from a known time-invariant distribution F with associated density function f. I allow for the possibility of a deadline: after T periods, the opportunity to do the task disappears. When $T = \infty$, there is no deadline.

I model limited memory as an exponential decay in the probability the task will be recalled (see Levy and Loftus 1984). Each period, an individual with imperfect memory will forget about the task with some probability; If the individual forgets about the task, they cannot act. If she remembers in one period, the probability she remembers in the next period is ρ. Thus, if the individual remembers about the task today, she will remember it m periods from now with probability ρ^m. In this model, forgetting is an absorbing state: once she forgets about the task, she does not remember again. This form of limited memory accords with psychological evidence on memory (Mullainathan 2002, Holman and Zaidi 2010): once something is forgotten, it is much less likely to be remembered in the future. Modeling forgetting as a fully absorbing state (as opposed to generalizing the model to allow a positive probability of moving from a state of forgetting to remembering) is a simplification that makes analysis more tractable, akin to how the quasi-hyperbolic discounting model is a simplification of hyperbolic discounting. Moreover, my model of memory is distinct from "slipping the mind", in which an individual forgets this period and does not act but

[8] In some contexts, the use of commitment devices is possible, but here I consider either non-contractible behaviors or an environment with transactions costs high enough to rule out commitment devices. Note that in any case, individuals who are naive about their present-bias would not pay to take up commitment devices.

[9] The value of y may represent a one-time benefit that is received tomorrow, or a flow of benefits received in the future, beginning tomorrow.

remembers next period. Slipping the mind is transitory and does not impact the probability of remembering in future periods; it is easily accommodated in the distribution of task costs F as some probability of drawing $c_t = \infty$.

Present-bias is captured by quasi-hyperbolic discounting (Laibson 1997, O'Donoghue and Rabin 1999) in which discounted utility is $u_0 + \beta \left(\sum_{m=1}^{\infty} \delta^m u_m\right)$, where u_t is the flow utility in each period t. All future periods beyond the present period are discounted by $\beta \leq 1$, in additional to the standard per period discount factor $\delta \leq 1$.When $\beta = 1$, the individual is time consistent (TC) and discounts the future exponentially, but when $\beta < 1$ the individual has time inconsistent preferences.

Because individuals may be present-biased and have incorrect beliefs, the choice of welfare criterion must be explicit. I define welfare the way an individual with present bias and correct beliefs would judge their welfare a period before having the opportunity to do the task. That is, welfare is ex ante welfare as judged from the perspective of $\beta = 1$ by an individual who has correct beliefs about their future behavior and memory. Hence, welfare is given by $U_i = \sum_{t=0}^{\infty} \delta^t u_t$. This definition is natural and is useful for evaluating how individuals choose (or would like to choose) task environments; it is often used (Gruber and Koszegi 2002, O'Donoghue and Rabin 2006, Heidhues and Koszegi 2010). However, it differs from other proposed welfare criteria for individuals with present bias (see e.g. Bernheim and Rangel 2009).

2.2 Equilibrium Behavior for Sophisticates and Naifs

An individual's behavior today will depend on what she expects to do in the future, about which she may have correct or incorrect beliefs. Specifically, let individuals believe that they will act in the future as though they had present-bias parameter $\hat{\beta}$. If $\hat{\beta} = \beta$ the individual correctly anticipates her future present bias, and I call her sophisticated. If $\hat{\beta} > \beta$, the individual believes she will not be as present-biased in the future as she is now. I analyze $\beta < \hat{\beta} = 1$ individuals, who display present-bias in the present period, but who think each period that they will act like a time-consistent ($\beta = 1$) individual in the future. I always assume individuals correctly perceive δ.

I also allow for the possibility that individuals have incorrect beliefs about memory ability: the individual believes her probability of remembering from period to period be $\hat{\rho} \geq \rho$. When $\hat{\rho} = \rho$ I call the individual and her beliefs "calibrated", and when $\hat{\rho} > \rho$, I call them "overconfident". This language allows a distinction between beliefs in this domain and in the present-bias domain, where $\hat{\beta} = \beta$ individuals are "sophisticated" and $\hat{\beta} > \beta$ individuals are "naive".

To determine how individuals behave, I follow O'Donoghue and Rabin (1999a, 1999b,

2001) and require that individuals follow *perception-perfect strategies.* Perception perfect strategies require that each period's behavior maximizes that period's preferences given beliefs about future strategies, and requires that beliefs be dynamically consistent. With dynamically consistent beliefs, individuals think they will act optimally in the future, given their beliefs about future preferences and strategies. The formal definition is given in the appendix; here I describe the implications of perception-perfect strategies for the types in the model. A strategy s_t is a plan of action for period t, and I restrict strategies to take two forms: a strategy is a cutoff c_t^* so that the individual who remembers acts if $c_t \leq c_t^*$, and a probability of action $\pi \in [0, 1]$ if $c = c_t^*$. I further require that strategies be dependent only on the state, not the history, a restriction that is only relevant for present-biased sophisticates; it is intuitive and simplifies the analysis substantially.[10]

I examine three types of individuals (TC, N, and S), distinguished by their degree of their present-bias and sophistication level:

TC: Time Consistent ($\beta = \hat{\beta} = 1$)
N: Naif ($\beta < \hat{\beta} = 1$)
S: Sophisticated ($\beta = \hat{\beta} < 1$)

Define the current value function W in period t, for type $i \in \{TC, N, S\}$, given memory beliefs $\hat{\rho}$ and anticipated future strategies $\hat{S}^t = \left\{\hat{s}_{t+1}^t, \hat{s}_{t+2}^t, ...\right\}$ as:

$$W_{it}^{\hat{\rho}}|\hat{S}^t = u_t + \beta\left(\sum_{m=1}^{\infty}\hat{\rho}^m\delta^m u_{t+m}\left(\hat{s}_{t+m}^t\right)\right)$$

where $\hat{s}_{t+m}^t$ is the individual's belief in period t about the strategy she will follow in period $t+m$. Note that the individual only is able to implement the strategy in period $t+m$ if she remembers, which she anticipates happens with probability $\hat{\rho}^m$; if she forgets, she cannot act and gets $u_{t+m} = 0$. (However, once she acts, she gets the benefit of the task next period regardless of whether she would have remembered next period.) If an individual acts m periods in the future, $u_{t+m} = \delta y - c_{t+m}$; if the individual acts in the current period t, I make a slight abuse of notation and write $u_t = \beta\delta y - c_t$.

A perception-perfect strategy for a time-consistent individual with beliefs $\hat{\rho}$ is that of a standard optimizing individual: their maximize their expected utility $W_{TC}^{\hat{\rho}}$, given perceived memory $\hat{\rho}$ and given that they expect the maximize expected utility in each future period. A time-consistent individual always predicts her behavior correctly, conditional on remembering, but may mispredict her probability of remembering.

[10] See the discussion in O'Donoghue and Rabin (1999b) for more detail, particularly footnotes 9 and 31.

Naifs maximize their perceived expected utility $W_N^{\hat{\rho}}$, given their perceived memory $\hat{\rho}$, and given that they expect to act like a time-consistent individual in all future periods. Naifs therefore mispredict their behavior, conditional on remembering, and may also mispredict their probability of remembering. Sophisticates recognize that they will have present-bias in the future. They maximize $W_S^{\hat{\rho}}$ and correctly perceive that each $s_\tau \in \arg\max W_S^{\hat{\rho}} | \hat{S} \left(\hat{\beta} = \beta, \hat{\rho} \right)$.

3 Simple Case: No Benefit to Delay

Before moving to the general model of stochastic task completion, I analyze behavior in a context in which there is no benefit to delaying completing the task: the cost of completing the task is constant across all periods with $c_t = c$ for all t. This case captures environments in which the effort cost does not vary much over time. Comparing this case to that of a stochastic cost of action (and hence potential benefit to delay) illustrates how the effect of the interaction between imperfect memory and present bias depends on the benefit to delay.

The results for time-consistent individuals are simple: so long as ever acting is optimal ($\delta y > c$), they act immediately. When the task is a net gain, delaying this gain lowers discounted utility: $\delta y - c > \delta \left(\delta y - c \right)$. Thus, memory and beliefs about memory have no effect on time-consistent individuals.

For individuals with present bias, I make two assumptions. First, I assume that the task is "$\beta-$worthwhile": it is worth doing eventually even for someone with present bias, requiring that $\beta \delta y > c$. Thus they would rather act today than never; if this were not the case, then a present-biased individual would never do the task. I also assume that the task has high enough cost that procrastination is desirable for an individual who believes they have perfect memory ($\hat{\rho} = 1$); this is a minimal requirement, as if procrastination is not desirable with perfect memory it will not be with imperfect memory either. This assumption requires that cost be high enough, relative to discount benefits: $c > \frac{(1-\delta)}{(1-\beta\delta)} \left(\delta \beta y \right)$, and is satisfied for all $c > 0$ when $\delta = 1$. Since δ is the exponential discount factor, δ near 1 is likely.

Assumption 1. *The task is $\beta-$worthwhile ($\beta \delta y > c$) and procrastination is desirable with perfect memory: $c \left(1 - \beta\delta\right) > \left(1 - \delta\right) \delta \beta y$.*

Then, the following proposition describes the behavior of the various individuals:

Proposition 1. *In the no-benefit to delay case:*

- *Time consistent individuals act immediately.*

- *A naif acts immediately if perceived memory is low enough ($\frac{\beta\delta y-c}{\beta\delta(\delta y-c)} > \hat{\rho}$). If the naif does not act immediately, they never act if there is no deadline; with a deadline, they procrastinate until the deadline and then act conditional on remembering.*

- *Sophisticates facing a deadline use a cyclical strategy, in which they plan to act at the deadline and every d^*+1 periods before the deadline (conditional on remembering), where the maximum tolerable delay d^* is the lowest integer such that $\beta\delta y - c > \beta\left(\delta\hat{\rho}\right)^{d^*+1}(\delta y - c)$. When the task has no deadline ($T=\infty$), the sophisticate can either use a cyclical strategy acting every d^*+1 periods or a mixed strategy. With the mixed strategy, she plans to act with probability $\pi_{\hat{\rho}} = \frac{1}{c}\frac{\beta\delta y-c}{(1-\beta)}\frac{(1-\hat{\rho}\delta)}{\hat{\rho}\delta}$ each period she remembers.*

The naive agent believes she will act tomorrow (conditional on remembering) if she does not act today, because she believes she will behave as a time consistent agent would in the future. A naif would only act today if $\beta\delta y - c > \hat{\rho}\beta\delta\left(\delta y - c\right)$, as she discounts the future by β and believes she will only remember with probability $\hat{\rho}$. By assumption, the naif will not act today when $\hat{\rho} = 1$, always expecting to act the next day. If there is a deadline, the naif will act in that period, if she remembers and hasn't acted yet: in the deadline period, she compares acting in that period versus not acting at all. Since the task is $\beta-$worthwhile, she will act. The probability she remembers in the deadline period is ρ^T.

Individuals who are sophisticated about their present-bias act differently from naifs or time-consistent individuals. When memory is perfect ($\rho = 1$), the strategy "never do the task" is not a perception perfect equilibrium: a sophisticate would act today if she knew she would never act in the future, since the task is $\beta-$worthwhile. Similarly the strategy "do the task each period" is not a perception-perfect equilibrium, as the sophisticate would rather delay to tomorrow than do it today (see O'Donoghue and Rabin 1999a). There are two types of strategies a sophisticated agent can follow: cyclical strategies (with a maximum tolerable delay of d^* periods, so act every d^*+1 periods) or mixed strategies (act with probability π each period). I emphasize the mixed strategy results, since cyclical strategies are sensitive to when the cycle is begun. (Mixed strategies are also the limit strategy for the stochastic case described in the next section).

Having established how individuals behavior, we can examine the effects of memory:

Proposition 2. *In the no-benefit to delay case, the effects of memory are as follows:*

- *Changing memory ρ or perceived memory $\hat{\rho}$ has no effect on time-consistent individuals.*

- *Increasing memory ρ without changing beliefs $\hat{\rho}$ raises the welfare of naifs (weakly) and sophisticates (strictly).*

- *Raising $\hat{\rho}$ holding fixed ρ weakly lowers the welfare of naifs and strictly lowers the welfare of sophisticates playing non-degenerate mixed strategies.*

- *Increasing memory ρ while maintaining calibrated beliefs ($\hat{\rho} = \rho$) lowers the welfare of sophisticates playing non-degenerate mixed strategies by lowering π, but has an ambiguous effect on naifs. Increasing memory raises welfare if the naif was originally procrastinating, but decreases welfare if doing so enables procrastination by a naif who originally acted at the start.*

Changing memory has important effects on present-biased individuals, but not on time consistent individuals in the no-benefit to delay case, since time consistent individuals act immediately. For present-biased individuals, increasing memory ability holding fixed beliefs raises welfare: strategies don't respond since they are based on beliefs, but individuals are less likely to forget the task.

Limited memory has a cost, but for individuals with self-control problems, awareness of limited memory has a benefit: it can act as a commitment device. Increasing perceived memory $\hat{\rho}$ (i.e. adding overconfidence) while holding fixed actual memory is always (weakly) harmful to present-biased individuals, since doing so enables delay. For naifs, increasing $\hat{\rho}$ reduces the range of parameters for which they will act immediately. When $\hat{\rho} = 0$, the naif will act immediately, since if she were to delay, she would be sure to forget; imperfect memory can therefore emulate the effect of a deadline. For sophisticates, a higher $\hat{\rho}$ leads to a lower probability of action π without any compensating increased ability to remember.

Increasing memory ability and beliefs together combines the benefits of increased memory with the costs of strategies that lead to delay. For naifs, the effect is ambiguous. If the naif was already procrastinating before memory was increased, they still procrastinate; as a result, they on net benefit from increased ability to remember. However, if $\rho = \hat{\rho}$ was low enough that the naif acted immediately, increasing ρ and $\hat{\rho}$ can lead to delay, and lower welfare. For the sophisticate, the net effect on welfare is negative. The mixed strategy π is chosen so that an individual is indifferent between acting and not: $\beta\delta y - c = \beta\delta\hat{\rho}\,(\delta y - c)\sum_{t=0}^{\infty}\delta^t \Pr(\text{act in } t)$. This can be rearranged to give that the expected welfare for the sophisticate is $EU_S = \frac{\beta\delta y - c}{\beta\delta\hat{\rho}}$, decreasing in $\hat{\rho}$. (Welfare under cyclical strategies depends on where the cycle starts, but lowering $\hat{\rho}$ shortens the length of the cycle.)

Memory technologies can modeled in a variety of ways. For instance, a one-time reminder will brings the task to mind if it has been forgotten; I consider these in Section 6. Here, consider "reminder systems": technologies that lower the probability of forgetting each period, thereby raising ρ. For instance, an individual may keep a to-do list (raising the probability the task is remembered each period) but may also lose or cease paying attention to the list, leading to forgetting. A reminder system can be totally effective (raising ρ to 1) or partially effective. Anticipated reminder systems change beliefs $\hat{\rho}$ as well as actual memory, while unanticipated reminders hold $\hat{\rho}$ fixed. Propositions 1 and 2 imply the following corollary.

Corollary 1. *Suppose reminder systems are costless. When there is no benefit of delay, unanticipated reminder systems raise the welfare of a naif or sophisticate. Anticipated reminder systems have an ambiguous effect on welfare: anticipated reminders lower the probability the naif will act immediately, but conditional on delay, they raise the probability the task will be completed. For a sophisticate playing a mixed strategy, anticipated reminder systems lower the probability of action each period and lower welfare; anticipated reminder systems also increase the maximum tolerable delay d^* in cyclical strategies.*

4 Task Completion With Stochastic Costs

4.1 Behavior

In this section, I allow for the possibility of a benefit to delaying action by letting the cost of acting in each period be stochastic: the cost c_t is drawn independently each period from a distribution F that I assume is continuous, differentiable and has positive density throughout the range 0 to δy. I restrict results here to the infinite horizon ($T = \infty$) case for clarity, as individuals will have time varying strategies in the presence of a deadline.

Strategies are cutoffs c_i^* such that below c_i^* an individual of type i acts and above c^* the individual does not act.[11] This cutoff maximizes their current value function $W_i^{\hat{\rho}}(c)$, which represents an individual's preferences and perceived future strategy. The cutoff will depend on the expected continuation value $EV_i^{\hat{\rho}}(c)$, which in turn depends on distribution of c, the expected future strategies of type i, and perceived memory ability $\hat{\rho}$. Given the recursive structure of the task, we can define the current value functions and perceived continuation value functions. The current value function $W_i^{\hat{\rho}}(c) = \begin{cases} \beta\delta y - c & \text{if act} \\ \hat{\rho}\beta\delta EV_S^{\hat{\rho}} & \text{if do not act} \end{cases}$, with values of $\beta = 1$ for time consistent individuals and $\beta < 1$ for naifs and sophisticates. The cutoff strategies are then:

$$\begin{aligned} c_{TC}^* &= \delta\left(y - \hat{\rho}EV_{TC}^{\hat{\rho}}\right) \\ c_N^* &= \beta c_{TC}^* \\ c_S^* &= \beta\delta\left(y - \hat{\rho}EV_S^{\hat{\rho}}\right) \end{aligned}$$

[11] I assume that there is no mass of probability at c^*, so that it does not matter what an individual does when $c = c^*$.

Present-biased individuals' time inconsistent preferences result in their the current value function differing from the perceived continuation value function $V_i^{\hat{\rho}}$. An individual's continuation value function discounts utils exponentially (since all these periods are in the future), even if he is present-biased. It is a *perceived* continuation value function because it depends on perceived memory $\hat{\rho}$, as well as perceived future strategies. Naifs believe they will follow the time consistent strategy in the future, and so their perceived continuation value is the same as that of the time consistent individual: $V_N^{\hat{\rho}}(c) = V_{TC}^{\hat{\rho}}(c)$. Sophisticates correctly recognize their future cutoffs depend on β and so correctly forecast their future action. The perceived continuation value functions $V_i^{\hat{\rho}}$ for each type are then given as follows:

$$\begin{aligned}
V_{TC}^{\hat{\rho}}(c) &= \max\left\{\delta y - c, \hat{\rho}\delta E V_{TC}^{\hat{\rho}}\right\} \\
V_N^{\hat{\rho}}(c) &= V_{TC}^{\hat{\rho}}(c) \\
V_S^{\hat{\rho}}(c) &= \begin{cases} \delta y - c & \text{if } c \leq \beta\left[\delta y - \hat{\rho}\delta E V_S^{\hat{\rho}}\right] \\ \hat{\rho}\delta V_S^{\hat{\rho}}(c) & \text{if } c > \beta\left[\delta y - \hat{\rho}\delta E V_S^{\hat{\rho}}\right] \end{cases}
\end{aligned}$$

Recall that ex ante welfare is defined from the perspective of $\beta = 1$. Only when $\rho = \hat{\rho}$ and $\hat{\beta} = \beta = 1$ are the current value, continuation value, and welfare functions the same.

Note that beliefs about memory only affect welfare via strategies. Lemma 1 shows that increasing perceived memory $\hat{\rho}$ makes all types more demanding regarding getting a low cost draw in order to act. This is intuitive, as increasing perceived memory increases the perceived continuation value. However, $\hat{\rho}$ has a subtle effect on sophisticates, as they recognize that increasing $\hat{\rho}$ will enable procrastination in the future, which is inefficient. They trade off this induced procrastination with the actual benefits from action, and the net effect of raising $\hat{\rho}$ on all types is still to decrease c_i^*, and therefore decrease the probability of action in a given period (conditional on remembering and not having acted yet).

Lemma 1. *Time consistent, naive, and sophisticated individuals all have* $\frac{dc_i^*}{d\hat{\rho}} < 0$: *increasing perceived memory ability lowers action cutoffs and enables delay.*

4.2 Results

Proposition 3 describes the effects of increasing memory. Improving memory has ambiguous effects on the welfare of present-biased individuals if beliefs also adjust. For present-biased individuals, there is a trade-off that comes from raising both memory ability and beliefs together. Improving memory itself has a welfare gain when there is a potential benefit of delay due to stochastic tasks costs: the individual is less likely to forget when she waits. Yet when beliefs adjust, better memory also has a cost, because strategies adjust and delay is more likely. (Improving memory ρ while holding beliefs $\hat{\rho}$ fixed is unambiguously good for welfare: if strategies do not respond, increasing ρ simply raises the probability the task is remembered and completed.) When an individual is already very likely to act immediately (e.g. when $\hat{\rho}$ is near 0), increasing memory has a strategy response that leads to additional delay, which is a welfare cost, but there is limited direct gain, since the individual was not delaying often. Conversely, when β is close to 1, they are near the optimum from a welfare perspective, and so lose less from strategies adjusting; the gain from increasing the probability of remembering to act dominates.

Proposition 3. *Increasing memory while maintaining calibrated beliefs ($\hat{\rho} = \rho$) raises the welfare of time consistent individuals, but has an ambiguous effect on the welfare of naive and sophisticated present-biased individuals. The effect on present-biased individuals is negative if the probability of action in a period $F(c_i^*)$ is high enough and is positive if β is close to 1. Increasing memory alone (holding beliefs constant) has a first order positive effect on the welfare of all types.*

These results about memory imply a corollary about reminder systems.

Corollary 2. *Unanticipated reminder systems raise the welfare of all types. For all types, anticipated reminder systems lower action cutoffs and enable delay. Anticipated reminder systems raise the welfare of time consistent individuals, but have an ambiguous effect on present-biased individuals.*

I now consider how biases interact. Small amounts of overconfidence do not have a first order welfare loss for time consistent individuals, even though they need not act immediately and forget with positive probability. However, because of the effects of procrastination, small amounts of overconfidence have a first order welfare loss for present-biased individuals. The proposition below formalizes this:

Proposition 4. *For time consistent individuals, adding small amounts of overconfidence ($\hat{\rho} > \rho$) about memory has no first order welfare effect. For present-biased individuals, both naive and sophisticated, adding small amounts of overconfidence about memory leads to a first order welfare loss.*

Overconfidence about memory induces all types to delay more than would be optimal, since they underestimate the costs of delay. Yet time consistent individuals are indifferent at the margin c_{TC}^* between acting and delaying (an application of the Envelope Theorem). For present-biased individuals, the situation differs. Although they too (in a given period) are indifferent between acting and delaying at c_i^*, from a welfare perspective their action cutoff is already suboptimal due to present bias. We can see that taking the derivative of welfare with respect to $\hat{\rho}$, holding ρ fixed and evaluating at $\hat{\rho} = \rho$, we have

$$\frac{d}{d\hat{\rho}} EU_i|_{\rho} = \frac{1}{1 - \delta\rho\left[1 - F\left(c_i^*\right)\right]} \underbrace{\left[\left(\delta y - c_i^*\right) - \delta\hat{\rho} EU_i\right]}_{\text{Non-zero with present bias}} f\left(c_i^*\right) \frac{dc_i^*}{d\hat{\rho}}$$

From the definition of c_i^* for present-biased individuals, $(\delta y - c_i^*) > \delta\hat{\rho}EU_i$. Overconfidence therefore has a first order welfare loss when $\beta < 1$, even for small amounts of present bias, because there is a preexisting distortion in the individual's choice of action.

5 Investing in Reminder Systems: "Writing it Down"

Individuals have the ability to manipulate their memory ability through the use of memory technologies, which include calendars, lists, reminder services, and others. Since there are many apparently cheap calendar technologies, it is a puzzle that limited memory still affects economic outcomes (Ericson 2011; Karlan, et al. 2010). This section considers whether individuals will invest in using a memory technology. Three factors combine to lead individuals to fail to use memory technologies. First, they simply may be overconfident about their memory ability, leading them not to invest in memory technologies. Second, present-biased individuals may procrastinate in setting up memory technologies, since they involve costs today but benefits in the future. Third, sophisticated present-biased individuals may optimally reject memory technologies, in order to commit themselves to act sooner.

I consider a simple and stark reminder system: "writing it down". Writing it down entails a cost of w today, which in turn guarantees perfect memory ($\rho = \hat{\rho} = 1$) in all future periods. As in Section 4, the individual chooses whether to act each period based on their draw of the task cost c_t. However, the individual also has another option each period: if she does not act, she has the choice of whether to "write it down". The timeline is as follows: within each period, the individual chooses first whether to act or not and then chooses between writing or not-writing. A strategy is then $s_i = \{c_i^*, \omega_i\}$ where c_i^* is the cutoff for acting and $\omega_i \in [0, 1]$ is the probability that the individual will write in a given period if she does not act. I continue to require that individuals of each type play perception perfect strategies, including the decision of whether to write.

First, consider a time consistent agent with correct memory beliefs. She will write immediately or not all,[12] and will write whenever it is optimal from a welfare perspective. Hence, for such an agent, the welfare loss of limited memory is bounded by the cost of

[12] Writing immediately results from the assumption that the cost of writing is constant. This is likely to be roughly true, but some delay in writing could result if the cost of writing fluctuated substantially.

memory aids (cf. Holman and Zaidi 2010).

Proposition 5. *The welfare loss to a time consistent individual with correct memory beliefs ($\hat{\rho} = \rho$) is limited by the cost w of writing. She will write if $w < w^*_{TC}$ where $w^*_{TC} \equiv \delta EV^1_{TC} - \hat{\rho}\delta EV^{\hat{\rho}}_{TC}$. If she ever writes, she will write immediately.*

Since a time-consistent individual with correct beliefs will write down if it is optimal to do so, it is never beneficial to force them to write, or to subsidize reminder systems. Since the welfare loss is limited by the cost of memory aids, limited memory should not play an important role for calibrated, time-consistent individuals. Yet if individuals are overconfident about their memory ability, they will underinvest in memory technology. The cost of limited memory in the presence of overconfidence is bounded only by the benefit of doing the task. In the limit, a person may think he has perfect memory, and so choose never to write. He may in fact be certain to forget, and thereby forego the benefit of the task (see also Ericson 2011, Holman and Zaidi 2010).

A number of factors combine to reduce present-biased individuals' use of memory technologies. First, note that writing things down may not be optimal for a present-biased individual. Recall the previous propositions show that improving memory may reduce the welfare of a present-biased individual. In such cases, individuals who are sophisticated about their present-biased preferences would not pay to improve their memory and might welcome lower memory as a commitment device. (However, naifs might still invest in improving their memory, not realizing that doing so will make them worse off.)

Even when improving memory does improve their welfare, present-biased individuals still will not write down often enough from an ex ante perspective: using the memory technology is a costly effort task with benefits in the future but costs today, and hence subject to the same procrastination problems as other tasks. For some costs of writing, present bias will lead sophisticates and naifs never to write when they would have chosen to commit to write tomorrow; in other cases, naifs may plan to write tomorrow but never do so, and sophisticates will delay writing, playing a mixed strategy and writing with some probability each period.[13]

Consider just the first effect by removing the ability to procrastinate on writing it down: let individuals have a one-shot option to write it down, after which the opportunity to write disappears. With a one-shot opportunity, an individual will write if $\delta EV^1_i - \rho\delta EV^\rho_i > \frac{1}{\beta}w$, while from a welfare perspective it is optimal for sophisticates to write whenever $\delta EV^1_S - \rho\delta EV^\rho_S > w$. Thus, in the one-shot option, sophisticates will write so long as the $\beta-$discounted benefit of the project with perfect memory exceeds that of the project with imperfect memory by more than the cost of writing. (Naifs should optimally write when

[13]The individual may also play a cyclical strategy. I focus on the mixed strategies only.

$\delta EU_N^1 - \rho\delta EU_N^\rho > w$. Their perceived continuation values are not equal to their welfare, as they misperceive their future strategies.)

The more striking results come when individuals can procrastinate on writing it down. Then, naifs will never write if the cost of writing is more than the cost of going one period with imperfect memory– because they misperceive their future actions and plan to act tomorrow.

Proposition 6. *Naifs will plan to write tomorrow whenever a time consistent agent would write. When $\beta w_{TC}^* < w < w_{TC}^*$, a naif will never write it down, but always plan to do so the next day. If $w < \beta w_{TC}^*$, then a naif will write it down today.*

The naif compares writing down today, and thereby remembering tomorrow for sure (getting $\beta\delta EV_N^1 - w$) with not writing today, and potentially forgetting before tomorrow. Hence, whenever $w > w_L$, the individual will not write today, where

$$w_L = \beta\left[\delta EV_N^1 - \rho\delta EV_N^\rho\,(\text{plan to write tomorrow})\right]$$

It is easily seen that w_L decreases in β, since for a naif, the perceived continuation values do not depend on β. From Proposition 3, we see that increasing memory can lower welfare; a naif may write it down when doing so lowers their welfare. Moreover, the cost of limited memory is not bounded by the cost of memory aids for naifs, since they may never use a memory aid even if they plan to.

Sophisticates' strategies for writing are similar to those for completing the task in Proposition 1, and follow the same intuition. Sophisticates may not want to write (since they recognize it could reduce their welfare). Suppose they do want to write. As β approaches 1, the sophisticates will write immediately, since they will act like a time consistent individual. But when β is low enough, the sophisticate will not write. Thus, as for a naif, the sophisticate may want to use a memory aid, but because they do not do so immediately, they may forget before they write or complete the task.

Proposition 7. *Suppose sophisticates would like to commit to write tomorrow. There is a range of costs of writing such that either 1) sophisticates will play a mixed strategy in which they will write with some probability π_w each period or 2) sophisticates will play a cyclical strategy in which they write every d_w^* periods.*

Hence, this model in which individuals can write things down at an effort cost w is akin to overlaying one procrastination problem (the "no benefit to delay case", for the choice of whether to write) on top of a stochastic cost task completion problem.

6 One-Shot Reminders Before a Deadline

How should one-shot reminders be designed so that they are most effective? For time-consistent, calibrated individuals, it is always optimal to tell them in advance that a reminder will be provided. With these anticipated reminders, strategies will adjust appropriately: a future reminder will bring the task back to mind, and enable optimal delay if early cost-draws are high. However, for present-biased individuals, ancitipated reminders may actually lower their welfare by enabling procrastination; Propositions 2 and 3 showed that increasing memory may lower welfare.

Unanticipated reminders, however, can only increase welfare– they hold perceived memory $\hat{\rho}$ constant while increasing actual memory ρ. Here, I consider when to provide an unanticipated reminder. The optimal time to provide a reminder trades off two effects. First, the probability the reminder will be useful increases as time goes on, since the individual is more likely to forget. Second, the value of the reminder (conditional on it being useful) depends on the number of periods until the deadline. If the reminder comes too close to the deadline, the individual may not have time to act after being reminded (i.e. if they are unlucky and get a high task cost draw); if the reminder comes too early, the individual may delay action (due to procrastination or memory overconfidence) and forget before acting.

A time-consistent individual's welfare always increases in the length to the deadline, but this is not true if individuals are present-biased or have incorrect beliefs.[14] Define $\tau = T - t$ to be the number of periods until the deadline. I write $EU_{\tau,i}$ to be the expected utility (welfare) of getting the task with τ periods before the deadline. We have for each type i:

$$EU_{\tau,i} = \int_0^{c^*_{i,T-\tau}} (\delta y - c)\, dF(c) + \left[1 - F\left(c^*_{i,T-\tau}\right)\right] \delta\rho EU_{\tau-1,i}$$

Figure 1, Panel C shows a combination of parameters for which a shorter deadline increases the welfare of the naif: with a long deadline, the naif inefficiently delays, which leads to forgetting. Of course, for the calibrated time-consistent individual, longer deadlines are better, and the net value of the task decreases as the deadline approaches.

Reminders are only potentially useful[15] when the individual has forgotten and not yet acted. The value of the reminder in this case is the value of being given the task with a deadline τ periods from now: $EU_{\tau,i}$. Then, the discounted expected value of a reminder with

[14] Empirically, Shu and Gneezy (2010) find that longer deadlines for redeeming a gift certificate lowered redemption probability; however, a lower redemption probability need not imply lower welfare.

[15] I think of the reminder as being "potentially useful" even if the individual does not act after having received the reminder. The usefulness comes from moving the individual from a state of forgetting to a state of remembering.

τ periods remaining in the task (and t periods into the task) is:

$$Value_i(\tau, t) = \delta^t EU_{\tau,i} \cdot (1-\rho) \sum_{j=1}^{(T-t)} \rho^{j-1} \prod_{k=0}^{j-1} (1 - F(c_k))$$

The value of getting a potentially useful reminder $EU_{\tau,i}$ may not monotonically decline in τ (unless the individual is calibrated and time-consistent). However, the probability a reminder is potentially useful (which comprises the remaining terms in the equation above) increases as time passes, and can be recursively defined:

$\Pr(useful_t) = \Pr(useful_{t-1}) + (1-\rho)\left[1 - F\left(c^*_{t-1}\right)\right]$ Pr(Remember$_{t-1}$ and did not act before$_{t-1}$)

A reminder is always potentially useful if it was potentially useful in the previous period: in these cases, you have forgotten, implying you would not be able to act. The increase in probability of the reminder being useful comes from the state of the world in which you do not act but do forget: this occurs $(1-\rho)[1 - F(c_{t-1})]$ of the time that you had been in state of remembering but not acting: ρ raised to a power times the product of all the previous probabilities you hadn't acted.

Below, Figure 1, top panel, plots how the value of one-shot reminders varies by when they are given. The task considered has a deadline in period $T = 15$. For a time consistent individual, a reminder is most valuable in period 4, but does not vary that much over time. This individual is very likely to act early (their probability of acting in the first period is about 74%), and so if they have not acted by period 4 it is likely they have forgotten, in which case it is valuable to give them a reminder quickly so that they can act without further delay.

In contrast, the best time to give a reminder to a presented-biased naif (for this set of parameters) is in the very last period. The naif has a low probability of acting early on (only 19% in the first period), so even though the reminder is potentially useful because they may have forgotten (Figure 1, middle panel), it will not actually be used because the naif continues to procrastinate. The high value of the reminder in the final period then comes from the fact that the naif is better off having a short deadline, without the ability to procrastinate and forget: it is better to get the task with a short deadline than a long deadline (Figure 1, bottom panel). By contrast, the value of the task to a time consistent individual increases as the deadline lengthens, driving the result that their reminder should be early.

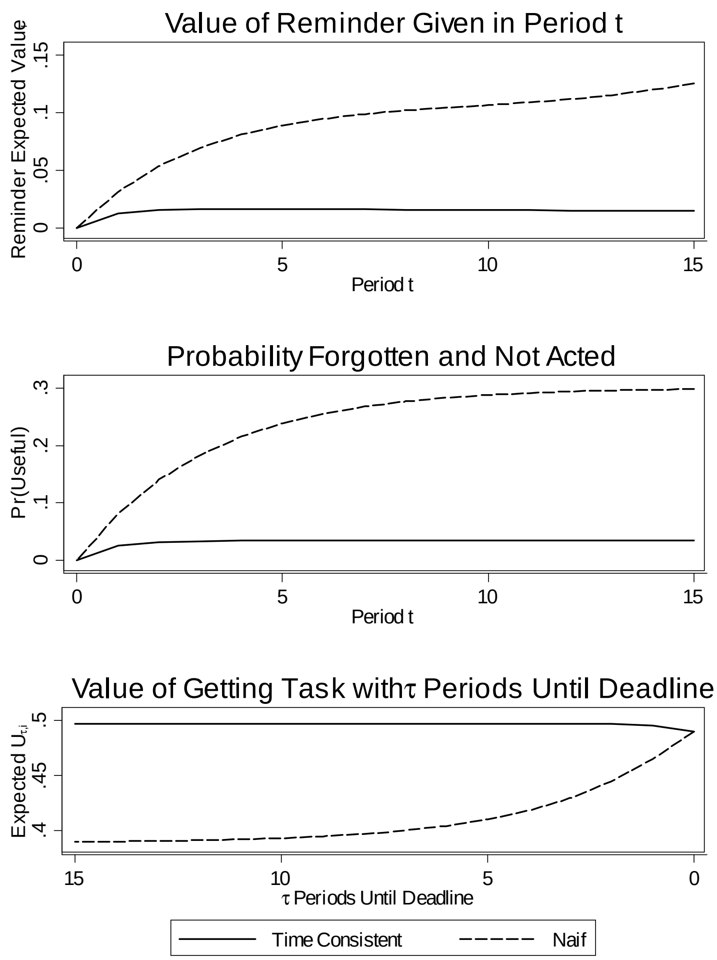

Figure 1: One Shot Reminders, with and without Present-Bias. Assumes reminder is unanticipated, $\hat{\rho} = \rho = 0.9$, $\beta = 0.8, \delta = 0.99$, $y = 1$, c is uniformly distributed between 0.4 and 0.6, and deadline $T = 15$.

7 Conclusion

Psychology paints a rich picture of individual decision making. Memory and present bias interact in ways that are interesting, welfare relevant, and affect reminder design. Considering a single phenomenon in isolation may be misleading—for instance, overconfidence regarding memory ability as a larger effect on the welfare of a present-biased individual than a time-consistent individual. Yet it is also the case that the limitations of memory may help present-biased individuals in some situations. Researchers calibrating models of present bias or making inferences about naiveté or sophistication should take into account the effects limited memory may have on individuals' behavior. Limited memory also has implications for other aspects of task design, such as deadlines. When calibrating a model for an optimal deadline, the both memory and present-bias should be considered.

An individual's memory ability is endogenous to her decision to invest in memory technologies. This paper has shown that present-biased individuals–especially naifs– will invest less in reminders and procrastinate in setting up memory devices, suggesting that present-biased individuals will have lower rates of effective memory ability.

The way to deliver a reminder depends on whether an individual is present-biased. Time-consistent individuals should be given anticipated reminders so that they can optimally respond. For present-biased individuals, anticipated reminders can lower welfare but unanticipated reminders are always beneficial. This paper also showed that the timing of reminder delivery also depends on the degree of present bias.

REFERENCES

Acland, Daniel and Matthew Levy. 2013. "Naivete, Projection Bias, and Habit Formation in Gym Attendance." SSRN Working Paper.

Altmann, S. and C. Traxler. 2012. "Nudges at the Dentist." IZA Discussion Paper 6699.

Angeletos, George-Marios, David Laibson, Andrea Repetto, Jeremy Tobacman, and Stephen Weinberg. 2001. "The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation." *The Journal of Economic Perspectives* 15:47-68.

Ariely D, Wertenbroch K. 2002. "Procrastination, Deadlines, and Performance: Self-Control by Precommitment." *Psychological Science*, 13, 219–224.

Augenblick, Ned, Muriel Niederle, Charles Sprenger. 2013. "Working Over Time: Dynamic Inconsistency in Real Effort Tasks". *NBER Working Paper 18734.*

Bernheim, B. Douglas and Antonio Rangel. 2009. "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics," *Quarterly Journal of Economics* 124:51-104.

Bernheim, Douglas, Andrey Fradkin, and Igor Popov. 2011. "The Welfare Economics of Default Options: A Theoretical and Empirical Analysis of 401(K) Plans." NBER Working Paper No. 17587.

Cadena, Ximena and Antionette Schoar. 2011. "Remembering to Pay? Reminders vs. Financial Incentives for Loan Payments". NBER Working Paper 17020.

Carroll, Gabriel, James Choi, David Laibson, Brigitte Madrian, and Andrew Metrick. 2009. "Optimal Defaults and Active Decisions." *Quarterly Journal of Economics* 124,1639-1674.

Damgaard, Mette and Christina Gravert. 2014. "Now or Never! The Effect of Deadlines on Charitable Giving: Evidence from a Natural Field Experiment." Aarhus University Working Paper.

Della Vigna, Stefano, and Ulrike Malmendier. 2006. "Paying Not to Go to the Gym." *American Economic Review* 96:694-719.

Ericson, Keith M. 2011. "Forgetting We Forget: Overconfidence and Memory." *Journal of the European Economic Association*, 9(1):43-60.

Ericson, Keith M. 2014. "When Consumers Do Not Make an Active Decision: Dynamic Default Rules and their Equilibrium Effects." *NBER Working Paper* 20127.

Frederick, Shane, George Loewenstein and Ted O'Donoghue. 2002. "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature*, 40: 351-401.

Gilbert, Ben and Joshua Graff Zivin. 2013. "Dynamic Salience with Intermittent Billing: Evidence from Smart Electricity Meters." NBER Working Paper 19510.

Gruber, Jonathan, and Botond Koszegi. 2004. "Tax Incidence when Individuals are Time-Inconsistent: The Case of Cigarette Excise Taxes." *Journal of Public Economics*, 88(9–10): 1959–87.

Guy, R. Hocking J, Wand H, Stott S, Ali H, and Kaldor J. 2012. "How effective are short message service reminders at increasing clinic attendance? A meta-analysis and systematic review." *Health Services Research* 47(2): 614-32.

Heidhues, Paul and Botond Koszegi. 2010. "Exploiting Naïvete about Self-Control in the Credit Market" *American Economic Review* 100:2279-2303.

Holman, J. and F. Zaidi. "The Economics of Prospective Memory." Working Paper.

Karlan, Dean, Margaret McConnell, Sendhil Mullainathan, and Jonathan Zinman. 2010. "Getting to the Top of Mind: How Reminders Increase Saving." *NBER Working Paper* No. 16205.

Kirby, Kris N. 1997. "Bidding on the future: Evidence against normative discounting of delayed rewards." *Journal of Experimental Psychology: General* 126:54-70.

Laibson, David. "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, May 1997, 112 (2) 443-477.

Letzler, Robert and Joshua Tasoff. 2013. "Everyone Believes in Redemption: Nudges and Overoptimism in Costly Task Completion." SSRN Working Paper.

Madrian, Brigitte C., and Dennis F. Shea. 2001. "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior." *Quarterly Journal of Economics,* 116:1149-1187.

McClure, S., Ericson, K., Laibson, D., Loewenstein, G., and Cohen, J. 2007. "Time discounting for primary rewards," *Journal of Neuroscience*, 27:5796-804.

McDaniel, Mark and Gilles Einstein. 2007. *Prospective Memory: An Overview and Synthesis of an Emerging Field.* Sage Publications.

McDonald, Heather, Amit Garg and R. Haynes. 2002. "Interventions to Enhance Patient Adherence to Medication Prescriptions." *Journal of the American Medical Association* 288(22): 2868-2879.

O'Donoghue, Ted and Matthew Rabin. 1999a. "Doing it Now or Later," American Economic Review, 89(1), pp. 103-124.

O'Donoghue, Ted, and Matthew Rabin. 1999b. "Incentives For Procrastinators." Quarterly Journal of Economics 114:769-816.

O'Donoghue , Ted and Matthew Rabin. 2001. "Choice and Procrastination," Quarterly Journal of Economics, 121-160.

O'Donoghue, Ted, and Matthew Rabin. 2006. "Optimal Sin Taxes." Journal of Public Economics, 90(10-11): 1825–49

Read, Daniel and Barbara van Leeuwen. 1998. "Predicting Hunger: The Effects of Appetite and Delay on Choice." *Organizational Behavior and Human Decision Processes* 76(2):189–205.

Reis, Ricardo. 2006. "Inattentive Consumers". *Journal of Monetary Economics*, 53 (8), 1761-1800

Shu, Suzanne and Ayelet Gneezy. 2010. "Procrastination of Enjoyable Experiences." *Journal of Marketing Research*, 47(5): 933-944.

Taubinsky, Dmitry. 2013. "From Intentions to Actions: A Model and Experimental Evidence of Inattentive Choice", Working Paper.

Supplementary Material for "On the Interaction of Memory and Procrastination: Implications for Reminders"

Keith M Marzilli Ericson

A.I Proofs of Propositions in the Text

Proposition 1. *In the no-benefit to delay case:*

- *Time consistent individuals act immediately.*
- *A naif acts immediately if perceived memory is low enough ($\frac{\beta\delta y-c}{\beta\delta(\delta y-c)} > \hat{\rho}$). If the naif does not act immediately, they never act if there is no deadline; with a deadline, they procrastinate until the deadline and then act conditional on remembering.*
- *Sophisticates facing a deadline use a cyclical strategy, in which they plan to act at the deadline and every d^*+1 periods before the deadline (conditional on remembering), where the maximum tolerable delay d^* is the lowest integer such that $\beta\delta y - c > \beta\left(\delta\hat{\rho}\right)^{d^*+1}\left(\delta y - c\right)$. When the task has no deadline ($T=\infty$), the sophisticate can either use a cyclical strategy acting every d^*+1 periods or a mixed strategy. With the mixed strategy, she plans to act with probability $\pi_{\hat{\rho}} = \frac{1}{c}\frac{\beta\delta y-c}{(1-\beta)}\frac{(1-\hat{\rho}\delta)}{\hat{\rho}\delta}$ each period she remembers.*

Proof. Time consistent individuals act immediately since $\delta y - c > \delta\hat{\rho}\left(\delta y - c\right) > 0$.

The naif believes she will act tomorrow (conditional on remembering), and so compares acting today and getting with waiting, and acts if

$$\beta\delta y - c > \hat{\rho}\beta\delta\left(\delta y - c\right)$$

If this condition does not hold, the naif plans to act next period if she remembers. But next period, she faces the same decision and chooses to delay again each period until the deadline. At the deadline, if she remembers, she will act, since $\beta\delta y - c > 0$ by the $\beta-$worthwhile assumption.

For the sophisticate, neither "act today" nor "never act" are equilibrium strategies. The sophisticate prefers acting today to never if $\beta\delta y - c > 0$, which is true by assumption. The sophisticate prefers acting tomorrow to acting today if $\left(\beta\delta y - c\right) < \beta\hat{\rho}\left(\delta y - c\right)$, which is always true (by assumption) when $\hat{\rho} = 1$.

For mixed strategies with $T=\infty$, the agent acts with probability π each period, conditional on remembering. The probability of action is chosen so that the agent is indifferent between acting today or waiting, so π is defined by:

$$\begin{aligned}
\beta\delta y - c &= \beta\left[(\delta y - c)\right]\left\{\delta\hat{\rho}\pi + (\delta\hat{\rho})^2(1-\pi)\pi + (\delta\hat{\rho})^3(1-\pi)^2\pi + ...\right\} \\
&= \beta(\hat{\rho}\delta)(\delta y - c)\pi\left\{\sum_{t=0}^{\infty}\left[\delta\hat{\rho}(1-\pi)\right]^t\right\} \\
&= \beta(\hat{\rho}\delta)(\delta y - c)\frac{\pi}{1-\delta\hat{\rho}(1-\pi)}
\end{aligned}$$

Giving $\pi = \frac{1}{c}\frac{\beta\delta y - c}{(1-\beta)}\frac{(1-\hat{\rho}\delta)}{\hat{\rho}\delta}$. Note that π decreases in $\hat{\rho}$,as $\frac{d\pi}{d\hat{\rho}} = \frac{1}{c}\frac{\beta\delta y - c}{(1-\beta)}\left(-\frac{1}{\delta\hat{\rho}^2}\right) < 0$.

Now, consider the sophisticate's other type of strategy, a cyclical strategy. In this strategy, the agent acts every $d^* + 1$ periods, where d^* is the lowest integer such that

$$\beta\delta y - c > \beta(\delta\rho)^{d^*+1}[\delta v - c]$$

Hence, when $d^* = 2$,the individual plans to act in period x, period $x + 3$, period $x + 6$, etc., tolerating a maximal delay of 2 periods in between each action. Increasing memory increases d^*. When there is a deadline, the sophisticate knows she will act in the last period. She therefore acts every d^* periods beforehand.

Proposition 2. *In the no-benefit to delay case, the effects of memory are as follows:*

- *Changing memory ρ or perceived memory $\hat{\rho}$ has no effect on time-consistent individuals.*
- *Increasing memory ρ without changing beliefs $\hat{\rho}$ raises the welfare of naifs (weakly) and sophisticates (strictly).*
- *Raising $\hat{\rho}$ holding fixed ρ weakly lowers the welfare of naifs and strictly lowers the welfare of sophisticates playing non-degenerate mixed strategies.*
- *Increasing memory ρ while maintaining calibrated beliefs ($\hat{\rho} = \rho$) lowers the welfare of sophisticates playing non-degenerate mixed strategies by lowering π, but has an ambiguous effect on naifs. Increasing memory raises welfare if the naif was originally procrastinating, but decreases welfare if doing so enables procrastination by a naif who originally acted at the start.*

Proof. Time consistent individuals act immediately in this case, so memory is irrelevant.

If the naif acts immediately, her welfare is $\delta y - c$,while if she delays, her welfare is $\delta^T\rho^T(\delta y - c)$, so acting immediately is optimal. Holding beliefs fixed, the naif's strategy will not change. Hence, increasing ρ holding $\hat{\rho}$ constant has no effect if the agent was going to act immediately, and increases welfare if she procrastinated until the deadline by increasing the probability she remembers at that point. Increasing $\hat{\rho}$ either has no effect, or leads the naif to delay action, thereby lowering welfare.

For the naif, increasing memory ρ while maintaining calibrated beliefs ($\hat{\rho} = \rho$) has one of two effects on welfare. By increasing $\hat{\rho}$, the agent may choose to delay instead of acting immediately, lowering welfare. But if the agent was going to delay originally, increasing ρ raises the probability she will remember, increasing welfare.

For the sophisticate, define a_t as the (unconditional) probability the agent acts in t.

$$a_t = \pi_{\hat{\rho}} \rho^t \left(1 - \pi_{\hat{\rho}}\right)^t = \pi_{\hat{\rho}} \operatorname{Pr}(\text{remember}) \operatorname{Pr}\left(\text{not acted yet}|\text{remember}\right)$$

and analogously, let $\hat{a}_t = \pi_{\hat{\rho}} \hat{\rho}^t \left(1 - \pi_{\hat{\rho}}\right)^t$ be the perceived probability the agent acts in period t. The ex ante welfare of the sophisticate $EU_S = (\delta y - c) \sum_{t=0} \delta^t a_t$.Note that a_t depends on both π (and hence perceived memory) but also on actual memory ρ. Holding beliefs fixed, increasing ρ increases the probability of remembering each period, raising welfare.

Imposing $\hat{\rho} = \rho$ so beliefs always stay calibrated, we have $a_t = \hat{a}_t$,and $EU_S = \frac{\beta\delta y - c}{\beta\delta\rho}$ since π is defined by $\beta\delta y - c = \beta\delta\hat{\rho}\left(\delta y - c\right) \sum_{t=0}^{\infty} \delta^t \hat{a}_t$.Then it is easy to see that increasing memory leads to lower welfare when beliefs remain calibrated, as $\frac{dEU_S}{d\rho} < 0$. This results because while increasing from the lower probability of action (π decreases in $\hat{\rho}$). Note that increasing $\hat{\rho}$ holding fixed ρ decreases π, and $\frac{dEU_S}{d\pi} > 0$, so increasing overconfidence lowers welfare.

Finally, for cyclical strategies, welfare results are sensitive to where the cycle starts.

Lemma 2. *Time consistent, naive, and sophisticated individuals all have* $\frac{dc_i^*}{d\hat{\rho}} < 0 :$ *increasing perceived memory ability lowers action cutoffs and enables delay.*

Proof. For time-consistent individuals, note

$$\frac{dc_{TC}^*}{d\hat{\rho}} = -\delta EV_{TC}^{\hat{\rho}}\left(c_{+1}\right) - \hat{\rho}\delta \frac{d}{d\hat{\rho}} EV_{TC}^{\hat{\rho}} < 0$$

as $\frac{d}{d\hat{\rho}} EV_{TC}^{\hat{\rho}} > 0$. For naifs, then$\frac{dc_N^*}{d\hat{\rho}} = \beta \frac{dc_{TC}^*}{d\hat{\rho}} < 0$. For sophisticates, we have

$$\text{(A1)} \qquad \frac{dc_s^*}{d\hat{\rho}} = -\beta\delta EV_S^{\hat{\rho}}\left(c_{+1}\right) - \beta\delta\hat{\rho} \frac{d}{d\hat{\rho}} EV_S^{\hat{\rho}}\left(c_{+1}\right)$$

Now, we need to examine the second term, as it is not yet guaranteed that $\frac{d}{d\hat{\rho}} EV_S^{\hat{\rho}} > 0$. Recall $EV_S^{\hat{\rho}} = \int_0^{c^*} \left(\delta y - c\right) dF\left(c\right) + \left(1 - F\left(c^*\right)\right) \delta\hat{\rho} EV_S^{\hat{\rho}}$, so taking the derivative and rearranging gives $\frac{d}{d\hat{\rho}} EV_S^{\hat{\rho}} = \frac{\left[\left(\delta y - c^*\right) - \delta\hat{\rho} EV_S^{\hat{\rho}}\right] f\left(c^*\right) \frac{dc^*}{d\hat{\rho}} + \left(1 - F\left(c^*\right)\right) \delta EV_S^{\hat{\rho}}}{\left[1 - \left(1 - F\left(c^*\right)\right) \delta\hat{\rho}\right]}$. Substituting back into the equation above gives

$$\begin{aligned}
\frac{dc_s^*}{d\hat{\rho}} &= -\beta\delta EV_S - \beta\delta\hat{\rho}\frac{\left[(\delta y - c^*) - \delta\hat{\rho}EV_S^{\hat{\rho}}(c)\right] f(c^*) \frac{dc^*}{d\hat{\rho}} + (1 - F(c^*))\,\delta EV_S^{\hat{\rho}}(c)}{[1 - (1 - F(c^*))\,\delta\hat{\rho}]} \\
\frac{dc_s^*}{d\hat{\rho}} &= -\left[1 + \beta\delta\hat{\rho}\frac{\left[(\delta y - c^*) - \delta\hat{\rho}EV_S^{\hat{\rho}}(c)\right] f(c^*)}{[1 - (1 - F(c^*))\,\delta\hat{\rho}]}\right]\left[1 + \hat{\rho}\frac{1 - F(c^*)}{1 - (1 - F(c^*))\,\delta\hat{\rho}}\right]\beta\delta EV_S(c_{+1})
\end{aligned}$$

and so $\frac{dc_s^*}{d\hat{\rho}} < 0$, since $(\delta y - c^*) - \delta\hat{\rho}EV_S^{\hat{\rho}}(c) > 0$ by the definition of c^*.

Proposition 3. *Suppose individuals originally held calibrated beliefs about memory ($\hat{\rho} = \rho < 1$). Then introduce overconfidence by holding ρ fixed but increasing $\hat{\rho}$. For time consistent individuals, introducing overconfidence about memory has no first order welfare effect. For present-biased individuals, both naive and sophisticated, introducing overconfidence about memory leads to a first order welfare loss.*

Proof. For each type i, we have welfare recursively defined:

$$\begin{aligned}
EU_i &= \int_0^{c_i^*} (\delta y - c)\, dF(c) + [1 - F(c_i^*)]\,\delta\rho EU_i \\
EU_i &= \frac{1}{1 - \delta\rho\,[1 - F(c_i^*)]}\int_0^{c_i^*} (\delta y - c)\, dF(c)
\end{aligned}$$

Then,

$$\frac{d}{d\hat{\rho}}EU_i|_\rho = \frac{1}{1 - \delta\rho\,[1 - F(c_i^*)]}\,[(\delta y - c_i^*) - \delta\hat{\rho}EU_i]\, f(c_i^*)\,\frac{dc_i^*}{d\hat{\rho}}$$

Note that by the envelope theorem, $[(\delta y - c_{TC}^*) - \delta\hat{\rho}EU_{TC}] = 0$,and thus, time-consistent individuals do not have a first-order welfare loss from changes in $\hat{\rho}$: $\frac{d}{d\hat{\rho}}EU_{TC}|_\rho = 0$. However, for a naive individual, $[(\delta y - c_N^*) - \delta\rho EU_N] > 0$ since by the definition of c_N^*, $c_N^* = \beta\delta y - \hat{\rho}\beta\delta EV_{TC}^{\hat{\rho}}$. We have $\frac{1}{\beta}c_N^* > c_N^*$, and $EU_N < EV_{TC}^{\hat{\rho}} = EU_{TC}$. From Lemma 1, $\frac{dc_N^*}{d\hat{\rho}} < 0$,and thus $\frac{d}{d\hat{\rho}}EU_i|_\rho < 0$.

For sophisticates, we have $[(\delta y - c_S^*) - \delta\rho EU_S] > 0$. Recall, that when beliefs are correct, sophisticates have $EU_S = EV_S$, and note that by the definition of c_S^*, we have again $0 = \left(\delta y - \frac{1}{\beta}c_S^*\right) - \hat{\rho}\delta EV_S^{\hat{\rho}}$. The interior is positive, and Lemma 1 indicates $\frac{dc_S^*}{d\hat{\rho}} < 0$,so $\frac{d}{d\hat{\rho}}EU_S(c)\,|_\rho < 0$.

Proposition 4. *Suppose individuals have calibrated beliefs ($\hat{\rho} = \rho$). Improving memory while maintaining accurate beliefs raises the welfare of time consistent individuals. Doing so has an* *ambiguous effect on the welfare of present-biased individuals, both naive and sophisticated: the effect is negative if the probability of action in a period $F(c_i^*)$ is high enough and is positive if β is close to 1. Increasing memory alone (holding beliefs constant) has a first order positive effect on the welfare of all types.*

Proof. First, take the time-consistent agent and see that

$$\begin{aligned}
\frac{d}{d\hat{\rho}}EU_{TC}|_{\hat{\rho}=\rho} &= \frac{d}{d\hat{\rho}}\int_0^{c_{TC}^*}(\delta y - c)\,dF(c) + \frac{d}{d\hat{\rho}}\left[1-F\left(c_{TC}^*\right)\right]\delta\rho EU_{TC} \\
&= \frac{1}{1-\delta\rho\left[1-F\left(c_{TC}^*\right)\right]}\left[\left[\left(\delta y - c_{TC}^*\right) - \delta\hat{\rho}EU_{TC}\right] f\left(c_{TC}^*\right)\frac{dc_{TC}^*}{d\hat{\rho}} + \left[1-F\left(c_{TC}^*\right)\right]\delta EU_{TC}\right]
\end{aligned}$$

Note that at the margin, the individual is indifferent between continuing and acting, so $\left(\delta y - c_{TC}^*\right) = \delta\hat{\rho}EV_{TC}(c)$ and $EV_{TC}(c) = EU_{TC}$, we have

$$\frac{d}{d\hat{\rho}}EU_{TC}|_{\hat{\rho}=\rho} = \frac{1-F\left(c_{TC}^*\right)}{1-\delta\rho\left[1-F\left(c_{TC}^*\right)\right]}\delta EU_{TC} > 0 \tag{A2}$$

which has a first order positive effect. Also note that $\frac{d}{d\rho}EU_{TC}|_{\hat{\rho}} = \frac{\left[1-F\left(c_{TC}^*\right)\right]}{1-\delta\rho\left[1-F\left(c_{TC}^*\right)\right]}\delta EU_{TC} > 0$, so increasing memory holding beliefs constant increases welfare.

Now, take a naive present-biased agent. Then

$$\frac{d}{d\hat{\rho}}EU_N|_{\hat{\rho}=\rho} = \frac{1}{1-\delta\rho\left[1-F\left(c_N^*\right)\right]}\left[\left[\left(\delta y - c_N^*\right) - \delta\rho EU_N\right]\frac{dc_N^*}{d\hat{\rho}} f\left(c_N^*\right) + \delta\left[1-F\left(c_N^*\right)\right]EU_N\right]$$

From the previous proposition, we know the first term in the brackets is negative. The second is positive. As $F\left(c_N^*\right) \to 1$, the negative term dominates, and increasing memory reduces welfare. But as $\lim_{\beta\to1}\left[\left(\delta y - c_N^*\right) - \delta\rho EU_N\right] = 0$, the positive term dominates.

Finally, holding beliefs constant and increasing memory improves welfare, as

$$\frac{d}{d\rho}EU_N|_{\hat{\rho}} = \frac{\delta\left[1-F\left(c_N^*\right)\right]EU_N}{1-\delta\rho\left[1-F\left(c_N^*\right)\right]} > 0.$$

The same logic applies for the sophisticated present-biased agent.

Proposition 5. *The welfare loss to a time consistent agent with $\hat{\rho} = \rho$ is limited by the cost w of writing. She will write if $w < w_{TC}^*$ where $w_{TC}^* \equiv \delta EV_{TC}^1 - \hat{\rho}\delta EV_{TC}^{\hat{\rho}}$. If she ever writes, she will write immediately.*

Proof. The agent's cutoff strategy will depend on whether it is optimal to write

$$\begin{aligned} c^*(\omega) &= \begin{array}{ll} \text{If write} & \delta y - \left(EV^1_{t+1} - w\right) \\ \text{If not write} & \delta y - \delta\rho EV^\rho_{t+1} \end{array} \\ &= \delta y - \max\left\{\left(\delta EV^1_{TC} - w\right), \delta\rho EV^\rho_{TC}\right\} \end{aligned}$$

It is immediately apparent that if it is ever optimal to write, it is optimal to write immediately (or act immediately): $(\delta EV^1_{TC} - w) > \delta\hat{\rho}EV^\rho_{TC}$ is necessary and sufficient for both.

Proposition 6. *Naifs will plan to write tomorrow whenever a time consistent agent would write. When $\beta w^*_{TC} < w < w^*_{TC}$, a naif will never write it down, but always plan to do so the next day. If $w < \beta w^*_{TC}$, then a naif will write it down today.*

Proof. A naif plans to act like a time-consistent individual in the future, so plans to write tomorrow (conditional on not acting tomorrow) so long as $(\delta EV^1_{TC} - w) > \delta\hat{\rho}EV^\rho_{TC}$. Assume this is the case, so that $w < w^*_{TC}$. Now, suppose the naif does not act this period. Her condition for writing is then:

$$\beta\delta EV^1_{TC} - w > \beta\delta\hat{\rho}EV^\rho_{TC}$$

and so $w^*_N = \beta\left(\delta EV^1_{TC} - \hat{\rho}\delta EV^{\hat{\rho}}_{TC}\right) = \beta w^*_{TC}$

Proposition 7. *Suppose sophisticates would like to commit to write tomorrow. There is a range of costs of writing such that either 1) sophisticates will play a mixed strategy in which they will write with some probability π each period or 2) sophisticates will play a cyclical strategy in which they write every d^* periods.*

Proof. Suppose the sophisticate prefers wring now to never, so that:

$$\delta EV^1_S - \rho\delta EV^\rho_S\,(\text{never}) > \frac{w}{\beta}$$

where I denote the continuation value without the option to write in the future as $EV^\rho_S\,(\text{never})$. Under the following condition, the sophisticate would prefer writing tomorrow (if they did not act) over writing today:

$$\left[\delta EV^1_S - \rho\delta EV^\rho_S\,(\text{tomorrow})\right] < \frac{w}{\beta}$$

where EV^ρ_S (tomorrow)indicates the continuation value under the strategy of writing tomorrow given that they do not act. Now, there is a range such that

$$\delta EV_S^1 - \rho\delta EV_S^\rho\,(\text{never}) > \frac{w}{\beta} > \delta EV_S^1 - \rho\delta EV_S^\rho\,(\text{tomorrow})$$

so long as $EV_S^\rho\,(\text{never}) < EV_S^\rho\,(\text{tomorrow})$. But this must be true. Write $c_1^* = \delta y - \delta EV_S^1 - w$,and $c_2^* = \delta y - \rho\delta EV_S^\rho$, so

$$\begin{aligned} EV_S^\rho\,(\text{tomorrow}) &= \int_0^{c_1^*} (\delta y - c)\, dF(c) + [1 - F(c_1^*)]\left[\delta EV_S^1 - w\right] \\ EV_S^\rho\,(\text{never}) &= \int_0^{c_2^*} (\delta y - c)\, dF(c) + [1 - F(c_2^*)]\,\rho\delta EV_S^\rho\,(\text{never}) \end{aligned}$$

and since the individual prefers writing now to never, $EV_S^\rho\,(\text{never}) < EV_S^\rho\,(\text{tomorrow})$.Hence, such a range exists.

In this range, "act never" is not an equilibrium, and "act always" is not an equilibrium. I solve for the mixed strategy equilibrium. Sophisticates choose the probability π of acting each period (conditional on remembering), such that they are indifferent between acting and not:

(A3) $$EV_S^1 - \rho EV_S^{\rho|\pi} = \frac{1}{\beta\delta} w$$

where $EV_S^{\rho|\pi}$ is the continuation value under the strategy π. Now, we have

$$\begin{aligned} EV_S^{\rho|\pi} &= \int_0^{c_S^*} (\delta y - c)\, dF(c) + [1 - F(c^*)]\left[\pi\left[-w + EV_S^1\right] + (1-\pi)\,\rho\delta EV_S^{\rho|\pi}\right] \\ &= \frac{1}{1 - [1 - F(c^*)](1-\pi)\rho\delta}\left[\int_0^{c_S^*} (\delta y - c)\, dF(c) + \pi\,[1 - F(c^*)]\left[-w + \delta EV_S^1\right]\right] \end{aligned}$$

Choosing π to satisfy Equation A3, we get

$$\begin{aligned} \left[\int_0^{c_S^*} (\delta y - c)\, dF(c) + \pi\,[1 - F(c^*)]\left[-w + \delta EV_S^1\right]\right]\frac{\rho\delta}{1 - [1 - F(c^*)](1-\pi)\rho\delta} &= -\frac{w}{\beta} + \delta EV_S^1 \\ \frac{\rho\delta}{1 - [1 - F](1-\pi)\rho\delta}\left[C + \pi\,[1 - F]\left[-w + \delta EV_S^1\right]\right] &= -\frac{w}{\beta} + \delta EV_S^1 \end{aligned}$$

where $C = \int_0^{c_S^*} (\delta y - c)\, dF(c)$. Solving for π gives

$$\pi = \frac{(\beta\delta EV_S^1 - w)(1 - (1 - F(c^*))\,\delta\rho) - \beta\delta(\rho C)}{(1 - F(c^*))(w(1-\beta))\,\delta\rho}$$

A.II Definitions: Perception-Perfect Strategies

Definition 2. *Given $\hat{\beta} \leq 1, \hat{\rho} \leq 1,$ and $\delta,$ a set of beliefs $\left\{\hat{S}^t, \hat{S}^{t+1}, \hat{S}^{t+2}...\right\}$ is dynamically consistent if:*
1) for all $\hat{S}^t, \hat{s}^t_\tau = \arg\max_{a \in A} \hat{W}_{\tau i}\left(\hat{S}^\tau, \hat{\beta}, \hat{\rho}\right)$ for all τ and
2) for all $\hat{S}^t$ and $\hat{S}^{t'}$ with $t < t'$, $\hat{s}^t_\tau =, \hat{s}^{t'}_\tau$ for all $\tau > t'$.
where $\hat{W}_{\tau i} = u_\tau + \hat{\beta} \sum_{m=1}^{\infty} \delta^m \hat{\rho}^m u_{\tau+m}$

Definition 3. *A Perception Perfect Strategy is $S^*\left(\beta, \hat{\beta}, \hat{\rho}\right) = \left(s^*_t\left(\beta, \hat{\beta}, \hat{\rho}\right), s^*_{t+1}\left(\beta, \hat{\beta}, \hat{\rho}\right), ..\right)$ such that there exists dynamically consistent beliefs $\hat{S}\left(\hat{\beta}, \hat{\rho}\right)$ such that $s_t = \arg\max_{a \in A} W_t\left(a, \hat{S}\left(\hat{\beta}, \hat{\rho}\right)\right)$ $\forall t$.*