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ARTIFICIAL INTELLIGENCE

https://www.mif.vu.lt/~cyras/AI/ai-cyras.pdf

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Preface

This course-book views artificial intelligence (AI) from the standpoint of programming. Fundamental concepts of classical AI are presented: problem solving by search, solver, planner, etc. A purpose is to understanding the spirit of a discipline of artificial intelligence.

The course-book is available at https://www.mif.vu.lt/~cyras/AI/ai-cyras.pdf and presents primarily figures and text excerpts which are comprised in the broader Lithuanian edition[1].

The following themes are presented:

- History of artificial intelligence
- Philosophical questions
- The Turing test
- A system of artificial intelligence according to Nils Nilsson: 1) a global database, 2) a set of production rules, and 3) a control system
- Examples: the 8 queens puzzle, the knight's tour, path search in a labyrinth
- Problem spaces
- Backtracking
- Depth-first search and breadth-first search, Dijkstra's algorithm, A*
- The role of heuristics
- Forward chaining and backward chaining
- Knowledge-based reasoning, deduction, resolution technique
- Hill climbing
- Elements of expert systems architecture: facts, rules, and an inference engine
- Knowledge representation
- Structured representation, frames and objects
- Semantic networks
- Artificial intelligence and law
- Extensional relational structure and what is an ontology

Acknowledgements. Sincere thanks to students who contributed to this course-book. Edgaras Abromaitis' work was essential in shaping the text and producing the figures.

[1] V. Čyras. Intelektualios sistemos. e-Book. ISBN 978-9955-33-561-0. <http://www.mif.vu.lt/~cyras/AI/konspektas-intelektualios-sistemos.pdf>.

1. Introduction

American scientist Marvin Minsky defined AI as "the science of making machines do things that would require intelligence if done by men."[2] A similar definition is "the science of designing computer systems to perform tasks that would normally require human intelligence" (Sowa 2000, p. XI). The term "artificial intelligence" was coined in a research project proposed for the 1956 summer workshop at Dartmouth College in Hanover, New Hampshire, United States.[3] The term is attributed to John McCarthy.[4]

The term "artificial intelligence" is used to describe machines that mimic "cognitive" functions that humans associate with other human minds, such as "learning" and "problem solving", see (Russell, Norvig 2010, p. 2).

1.1. The subject matter of artificial intelligence

The subject matter of AI also shows the place of AI within the discipline of computer science. Following is a classification of *computing* according to the ACM Computing Classification System in US; see https://www.acm.org/publications/class-2012 by the Association for Computer Machinery.

General Literature
Hardware
Computer systems organization
Networks
Software
Data
Theory of computation
Mathematics of computing
Information systems
Security and privacy
Human-centered computing
Computing methodologies
Computer applications
Social and professional topics

Artificial intelligence is comprised by the branch Computing methodologies above which is classified further:

Symbolic and algebraic manipulation

[2] See generally Marvin L. Minsky (1968) Introduction, in: Semantic Information Processing, M. L. Minsky (ed.), MIT Press, Cambridge (Massachusetts). See also https://www.britannica.com/biography/Marvin-Lee-Minsky.
[3] John McCarthy, Marvin Minsky, Nathaniel Rochester, Claude Shannon, A Proposal for the Dartmouth Summer Research Project on Artificial Intelligence (31 August 1955), AI Magazine, vol. 27 (2006), no. 4, pp. 12–14, https://www.aaai.org/ojs/index.php/aimagazine/article/view/1904/1802.
[4] John McCarthy's answer to the question "What is artificial intelligence?" is "It is the science and engineering of making intelligent machines, especially intelligent computer programs. It is related to the similar task of using computers to understand human intelligence, but AI does not have to confine itself to methods that are biologically observable." McCarthy continues answering what intelligence is: "Intelligence is the computational part of the ability to achieve goals in the world." McCarthy is sceptical about parallel machines: "Parallelism itself presents no advantages, and parallel machines are somewhat awkward to program" (see McCarthy 2007, pp. 2–4, http://www-formal.stanford.edu/jmc/whatisai.pdf).

Parallel computing methodologies
Artificial intelligence
Machine learning
Modeling and simulation
Computer graphics
Distributed computing methodologies
Concurrent computing methodologies

Artificial intelligence is classified further:

- Natural language processing
  - Information extraction; Machine translation; Discourse, dialogue and pragmatics; Natural language generation; Speech recognition; Lexical semantics; Phonology / morphology; Language resources
- **Knowledge representation and reasoning**
  - Description logics; Semantic networks; Nonmonotonic, default reasoning and belief revision; Probabilistic reasoning; Vagueness and fuzzy logic; Causal reasoning and diagnostics; Temporal reasoning; Cognitive robotics; Ontology engineering; Logic programming and answer set programming; Spatial and physical reasoning; Reasoning about belief and knowledge
- Planning and scheduling
  - Planning for deterministic actions; Planning under uncertainty; Multi-agent planning; Planning with abstraction and generalization; Robotic planning
- Search methodologies
  - Heuristic function construction; Discrete space search; Continuous space search; Randomized search; Game tree search; Abstraction and micro-operators; Search with partial observations
- Control methods
  - Robotic planning; Computational control theory; Motion path planning
- Philosophical/theoretical foundations of artificial intelligence
  - Cognitive science; Theory of mind
- Distributed artificial intelligence
  - Multi-agent systems; Intelligent agents; Mobile agents; Cooperation and coordination
- Computer vision
  - Computer vision tasks
    - Biometrics; Scene understanding; Activity recognition and understanding; Video summarization; Visual content-based indexing and retrieval; Visual inspection; Vision for robotics; Scene anomaly detection
  - Image and video acquisition
    - Camera calibration; Epipolar geometry; Computational photography; Hyperspectral imaging; Motion capture; 3D imaging; Active vision
  - Computer vision representations
    - Image representations; Shape representations; Appearance and texture representations; Hierarchical representations
  - Computer vision problems

- Interest point and salient region detections; Image segmentation; Video segmentation; Shape inference; Object detection/recognition/identification; Tracking; Reconstruction; Matching

What means "artificially intelligent"? Intelligent entities have the abilities to perceive, understand, predict, and manipulate. AI is related with logic. Logic (from the Ancient Greek logikḗ, "the word" or "what is spoken" (but coming to mean "thought" or "reason"), is generally held to consist of the systematic study of the form of arguments. Also related to logos, "word, thought, idea, argument, account, reason, or principle" (https://en.wikipedia.org/wiki/Logic).

1.2. The Tower of Hanoi

See https://en.wikipedia.org/wiki/Tower_of_Hanoi.

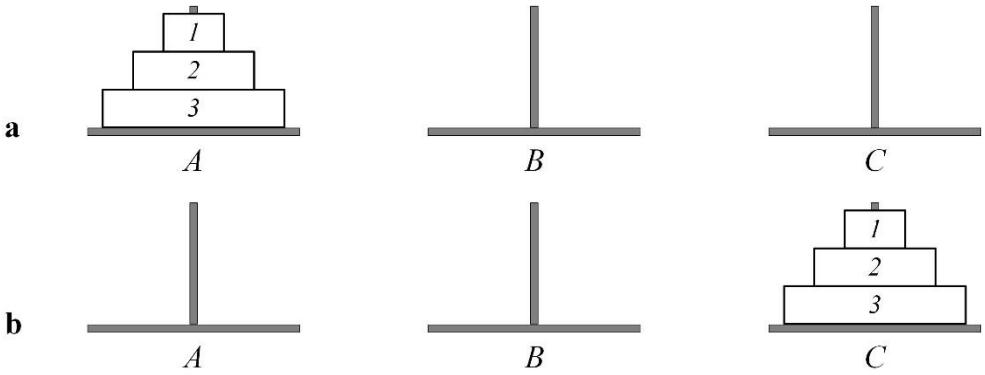


Fig. 1.1. a) Initial state *A*=(*3*,*2*,*1*), *B*=(), *C*=(). b) Terminal state *A*=(), *B*=(), *C*=(*3*,*2*,*1*)

Finding a sequence of moves is an intelligent task. Intelligence is hidden in the recursive algorithm below (Fig. 1.2):

Part 1. Move *n–1* disks from *A* to *B*;

Part 2. Move disk *n* from *A* to *C*;

Part 3. Move *n–1* disks from *B* to *C*.

The sequence of moves for *n*=*3*:

Initial state *A*=(*3*,*2*,*1*), *B*=(), *C*=().

1. Move disk *1* from *A* to *C*. *A*=(*3*,*2*), *B*=(), *C*=(*1*).
2. Move disk *2* from *A* to *B*. *A*=(*3*), *B*=(*2*), *C*=(*1*).
3. Move disk *1* from *C* to *B*. *A*=(*3*), *B*=(*2*,*1*), *C*=().
4. Move disk *3* from *A* to *C*. *A*=(), *B*=(*2*,*1*), *C*=(*3*).
5. Move disk *1* from *B* to *A*. *A*=(*1*), *B*=(*2*), *C*=(*3*).
6. Move disk *2* from *B* to *C*. *A*=(*1*), *B*=(), *C*=(*3*,*2*).
7. Move disk *1* from *A* to *C*. *A*=(), *B*=(), *C*=(*3*,*2*,*1*).

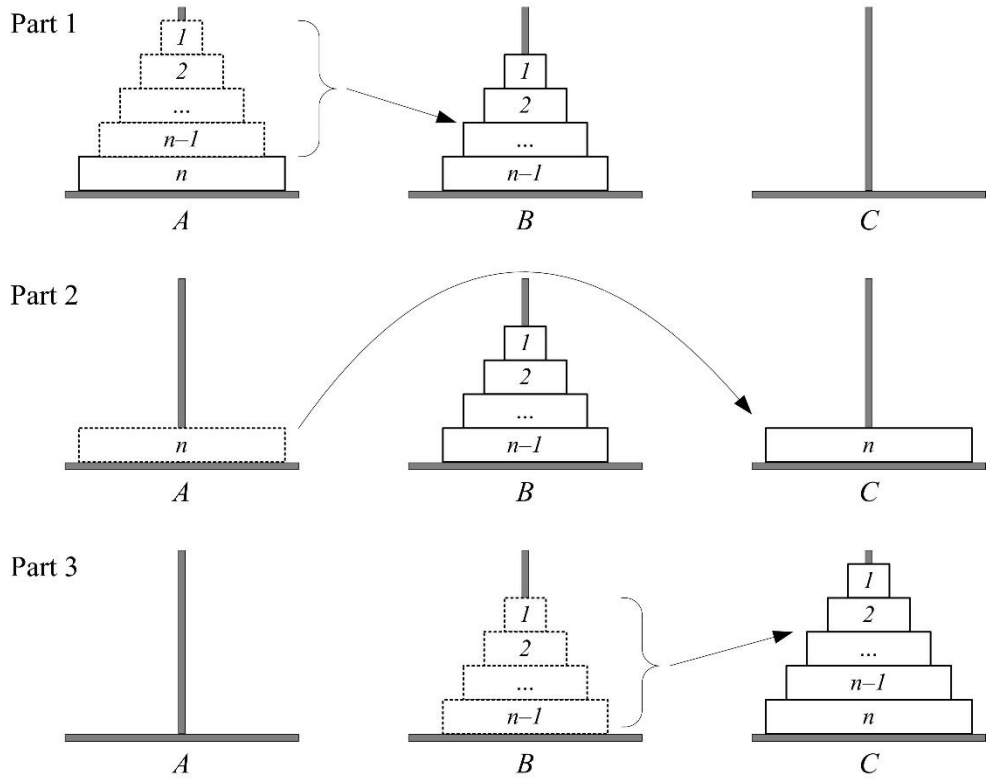


Fig. 1.2. Three parts of the recursive algorithm

Following is the recursive procedure:

```
procedure ht(x, y, z: char; n: integer);
{x, y, z – rod names; n – number of disks.}
{x – from, y – intermediary, z – onto.}
begin
  if n > 0 then
  begin
    ht(x, z, y, n-1); {1. Move n-1 disks onto intermediary.}
    writeln(' Move disk ', n, ' from ', x,' to ', z);  {2.}
    ht(y, x, z, n-1); {3. Move n-1 disks onto target.      }
  end
end
```

Invoking the above procedure with *n=3*:

```
ht('A', 'B', 'C', 3)
```

The number of moves is exponential: $2^n - 1$.

Further we discuss state transition in *state transition graphs*. Fig. 1.3 shows GRAPHSEARCH-DEPTH-FIRST search tree for the Hanoi tower problem *n=3*.

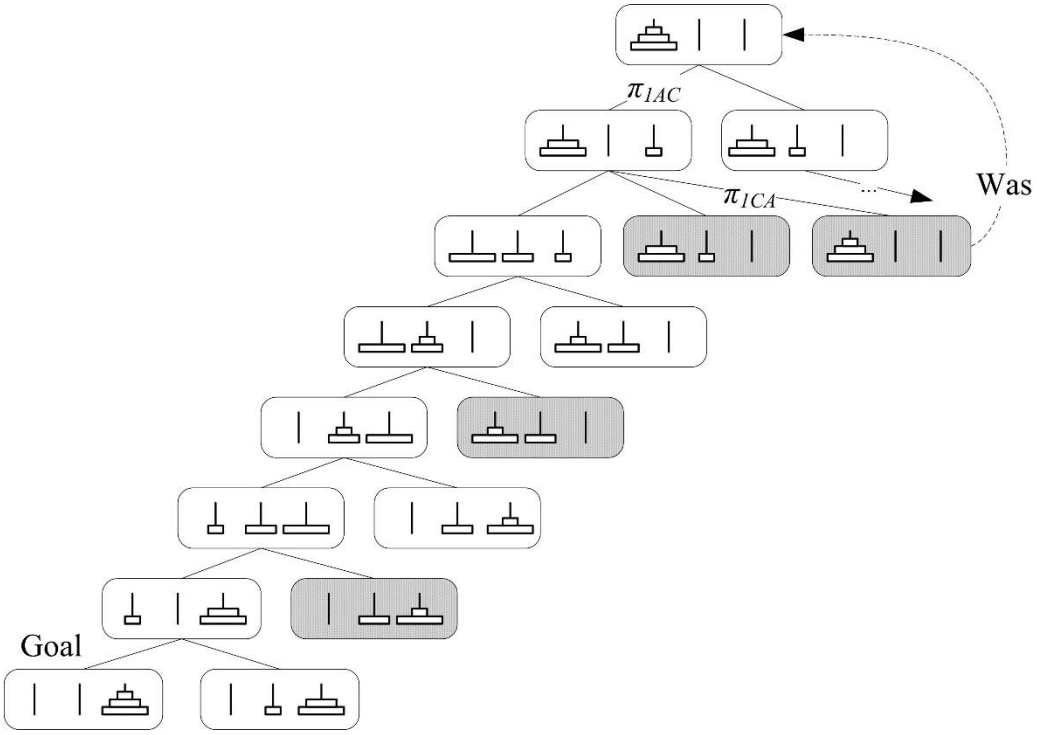


Fig. 1.3. A search tree of the algorithm GRAPHSEARCH_ DEPTH_FIRST for the Hanoi tower problem $n=3$. The grey nodes appear in the list OPEN or CLOSED

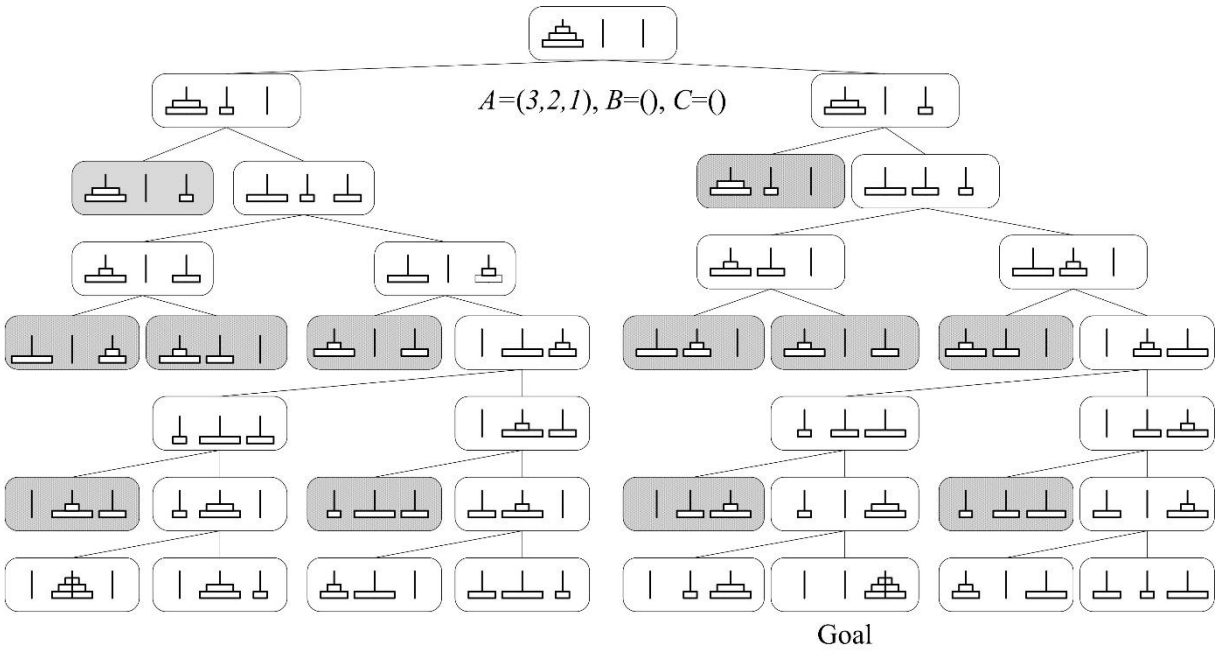


Fig. 1.4 . A search tree of the GRAPHSEARCH_BREADTH_FIRST algorithm for the Hanoi tower problem. The grey nodes appear in the list OPEN or CLOSED

Exercise 1. The iterative Hanoi tower solution. The iterative algorithm for the Hanoi tower problem is less well-known as the recursive one. First, we arrange the pegs in a circle, so that clockwise we have *A*, *B*, *C*, and then *A* again. Following this, assuming we never move the same disk twice in a row, there will always only be one disk that can be legally moved, and we transfer it to the first peg it can occupy, moving in a clockwise direction, if *n* is even (2, 4, 6,…), and counterclockwise, in *n* is odd (1, 3, 5,…). (See Brachman & Levesque (2004), *"Knowledge representation and reasoning"*, p. 133–134, Exercise 2). Write a program.

Exercise 2. On the complexity of the Hanoi tower problem. Suppose one move takes 1 second. How many years will it take to solve the Hanoi tower problem for *n*=64?

Exercise 3. Monkey and banana problem. A monkey is in a room. Suspended from the ceiling is a bunch of bananas, beyond the monkey's reach. However, in the room there are also a chair and a stick. The ceiling is just the right height so that a monkey standing on a chair could knock the bananas down with the stick. The monkey knows how to move around, carry other things around, reach for the bananas, and wave a stick in the air. What is the best sequence of actions for the monkey? (https://en.wikipedia.org/wiki/Monkey_and_banana_problem)

1.3. Dynamic nature of artificial intelligence

Research is at the forefront of AI. "Intelligence is whatever machines haven't done yet" (http://www.nomodes.com/Larry_Tesler_Consulting/Adages_and_Coinages.html). Programmers may do tasks not knowing that they are assigned to AI.

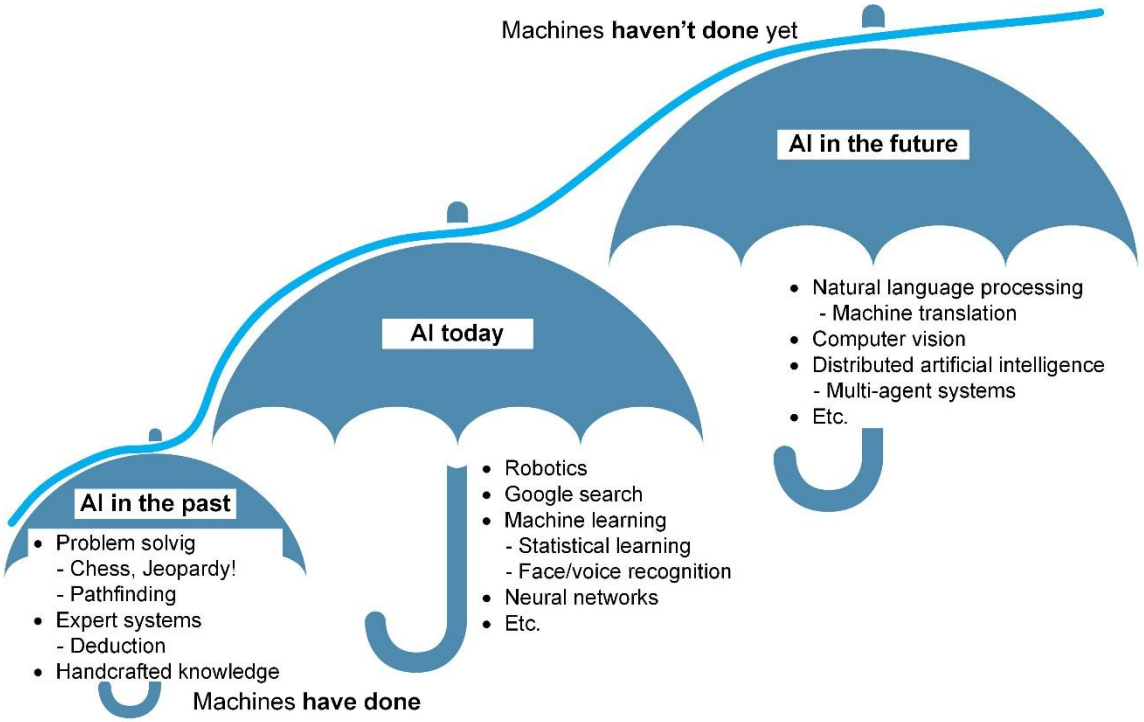


Fig. 1.5. As machines become increasingly capable, tasks considered to require "intelligence" are often removed from the definition of AI, a phenomenon known as the AI effect.

1.4. Methods of problem solving

Following are two classifications in artificial intelligence:

Tasks in artificial intelligence	Problem solving methods
1. interpret (analysis) 1.1. identify (recognise) 1.1.1. monitor (audit, check) 1.1.2. diagnose (debug) 1.2. predict (simulate) 1.3. control 2. construct (synthesis) 2.1. specify (constrain) 2.2. design 2.2.1. plan (process) 2.2.2. configure (structure) 2.3. assemble (manufacture) 2.3.1. modify (repair)	1. classification) 1.1. certain classification 1.1.1. decision trees 1.1.2. decision tables 1.2. heuristic classification 1.3. model-based classification 1.3.1. set covering classification 1.3.2. functional classification 1.4. statistical classification 1.5. case-based classification 2. construction 2.1. heuristic construction 2.1.1. skeletal construction 2.1.2. propose-and-revise strategy 2.1.3. propose-and-exchange strategy 2.1.4. least-commitment strategy 2.2. model-based construction 2.3. case-based construction 3. simulation 3.1. one-step simulation 3.2. multiple-step simulation 3.2.1. numerical multiple-step simulation 3.2.2. qualitative multiple-step simulation

What differentiates a method and a principle? A method is applied in a clear situation but a principle – in a situation that needs not to be clear. A principle is applied in the case a method is not known in a given situation. This is similar to a difference between a legal norm and a principle. Following are short descriptions by Albertas Čaplinskas, a scholar and practitioner in software engineering:

Method = schema + procedures
Methodology = method + recommendations

There are other characterisations of the term "method" such as "Method = notation + process model". An example is UML + RUP, i.e. Rational's Unified Modeling Language + Process (Hesse, Verrijn-Stuart 2001, p. 1).

2. Artificial intelligence system as a production system

This chapter follows (Nilsson 1982, Section 1.1)
DEFINITION 2.1. A *production system* is a triple:

1. A global database (GDB);
2. A production (rule) set $\{\pi_1, \pi_2, \ldots, \pi_m\}$;
3. A control system.

The basic production system algorithm for solving a problem such as the 8-puzzle, the knight's tour, the 8 queens puzzle, etc. can be written in nondeterministic form as follows:

```
procedure PRODUCTION
{1} DATA := initial GDB;
{2} until DATA will satisfy the termination condition, do
{3} begin
{4}     select  some rule, π, in the set of rules
                that can be applied to DATA
{5}     DATA := π(DATA)        {Result of applying π to DATA}
{6} end
```

The output is a sequence of productions $\langle \pi_{i1}, \pi_{i2}, \ldots, \pi_{in} \rangle$. This is a non-determinate procedure. PRODUCTION performs depth-first search. The procedure is treated as a *paradigm* of a control system. A meaning of the word "paradigm" can be found, for instance, in Oxford English Dictionary, https://www.oed.com/: "A pattern or model, an exemplar; (also) a typical instance of something, an example."

The Knight's Tour problem (https://en.wikipedia.org/wiki/Knight%27s_tour) is examined to demonstrate problem solving with the PRODUCTION procedure. A knight's tour is a sequence of moves of a knight on a chessboard such that the knight visits every square only once. Eight productions $\{\pi_1, \ldots, \pi_8\}$ are shown in Fig. 2.1 a. Suppose the knight starts from the square [*1,1*] (Fig. 2.1 b). Variations involve chessboards $N \times N$ and the simplest case is N=5. The first terminal state is shown in Fig. 2.1 c. There are several solutions. The first deadend is reached on the square 21 [X=4,Y=2] (Fig. 2.2 b). The search tree is shown in Fig. 2.2.

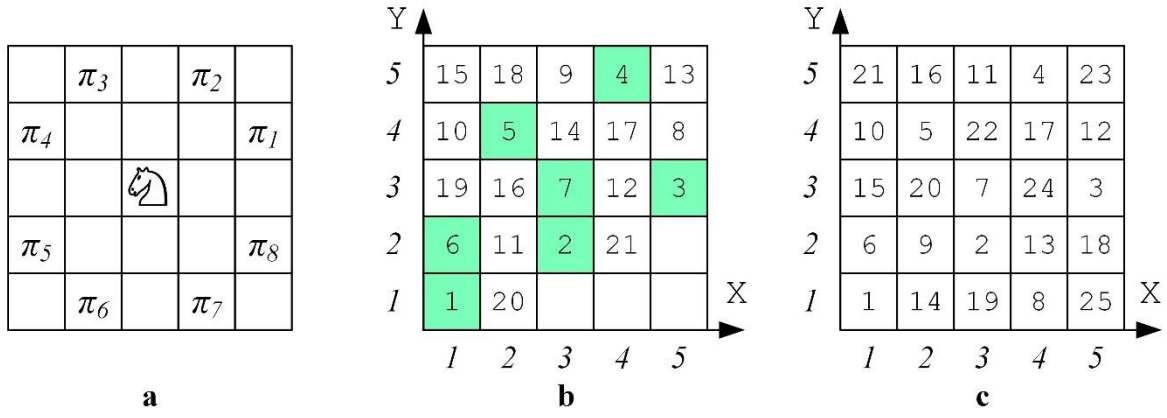


Fig. 2.1. **a** Eight moves of a knight. **b** The knight starts from the square [*1,1*]. On a 5×5 board, see coloured seven foremost moves that lead to success; move 8 fails. **c** The first terminal state

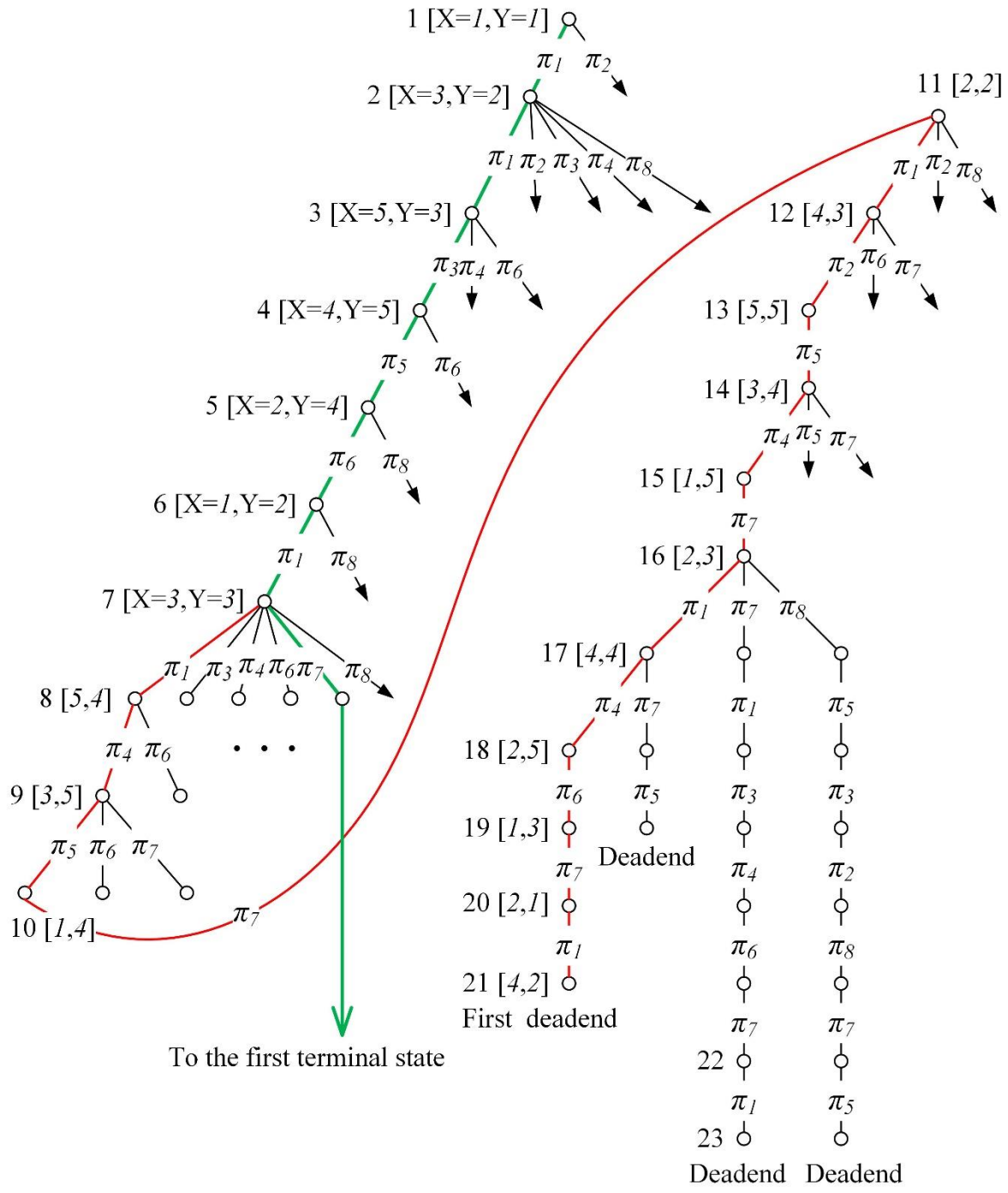


Fig. 2.2. The search tree which leads to the first deadend on a 5×5 board. The state at this deadend is shown in Fig. 2.1 b

What do the variables represent in the program which is shown below?

1. BOARD, X and Y – the global database;
2. CX and CY – the production set;
3. Procedure TRY – the control system.

A solution is a sequence of 24 moves $\langle \pi_{i1}, \pi_{i2}, \pi_{i3}, \ldots, \pi_{i24} \rangle$, where $\pi_{ij} \in \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8\}$, i.e. $i_j \in \{1, 2, \ldots, 8\}$.

```
program KNIGHTS_TOUR;
const N     = 5;     {Length of the board}
      NN    = 25;    {Number of squares 5*5}
type  INDEX = 1..N;
var
   BOARD  : array[INDEX, INDEX] of integer;  {Global database}
   CX, CY : array[1..8] of integer;          {8 production rules}
   I, J   : integer; YES: boolean;

procedure INITIALIZE;
var   I, J : integer
begin
   {1) Production set is formed}
   CX[1] :=  2;    CY[1] :=  1;
   CX[2] :=  1;    CY[2] :=  2;
   CX[3] := -1;    CY[3] :=  2;
   CX[4] := -2;    CY[4] :=  1;
   CX[5] := -2;    CY[5] := -1;
   CX[6] := -1;    CY[6] := -2;
   CX[7] :=  1;    CY[7] := -2;
   CX[8] :=  2;    CY[8] := -1;

   {2) Initialize the global database}
   for I := 1 to N do
      for J := 1 to N do
         BOARD[I, J] := 0;
end; {INITIALIZE}

procedure TRY (L : integer; X, Y : INDEX; var YES : boolean);
{Input parameters: L - move number; X, Y - knight's last position.}
{Output parameter (i.e. the result): YES.}
var
   K     : integer; {Production number.}
   U, V  : integer; {New knight's position.}
begin
   K := 0;
   repeat  {Select each of 8 productions.}
      K := K + 1;
      U := X + CX[K]; V := Y + CY[K];
      {Check if the condition of the production is satisfied.}
      if (U >= 1) and (U <= N) and (V >= 1) and (V <= N)
      then {Within the board.}
         {Check if the square is empty.}
         if BOARD[U, V] = 0
         then
           begin
                {New position.}
                BOARD[U, V] := L;
                {Check if all squares are visited.}
                if L < NN
                then
                  begin {If not all are visited.}
                    TRY(L+1, U, V, YES);
                    {If no success,}
                    {then backtrack and free the position.}
                    if not YES then BOARD[U, V] := 0;
                  end
                else YES := true; {When L=NN.}
           end;
   until YES or (K = 8); {Either success or all productions were tried.}
end; {TRY}
```

```
begin {Main program)}
   {1. Initialize.}
   INITIALIZE; YES := false;
   {2. Initial position [1,1].}
   BOARD [1, 1] := 1;
   {3. Make move no.2 from X=1 and Y=1 and obtain the answer YES.}
   TRY(2, 1, 1, YES);
   {4. If a solution found then print it.}
   if YES then
      for I := N downto 1 do
      begin
         for J := 1 to N do
            write(BOARD[J,I]);
         writeln;
      end;
   else writeln('Path does not exist.');
end.
```

Two elements are distinguished in *knowledge-based system* architecture:

1. *reasoning*. This stands for a control system. The inference engine does not depend on a domain and can be purchased.
2. *knowledge base*. It is comprised of the global database and the production set. It depends on the domain and can hardly be purchased. “God is in the detail.”

2.1. Testing

Modify your Knight’s Tour solver to print the trace below.

```
PART 1. Data
    1) Board: 5x5.
    2) Initial position: X=1, Y=1. L=1.

PART 2. Trace
      1) R1. U=3, V=2. L=2. Free. BOARD[3,2]:=2.
      2) -R1. U=5, V=3. L=3. Free. BOARD[5,3]:=3.
      3) --R1. U=7, V=4. L=4. Out.
      4) --R2. U=6, V=5. L=4. Out.
      5) --R3. U=4, V=5. L=4. Free. BOARD[4,5]:=4.
      6) ---R1. U=6, V=6. L=5. Out.
      7) ---R2. U=5, V=7. L=5. Out.
      8) ---R3. U=3, V=7. L=5. Out.
      9) ---R4. U=2, V=6. L=5. Out.
     10) ---R5. U=2, V=4. L=5. Free. BOARD[2,4]:=5.
     11) ----R1. U=4, V=5. L=6. Thread.
       And so on until the deadend at L=21. Then backtrack one level, i.e. pop one hyphen.
    ...) --------------------R1. U=4, V=2. L=21. Free. BOARD[4,2]:=21.
    ...) --------------------R1. U=6, V=3. L=22. Out.
    ...) --------------------R2. U=5, V=4. L=22. Thread.
    ...) --------------------R3. U=3, V=4. L=22. Thread.
    ...) --------------------R4. U=2, V=3. L=22. Thread.
    ...) --------------------R5. U=2, V=1. L=22. Thread.
    ...) --------------------R6. U=3, V=0. L=22. Out.
    ...) --------------------R7. U=5, V=0. L=22. Out.
    ...) --------------------R8. U=6, V=1. L=22. Out. Backtrack.
    ...) -------------------R2. U=3, V=3. L=21. Thread.
    ...) -------------------R3. U=1, V=3. L=21. Thread.
    ...) -------------------R4. U=0, V=2. L=21. Out.
    ...) -------------------R5. U=0, V=0. L=21. Out.
    ...) -------------------R6. U=1, V=-1. L=21. Out.
    ...) -------------------R7. U=3, V=-1. L=21. Out.
    ...) -------------------R8. U=4, V=0. L=21. Out. Backtrack.
    ...) ------------------R8. U=3, V=2. L=20. Thread. Backtrack
    ...) -----------------R7. U=3, V=3. L=19. Thread.
    ...) -----------------R8. U=4, V=4. L=19. Thread. Backtrack.
```

```
   ...) ----------------R5. U=2, V=3. L=18. Thread.
   ...) ----------------R6. U=3, V=2. L=18. Thread.
   ...) ----------------R7. U=5, V=2. L=18. Free. BOARD[5,2]:=18.
   ...) ----------------R1. U=7, V=3. L=19. Out.
```

And so on.

```
 70611) ---------------------R1. U=5, V=5. L=23. Free. BOARD[5,5]:=23.
 70612) ----------------------R1. U=7, V=6. L=24. Out.
 70613) ----------------------R2. U=6, V=7. L=24. Out.
 70614) ----------------------R3. U=4, V=7. L=24. Out.
 70615) ----------------------R4. U=3, V=6. L=24. Out.
 70616) ----------------------R5. U=3, V=4. L=24. Thread.
 70617) ----------------------R6. U=4, V=3. L=24. Free. BOARD[4,3]:=24.
 70618) -----------------------R1. U=6, V=4. L=25. Out.
 70619) -----------------------R2. U=5, V=5. L=25. Thread.
 70620) -----------------------R3. U=3, V=5. L=25. Thread.
 70621) -----------------------R4. U=2, V=4. L=25. Thread.
 70622) -----------------------R5. U=2, V=2. L=25. Thread.
 70623) -----------------------R6. U=3, V=1. L=25. Thread.
 70624) -----------------------R7. U=5, V=1. L=25. Free. BOARD[5,1]:=25.

PART 3. Results
  1) Path is found. Trials=70624.
  2) Path graphically:

  Y, V ^
     5 | 21 16 11  4 23
     4 | 10  5 22 17 12
     3 | 15 20  7 24  3
     2 |  6  9  2 13 18
     1 |  1 14 19  8 25
       ------------------> X, U
          1  2  3  4  5
```

Seven tests:

1) N=5, X=1, Y=1. Trials=70 624.
2) N=5, X=5, Y=1. Trials=236.
3) N=5, X=1, Y=5. Trials= 111 981.
4) N=5, X=2, Y=1. No tour. Trials=14 635 368.
5) N=6, X=1, Y=1. Trials=1 985 212.
6) N=7, X=1, Y=1. Trials=57 209 233.
7) N=4, X=1, Y=1. No tour. Trials=17 784.

Print the output of each test to two files as follows:

a) PART 1 and PART 3 – to screen and the file *out-short.txt*;
b) all three parts – to the file *out-long.txt*.

3. Control with backtracking and procedure BACKTRACK

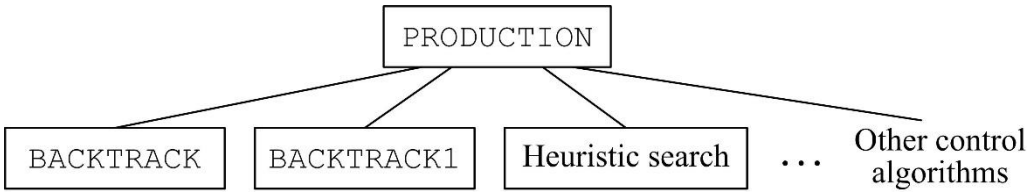


Fig. 3.1. BACKTRACK in the context of control algorithms

BACKTRACK is used to solve the 8-queens problem:

1. Global database – the chessboard.
2. A set of 64 productions $\{\pi_{i,j}, i,j = 1, 2,\ldots, 8\}$, where $\pi_{i,j}$ denotes placing a queen into position $[i,j]$. We reduce this set to 8 productions $\{\pi_k, k = 1, 2,\ldots, 8\}$. Each queen i is placed in row i one by one, i=1, 2 and so on until 8. π_k denotes placing a queen into column k. The first queen is placed in row 1 and column *k2*, the second – in row 2 and column *k2* and so on.
3. Control system – procedure BACKTRACK.

An empty board stands for the initial state. A solution is $\langle\pi_{k1}, \pi_{k2},\ldots, \pi_{k8}\rangle$, where *k1*, *k2*,…, *k8* $\in \{1, 2,\ldots, 8\}$.

Below we adapt from (Nilsson 1982, Section 2.1). A general description of the backtracking control strategy was presented earlier as procedure PRODUCTION. Compared with graph-search control regimes, backtracking strategies are typically simpler to implement and require less storage.

A simple recursive procedure captures the essence of the operation of a production system under backtracking control. This procedure, which we call BACKTRACK, takes a single argument, Data, initially set equal to the global database of the production system. Upon successful termination, the procedure returns a list of rules, that, if applied in sequence to the initial database, produces a database satisfying the termination condition. If the procedure halts without finding such a list of rules, it returns FAIL. The BACKTRACK procedure is defined as follows:

```
recursive procedure BACKTRACK(DATA) {return list of rules}
{DATA – a current state of the GDB. Returns a list of rules.}
    { 1}      if TERM(DATA) then return NIL; {TERM is a
                   predicate true for arguments that satisfy
                   the termination condition of the production
                   system. Upon successful termination, NIL,
                   the empty list is returned.}
    { 2}      if DEADEND(DATA) then return FAIL; {DEADEND is a
                   predicate true for arguments that are known
                   not to be on a path to a solution. In this
                   case, the symbol FAIL is returned.}
    { 3}      RULES := APPRULES(DATA); {APPRULES is a function
                   that computes the rules applicable to its
                   argument and orders them (either arbitrari-
                   ly or according to heuristic merit).}
```

```
{ 4}      LOOP: if NULL(RULES) then return FAIL;
                    {If there is no (more) rules to apply,
                    the procedure fails.}
{ 5}            R     := FIRST(RULES); {The best of the
                         applicable rules is selected.}
{ 6}            RULES := TAIL(RULES); {The list of
                    applicable rules is diminished by
                    removing the one just selected.}
{ 7}            RDATA := R(DATA); {Rule R is applied to
                                  produce a new database.}
{ 8}            PATH  := BACKTRACK(RDATA); {Recursive call
                                       on the new database.}
{ 9}            if PATH = FAIL then goto LOOP; {If the re-
                    cursive call fails, try another rule.}
{10}      return CONS(R, PATH); {Otherwise, pass the
                    successful list of rules up, by adding
                    R to the front of the list.}
end
```

3.1. An example of the DEADEND predicate

a

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

b

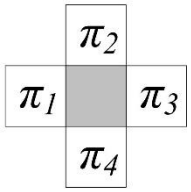


Fig. 3.2. a) The terminal state in the 15-puzzle. b) Four moves

a

2	1	3	4
5	6	7	8
9	10	11	12
13	14	15	

b

2	1	3	4
5	6	8	7
9	10	11	12
13	14	15	

c

Fig. 3.3. a) A sample initial state in the 15-puzzle with one permutation, an odd number. This is a failure state. b) The number of permutations is 2, an even number. Success can be reached from this state

4. The 8-queens puzzle

To see an example of multilevel backtracking phenomenon, consider using BACKTRACK to solve the 8-queens problem. In this problem, we must place 8 queens on an 8×8 board so that none of them can capture any others (Nilsson 1982, p. 60).

Below we follow (Nilsson 1982, Section 2.1). For our global database, we use an 8×8 array with marked cells corresponding to squares occupied by queens. The terminal condition, expressed by the predicate TERM, is satisfied for a database if and only if it has precisely 8 queen marks and the marks correspond to queens located so that they cannot capture each other.

There are many alternative formulations possible for the production rules. A useful one for our purposes involves the following rule schema, for $1 \le i, j \le 8$:

R_{ij} Precondition:
$i = 1$: There are no marks in the array.
$1 < i \le 8$: There is a queen mark in row $i - 1$ of the array.
Effect: Puts a queen mark in row i, column j of the array.

Thus, the first queen mark added to the array must be in row 1, the second must be in row 2, etc. To use the BACKRACK procedure to solve the 8-queens problem, we have still to specify both the predicate DEADEND and an ordering relation for applicable rules. Suppose we arbitrarily say that R_{ij} is ahead of R_{ik} in the ordering only when $j < k$. The predicate DEADEND might be defined so that it is satisfied for databases where it is obvious that no solution is possible; for example, certainly no solution is possible for any database containing a pair of queen marks in mutually capturing positions. The algorithm backtracks before finding a solution. A search tree consists of 876 edges

On a 4×4 board, the algorithm backtracks 22 times before finding a solution; even the very first rule applied must ultimately be taken back (Nilsson 1982, p. 57–58). A search tree with 26 edges is shown in Fig. 5.5.

Placing 8-queens is encoded in the array `X`, where `X[I]=K` denotes placing a queen in row `I`, column `K`. A solution is shown in Fig. 4.1: `X[1]=1`, `X[2]=5`, `X[3]=8`, `X[4]=6`, `X[5]=3`, `X[6]=7`, `X[7]=2`, `X[8]=4`.

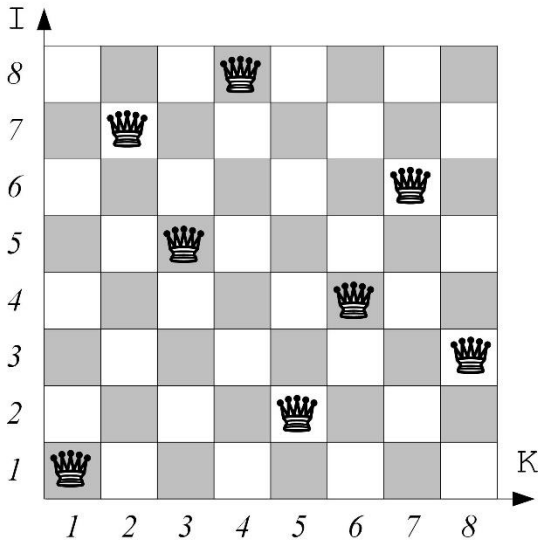


Fig. 4.1. A solution of the 8-queens puzzle

A *brute-force* algorithm searches among 8! = 8·7·6·…·2·1 = 40320 permutations. A solution is as follows:

$$\texttt{PATH}=\langle \pi_1, \pi_5, \pi_8, \pi_6, \pi_3, \pi_7, \pi_2, \pi_4 \rangle$$

Here the `I`-th element is $\pi_{X[I]}$, i.e. `PATH[I]` = $\pi_{X[I]}$, where `I` = 1, 2,…, 8. There are *N!* permutations in the case of an $N \times N$ board. As *N* grows, the factorial *N*! increases faster than all polynomials. The factorial *N*! is an exponential function: $N! = 2^{\alpha \cdot N}$, where α is a real number that depends on *N*.

The program below follows (Dagienė, Grigas, Augutis, 1986); see also https://en.wikipedia.org/wiki/Eight_queens_puzzle.

In the case `I=2`, `K=3`, the column is `K=3`, the ascending diagonal is `K–I = 3–2 = 1`, and the descending diagonal is `I+K–1 = 2+3–1 = 4`.

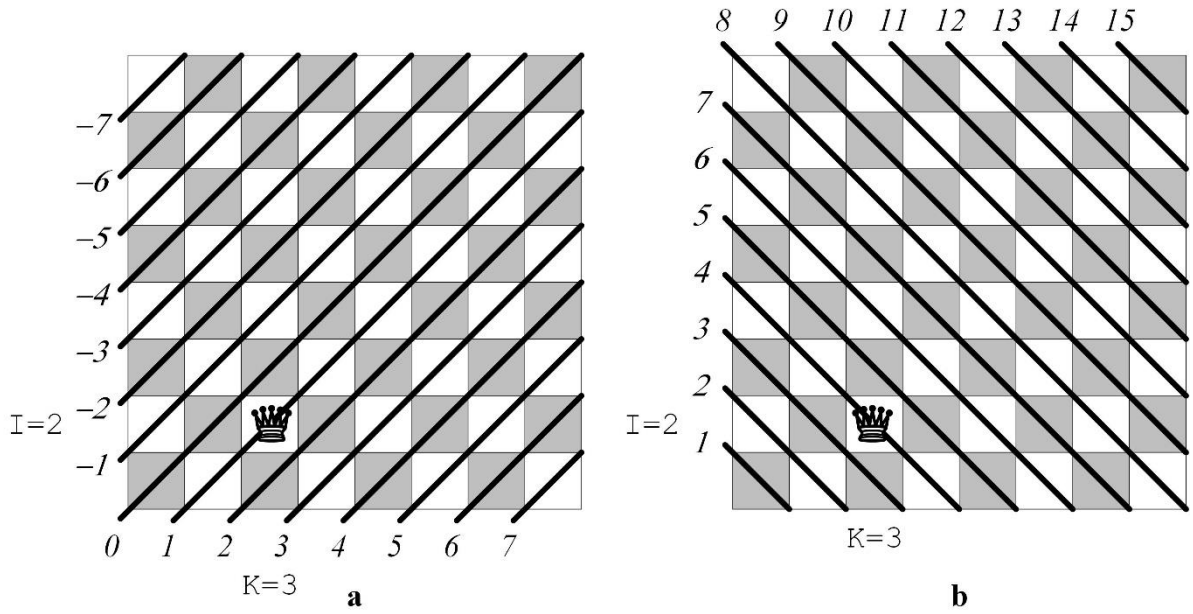


Fig. 4.2. **a** Ascending diagonals are numbered from –7 to 7.
b Descending diagonals are numbered from 1 to 15

```
program QUEENS;
const   N    =  8;
        NM1  =  7; {N-1}
        N2M1 = 15; {2*N-1}
var
   X      : array [1 .. N] of integer;
   VERT   : array [1 .. N] of boolean;      {Columns.}
   UP     : array [-NM1 .. NM1] of boolean; {Moving up diagonals.}
   DOWN   : array [1..N2M1] of boolean;     {Moving down diagonals.}
   YES    : boolean;
   I, J   : integer;
   TRIALS : longint;

procedure TRY (I : integer; var YES : boolean);
{Input I - trial's number. Output YES - succeed or no.}
   var K : integer;
begin
   K := 0;
   repeat
      K := K + 1; TRIALS := TRIALS + 1;
      if VERT[K] and UP[K - I] and DOWN[I + K - 1]
      then {The column, the up and the down diagonals are free.}
      begin
         X[I] := K;
         VERT[K] := false; UP[K - I] := false; DOWN[I + K - 1] := false;
         if I < N then {Continue.}
            begin
              TRY(I + 1, YES);
              if not YES
              then {Path further is not found.}
                begin
                  VERT[K] := true; UP[K - I] := true;
                  DOWN[I + K - 1] := true; {The square is vacated.}
                end;
            end
         else YES := true; {The last row I = N.}
      end;
   until YES or (K = N);
end; {TRY}

begin {Main program.}
   {1. Initialize.}
   for J := 1    to N    do VERT[J] := true;
   for J := -NM1 to NM1  do UP[J]   := true;
   for J := 1    to N2M1 do DOWN[J] := true;
   YES := false; TRIALS := 0;
   {2. Invoke the procedure.}
   TRY(1, YES);
   {3. Print the board.}
   if YES then
     begin
       for I := N downto 1 do
       begin
         for J := 1 to N do
            if X[I] = J then write(1 : 3) else write(0 : 3);
         writeln;  {Write either number 1 or 0 in 3 positions.}
       end;
       writeln('The number of trials: ', TRIALS);
     end
   else writeln('No solution.');
end.
```

5. Heuristic

A heuristic is a kind of a rule of thumb on which rule to apply; see (Russell, Norvig, 2003, p. 94). The term “heuristic” see Oxford English Dictionary, http://www.oed.com/:

adjective

a. Serving to find out or discover…

c. Under an ‘heuristic’ programming procedure the computer searches through a number of possible solutions at each stage of the programme, it evaluates a ‘good’ solution for this stage and then proceeds to the next stage. Essentially heuristic programming is similar to the problem solving techniques by trial and error methods which we use in everyday life. (1964 T. W. McRae)

noun

b. A process that may solve a given problem, but offers no guarantees of doing so, is called a heuristic for that problem. *Ibid.* For conciseness, we will use ‘heuristic’ as a noun synonymous with ‘heuristic process’. (1957 A. Newell et al.)

5.1. Heuristic search in N-queens problem

As an example, we examine the heuristic which is proposed in (Nilsson 1982, Section 2.1, p. 58) to solve the 4-queens problem. A solution exists (see Fig. 5.1). The problem can be generalised to *N*-queens on an *N*×*N* board. The previous section was devoted to obtaining a solution with the BACKTRACK procedure and involved no heuristics.

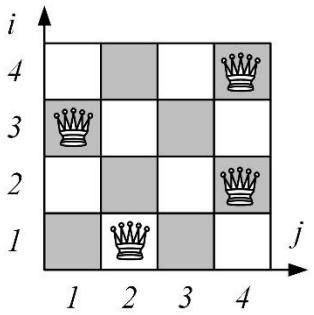


Fig. 5.1. A solution for the N-queens problem on a 4×4 board

A more efficient algorithm (with less backtracking) can be obtained if we use a more informed rule ordering. One simple, but useful ordering for this problem involves using the function *diag(i,j)*, defined to be the length of the longest diagonal passing through cell *(i,j)*. Let π_{ij} be ahead of π_{mn} in the ordering if *diag(i,j)* < *diag(m,n)*. (For equal values of *diag*, use the same orders as before.) Below we verify that this ordering scheme solves the 4-queens problem with only 2 backtracks (see Nilsson 1998, p. 58).

A corresponding search tree consists of 6 edges and is shown in Fig. 5.4. Let us examine the heuristics in detail. The heuristic is formalised

$$\pi_{i,j} < \pi_{i,k}, \text{ if } diag(i,j) < diag(i,k),$$

where *diag(i,j) = max(up(i,j), down(i,j))*. First,

$$diag(1,1)=max(4,1)=4$$

Similarly, other row 1 diagonals *diag(1,j), j=1,…,4* are calculated:

diag(1,1)=4; diag(1,2)=max(3,2)=3; diag(1,3)=max(2,3)=3; diag(1,4)=max(1,4)=4

Hence the ordering of the row 1 diagonals is as follows:

$$diag(1,2) \le diag(1,3) \le diag(1,1) \le diag(1,4)$$

Row 1 productions are ordered correspondingly:

$$\pi_{1,2} < \pi_{1,3} < \pi_{1,1} < \pi_{1,4}$$

Row 2 diagonals *diag(2,j), j=1,…,4*:

diag(2,1)=max(3,1)=3; diag(2,2)=max(4,3)=4; diag(2,3)=max(3,4)=4; diag(2,4)=max(2,3)=3

Row 2 productions are ordered:

$$\pi_{2,1} < \pi_{2,4} < \pi_{2,2} < \pi_{2,3}$$

Row 3 diagonals *diag(3,j), j=1,…,4*:

diag(3,1)=max(2,3)=3; diag(3,2)=max(3,4)=4; diag(3,3)=max(4,3)=4; diag(3,4)=max(3,2)=3

Row 3 productions are ordered:

$$\pi_{3,1} < \pi_{3,4} < \pi_{3,2} < \pi_{3,3}$$

Row 4 diagonals *diag(4,j), j=1,…,4*:

diag(4,1)=max(1,4)=4; diag(4,2)=max(2,3)=3; diag(4,3)=max(3,2)=3; diag(4,4)=max(4,1)=4

Row 4 productions are ordered:

$$\pi_{4,2} < \pi_{4,3} < \pi_{4,1} < \pi_{4,4}$$

To sum up, *diag(i,j)* values are shown in Fig. 5.2 below.

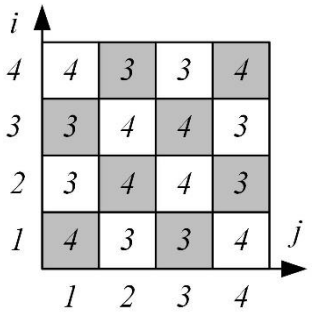


Fig. 5.2. *diag(i,j)* on a 4×4 board

The ordering of production rules is shown in the table below.

Table 5.3. Ordering productions on a 4×4 board

$\pi_{1,2} < \pi_{1,3} < \pi_{1,1} < \pi_{1,4}$	– row 1
$\pi_{2,1} < \pi_{2,4} < \pi_{2,2} < \pi_{2,3}$	– row 2
$\pi_{3,1} < \pi_{3,4} < \pi_{3,2} < \pi_{3,3}$	– row 3
$\pi_{4,2} < \pi_{4,3} < \pi_{4,1} < \pi_{4,4}$	– row 4

The search tree is shown in Fig. 5.4 below.

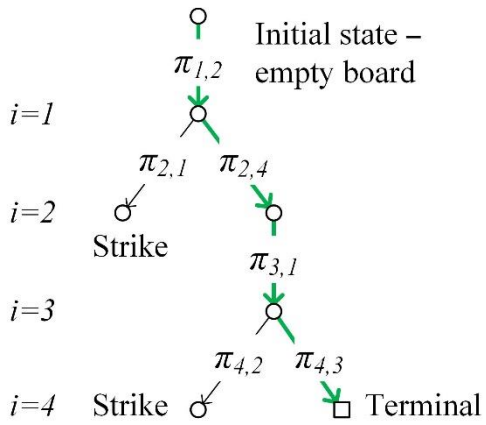


Fig. 5.4. Heuristic search produces a tree of 6 edges. Two backtracks only. The path is is $\langle \pi_{2,1}, \pi_{2,4}, \pi_{3,1}, \pi_{4,3} \rangle$

Brute-force search (i.e. no heuristics – $\pi_{i,1} < \pi_{i,2} < \pi_{i,3} < \pi_{i,4}$) produces the search tree which shown in Fig. 5.5.

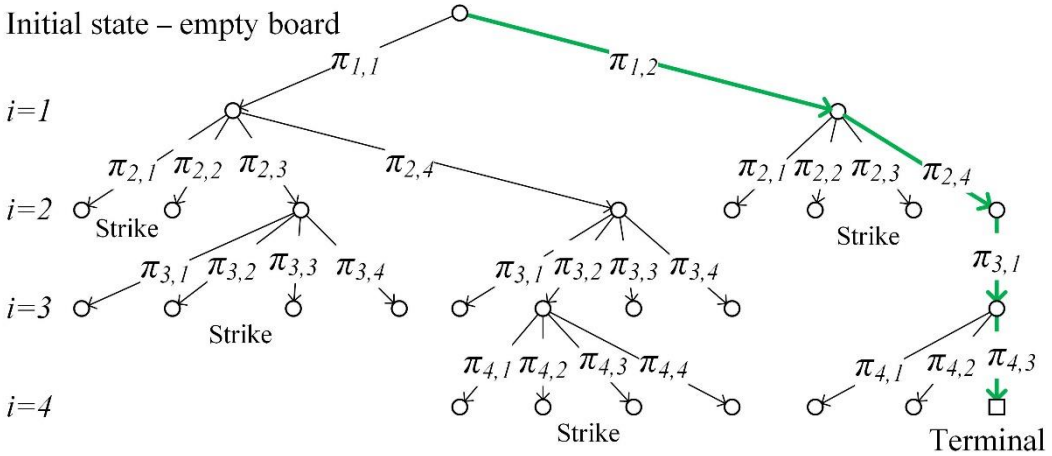


Fig. 5.5. Brute-force search on a 4×4 board. 26 edges is significantly more than 6 edges in the case of heuristic search

Search on an 8×8 board with the shortest diagonal heuristic consists of 204 states. First six moves are shown in Fig. 5.6. Further moves require backtrack.

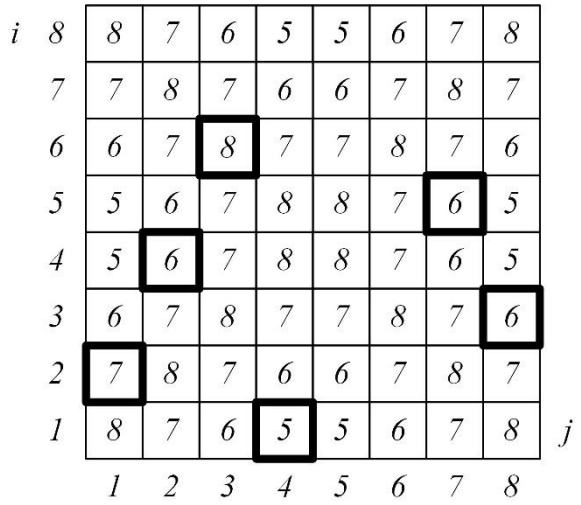


Fig. 5.6. Ordering productions according to the shortest diagonal heuristic on an 8×8 board

5.2. Knight's move heuristic in the N-queens problem

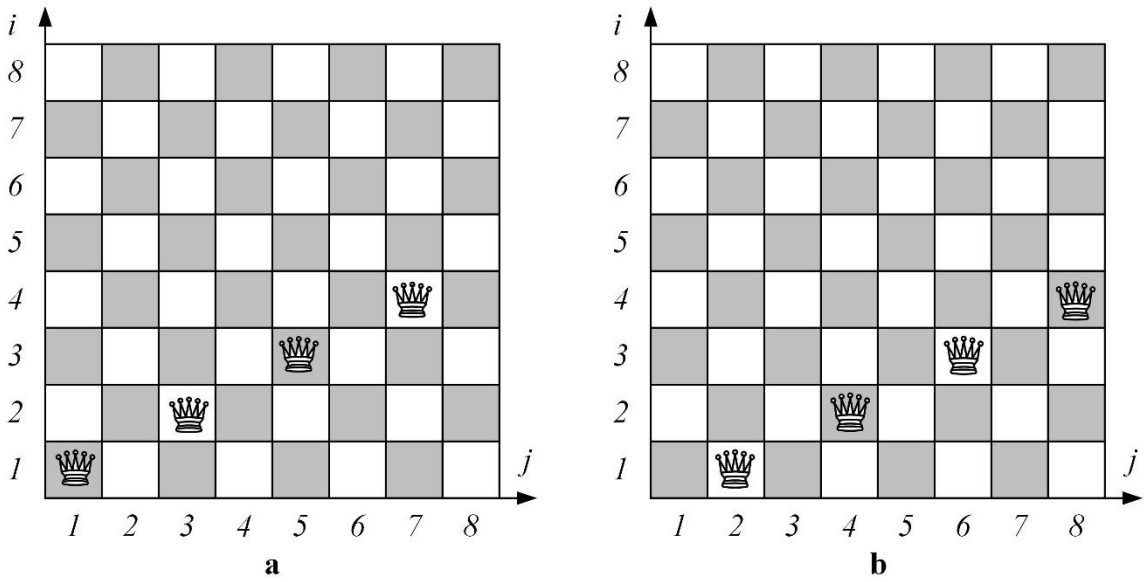


Fig. 5.7. Knight's move starting from: (a) [*i=1,j=1*] and (b) [*i=1,j=2*]

The knight's move is expressed with the production rule π_2 below. It is similar to π_1 in brute-force search.

π_2 := { • [*i+1,j+2*], if within the board. Here [*i,j*] is the last queen in the previous row
• [*i+1,j=1*], if [*i+1,j+2*] is out of the board

Let's start from the square [*i=1,j=2*] for $N \neq 6n+3$, i.e. boards excluding *N=9, 15, 21,…* Let's start from [*i=1,j=4*] for boards *N=9, 15, 21,…* This action is expressed with the production rule π_1 as follows:

$\pi_1 :=$ { • $[i=1,j=2]$ if $N \neq 6n+3$ • $[i=1,j=4]$ if $N = 6n+3$

After backtracking to square $[i,j]$, the subsequent square $[i,j+1]$ is taken in a cycle. In other words, if $j+1 > N$ (i.e. out of board) then the first column, $j=1$, is taken. This action expressed with the rule π_s as follows:

$\pi_s :=$ $[i,j+1]$ in a cycle, i.e. $[i,j=1]$ if $j+1 > N$ (out of the board)

On a 4×4 board, a solution is obtained with no backtracking (see Fig. 5.8).

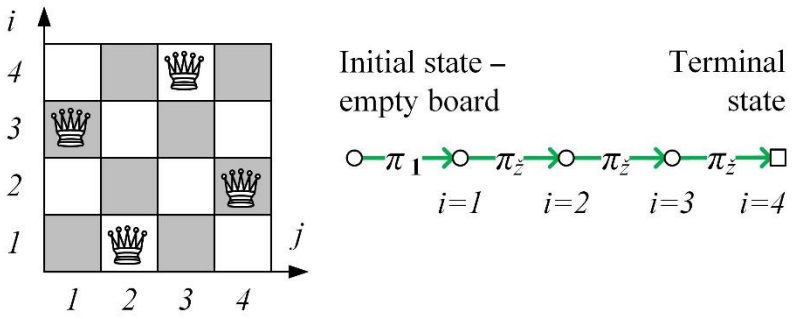


Fig. 5.8. A solution for the N-queens problem on a 4×4 board with the knight's move heuristic. No backtracking!

On a 5×5 board, a solution with the knight's move heuristic is obtained with no backtracking (see Fig. 5.9).

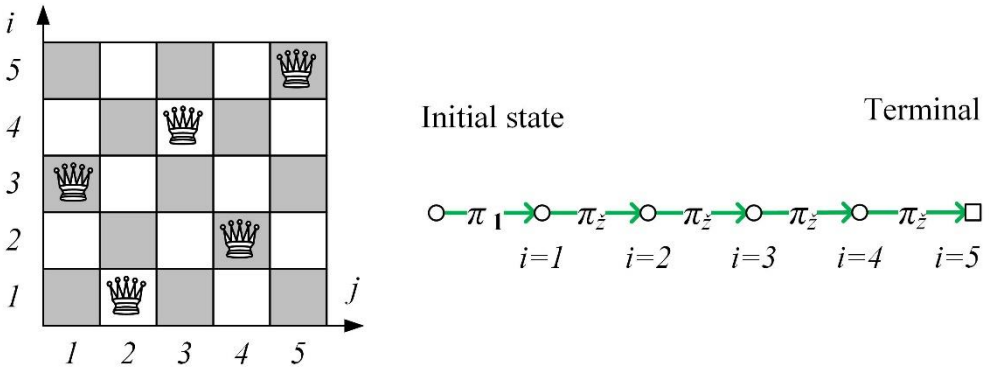


Fig. 5.9. A solution on a 5×5 board with the knight's move heuristic. No backtracking!

On a 5×5 board, brute-force search, however, needs backtracking; see the search tree in Fig. 5.10. The solution is $\langle \pi_1, \pi_3, \pi_5, \pi_2, \pi_4 \rangle$. It differs from the solution $\langle \pi_1, \pi_z, \pi_z, \pi_z, \pi_z \rangle$ which is obtained with the knight's move heuristic and is shown in Fig. 5.9 above.

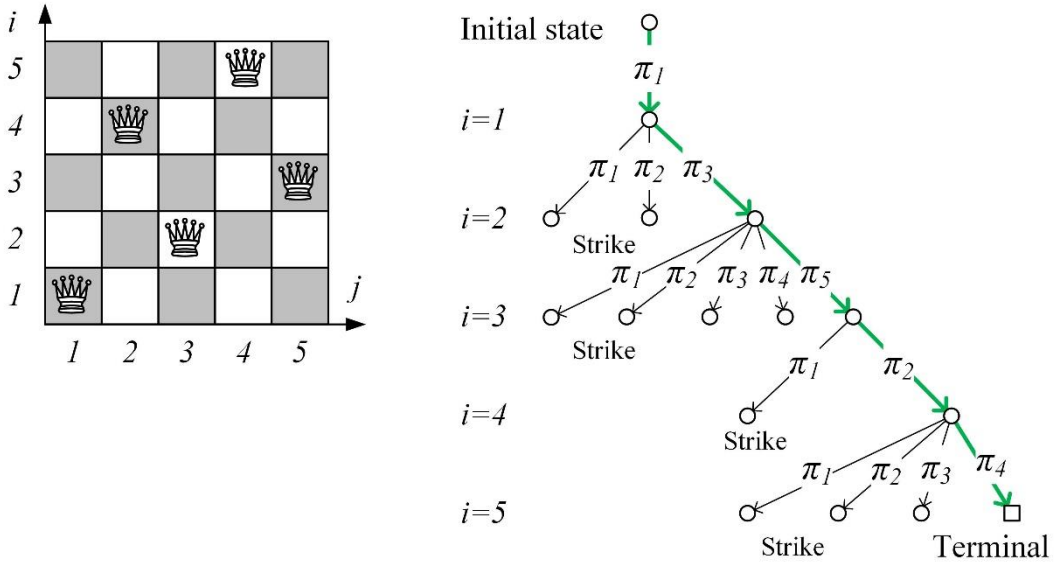


Fig. 5.10. A solution for the N-queens problem on a 5×5 board with brute-force search (i.e. no heuristic). Backtracking is needed. The search tree consists of 15 edges

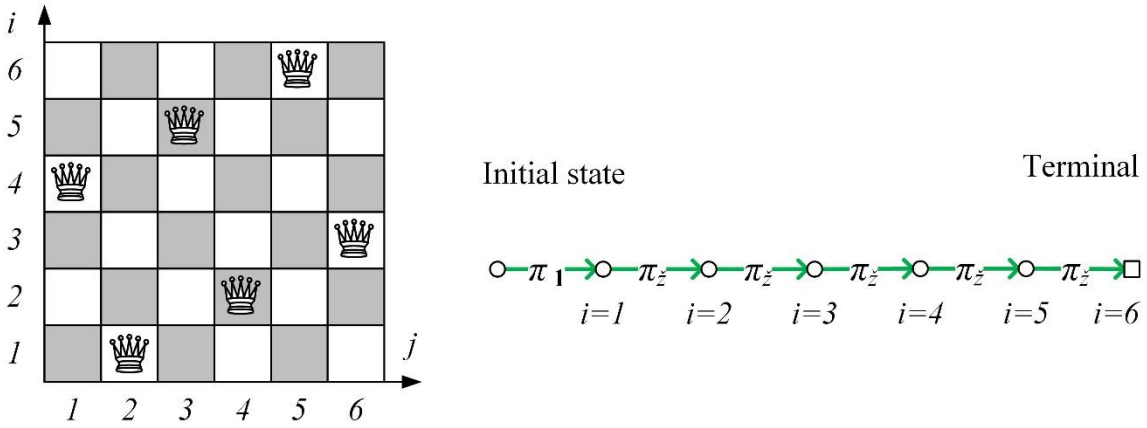


Fig. 5.11. A solution on a 6×6 board with the knight's move heuristic. No backtracking!

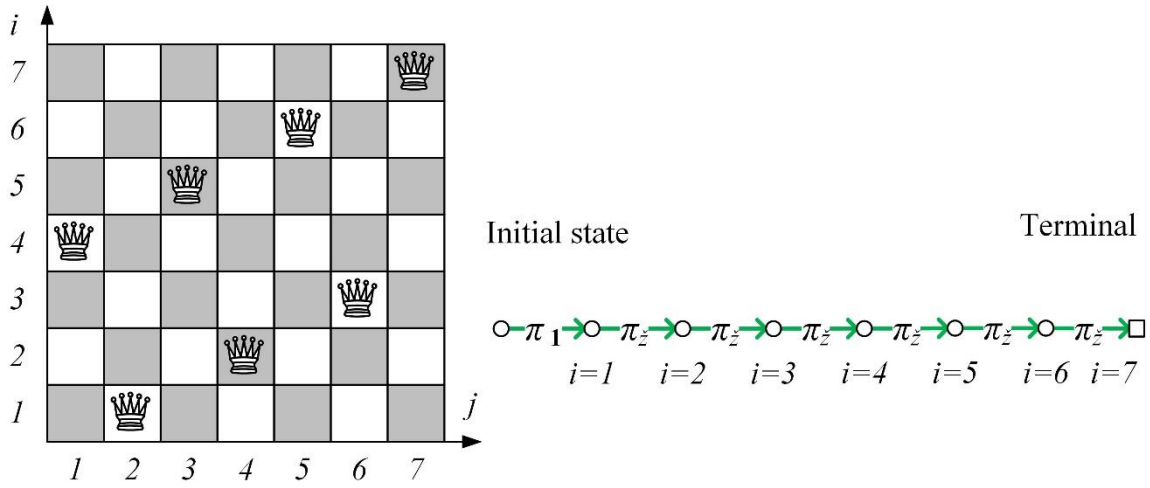


Fig. 5.12. A solution on a 7×7 board with the knight's move heuristic. No backtracking!

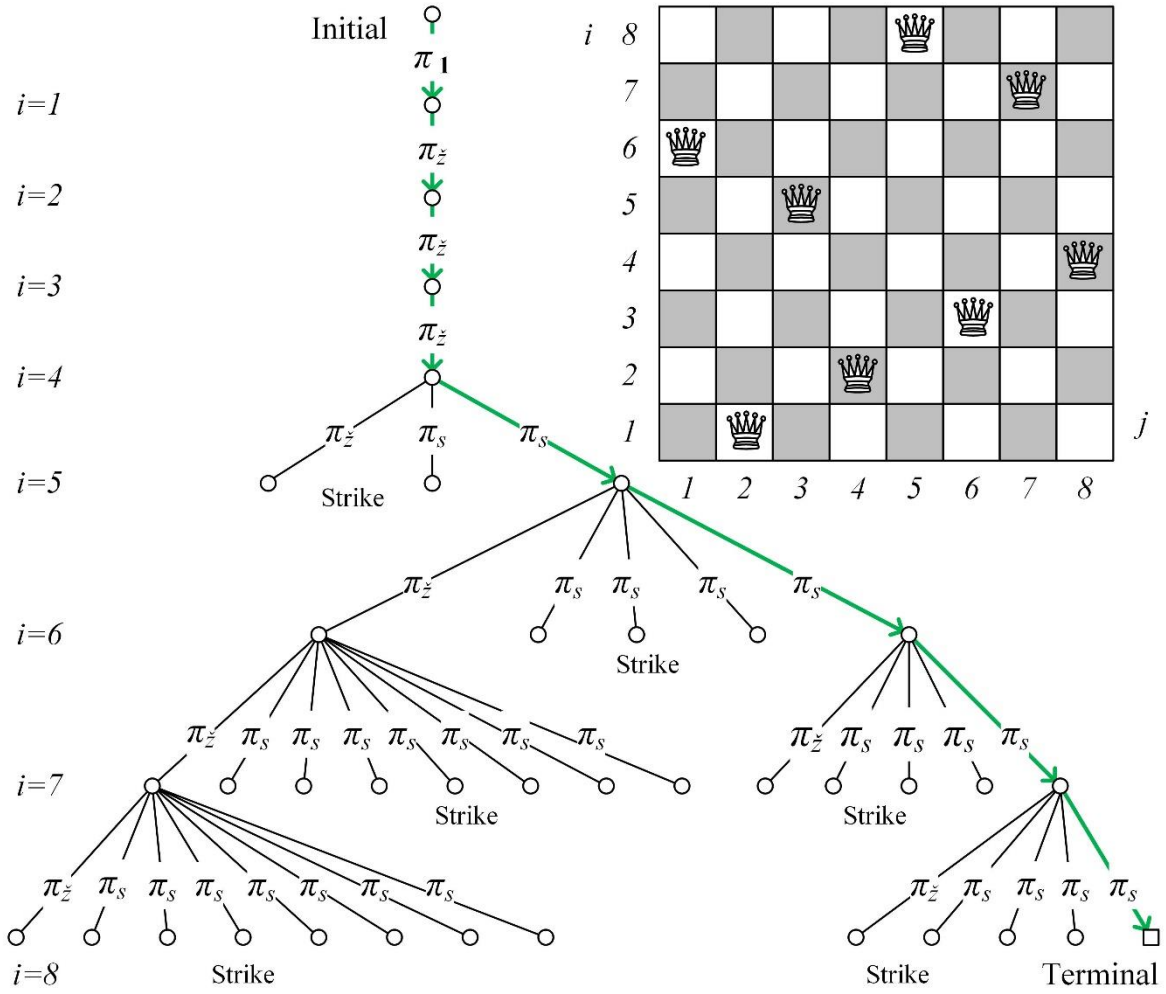


Fig. 5.13. A solution on an 8×8 board with the knight's move heuristic. The search space consists of 38 states. This is a significantly smaller number than 204 states in the shortest diagonal heuristic and 876 states in brute-force search

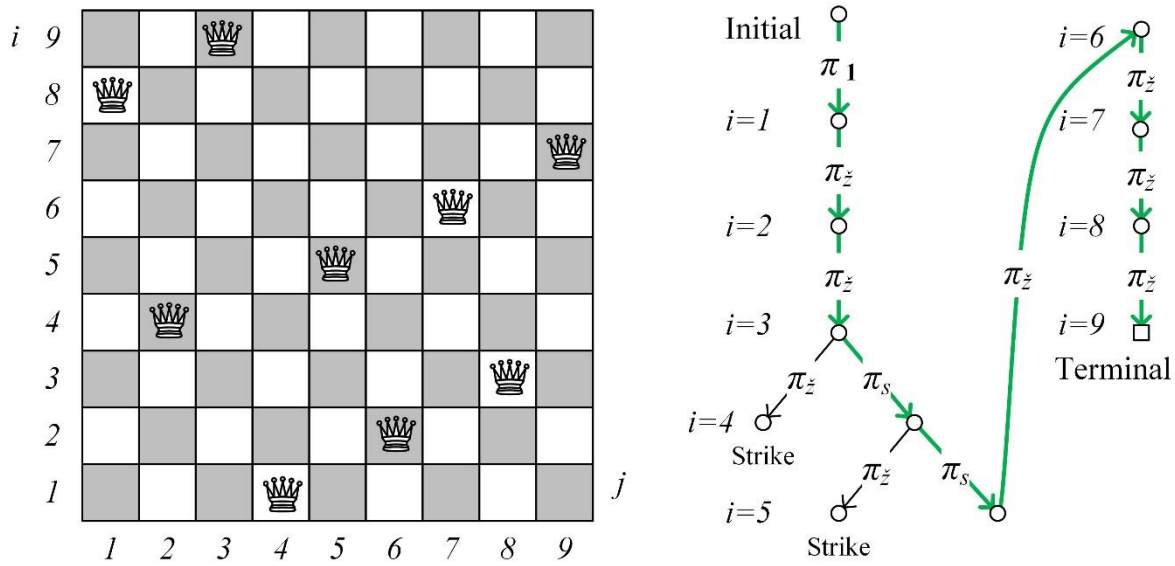


Fig. 5.14. A solution on a 9×9 board with the knight's move heuristic. Two backtracks only

Table 5.15. The number of states in the search space

N	Number of states in the search space		
	Brute force	**Shortest diagonal**	**Knight's move**
4	26	6	4
5	15	15	5
6	171	69	6
7	42	87	7
8	876	204	38
9	333	874	11
10	975	437	10
11	517	200	11
12	3066	297	12
13	1365	684	13
14	26495	1742	300
15	20280	487	17
16	160712	111	16
17	91222	294	17
18	743229	7130	18
19	48184	24714	19
20	3992510	918	372
21	179592	48222	23
22	38217905	40744	22
23	584591	10053	23
24	9878316	5723	24
25	1216775	2887	25
26	10339849	265187	2196
27	12263400	986476	29
28	84175966	2602283	28
29	44434525	1261296	29
30	1692888135	52601	30
31	Many	1850449	31
32	Many	2804692	2866
33	Many	2582396	35
34	Many	35784	34
35	Many	110473	35
36	Many	19605979	36
37	Many	135980	37
38	Many	642244758	30532
39	Many	193745	41
40	Many	4685041	40

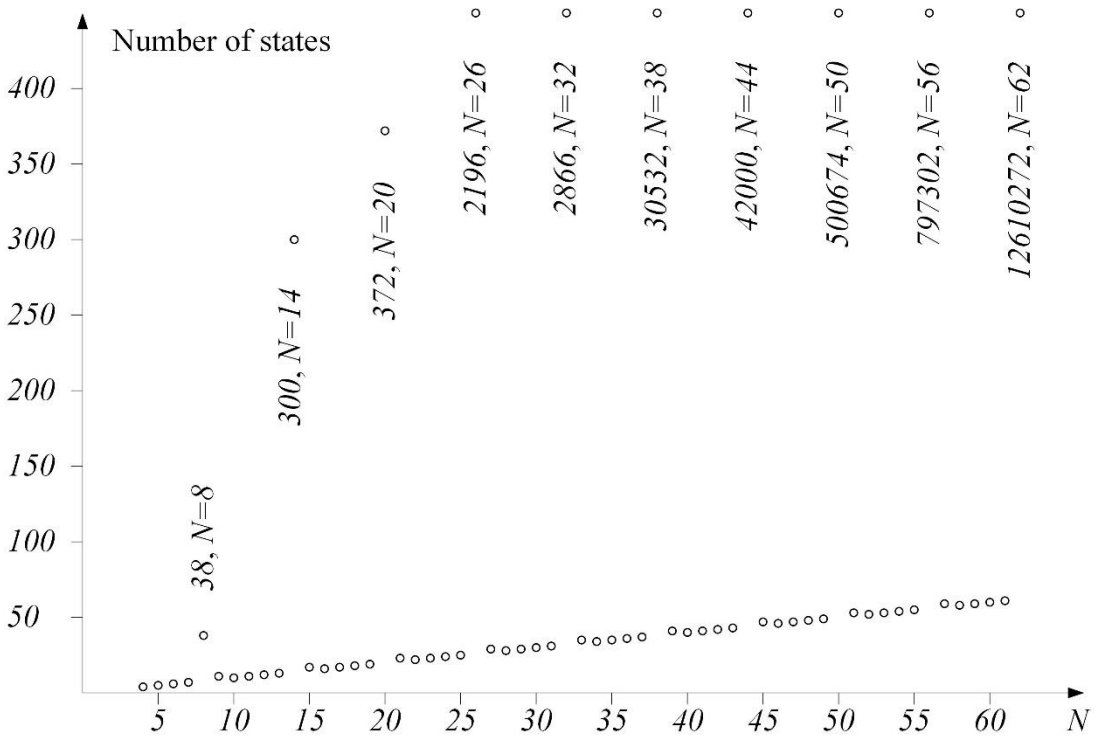


Fig. 5.16. Linear dependence – no backtrack in cases $N \in \{4, 5, 6, 7, 10, 11, 12, 13, 16, 17, 18, 19, 22, 23, 24, 25$ etc.$\}$. Backtrack in the remaining cases: $N \in \{8, 9, 14, 15, 20, 21, 26, 27$ etc.$\}$. Hypothesis: no backtrack in cases $N=6 \cdot n$, $N=6 \cdot n+1$, $N=6 \cdot n+4$, and $N=6 \cdot n+5$; backtrack in cases $N=6 \cdot n+2$ and $N=6 \cdot n+3$

A student formulated the following hypothesis. Prove it or disprove it.

$$N = \begin{cases} 6 \cdot n & \bullet \text{ No backtrack} \\ 6 \cdot n+1 & \bullet \text{ No backtrack} \\ 6 \cdot n+2 & \bullet \text{ Backtrack} \\ 6 \cdot n+3 & \bullet \text{ Backtrack} \\ 6 \cdot n+4 & \bullet \text{ No backtrack} \\ 6 \cdot n+5 & \bullet \text{ No backtrack} \end{cases}$$

6. BACKTRACK1 – a cycle-avoiding algorithm

Below we follow [Nilsson 1982 Section 2.1, p. 56–61]. Procedure BACKTRACK may never terminate; it may generate new nonterminal databases indefinitely or it may cycle. (Cycling is demonstrated with a LABYRINTH depth-first search program in the next section.) Both of these cases can be arbitrarily prevented by imposing a *depth bound* on the recursion. Cycling can be more straightforwardly prevented by maintaining a list of the databases produced so far and by checking new ones to see that they do not match any on the list.

Therefore we need a slightly more complex algorithm to avoid cycles. All databases on a path back to the initial one must be checked to insure that none are revisited. In order to implement this backtracking strategy as a recursive procedure, the entire chain of databases must be an argument of the procedure. Again, practical implementations of AI backtracking production systems use various techniques to avoid the need for explicitly listing all of these databases in their entirety.

BACKTRACK1 takes a list of databases as its argument; when first called, this list contains the initial database as its single element. Upon successful termination, BACKTRACK1 returns a sequence of rules that can be applied to the initial database to produce one that satisfies the termination condition.

```
procedure BACKTRACK1(DATALIST); {Returns a list of rules.}
{ 1}  DATA := FIRST(DATALIST);  {DATALIST – is a list of all
                databases on a path back to the initial one.
              DATA is the most recent one produced.}
{ 2}  if MEMBER(DATA, TAIL(DATALIST)) then return FAIL; {The
          procedure fails if it revisits an earlier database.}
{ 3}  if TERM(DATA) then return NIL;
{ 4}  if DEADEND(DATA) then return FAIL;
{ 5}  if LENGTH(DATALIST) > BOUND then return FAIL;
             {The procedure fails if too many rules have been
              applied. BOUND is a global variable specified
              before the procedure is first called.}
{ 6}  RULES := APPRULES(DATA);
{ 7}  LOOP:    if NULL(RULES) then return FAIL;
{ 8}    R     := FIRST(RULES);
{ 9}    RULES := TAIL(RULES); {E.g., TAIL(⟨a,b,c⟩)=⟨b,c⟩.}
{10}    RDATA := R(DATA);
{11}    RDATALIST := CONS(RDATA, DATALIST); {The list of data-
            bases visited so far is extended by adding RDATA.}
{12}    PATH := BACKTRACK1(RDATALIST);
{13}    if PATH = FAIL then goto LOOP;
{14}  return CONS(R, PATH);
end procedure;
```

The 8-puzzle example, which is examined in Chapter 15, uses BOUND = 7. Note that BACKTRACK1 does not remember *all* databases that it visited previously. Backtracking involves "forgetting" all databases whose paths lead to failures. BACKTRACK1 remembers only those databases on the *current* path back to the initial one, i.e. the thread. Hence, DATALIST plays the role of the stack.

7. Depth-first search in a labyrinth

This section demonstrates the BACKTRACK1 procedure in a labyrinth. The labyrinth is represented as a two-dimensional array. Wall tiles are marked with 1 and free ones with 0. Agent position is marked with 2; see Fig. 7.1. (See also https://en.wikipedia.org/wiki/Labyrinth; https://en.wikipedia.org/wiki/Maze_solving_algorithm.)

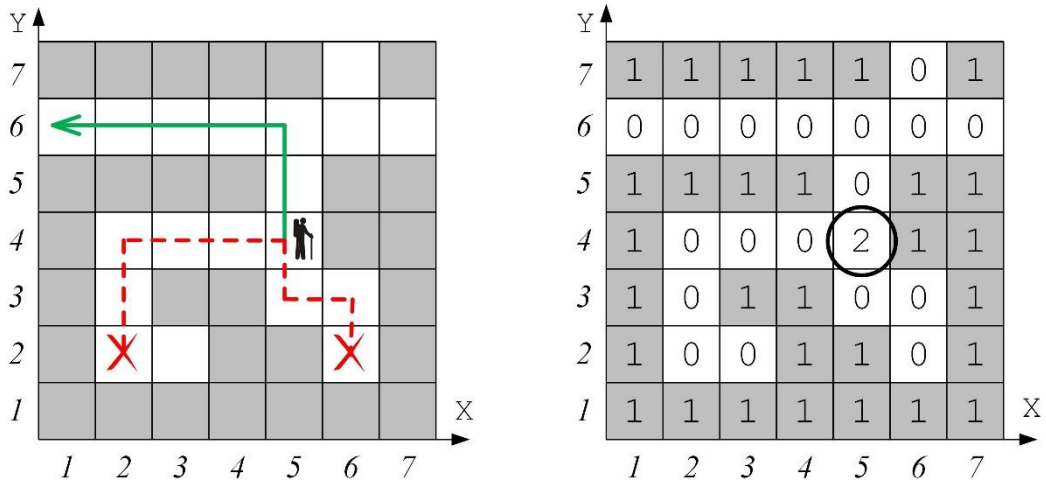


Fig. 7.1. A sample labyrinth and its representation with a two-dimensional array LAB. Wall tiles are marked 1 and free ones 0. Three exits are LAB[*1*,*6*], LAB[*7*,*6*] and LAB[*6*,*7*]. The agent starts in the position X=*5*, Y=*4* marked 2, i.e. LAB[*5*,*4*] = 2

The agent can move in four directions {π_1, π_2, π_3, π_4}; see Fig. 7.2. BACKTRACK1 program is shown below. It finds a path to an exit. The list DATALIST plays the stack's role. Metaphorically it corresponds to Ariadne's thread; see the Greek myth, https://en.wikipedia.org/wiki/Ariadne.

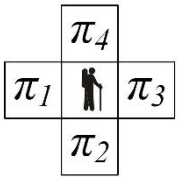


Fig. 7.2. Agent's four moves

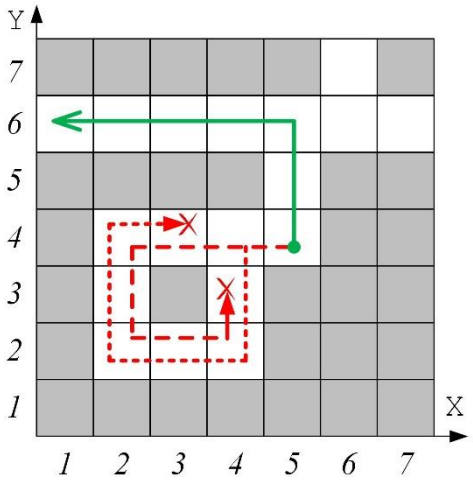


Fig. 7.3. A sample labyrinth with an island

In Fig. 7.3 the agent starts in X=*5*, Y=*4*. BACKTRACK runs into in infinite cycle around the island. However, BACKTRACK1 finds a path while travelling around the island two times. The BACKTRACK-WITH-CLOSED variant travels around the island once – only counterclockwise and finds the same path. The latter variant is identical with the variant GRAPHSEARCH-DEPTH-FIRST that is explored in chapters 13 and 14.

```
program LABYRINTH;  {BACKTRACK1, i.e. depth-first, no infinite cycle.}
const   M = 7; N = 7;                       {Dimensions.}
var LAB    : array[1..M, 1..N] of integer; {Labyrinth.}
    CX, CY : array[1..4] of integer; {4 production - shifts in X and Y.}
    L,        {Move's number. Starts from 2. Visited positions are marked.}
    X, Y,                {Agent's initial position.}
    I, J,                {Loop variables.}
    TRIAL : integer;     {Number of trials. To compare effectiveness.}
    YES   : boolean;     {True - success, false - failure.}

procedure TRY(X, Y : integer; var YES : boolean);
  var   K,               {The number of a production rule.}
        U, V : integer;  {Agent's new position.}
begin {TRY}
{K1} if  (X = 1) or (X = M) or (Y = 1) or (Y = N)
     then YES := true    {TERM(DATA} = true on the boarder.}
     else
     begin K := 0;
{K2}   repeat K := K + 1; {Next rule. Loop over production rules.}
{K3}     U := X + CX[K]; V := Y + CY[K]; {Agent's new position.}
{K4}     if LAB[U, V] = 0               {If a cell is free.}
         then
         begin TRIAL := TRIAL + 1;    {Number of trials.}
{K5}       L := L + 1; LAB[U,V] := L;{Marking the cell.}
{K6}       TRY(U, V, YES);           {Recursive call.}
           if not YES                {If failure}
{K7}       then begin
{K8}              LAB[U,V] := -1;    {then mark. (0 in case of BACKTRACK).}
                  L := L - 1;
                end;
         end;
       until YES or (K = 4);
     end;
end; {TRY}

begin {Main program.}
  {1. Reading the labyrinth.}
  for J := 1 to N do
  begin
     for I := 1 to M do read(LAB[I,J]);
     readln;
  end;
  {2. Reading agent's position.}
  read(X, Y); L := 2;  LAB[X,Y] := L;
  {3. Forming four production rules.}
  CX[1] := -1; CY[1] :=  0;  {Go West.      4   }
  CX[2] :=  0; CY[2] := -1;  {Go South.   1 * 3 }
  CX[3] :=  1; CY[3] :=  0;  {Go East.      2   }
  CX[4] :=  0; CY[4] :=  1;  {Go North.         }
  {4. Initialising variables.}
  YES := false;  TRIAL := 0;
  {5. Invoking the BACKTRACK1 procedure.}
  TRY(X, Y, YES);
  if YES
  then writeln('Path exists');          {Please also print the path found.}
  else writeln('Path does not exist');  {No paths exist.}
end.
```

Three variants:

V1) LAB[U,V] := –1. It's BACKTRACK-WITH-CLOSED, i.e. GRAPHSEARCH-DEPTH-FIRST.

V2) LAB[U,V] := 0 and LAB[U,V]:=L above. It's BACKTRACK1. Two routes around an island, but no cycles.

V3) LAB[U,V] := 0 and **no** LAB[U,V]:=L. It's the classical BACKTRACK. Infinite cycle.

The search tree is shown in Fig. 7.4. Note three variants V1, V2 and V3. Following is their the ordering according to time efficiency, where '<' denotes ordering:

BACKTRACK < BACKTRACK1 < BACKTRACK-WITH-CLOSED.

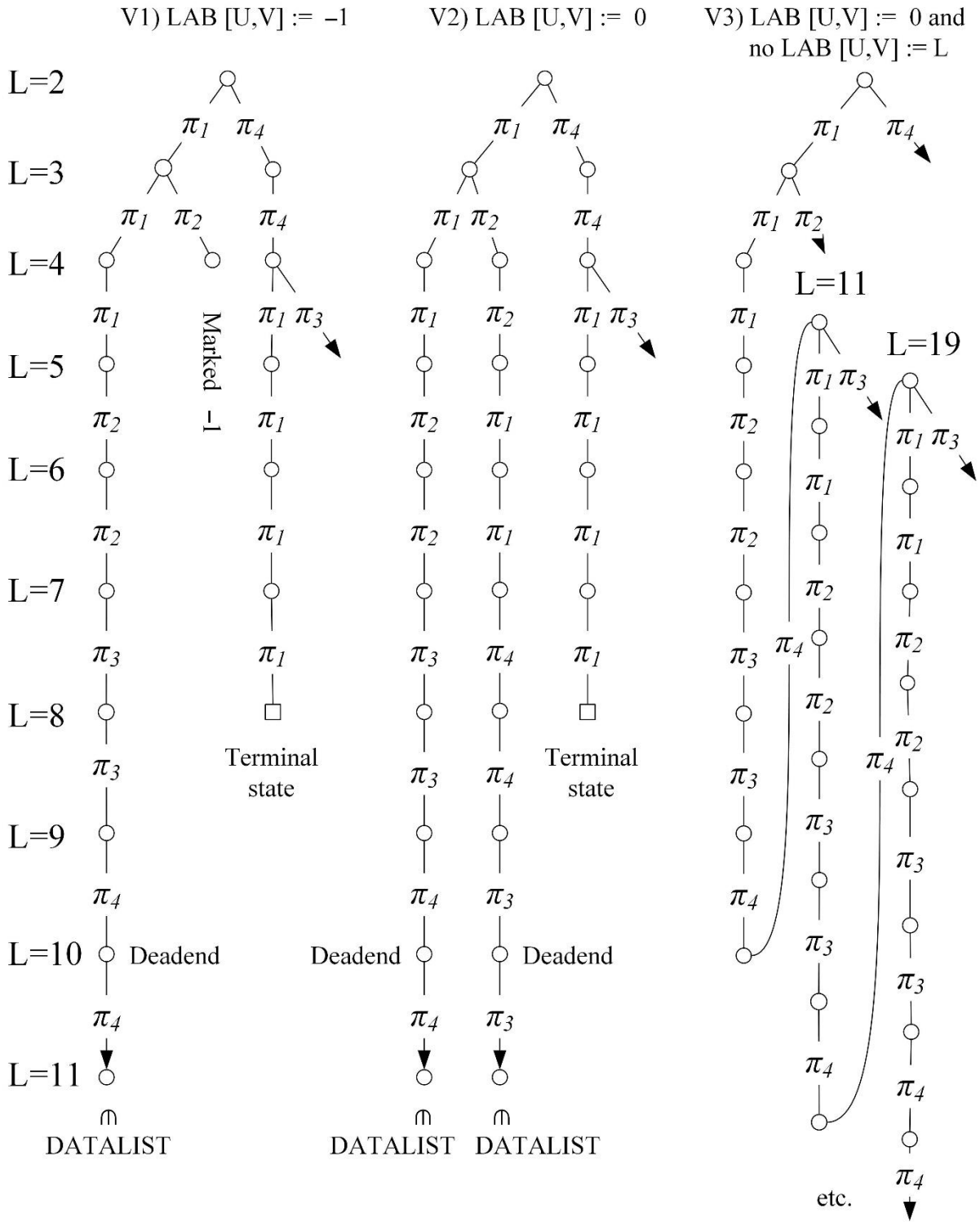


Fig. 7.4. Three variants of the search tree in the labyrinth in Fig. 7.3. V1) LAB[U,V] := –1. This is called BACKTRACK-WITH-CLOSED. V2) LAB[U,V] := 0 and LAB[U,V] := L. This is BACKTRACK1, i.e. backtracking with DATALIST in the role of the stack. An "island" is searched from two sides. No infinite cycles. V3) LAB[U,V] := 0 and no LAB[U,V] := L. This is the classical BACKTRACK that runs into infinite loop

7.1. Testing depth-first search in a labyrinth

Modify your labyrinth depth-first solver and test it. Test 1: the 7×7 labyrinth shown in Fig. 7.3; print the trace as below. Test 2: the 17×9 labyrinth and start at X=9, Y=6 as shown in Fig. 7.5. Test 3: build your 20×15 labyrinth with multiple islands, the resulting path length over 100 productions and multiple deadends (to be marked '–1' during the execution).

```
PART 1. Data
  1.1. Labyrinth

    Y, V
    ^
  7 |   1   1   1   1   1   0   1
  6 |   0   0   0   0   0   0   0
  5 |   1   1   1   1   0   1   1
  4 |   1   0   0   0   0   1   1
  3 |   1   0   1   0   1   1   1
  2 |   1   0   0   0   1   1   1
  1 |   1   1   1   1   1   1   1
    --------------------------------> X, U
        1   2   3   4   5   6   7
```

Read the labyrinth from file:

```
1   1   1   1   1   1   1    Line Y=1.
1   0   0   0   1   1   1    Line Y=2.
1   0   1   0   1   1   1    Line Y=3.
1   0   0   0   0   1   1    Line Y=4.
1   1   1   1   0   1   1    Line Y=5.
0   0   0   0   0   0   0    Line Y=6.
1   1   1   1   1   0   1    Line Y=7.
5   4                Start X=5, Y=4.
```

```
  1.2. Initial position X=5, Y=4. L=2.

PART 2. Trace
   1) R1. U=4, V=4. Free. L:=L+1=3. LAB[4,4]:=3.
   2) -R1. U=3, V=4. Free. L:=L+1=4. LAB[3,4]:=4.
   3) --R1. U=2, V=4. Free. L:=L+1=5. LAB[2,4]:=5.
   4) ---R1. U=1, V=4. Wall.
   5) ---R2. U=2, V=3. Free. L:=L+1=6. LAB[2,3]:=6.
   6) ----R1. U=1, V=3. Wall.
   7) ----R2. U=2, V=2. Free. L:=L+1=7. LAB[2,2]:=7.
   8) -----R1. U=1, V=2. Wall.
   9) -----R2. U=2, V=1. Wall.
  10) -----R3. U=3, V=2. Free. L:=L+1=8. LAB[3,2]:=8.
  11) ------R1. U=2, V=2. Thread.
  12) ------R2. U=3, V=1. Wall.
  13) ------R3. U=4, V=2. Free. L:=L+1=9. LAB[4,2]:=9.
  14) -------R1. U=3, V=2. Thread.
  15) -------R2. U=4, V=1. Wall.
  16) -------R3. U=5, V=2. Wall.
  17) -------R4. U=4, V=3. Free. L:=L+1=10. LAB[4,3]:=10.
  18) --------R1. U=3, V=3. Wall.
  19) --------R2. U=4, V=2. Thread.
  20) --------R3. U=5, V=3. Wall.
  21) --------R4. U=4, V=4. Thread.
      --------Backtrack from X=4, Y=3, L=10. LAB[4,3]:=-1. L:=L-1=9.
      -------Backtrack from X=4, Y=2, L=9. LAB[4,2]:=-1. L:=L-1=8.
  22) ------R4. U=3, V=3. Wall.
      ------Backtrack from X=3, Y=2, L=8. LAB[3,2]:=-1. L:=L-1=7.
  23) -----R4. U=2, V=3. Thread.
      -----Backtrack from X=2, Y=2, L=7. LAB[2,2]:=-1. L:=L-1=6.
  24) ----R3. U=3, V=3. Wall.
  25) ----R4. U=2, V=4. Thread.
      ----Backtrack from X=2, Y=3, L=6. LAB[2,3]:=-1. L:=L-1=5.
  26) ---R3. U=3, V=4. Thread.
  27) ---R4. U=2, V=5. Wall.
      ---Backtrack from X=2, Y=4, L=5. LAB[2,4]:=-1. L:=L-1=4.
  28) --R2. U=3, V=3. Wall.
  29) --R3. U=4, V=4. Thread.
  30) --R4. U=3, V=5. Wall.
      --Backtrack from X=3, Y=4, L=4. LAB[3,4]:=-1. L:=L-1=3.*8
  31) -R2. U=4, V=3. Thread.
  32) -R3. U=5, V=4. Thread.
  33) -R4. U=4, V=5. Wall.
```

```
      -Backtrack from X=4, Y=4, L=3. LAB[4,4]:=-1. L:=L-1=2.
  34) R2. U=5, V=3. Wall.
  35) R3. U=6, V=4. Wall.
  36) R4. U=5, V=5. Free. L:=L+1=3. LAB[5,5]:=3.
  37) -R1. U=4, V=5. Wall.
  38) -R2. U=5, V=4. Thread.
  39) -R3. U=6, V=5. Wall.
  40) -R4. U=5, V=6. Free. L:=L+1=4. LAB[5,6]:=4.
  41) --R1. U=4, V=6. Free. L:=L+1=5. LAB[4,6]:=5.
  42) ---R1. U=3, V=6. Free. L:=L+1=6. LAB[3,6]:=6.
  43) ----R1. U=2, V=6. Free. L:=L+1=7. LAB[2,6]:=7.
  44) -----R1. U=1, V=6. Free. L:=L+1=8. LAB[1,6]:=8. Terminal.

PART 3. Results

  3.1. Path is found.

  3.2. Path graphically:

    Y, V
  7 |   1   1   1   1   1   0   1
  6 |   8   7   6   5   4   0   0
  5 |   1   1   1   1   3   1   1
  4 |   1  -1  -1  -1   2   1   1
  3 |   1  -1   1  -1   1   1   1
  2 |   1  -1  -1  -1   1   1   1
  1 |   1   1   1   1   1   1   1
    --------------------------------> X, U
        1   2   3   4   5   6   7

  3.3. Rules: R4, R4, R1, R1, R1, R1.

  3.4. Nodes: [X=5,Y=4], [X=5,Y=5], [X=5,Y=6], [X=4,Y=6], [X=3,Y=6], [X=2,Y=6],
[X=1,Y=6].
```

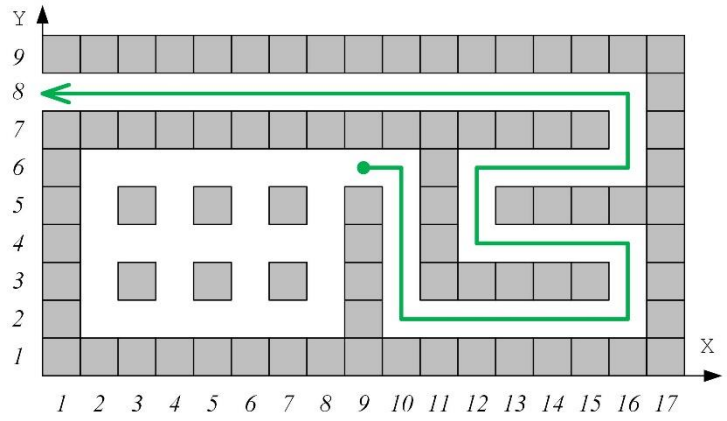


Fig. 7.5. A sample labyrinth with six islands. Six islands cause that the variant V2, BACKTRACK1, takes about 2^6=64 times more steps than V1, BACKTRACK-WITH-CLOSED

The number of islands, *n*, cause exponential growth, 2^n, in BACKTRACK1. To demonstrate this, modify your program to run two cases – both the variant V1 and V2. Your program should ask which variant to run: either V1 (that marks with '–1') or V2 (that marks with '0'). Therefore introduce a flag in your program. Then run it for V1 and subsequently for V2 to print different traces. Testing variants V1 and V2 serves to demonstrate your understanding that the number of steps of V2 depends exponentially on the number of islands. Design your 20×15 labyrinth with a big number of islands and run for V1 and V2.

8. Breadth-first search in a labyrinth

Suppose a labyrinth and four production rules {π_1, π_2, π_3, π_4}; see Fig. 8.1. The search tree for breadth-first search (BFS) is shown in Fig.8.2. The tree is also shown in layers in Fig. 8.3.

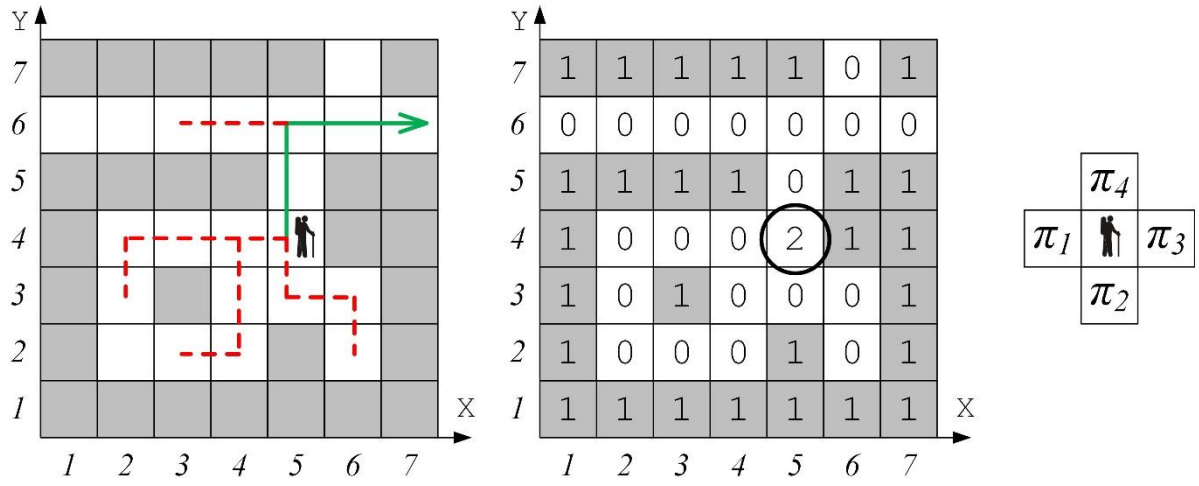


Fig. 8.1. A sample labyrinth and its representation to demonstrate breadth-first search. Agent starts from the position X=*5*, Y=*4*

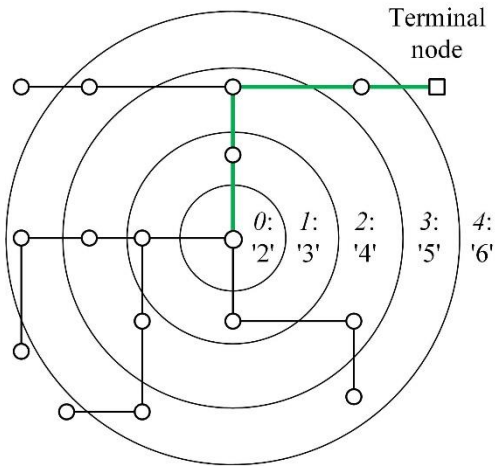


Fig. 8.2. Breadth-first search (BFS) tree. The nodes of each consecutive layer ("wave") are shown in a separate ring

A condition to apply BFS in an AI system is that the graph of states of the global datatabase system has to be known, e.g. a labyrinth map is provided. If the graph is not known in advance, BFS cannot be applied and the agent can apply depth-first search. BFS is an algorithm to plan a path and to use the synthesised path multiple times.

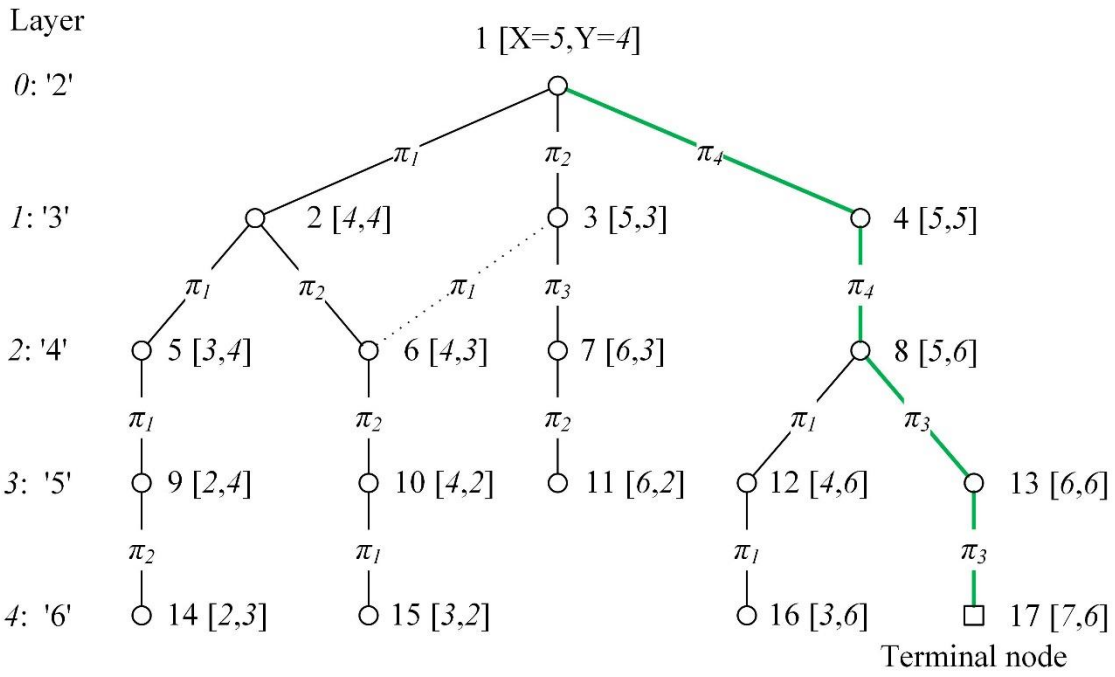


Fig. 8.3. A search tree with nodes sorted by layers ("waves")

The global database after procedure's completion is shown in Fig. 8.4 a. Array representation after the path is collected in order to print it is shown in Fig. 8.4 b.

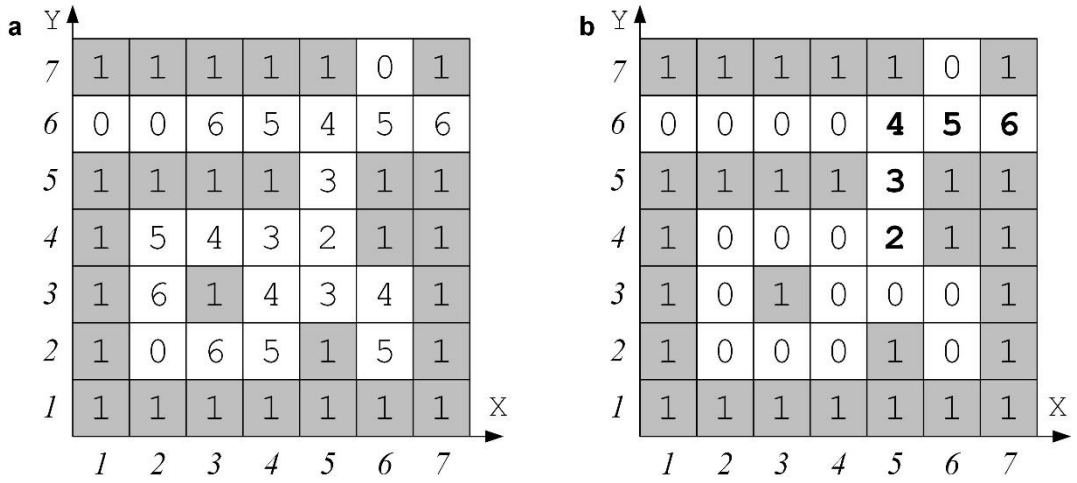


Fig. 8.4. a The final state of the global database – the procedure is completed. **b** Array representation after the path is collected

```
program LABYRINTH-BREADTH-FIRST (input, output);
  const
    M = 7; N = 7;                          {The dimensions of the labyrinth.  }
    MN = 49;                               {The number of cells M*N.          }
  var
    LAB, LABCOPY : array[1..M, 1..N] of integer; {Labyrinth and its copy.}
    CX, CY : array[1..4] of integer;       {4 production rules.               }
    FX, FY : array[1..MN] of integer;      {The "front" to store opened nodes.}
    CLOSE,                                 {The counter for a closed node.    }
    NEWN,                                  {The counter for an opened node.   }
```

```
    K,                                  {The counter for a production.     }
    X, Y,                               {The start position of the agent.  }
    U, V, I, J : integer;
    YES : boolean;

procedure BACK(U, V : integer); {Collect the path from the exit to start.}
    {INPUT: 1) U, V - the coordinates of the exit, and 2) global LABCOPY.}
    {OUTPUT: LAB.                                                        }
  var K, UU, VV : integer;
begin {BACK}
  LAB[U,V] := LABCOPY[U,V];           {The exit position is marked.      }
  K := 5;
  repeat {The search within 4 productions. Search for cell LABCOPY[UU,VV]}
         {with the mark which is 1 less than LABCOPY[U,V].               }
    K := K - 1; UU := U + CX[K]; VV := V + CY[K];
    if (1 <= UU) and (UU <= M) and (1 <= VV) and (VV <= N)
    then {Inside the boarders.}
        if LABCOPY[UU,VV] = LABCOPY[U,V] - 1
        then
        begin
          LAB[UU,VV] := LABCOPY[UU,VV] ;  {Marking a cell in LAB.        }
          U := UU; V := VV; K := 5;       {Swapping the variables.       }
        end;
  until LABCOPY[U, V] = 2;
end; {BACK}

begin {Main program}
  { 1. Reading the labyrinth.}
  for J := 1 to N do
  begin
    for I := 1 to M do
    begin read(LAB[I,J]);
          LABCOPY[I,J]:= LAB[I,J];
    end
    readln;
  end;
  { 2. Reading the starting position of the agent.}
  read(X,Y); LABCOPY[X,Y]:=2;

  { 3. Initialising 4 production rules.}
  CX[1] := -1; CY[1] :=  0;  {Go West.      2   }
  CX[2] :=  0; CY[2] := -1;  {Go South.   1 * 3 }
  CX[3] :=  1; CY[3] :=  0;  {Go East.      4   }
  CX[4] :=  0; CY[4] :=  1;  {Go North.         }

  { 4. Assigning initial values.}
  FX[1] := X;  FY[1] := Y;  CLOSE := 1;  NEWN := 1;  YES := false;

  { 5. Breadth-first search -- the "wave" algorithm.}

    if (X = 1) or  (X = M) or  (Y = 1) or  (Y = N)
    then {If an exit is reached then finish.}
         begin YES := true; U := X; V := Y;
         end;
```

```
    if (X > 1) and (X < M) and (Y > 1) and (Y < N)
    then
    repeat  {The loop through the nodes.}
      X := FX[CLOSE];  Y := FY[CLOSE]; {Coordinates of node to be closed.}
      K := 0;
      repeat  {The loop trough 4 production rules.}
        K := K + 1;  U := X + CX[K];  V := Y + CY[K];
        if LABCOPY[U, V] = 0 {The cell is free.}
        then begin
               LABCOPY[U,V] := LABCOPY[X,Y] + 1;       {New wave's number.}
               if (U = 1) or (U = M) or (V = 1) or (V = N) {Boarder.      }
               then YES := true; {Success. Here BACK(U,V) could be called.}
               else begin  {Placing a newly opened node into front's end. }
                      NEWN := NEWN + 1; FX[NEWN] := U; FY[NEWN] := V;
                    end;
             end;
      until (K = 4) or YES;  {Each of 4 productions is checked or success.}
      CLOSE := CLOSE + 1;    {Next node will be closed.}
    until (CLOSE > NEWN) or YES;

    { 6. Printing the path found.}
    if YES
    then begin
           writeln('The path exists.');
           BACK(U,V); {Collecting the path.}
           { Here a procedure should be called to print the path.}
         end
    else writeln('No path.');
end.
```

The solution is $\langle \pi_4, \pi_4, \pi_3, \pi_3 \rangle$. During program execution, the coordinates of newly opened nodes are put in the arrays FX and FY. This is shown in Table. 8.5.

Table 8.5. The frontier arrays FX[*i*] and FY[*i*] during the execution

I	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
Wave	'2'	'3'	'3'	'3'	'4'	'4'	'4'	'4'	'5'	'5'	'5'	'5'	'5'	'6'	'6'	'6'	'6'
FX	5	4	5	5	3	4	6	5	2	4	6	4	6	2	3	3	7
FY	4	4	3	5	4	3	3	6	4	2	2	6	6	3	2	6	6
CLOSE:=1	CLOSE=1	CLOSE=1	CLOSE=1	CLOSE=2	CLOSE=2	CLOSE=3	CLOSE=4	CLOSE=5	CLOSE=6	CLOSE=7	CLOSE=8	CLOSE=8	CLOSE=9	CLOSE=10	CLOSE=12	CLOSE=13	Stop
	NEWN:=2	NEWN:=3	NEWN:=4	NEWN:=5	NEWN:=6	NEWN:=7	NEWN:=8	NEWN:=9	NEWN:=10	NEWN:=11	NEWN:=12	NEWN:=13	NEWN:=14	NEWN:=15	NEWN:=16	NEWN:=17	

Breadth-first search visits the neighbour vertices before visiting the child vertices, and a queue is used. Depth-first search visits the child vertices before visiting the sibling vertices. A stack (often the program's call stack via recursion) is generally used when implementing the algorithm; see Wikipedia, https://en.wikipedia.org/wiki/Breadth-first_search, https://en.wikipedia.org/wiki/Depth-first_search, and also https://en.wikipedia.org/wiki/Graph_traversal.

8.1. Testing breadth-first search in a labyrinth

Modify your breadth-first solver to print the trace as below. Test 1: the labyrinth in Fig. 8.1. Test 2: the 17×9 labyrinth and start at X=*9*, Y=*6* as shown in Fig. 7.5. Test 3: build your 20×15 labyrinth with multiple islands, the resulting path length over 100 productions and multiple deadends.

```
PART 1. Data
  1.1. Labyrinth

    Y, V
  7 |   1   1   1   1   1   0   1
  6 |   0   0   0   0   0   0   0
  5 |   1   1   1   1   0   1   1
  4 |   1   0   0   0   2   1   1
  3 |   1   0   1   0   0   0   1
  2 |   1   0   0   0   1   0   1
  1 |   1   1   1   1   1   1   1
    -------------------------------> X, U
        1   2   3   4   5   6   7

  1.2. Initial position X=5, Y=4. L=2.

PART 2. Trace

WAVE 0, label L="2". Initial position X=5, Y=4, NEWN=1

WAVE 1, label L="3"
    Close CLOSE=1, X=5, Y=4.
        R1. X=4, Y=4. Free. NEWN=2.
        R2. X=5, Y=3. Free. NEWN=3.
        R3. X=6, Y=4. Wall.
        R4. X=5, Y=5. Free. NEWN=4.

WAVE 2, label L ="4"
    Close CLOSE=2, X=4, Y=4.
        R1. X=3, Y=4. Free. NEWN=5.
        R2. X=4, Y=3. Free. NEWN=6.
        R3. X=5, Y=4. CLOSED or OPEN.
        R4. X=4, Y=5. Wall.

    Close CLOSE=3, X=5, Y=3.
        R1. X=4, Y =3. CLOSED or OPEN.
        R2. X=5, Y =2. Wall.
        R3. X=6, Y =3. Free. NEWN=7.
        R4. X=5, Y =4. CLOSED or OPEN.

    Close CLOSE=4, X=5, Y=5.
        R1. X=4, Y=5. Wall.
        R2. X=5, Y=4. CLOSED or OPEN.
        R3. X=6, Y=5. Wall.
        R4. X=5, Y=6. Free. NEWN=8.

WAVE 3, label L="5"
    Close CLOSE=5, X=3, Y=4.
        R1. X=2, Y=4. Free. NEWN=9.
        R2. X=3, Y=3. Wall.
        R3. X=4, Y=4. CLOSED or OPEN.
        R4. X=3, Y=5. Wall.

    Close CLOSE=6, X=4, Y=3.
        R1. X=3, Y=3. Wall.
        R2. X=4, Y=2. Free. NEWN=10.
        R3. X=5, Y=3. CLOSED or OPEN.
        R4. X=4, Y=4. CLOSED or OPEN.
```

```
    Close CLOSE=7, X=6, Y=3.
        R1. X=5, Y=3. CLOSED or OPEN.
        R2. X=6, Y=2. Free. NEWN=11.
        R3. X=7, Y=3. Wall.
        R4. X=6, Y=4. Wall.

    Close CLOSE=8, X=5, Y=6.
        R1. X=4, Y=6. Free. NEWN=12.
        R2. X=5, Y=5. CLOSED or OPEN.
        R3. X=6, Y=6. Free. NEWN=13.
        R4. X=5, Y=7. Wall.

WAVE 4, label L="6"
    Close CLOSE=9, X=2, Y=4.
        R1. X=1, Y=4. Wall.
        R2. X=2, Y=3. Free. NEWN=14.
        R3. X=3, Y=4. CLOSED or OPEN.
        R4. X=2, Y=5. Wall.

    Close CLOSE=10, X=4, Y=2.
        R1. X=3, Y=2. Free. NEWN=15.
        R2. X=4, Y=1. Wall.
        R3. X=5, Y=2. Wall.
        R4. X=4, Y=3. CLOSED or OPEN.

    Close CLOSE=11, X=6, Y=2.
        R1. X=5, Y=2. Wall.
        R2. X=6, Y=1. Wall.
        R3. X=7, Y=2. Wall.
        R4. X=6, Y=3. CLOSED or OPEN.

    Close CLOSE=12, X=4, Y=6.
        R1. X=3, Y=6. Free. NEWN=16.
        R2. X=4, Y=5. Wall.
        R3. X=5, Y=6. CLOSED or OPEN.
        R4. X=4, Y=7. Wall.

    Close CLOSE=13, X=6, Y=6.
        R1. X=5, Y=6. CLOSED or OPEN.
        R2. X=6, Y=5. Wall.
        R3. X=7, Y=6. Free. NEWN=17. Terminal.

PART 3. Results

  3.1. Path is found.

  3.2. Path graphically:

LABCOPY
    Y, V
  7 |   1   1   1   1   1   0   1
  6 |   0   0   6   5   4   5   6
  5 |   1   1   1   1   3   1   1
  4 |   1   5   4   3   2   1   1
  3 |   1   6   1   4   3   4   1
  2 |   1   0   6   5   1   5   1
  1 |   1   1   1   1   1   1   1
    --------------------------------> X, U
        1   2   3   4   5   6   7

  3.3. Rules: R4, R4, R3, R3.

  3.4. Nodes: [X=5,Y=4], [X=5,Y=5], [X=5,Y=6], [X=6,Y=6], [X=7,Y=6].
```

9. Breadth-first search in a graph

The BFS algorithm is presented below according to the book by Earl Hunt (1978), section 10.1.1. The algorithm finds the path with fewest edges. See also https://en.wikipedia.org/wiki/Breadth-first_search.

INPUT: 1) a graph; 2) a start node *s*; 3) a terminal node *t*.
OUTPUT: the shortest path from *s* to *t*.

The algorithm operates with two lists, OPEN and CLOSED, which initially are empty.

1. Put the start node *s* on OPEN.
2. If OPEN is empty, return FAIL. There is no path. This happens in the case of a disconnected graph (see Fig. 9.1).
3. Close the **first** vertex *n* from OPEN: remove it from OPEN, and put it on CLOSED.
4. If *n* is a terminal node, exit successfully with the solution obtained by tracing a path backward along the arcs from *n* to *s*.
5. Expand node *n* generating a set *S*(*n*) of successors, i.e. all nodes adjacent to *n*. Add those nodes from *S*(*n*), which are neither in OPEN nor in CLOSED, to the **end** of OPEN. Formally, OPEN := OPEN ∪ *S*(*n*)/(OPEN∪CLOSED).
6. Go to step 2.

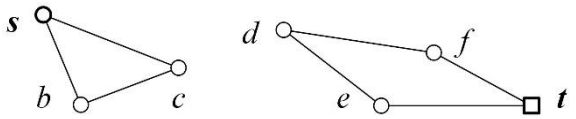


Fig. 9.1. A sample disconnected graph. No path between *s* and *t*

Searching the shortest path between *a* and *e* is shown in Fig. 9.2. Note that the word ‘node’ is usually interchangeable with the word ‘vertex’.

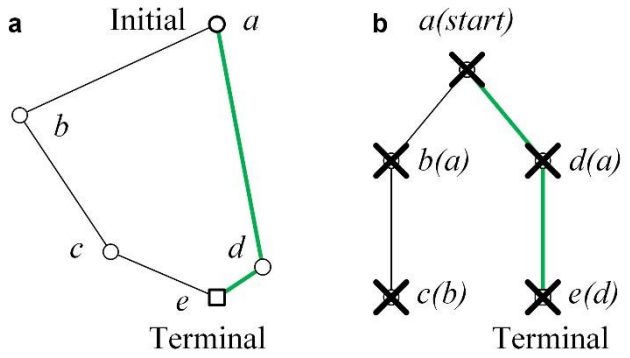


Fig. 9.2. a A graph for path search between the starting node *a* and the terminal node *e*. **b** The search tree. The shortest path ⟨*a*, *d*, *e*⟩ is of two edges

The lists OPEN and CLOSED are shown in Table 9.3 below.

Table 9.3. The lists OPEN and CLOSED in each step of the algorithm for shortest path search between *a* and *e* in Fig. 9.2.

	OPEN	CLOSED	Comment
1	*a*(*start*)	∅	Initial state
2	*b*(*a*), *d*(*a*)	*a*(*start*)	*S*(*a*) = {*b*,*d*}. *a* is closed; *b* and *d* opened
3	*d*(*a*), *c*(*b*)	*a*(*start*), *b*(*a*)	*S*(*b*) = {*a*, *c*} but *a*∈CLOSED
4	*c*(*b*), *e*(*d*)	*a*(*start*), *b*(*a*), *d*(*a*)	*S*(*d*) = {*a*, *e*} but *a*∈CLOSED

5	*e*(*d*)	*a*(*start*), *b*(*a*), *d*(*a*), *c*(*b*)	$S(c) = \{b, e\}$ but $b \in$CLOSED and $e \in$ OPEN. Therefore they are not put twice
6	∅	*a*(*start*), *b*(*a*), *d*(*a*), *c*(*b*), *e*(*d*)	The terminal node *e* is being closed. Its children are not analysed

A more complicated graph is shown in Fig. 9.4. Shortest path is searched between *a* and *f*.

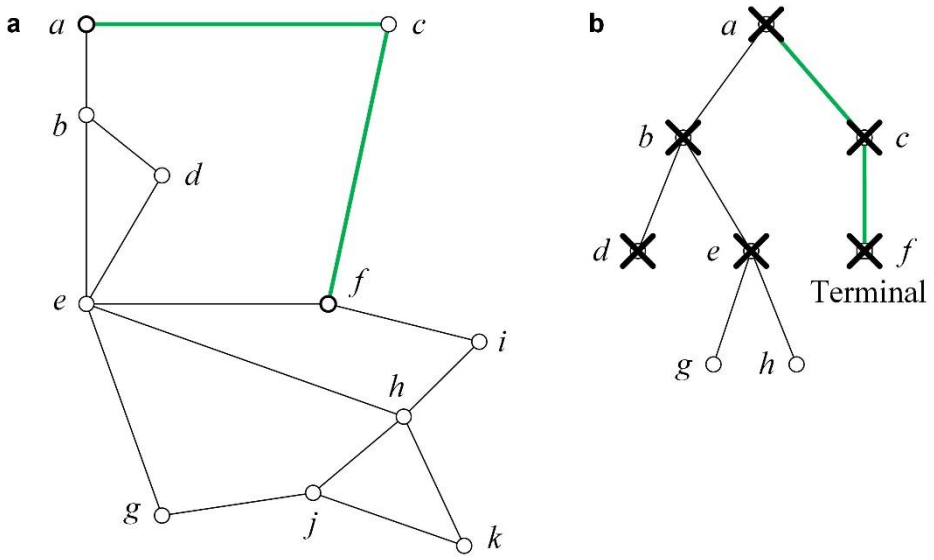


Fig. 9.4. a Shortest path search between *a* and *f*. **b** The search tree. The shortest path ⟨*a*, *c*, *f*⟩ is of two edges

Table 9.5. The lists OPEN and CLOSED in every step of the algorithm for shortest path search between *a* and *f* in Fig. 9.4.

	OPEN	CLOSED	Comment
1	*a*(*start*)	∅	
2	*b*(*a*), *c*(*a*)	*a*(*start*)	$S(a) = \{b,c\}$
3	*c*(*a*), *d*(*b*), *e*(*b*)	*a*(*start*), *b*(*a*)	$S(b) = \{a,d,e\}$ but $a \in$CLOSED
4	*d*(*b*), *e*(*b*), *f*(*c*)	*a*(*start*), *b*(*a*), *c*(*a*)	$S(c) = \{a,f\}$ but $a \in$CLOSED
5	*e*(*b*), *f*(*c*)	*a*(*start*), *b*(*a*), *c*(*a*), *d*(*b*)	$S(d) = \{b,e\}$ but $b \in$CLOSED
6	*f*(*c*), *g*(*e*), *h*(*e*)	*a*(*start*), *b*(*a*), *c*(*a*), *d*(*b*), *e*(*b*)	$S(e) = \{b,d,f,g,h\}$ but $b,d \in$CLOSED and $f \in$OPEN
7	*g*(*e*), *h*(*e*)	*a*(*start*), *b*(*a*), *c*(*a*), *d*(*b*), *e*(*b*), *f*(*c*)	The terminal node *f* is being closed. Its children are not analysed

One more graph is shown in Fig. 9.6.

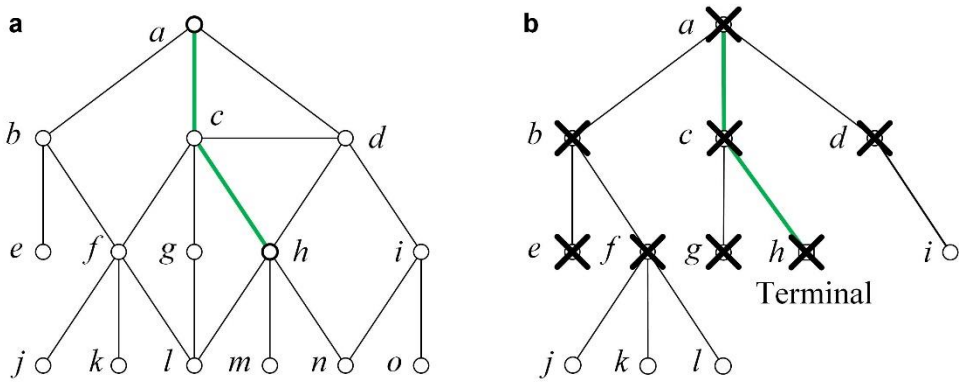


Fig. 9.6. a Shortest path search between *a* and *h*. An undirected graph $G=\langle V,E\rangle$, where $V = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o\}$, $E = \{(a,b), (a,c), (a,d), (b,e), (b,f), (c,d), (c,f), (c,g), (c,h), (d,h), (d,i), (f,j), (f,k), (f,l), (g,l), (h,l), (h,m), (h,n), (i,n), (i,o)\}$. The shortest path $\langle a, c, h\rangle$ is of two edges. **b** The search tree

Both lists, OPEN and CLOSED, can be written in the so called "frontier" notation:

~~*a*~~, ~~*b*~~, ~~*c*~~, ~~*d*~~, ~~*e*~~, ~~*f*~~, ~~*g*~~, ~~*h*~~, *i*, *j*, *k*, *l*

10. Shortest path problem in a graph with edge weights

Suppose a graph with nonnegative edge weights. We provide an algorithm to search for the shortest path from an initial node *s* to a terminal node *t*. This is a classic algorithm and is presented in textbooks, see e.g. (Hunt 1978, 10.1.2). When each edge in the graph has unit weight 1, this is equivalent to finding the path with fewest edges (see previous section).

This algorithm is a variant of Dijkstra's algorithm which is conceived by Dutch computer scientist Edsger Dijkstra in 1956. Dijkstra's algorithm to find the shortest path between *s* and *t* starts from *s*, picks the unvisited vertex with the lowest distance, calculates the distance through it to each unvisited neighbour, and updates the neighbour's distance if smaller, see https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm.

INPUT: 1) a graph with nonnegative edge weights; 2) a start node *s*; 3) a terminal node *t*.
OUTPUT: the shortest path from *s* to *t*.

Initially the lists OPEN and CLOSED are empty.

1. Put the start node *s* on OPEN.
2. If OPEN is empty, return FAIL. There is no path. This happens in the case of a disconnected graph.
3. Close the **first** node *n* from OPEN: remove it from OPEN, and put it on CLOSED. Here *n* is the node with the shortest distance from *s*. (OPEN is sorted in this way.)
4. If *n* is a terminal node, exit successfully with the solution obtained by tracing a path backward along the arcs from *n* to *s*.
5. Expand node *n* generating a set $S(n)$ of successors, i.e. all nodes adjacent to *n*. Add to OPEN those nodes, which are not in CLOSED; formally, OPEN := OPEN $\cup$ $S(n)$/CLOSED. For each n^* from $S(n)$/CLOSED, calculate its weight and assign it in OPEN. Formally, $\forall n^* \in S(n)$/CLOSED, assign $pathweight(s,n^*) := pathweight(s,n) + edgeweight(n,n^*)$. Sort OPEN. For each n^* which appears twice (with the old *pathweight* and a new one) update the weight for the smaller one.
6. Go to step 2.

Following we show the search for the shortest path between *a* and *e* in Fig. 10.1.

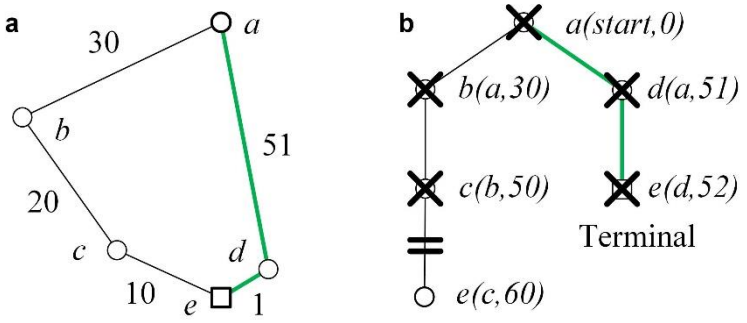


Fig. 10.1. a A graph with nonnegative edge weights. **b** The search tree for the shortest path between *a* and *e*. The path $\langle a, d, e \rangle$ is of length 52

The lists OPEN and CLOSED are shown in Table 10.2 below.

Table 10.2. The lists OPEN and CLOSED in each step of the algorithm for shortest path search between *a* and *e* in Fig. 10.1. The path ⟨*a(start,0), d(a,51), e(d,52)*⟩ is of length 52.

	OPEN	CLOSED	Comment
1	*a(start,0)*	∅	
2	*b(a,30), d(a,51)*	*a(start,0)*	$S(a) = \{b,d\}$
3	*c(b,50), d(a,51)*	*a(start,0), b(a,30)*	$S(b) = \{a,c\}$ but $a \in$ CLOSED
4	*d(a,51), e(c,60)*	*a(start,0), b(a,30), c(b,50)*	$S(c) = \{b,e\}$ but $b \in$ CLOSED
5	***e(d,52)***	*a(start,0), b(a,30), c(b,50), d(a,51)*	$S(d) = \{a,\mathbf{e}\}$ but $a \in$ CLOSED and $\mathbf{e} \in$ OPEN. New cost ***e(d,52)*** is better than the old one *e(c,60)*. Take it
6	∅	*a(start,0), b(a,30), c(b,50), d(a,51),* ***e(d,52)***	The terminal node ***e*** **is closed**

Suppose a more complicated graph which is shown in Fig. 10.3.

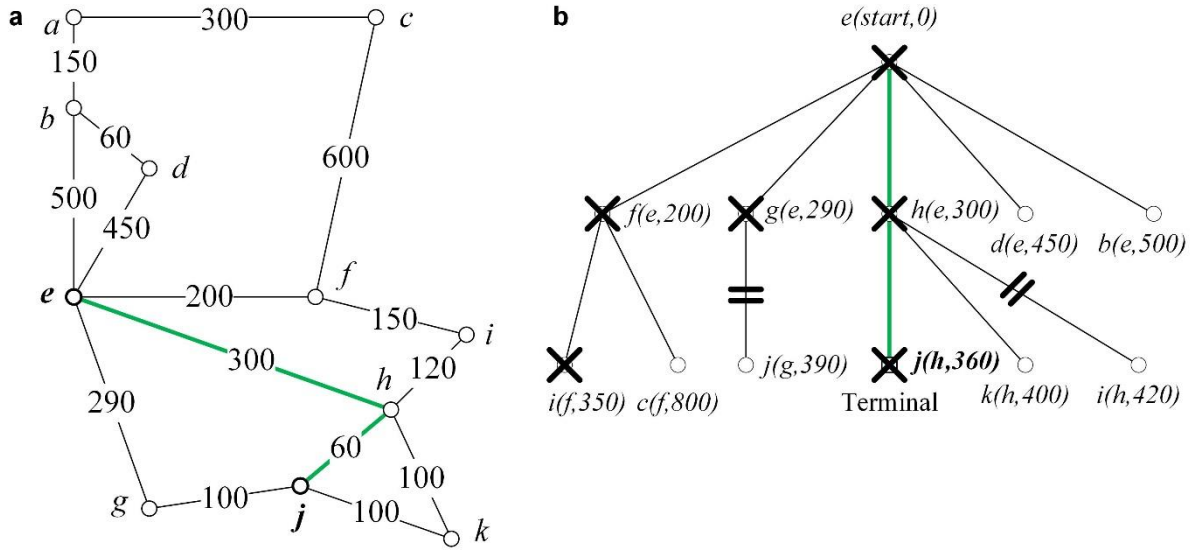


Fig. 10.3. a A sample graph. **b** The search tree for the shortest path between *e* and *j*. The path ⟨*e, h, j*⟩ is of length 360

The lists OPEN and CLOSED are shown in Table 10.4.

Table 10.4. The lists OPEN and CLOSED in each step of the algorithm for shortest path between *e* and *j* in Fig. 10.3. The path is ⟨*e(start,0), h(e,300), j(h,360)*⟩ is of length 360.

	OPEN	CLOSED	Comment
1	*e(start,0)*	∅	
2	*f(e,200), g(e,290), h(e,300), d(e,450), b(e,500)*	*e(start,0)*	$S(e) = \{b,d,f,g,h\}$
3	*g(e,290), h(e,300), i(f,350), d(e,450), b(e,500), c(f,800)*	*e(start,0), f(e,200)*	$S(f) = \{c,e,i\}$, but $e \in$ CLOSED

4	*h(e,300), i(f,350), j(g,390), d(e,450), b(e,500), c(f,800)*	*e(start,0), f(e,200), g(e,290)*	S(*g*) = {*e*,*j*}, but *e*∈CLOSED
5	*i(f,350),* ***j(h,360)****, k(h,400), d(e,450), b(e,500), c(f,800)*	*e(start,0), f(e,200), g(e,290), h(e,300)*	*S*(*h*) = {*e*,*i*,***j***,*k*}, but *e*∈CLOSED and *i*,***j***∈OPEN. New cost *i(h,420)* is worse than the old *i(f,350)*; therefore the old one is taken. New cost ***j(h,360)*** is better than the old *j(g,390)*; therefore the new one is taken
6	*j(h,360), k(h,400), d(e,450), b(e,500), c(f,800)*	*e(start,0), f(e,200), g(e,290), h(e,300), i(f,350)*	*S*(*i*) = {*f*,*h*}, but *f*,*h*∈CLOSED
7	*k(h,400), d(e,450), b(e,500), c(f,800)*	*e(start,0), f(e,200), g(e,290), h(e,300), i(f,350), j(h,360)*	The terminal node *j* is closed. Its children are not analysed

11. Depth-first search in a graph with no weights. The solver and the planner

Suppose an agent knows a graph. Does it make sense for the agent to use depth-first search and not breadth-first search to find a path between two nodes? Yes, it makes sense. Suppose the agent walks in the graph like in a park, does not need the shortest path and is satisfied with any long path.

The difference between depth-first search and breadth-first search is illustrated in Fig. 11.1 a. Here *s* stands for the initial node and *t* for the terminal node. Suppose that the edges are sorted contra-clockwise – similarly to corresponding production rules (Fig. 11.1 b). Depth-first search results in the path ⟨*s*,*a*,*e*,*t*⟩ of three edges (Fig. 11.1 c), whereas breadth-first search results in the path ⟨*s*,*t*⟩ of one edge only.

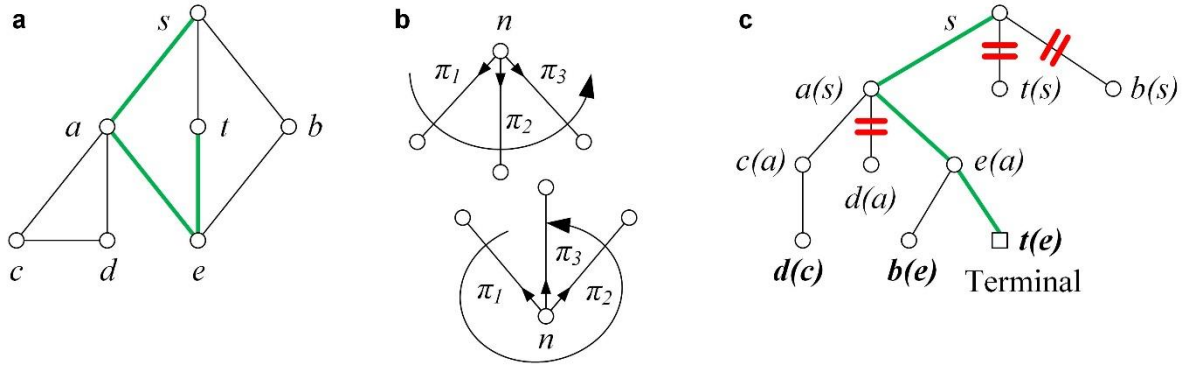


Fig. 11.1. a Depth-first search for a path between *s* and *t*. **b** Contra-clockwise ordering of edges: the "12 hours" edge is the last one. **c** The depth-first search tree. New links replace the old links of *d*, *b* and *t*: ***d(c)*** replaces *d(a)*, ***b(e)*** replaces *b(s)* and ***t(e)*** replaces *t(s)*. Three cut subtrees are denoted with big bold "="

INPUT: 1) a graph; 2) a start node *s*; 3) a terminal node *t*.
OUTPUT: The depth-first search path. It is not necessarily the shortest path.

1. Put the start node *s* on OPEN.
2. If OPEN is empty, return FAIL. There is no path. This happens in the case of a non-connected graph (see Fig. 9.1).
3. Close the **first** node *n* from OPEN: remove it from OPEN, and put it on CLOSED.
4. If *n* is a terminal node, exit successfully with the solution obtained by tracing a path backward along the arcs from *n* to *s*.
5. Expand node *n* generating a set *S*(*n*) of successors, i.e. all nodes adjacent to *n*. Add those nodes from *S*(*n*), which are not in CLOSED, to the **beginning** of OPEN (i.e. according to the depth-first principle). Formally, OPEN := (*S*(*n*)/CLOSED) ∪ OPEN. Delete duplicated nodes (i.e. the old ones) from the end of OPEN.
6. Go to step 2.

Table 11.2. The lists OPEN and CLOSED in each step of the algorithm for depth-first search of a path between *s* and *t* in Fig. 11.1. The path ⟨*s*,*a*,*e*,*t*⟩ of three edges is found

	OPEN	CLOSED	Comment
1	*s(start)*	∅	
2	*a(s), t(s), b(s)*	*s(start)*	$S(s) = \{a,t,b\}$.
3	*c(a), d(a), e(a), t(s), b(s)*	*s(start), a(s)*	$S(a) = \{c,d,e,s\}$. But $s \in$ CLOSED.
4	***d(c)***, *e(a), t(s), b(s)*	*s(start), a(s), c(a)*	$S(c) = \{d,a\}$. But $a \in$ CLOSED and ***d*** $\in$ OPEN. Therefore ***d(c)***, which is opened later, is put at the OPEN's head and *d(a)* is deleted from the OPEN's tail.
5	*e(a), t(s), b(s)*	*s(start), a(s), c(a), d(c)*	$S(d) = \{c,a\}$. But $c,a \in$ CLOSED
6	***b(e)***, ***t(e)***	*s(start), a(s), c(a), d(c), e(a)*	$S(e) = \{a,b,t\}$. But $a \in$ CLOSED and $b,t \in$ OPEN. Therefore ***b(e)*** and ***t(e)***, which are opened later, are put at the OPEN's head and *t(s)* and *b(s)* are deleted the from the OPEN's tail.
7	*t(e)*	*s(start), a(s), c(a), d(c), e(a), b(e)*	$S(b) = \{s,e\}$. But $s,e \in$ CLOSED.
8	∅	*s(start), a(s), c(a), d(c), e(a), b(e), t(e)*	The terminal node *t* is closed. The resulting path is collected backwards: *t* is reached from *e*; *e* is reached from *a*; and *a* is reached from *s*.

Solver and planner. The depth-first search strategy is usually used by solvers and the breadth-first search by planners. Solver is shorthand for problem-solver and a solving agent. Planner – a planning agent. Their differences:

	Solver	Planner
1	Does not have the map	Has the map
2	Normally does not find the shortest path	Can find the shortest path
3	Depth-first search. Breadth-first search is not possible	Normally breadth-first search. Depth-first search is also allowed
4	Onetime usage of the path	Multiple usage of the path

12. The prefix, infix and postfix order of tree traversal

Tree traversal refers to the problem of visiting all the nodes in a graph in a particular manner. Below we follow Jensen and Wirth (1982, Section 12.1). The tree in Fig. 12.1 is coded with the following string:

```
abc..de..fg...hi..jkl..m..n..
```

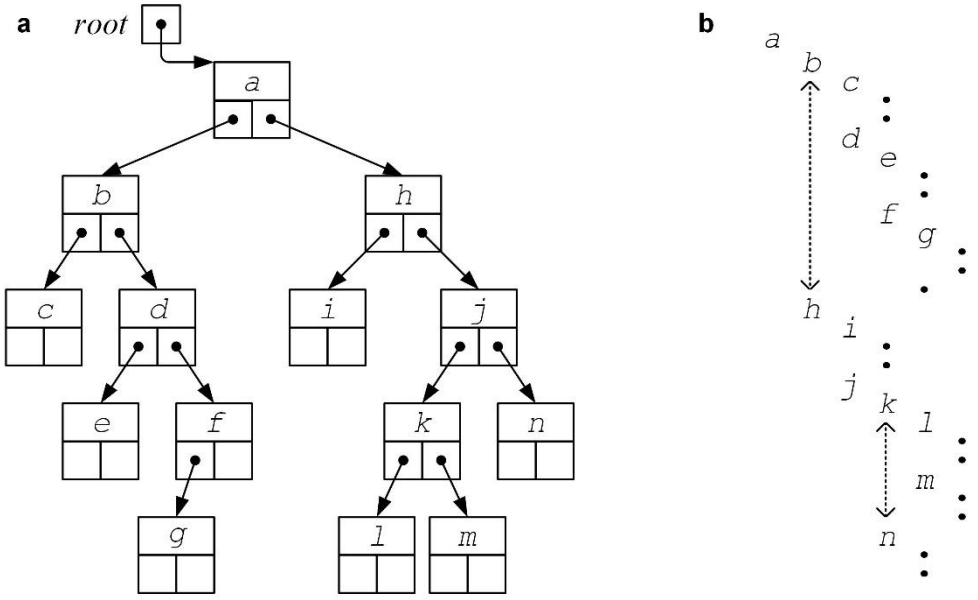


Fig. 12.1. a A sample binary tree from Jensen and Wirth (1982, p. 66), p. 66. It is encoded with the string `abc..de..fg...hi..jkl..m..n..` . **b** An isomorphic tree that is similar to a book's content map

Four lines below are printed by the program to traverse a binary tree in prefix-, infix- and postfix order:

```
abc..de..fg...hi..jkl..m..n..   – input data
abcdefghijklmn                  – preorder. Identical to depth-first search
cbedgfaihlkmjn                  – inorder
cegfdbilmknjha                  – postorder.
```

Four lines below are printed by the program for the input string `kl..m..`:

```
kl..m..   – input data
klm       – preorder. Identical to depth-first search
lkm       – inorder
lmk       – postorder.
```

The program is as follows.

```
program traversal(input, output);{Read the input string and form the tree.}
type ptr = ↑node;                {Data structure for a node.}
     node = record
              info : char;
              llink, rlink : ptr;
            end;
var root : ptr;                  {The root.}
    ch :   char;

procedure enter(var p:ptr);      {Read the input string and form the tree.}
begin
   read(ch);
   write(ch);
   if(ch ≠ '.') then
   begin
      new(p);
      p↑.info:=ch;
      enter(p↑.llink);
      enter(p↑.rlink);
   end;
   else p := nil;
end; {enter}

procedure preorder(p:ptr);       {Prefix order.}
begin
   if p ≠ nil then
   begin
      write(p↑.info);
      preorder(p↑.llink);
      preorder(p↑.rlink);
   end;
end; {preorder}

procedure inorder(p:ptr);        {Infix order.}
begin
   if p ≠ nil then
   begin
      inorder(p↑.llink);
      write(p↑.info);
      inorder(p↑.rlink);
   end;
end; {inorder}

procedure postorder(p:ptr);      {Postfix order.}
begin
   if p ≠ nil then
   begin
      postorder(p↑.llink);
      postorder(p↑.rlink);
      write(p↑.info);
   end;
end; {postorder}

begin                            {Main program.}
   write(' '); enter(root);     writeln;
   write(' '); preorder(root);  writeln;
   write(' '); inorder(root);   writeln;
   write(' '); postorder(root); writeln;
end.
```

Breadth-first search traversal, *abhcdijefknglm*, is not identical to any of three traversals.

Next, suppose the arithmetic expression

```
((A+B)*(C-D)+E)*F
```

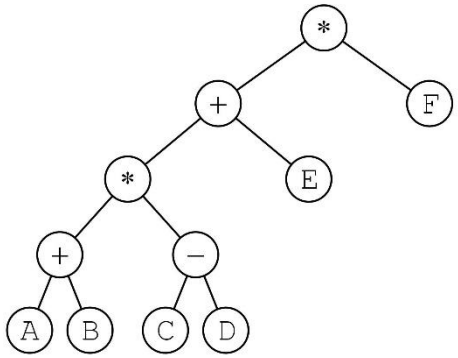
The representation in the prefix notation is

```
*+*+AB-CDEF
```

The representation in the postfix notation is

```
AB+CD-*E+F*
```

Compiling the expression into stack machine's code is illustrated below. The above expression `((A+B)*(C-D)+E)*F` is compiled into the code which is shown in Fig. 12.2.




```
Load A
Load B
Add
Load C
Load D
Subtract
Multiply
Load E
Add
Load F
Multiply
```

Fig. 12.2. The syntactic tree and the code of a stack machine for the arithmetic expression `((A+B)*(C-D)+E)*F`

The changes of the stack during the interpretation of the code are shown in Fig. 12.3.

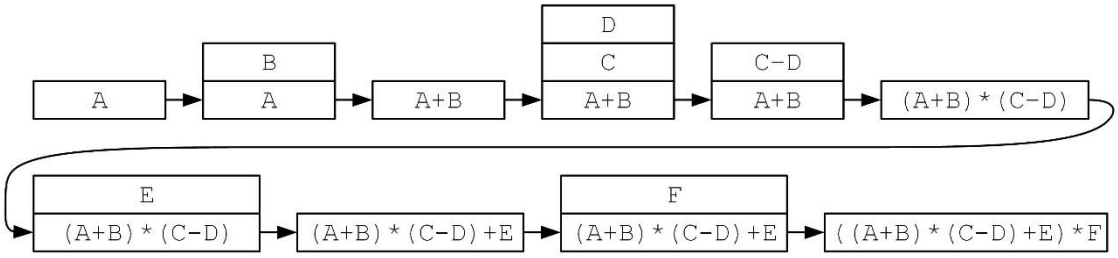


Fig. 12.3. State transitions of the stack while calculating the expression `((A+B)*(C-D)+E)*F`. Arrows denote state transitions from time *t* to *t*+1

Exercises

1. Write expressions `A*B+C/D`, `A*B+C-D` and `A+B*C-(D+E)*F-B` in prefix, infix and postfix notations. Write the code of a stack machine for the expressions above. Draw state transitions of the stack.

2. Write procedures for general trees: 1) enter a general tree, 2) prefix order traversal, and 3) postfix order traversal. For example, the string `ABC.DE...F.GHI.J.K.L` encodes a sample tree in Fig. 12.4. There are other languages to encode trees, for example, `A(B(CD(E))FG(H(IJKL)))`, where parentheses denote hierarchy.

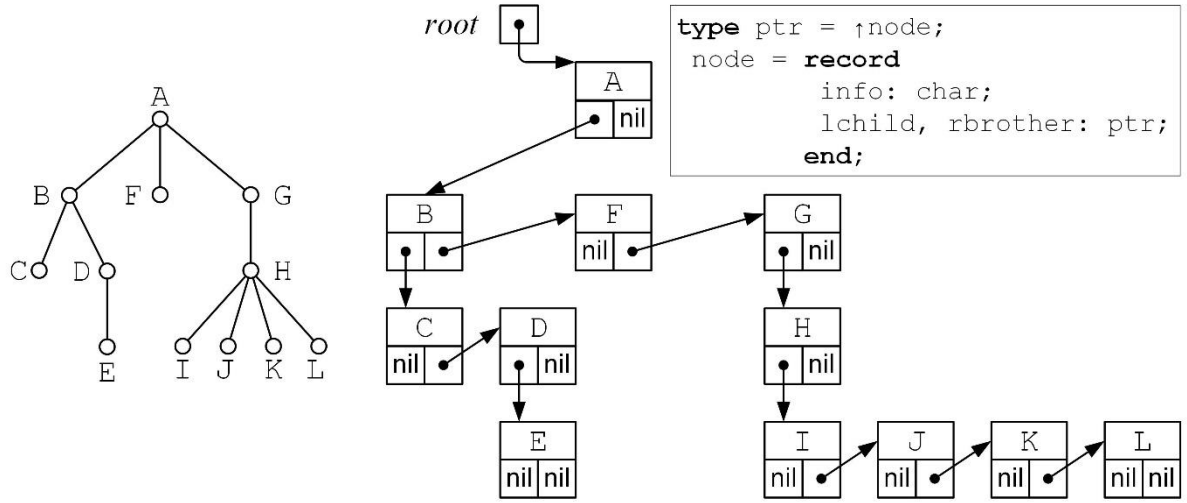


Fig. 12.4. A sample tree `ABC.DE...F.GHI.J.K.L` and its representation

Following is a pseudocode to enter a tree-coding string.

INPUT: *graph_string* – a tree-coding string.
OUTPUT: *Node* – a reference to the root.

```
function construct_tree(graph_string) : node
    current_node := nil
    for all character ∈ graph_string
        if character = dot_character
            new_node := node(character)
            set_parent(new_node, current_node)
            set_children(new_node, nil)
            if current_node ≠ nil
                add_child(current_node, new_node)
            end if
            current_node := new_node
        else
            current_node := get_parent(current_node)
        end if
    end for
    while get_parent(current_node) ≠ nil do
        current_node := get_parent(current_node)
    end while
    return current_node
end function
```

Next we show testing the above pseudocode with the tree which is shown in Fig. 12.5. The input of the test is `ABC.DE...F.GHI.J.K.L` . The output is shown in the table below.

```
PART 1. Data
    The tree is encoded with the string: "ABC.DE...F.GHI.J.K.L "

PART 2. Trace

     0)Read 'A'. Add child A to parent root. Down.
     1) A. Read 'B'. Add child B to parent A. Down.
     2) -B. Read 'C'. Add child C to parent B. Down.
     3) --C. Read '.'. Node C has no children. Backtrack.
        -B. Read 'D'. Add child D to parent B. Down.
     4) --D. Read 'E'. Add child E to parent D. Down.
     5) ---E. Read '.'. Node E has no children. Backtrack.
        --D. Read '.'. Node D has no children. Backtrack.
        -B. Read '.'. Node B has no children. Backtrack.
        A. Read 'F'. Add child F to parent A. Down.
     6) -F. Read '.'. Node F has no children. Backtrack.
        A. Read 'G'. Add child G to parent A. Down.
     7) -G. Read 'H'. Add child H to parent G. Down.
     8) --H. Read 'I'. Add child I to parent H. Down.
     9) ---I. Read '.'. Node I has no children. Backtrack.
        --H. Read 'J'. Add child J to parent H. Down.
    10) ---J. Read '.'. Node J has no children. Backtrack.
        --H. Read 'K'. Add child K to parent H. Down.
    11) ---K. Read '.'. Node K has no children. Backtrack.
        --H. Read 'L'. Add child L to parent H. Down.
    12) ---L. Read end of string symbol ' '.

PART 3. Results
    Tree:
     1) A
     2) -B
     3) --C
     4) --D
     5) ---E
     6) -F
     7) -G
     8) --H
     9) ---I
    10) ---J
    11) ---K
    12) ---L
```

A grammar in Backus-Naur form for our language to encode a tree

<tree> ::= <node> | <node> <branches>
<branches> ::= <branch> | <branches> <branch>
<branch> ::= <leaf> | <branching>
<leaf> ::= <node> <dot>
<branching> ::= <node> <branches> <dot>
<node> ::= ‘A’.. ‘Z’
<dot> ::= ‘.’

Contextual rule. The grammar above allows strings with dots at the end. These dots provide no meaning about the tree’s structure. Therefore these dots will be deleted.

The binary tree in Fig. 12.1 is coded *abc.de.fg....hi.jkl.m..n* . The latter string differs from *abc..de..fg...hi..jkl..m..n..* in (Jensen and Wirth 1982, p. 66).

13. A general graph-searching algorithm GRAPHSEARCH

GRAPHSEARCH is a general graph-searching algorithm that permits any kind of ordering OPEN the user might prefer – heuristic or uninformed (breadth-first or depth-first) (see Nilsson 1998, pp. 141–142). This algorithm can be used to perform breadth-first search, depth-first search, or heuristic search.

INPUT: 1) a graph with nonnegative edge costs; otherwise the cost is 1; 2) a start node *s*; 3) a terminal node *t*. (Several terminal nodes can be considered. They can be defined, for example, with a predicate.)

OUTPUT: a path between *s* and *t*.

Initially the lists OPEN and CLOSED are empty.

1. Put the start node *s* on OPEN and create a search tree *T*, consisting solely of *s*.
2. If OPEN is empty, return FAIL. There is no path.
3. Close the first node *n* from OPEN: remove it from OPEN, and put it on CLOSED.
4. If *n* is a terminal node, exit successfully with the solution obtained by tracing a path backward along the arcs in *T* from *n* to *s*.
5. Expand node *n* generating a set $S(n)$ of successors, i.e. all nodes adjacent to *n*. Add each $n^* \in S(n)$, which is not in CLOSED (i.e. n^* is not already a parent of *n* in *T*), to OPEN and *T*. Link n^* with *n*, $n^*(n)$. Formally, $\text{OPEN}_{\text{new}} := \text{OPEN}_{\text{old}} \cup (S(n)/\text{CLOSED})$.
6. Delete duplicated nodes in OPEN_{new}. In other words, for each $n^{**} \in S(n)$, which was in OPEN_{old}, i.e. $n^{**} \in (S(n) \cap \text{OPEN}_{\text{old}})$, decide to change the old link or not to change it. Decision is needed because of two links – the old link and a new one. Do not analyse the rest n^{***}, i.e. $n^{***} \in (S(n) \cap \text{CLOSED})$, because they are closed forever (as parents).
7. Reorder OPEN, either according to some arbitrary scheme or according to heuristic merit, for instance:
   a) cost;
   b) **"breadth-first"**. Put $S(n)$/CLOSED at the **end** of OPEN an delete duplicated nodes n^{**} from the beginning of OPEN. Formally, $\text{OPEN}_{\text{new}} := \text{OPEN}_{\text{old}} \cup (S(n)/\text{CLOSED})$. Hence, OPEN is ordered as a **queue** (first in, first out, or FIFO);
   c) **"depth-first"**. Put $S(n)$/CLOSED at the **beginning** of OPEN and delete duplicated nodes n^{**} from the end of OPEN. Formally, $\text{OPEN}_{\text{new}} := (S(n)/\text{CLOSED}) \cup \text{OPEN}_{\text{old}}$. (The beginning is understood to be to the left; the leftest node will be closed.) Hence, OPEN is ordered as a **stack** (last in, first out or LIFO).
8. Go to step 2.

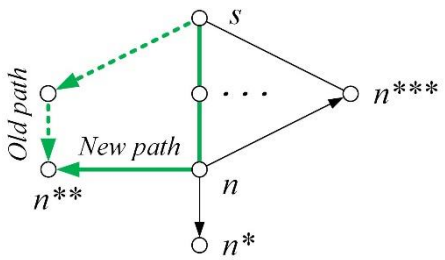


Fig. 13.1. Closing a node *n*

In breadth-first search, new nodes are simply put at the end of OPEN, and the nodes are not reordered. In depth-first-style search, new nodes are put at the beginning of OPEN. In heuristic search, OPEN is reordered according to the heuristic merit of the nodes.

14. Differences between BACKTRACK1 and GRAPHSEARCH-DEPTH-FIRST

BACKTRACK1 operates with DATALIST that plays the role of stack. GRAPHSEARCH-DEPTH-FIRST operates with two lists, OPEN and CLOSED. Two differences:

1. BACKTRACK1 operates with production rules and global database states, whereas GRAPHSEARCH-DEPTH-FIRST operates with OPEN and CLOSED.
2. BACKTRACK1 cycles twice around an island. Therefore multiple cycles appear in case of multiple islands. The reason is that DATALIST keeps only the current path, but no past paths. GRAPHSEARCH-DEPTH-FIRST prevents multiple cycles by saving nodes in CLOSED.

Therefore BACKTRACK1 normally takes more time, but less stack memory. GRAPHSEARCH-DEPTH-FIRST takes less time, but more memory (OPEN and CLOSED). Hence a trade-off of time and space is observed. This trade-off constitutes a principle of AI.

14.1. Searching in a graph

Consider the graph in Fig. 14.1 which is explored in Fig. 11.1 and path search between *s* and *t*.

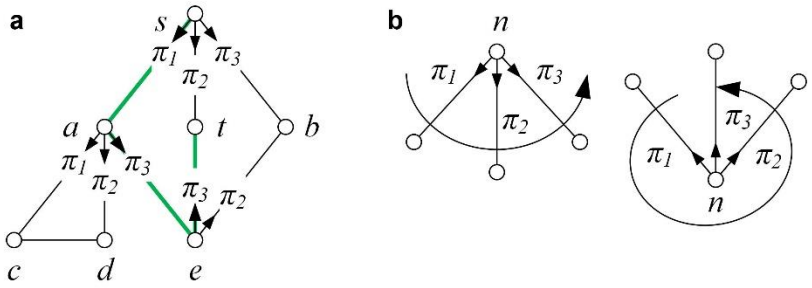


Fig. 14.1. a A graph for path search between *s* and *t*. **b** Contra-clockwise order of edges. "12 hours" edge is the last one

The search tree of GRAPHSEARCH-DEPTH-FIRST (Fig. 14.2 b) is smaller than that of BACKTRACK1 (Fig. 14.2 a) because of cut branches, most importantly *d(a)* which is shown in the dashed rounded rectangle.

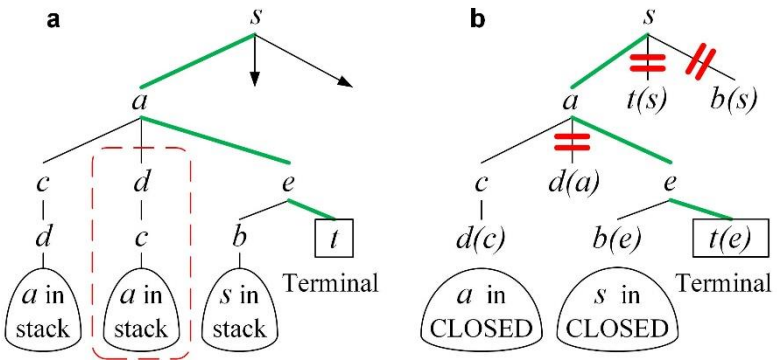


Fig. 14.2. Search trees. **a** BACKTRACK1. **b** GRAPHSEARCH-DEPTH-FIRST. The same path ⟨*s*,*a*,*e*,*t*⟩ is found

GRAPHSEARCH-DEPTH-FIRST produces OPEN and CLOSED which are shown below in the frontier notation. Strikethrough denotes a CLOSED node and a rectangle denotes a node which changed its parent link.

~~*s(start)*~~ {~~*a(s)*~~ *t(s)* *b(s)*} {~~*c(a)*~~ *d(a)* ~~*e(a)*~~} ~~***d(c)***~~ {~~***b(e)***~~ ~~***t(e)***~~}

The table shows three cuts of branches:

Situation	Opened earlier	Opened later. This is **chosen**
1	*d(a)*	***d(c)***
2	*b(s)*	***b(e)***
3	*t(s)*	***t(e)***

Both algorithms find the same path. However BACKTRACK1 takes more time. The search tree consists of 11 edges: 11 loads into the stack and 3 times to check if the loaded node is already present in the stack. GRAPHSEARCH-DEPTH-FIRST takes more memory: the search tree (and the frontier) consists of 10 nodes (including 3 duplicated names), whereas the stack in BACKTRACK1 takes depth 5.

14.2. Multiple cycles in a labyrinth

Suppose a more complicated example. A labyrinth is shown in Fig. 14.3 a. The starting node is *s*. There are two terminal nodes – exits *y* and *t*.

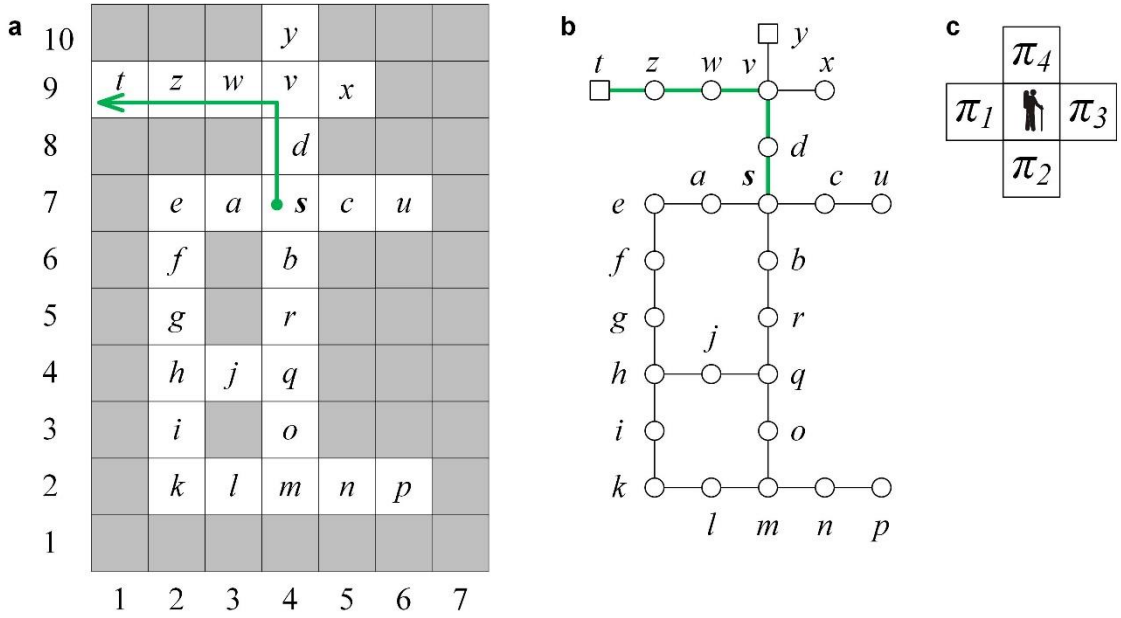


Fig. 14.3. a A labyrinth with the starting node is *s* and two exits *y* and *t*. **b** The representation in graph notation. **c** The order of edges

Both algorithms find the same path, but the search tree of GRAPHSEARCH-DEPTH-FIRST is smaller because of two cut branches; see Figures 14.4 and 14.5.

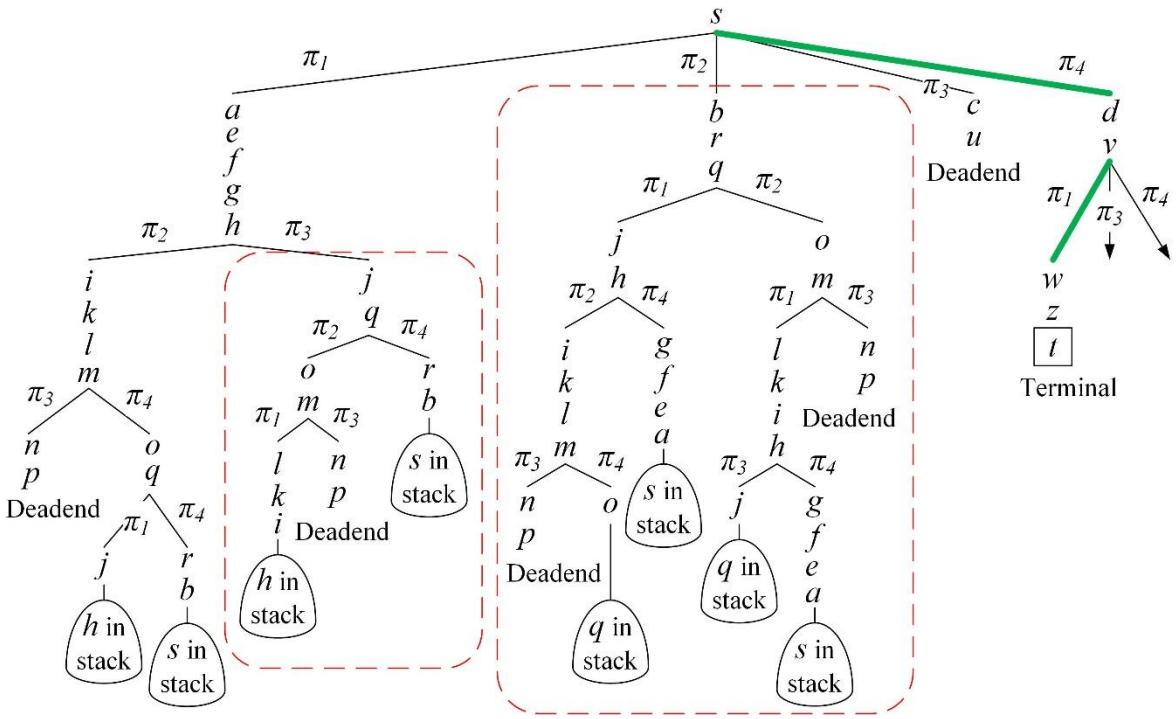


Fig. 14.4. The search tree of BACKTRACK1. It is implicit. The path is ⟨*s*,*d*,*v*,*w*,*z*,*t*⟩. Dashed rounded rectangles show two branches that are cut in GRAPHSEARCH-DEPTH-FIRST

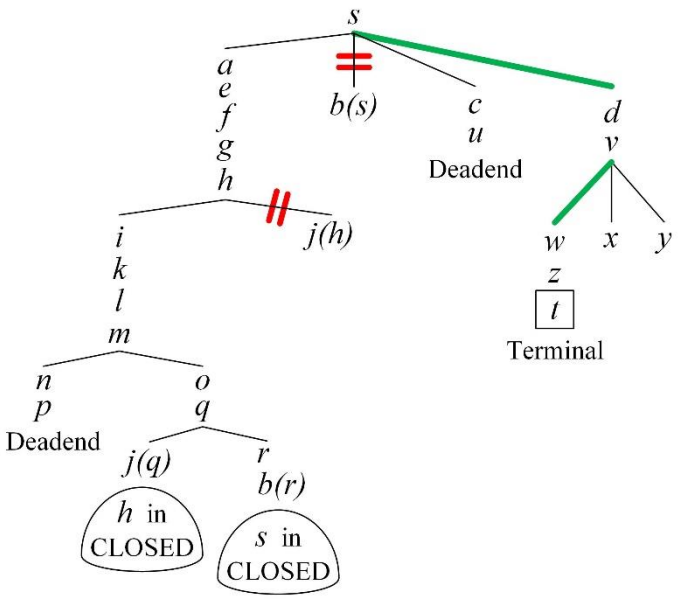


Fig. 14.5. The search tree of GRAPHSEARCH-DEPTH-FIRST. It is explicit due to OPEN and CLOSED. The path is ⟨*s*,*d*,v,*w*,*z*,*t*⟩

Following is the front notation for search:

~~*s(start)*~~ {~~*a(s)*~~ *b(s)* ~~*c(s)*~~ ~~*d(s)*~~} ~~*e(a)*~~ ~~*f(e)*~~ ~~*g(f)*~~ ~~*h(g)*~~ {~~*i(h)*~~ *j(h)*} ~~*k(i)*~~ ~~*l(k)*~~ ~~*m(l)*~~ {~~*n(m)*~~ ~~*o(m)*~~} *p(n)* *q(o)* {***j(q)*** ~~*r(q)*~~} ***b(r)*** ~~*u(c)*~~ ~~*v(d)*~~ {~~*w(v)*~~ *x(v)* *y(v)*} ~~*z(w)*~~ ~~*t(z)*~~

Following is the table of branches that were cut:

Situation	Opened earlier	Opened later. This is **chosen**
1	*j(h)*	***j(q)***
2	*b(s)*	***b(r)***

14.3. A counterexample of better performance with BACKTRACK1

Suppose a labyrinth in Fig. 14.6.

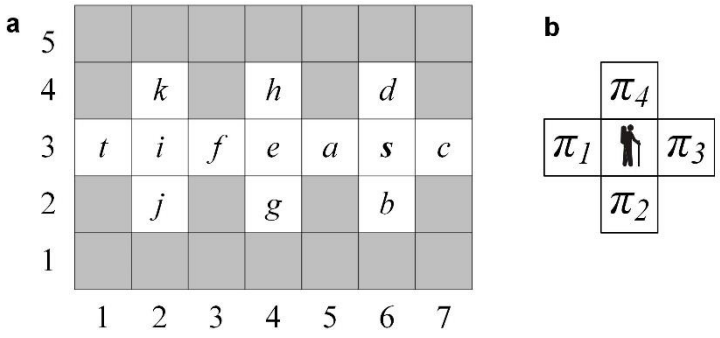


Fig. 14.6. a A labyrinth for path search from *s* to any exit. There are two exits: *t* and *c*. **b** The order of four production rules

BACKTRACK1 finds an exit straightforwardly to the left with no backtracks: ⟨*s*,*a*,*e*,*f*,*i*,*t*⟩. In order to compare two algorithms, suppose a cost criterion. Suppose the following costs in BACKTRACK1:

- 1 – the cost to put a production in the RULES list with the APPRULES function; see Section 2. Metaphorically, this is the cost to take a look at an applicable production rule;
- 100 – to put a node in the stack. This is the cost of a thread cell.

Suppose the following costs in GRAPHSEARCH-DEPTH-FIRST:

- 50 – to open a node. In other words, this is the cost to put a node in OPEN. This is the cost of a space cell.

Both algorithms find the same path. However, BACKTRACK1 costs less; see Fig. 14.7.

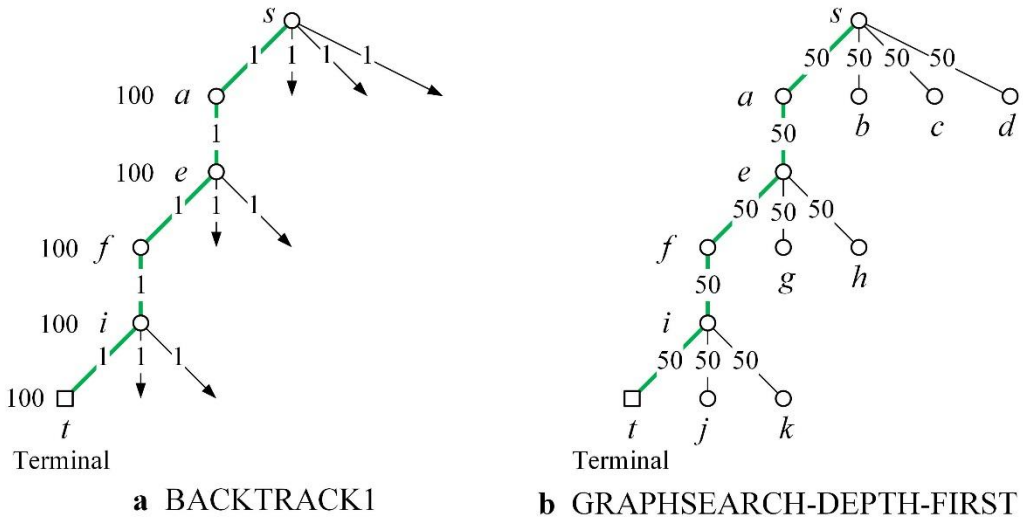


Fig. 14.7. a BACKTRACK1 search tree costs 5·100 + 12·1 = 512. **b** GRAPHSEARCH-DEPTH-FIRST costs more: 12·50 = 600

Exerxises

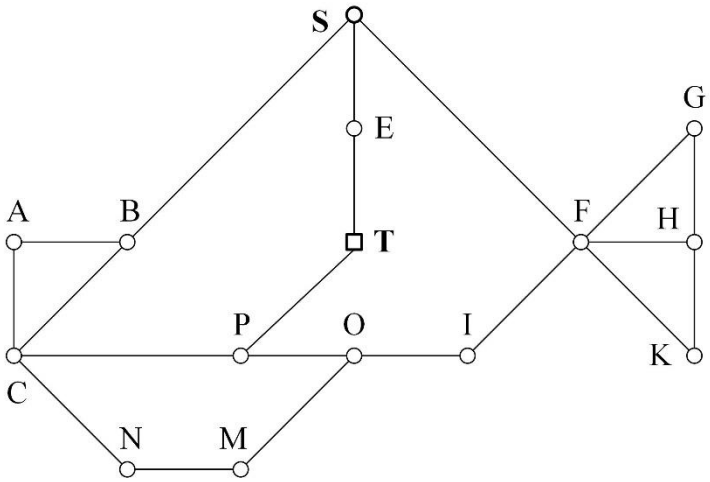
1. Which algorithm – GRAPHSEARCH-DEPTH-FIRST or BACKTRACK1 – performs better according to time and space in this labyrinth? The starting node is *s*. Draw the search tree with the costs: lookup – 1, thread – 100, and one memory cell in OPEN – 50.

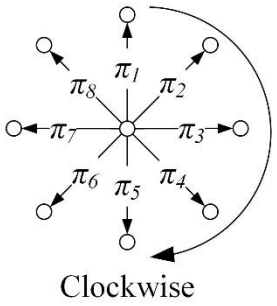
7		U					Z		
6		T	R	P			Q		
5				O		K	L	M	N
4		A	**S**	B		J			
3		C		D		I			
2		D	E	F	G	H			
1						V			
	1	2	3	4	5	6	7	8	9

2. List the path in terms of nodes between S and T and draw two search trees with the procedures:

- BACKTRACK1
- GRAPHSEARCH-DEPTH-FIRST. Write also: a) the front notation with CLOSED and OPEN nodes and b) the table of cuts.

Edges are ordered clockwise.





15. Hill-climbing strategy

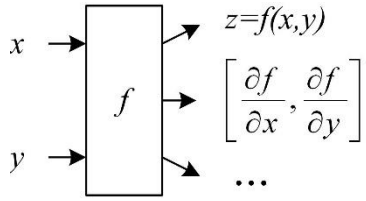


Fig. 15.1 A black-box that produces local information about a function *f* in a point *(x,y)*

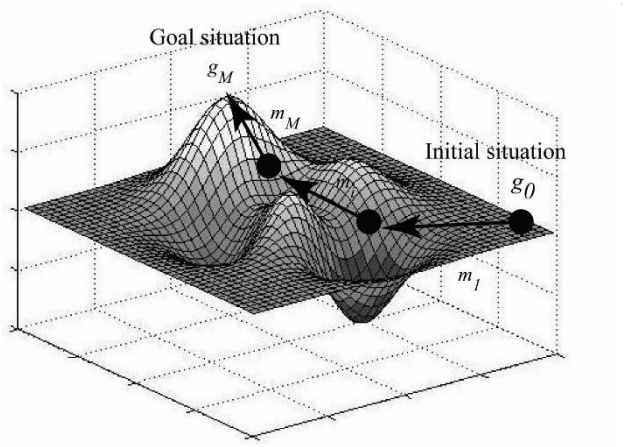


Fig. 15.2. Suppose maximisation of a function *z=f(x,y)*. Hill-climbing is modelled with a sequence ⟨[x_0,y_0], [x_1,y_1], [x_2,y_2], …⟩ in the direction of steepest gradient. The agent is blind – no ability to view the whole landscape

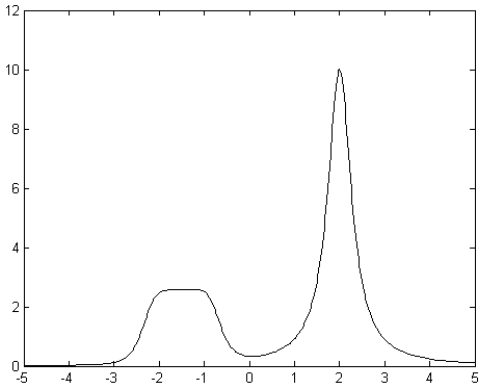


Fig. 15.3. Hill-climbing in 2-dimensional space is modelled with a sequence ⟨x_0, x_1, x_2, …⟩ in the direction of positive derivative $f'(x_i)$

Two control strategies are distinguished in artificial intelligence: irrevocable and tentative. Hill-climbing is an example of the irrevocable strategy. Dangers: local maximum (minimum) and plateau.

Hill climbing is demonstrated with the 8-puzzle (see Nilsson 1982, ch. 2.4.1; 1998, pp. 140–141). Here the evaluation function is *f(n) = –w(n)*, where *w(n)* is the number of misplaced tiles (comparing with the terminal state) in that database associated with node *n*.

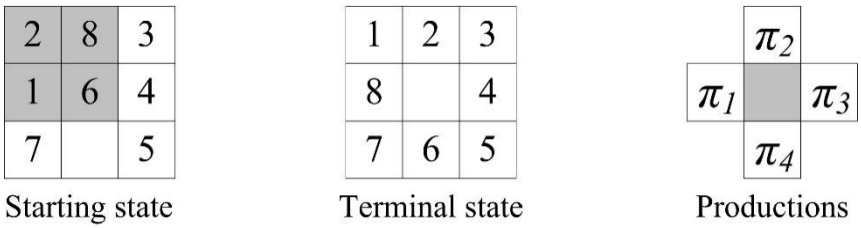


Fig. 15.4. a) A sample initial state *s* and the terminal state *t* in the 8-puzzle. *f(s) = –4*, *f(t) = 0*. The task is to maximize *f*. b) Four moves – a production set ⟨π_1, π_2, π_3, π_4⟩

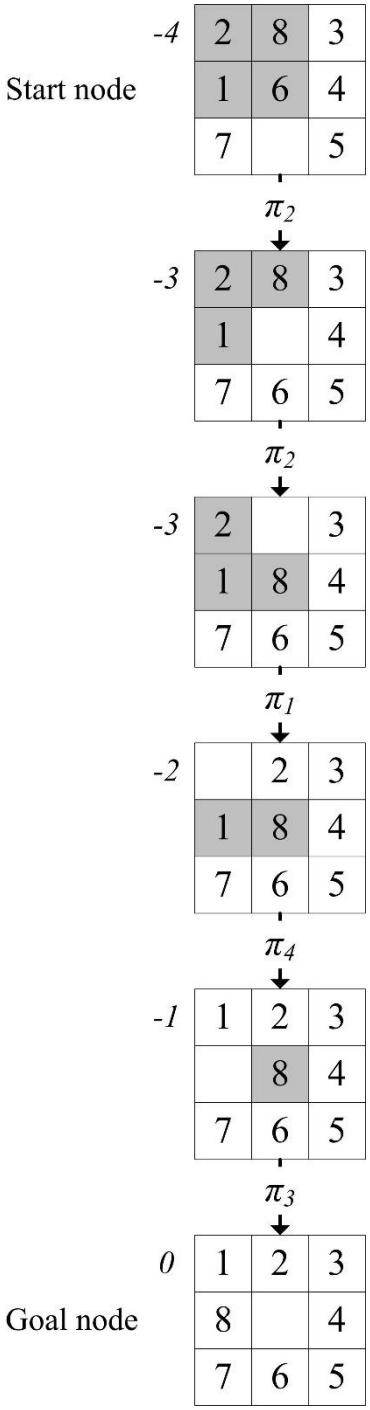


Fig. 15.5. Maximising $f(n) = -w(n)$, where $w(n)$ counts the number of misplaced tiles comparing with the goal node

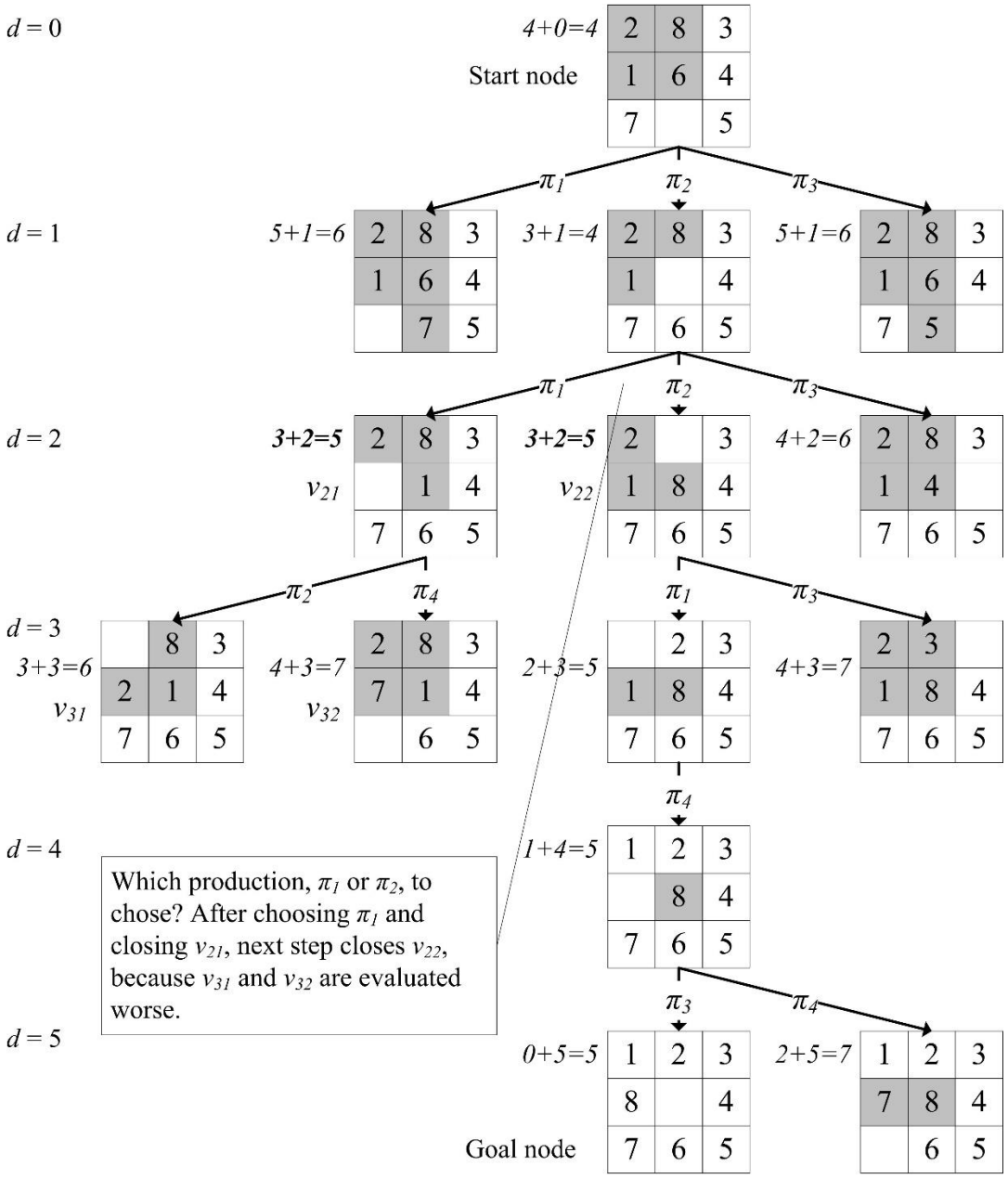


Fig. 15.6. Minimisation of $f(n) = w(n)+d(n)$, where $w(n)$ is the number of pieces in wrong places and $d(n)$ is the depth of the node

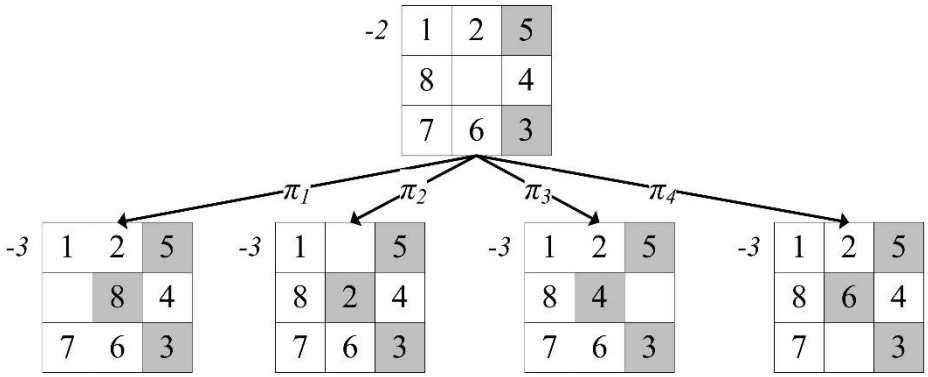


Fig. 15.7. The start state is in a local maximum. Each production makes worse

16. Manhattan distance

Suppose a big graph and breadth-firs search from a start node *s* to a terminal node *t*. The wave can be spread out from *s* to *t*, or from *t* to *s*, or two parallel waves (Fig. 14.1). Two parallel waves require considerably less time and memory.

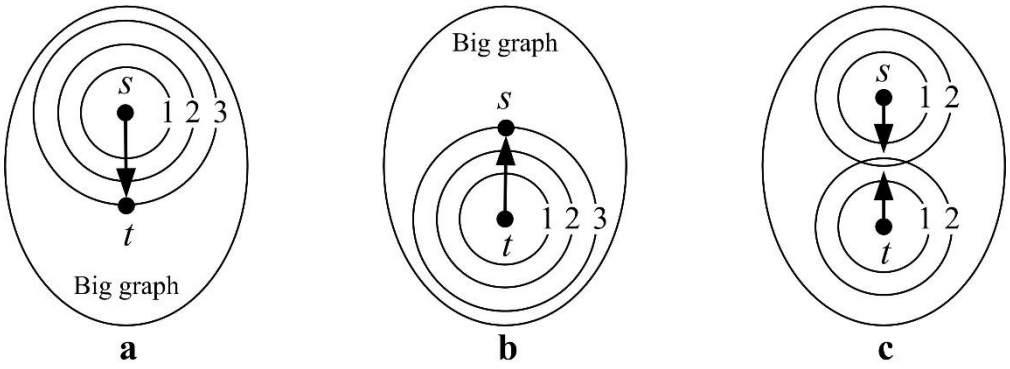


Fig. 16.1. Breadth-first search in a big graph: a) from *s* to *t*; b) from *t* to *s*; c) two parallel waves – from *s* and *t*

Proof. Let us denote *N* the length of the path. The number of nodes in the frontier is $O(N^{\alpha})$, where generally $\alpha>2$. This number is greater than in two parallel breadth-first searches: $O(N^{\alpha}) > O((N/2)^{\alpha}) + O((N/2)^{\alpha})$. For example, in 2-dimensional space, $\alpha = 2$, and two parallel frontiers take 2 times less memory: $N^2 > (N/2)^2 + (N/2)^2 = N^2/4 + N^2/4 = N^2/2$.
In 3-dimensional space, $\alpha = 3$, two parallel frontiers take 3 times less memory: $N^3 > (N/2)^3 + (N/2)^3 = N^3/8 + N^3/8 = N^3/4$.

The frontier in Fig. 16.2 (total 145 nodes) is listed in Table 16.3 a. Any path comprises 5 productions "go East" and 3 productions "go South". Therefore the search takes 8 waves.

Wave no.	Number of nodes
0	1 + 0
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
Total	145

a

Wave no.	Number of nodes
0	1 + 1 + 0 + 0
1	4 + 4
2	8 + 8
3	12 + 12
4	16 + 16
Total	82

b

Table 16.3. The number of opened nodes: a) total 145 nodes in Fig. 16.2, and b) total 82 nodes in Fig. 16.4.

The *N*-th wave captures total $2\cdot (N^2 + N) + 1$ nodes including the start node *s*. Therefore the complexity of breadth-first search in a Manhattan graph is quadratic, $O(N^2)$:

$$4 + 8 + \ldots + 4\cdot N = 4\cdot (1 + 2 + 3 + \ldots + N) = 4\cdot (N\cdot (N + 1)/2) = 2\cdot (N^2 + N) = O(N^2)$$

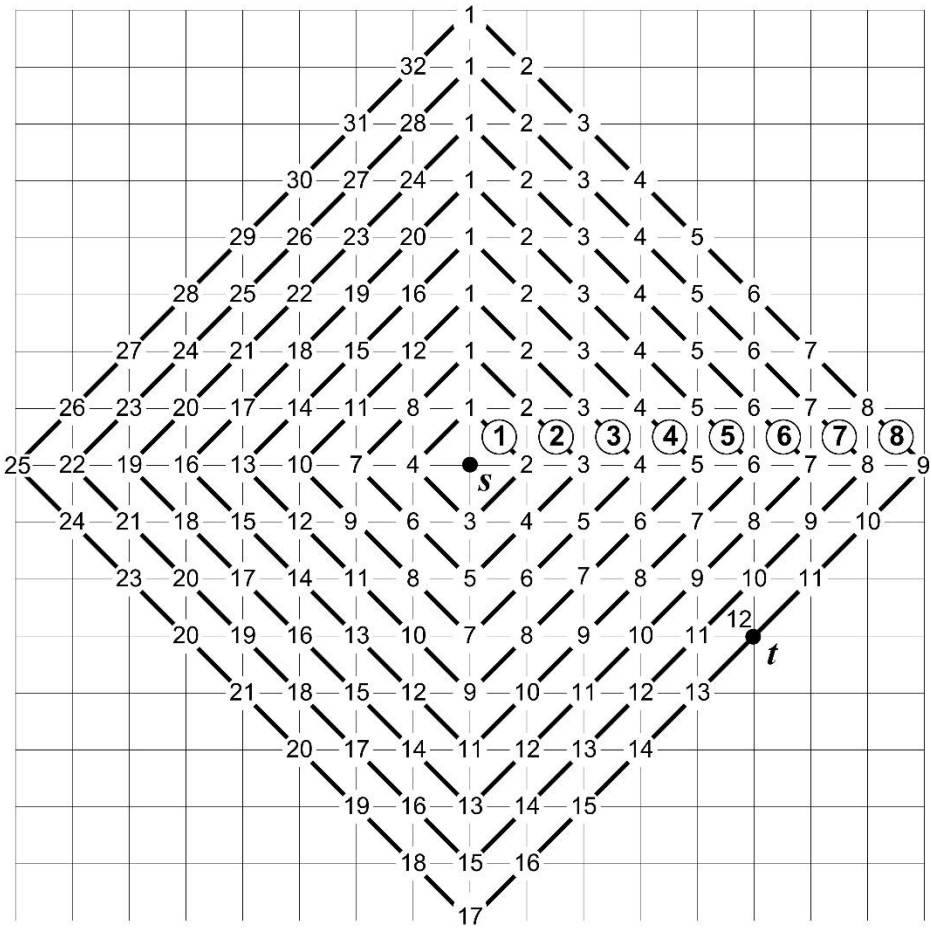


Fig. 16.2. From *s* to *t* requires 8 waves: 5 steps “go East” and 3 “go South”

Now suppose two parallel waves; see Fig 16.4. Total 82 nodes; see Table 16.3 b.

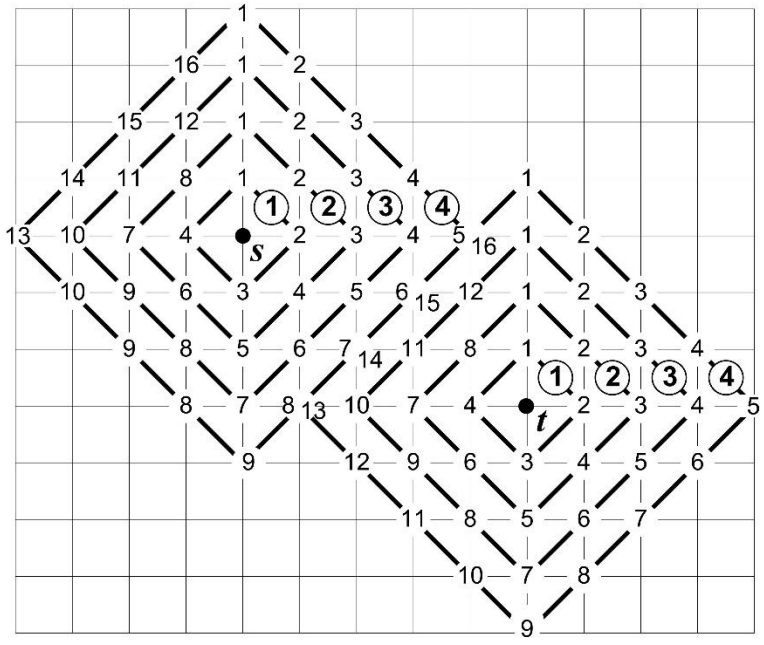


Fig. 16.4. Two parallel waves: from *s* to *t* and from *t* to *s*

17. A* search algorithm

A* (say A star) is the GRAPHSEARCH algorithm with the following evaluation function *f* of the current node *n*:

$$f(n) = g(n)+h(n) = distFactual(s,n) + distEstimate(n,t)$$

where

- *g(n)* is the shortest path from the starting node *s* to *n*, *distFactual(s,n)*,
- *h(n)* is an admissible heuristic estimate of the distance from *n* to the goal *t*, *distEstimate(n,t)*.

Details see in (Hart, Nilsson, Raphael 1968; Nilsson 1982), numerous textbooks e.g. (Russell, Norvig 2003; Luger 2005) and Wikipedia, https://en.wikipedia.org/wiki/A*_search_algorithm. The algorithm employs the lists OPEN and CLOSED.

17.1. Manhattan distance in the tile world

Further we present an example where the Manhattan distance stands for *h(n)*.

DEFINITION (Manhattan distance) Manhattan norm $d(n_1,n_2)$, i.e. distance between two points $n_1=(x_1,y_1)$ and $n_2=(x_2,y_2)$ is:

$$d(n_1,n_2)=|x_2-x_1|+|y_2-y_1|$$

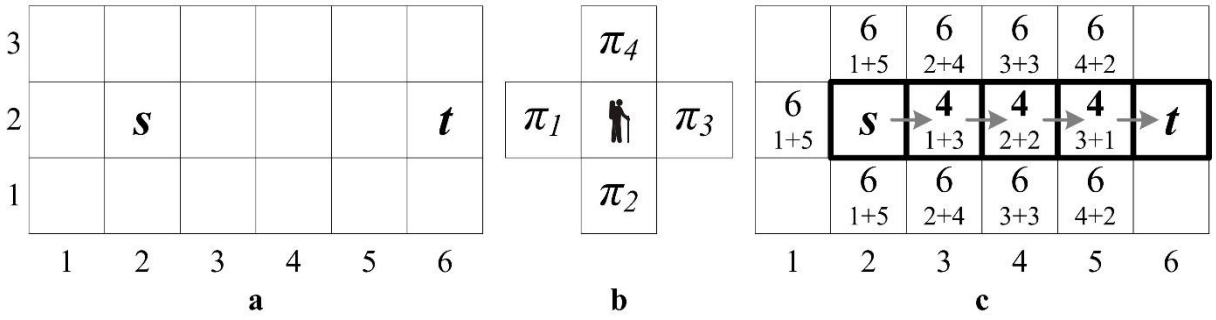


Fig. 17.1. a) A labyrinth with the start node *s* and the terminal node *t*. b) Four moves. c) Closed and opened nodes (frontier) and the path

A more complicated example is shown in Fig. 17.2.

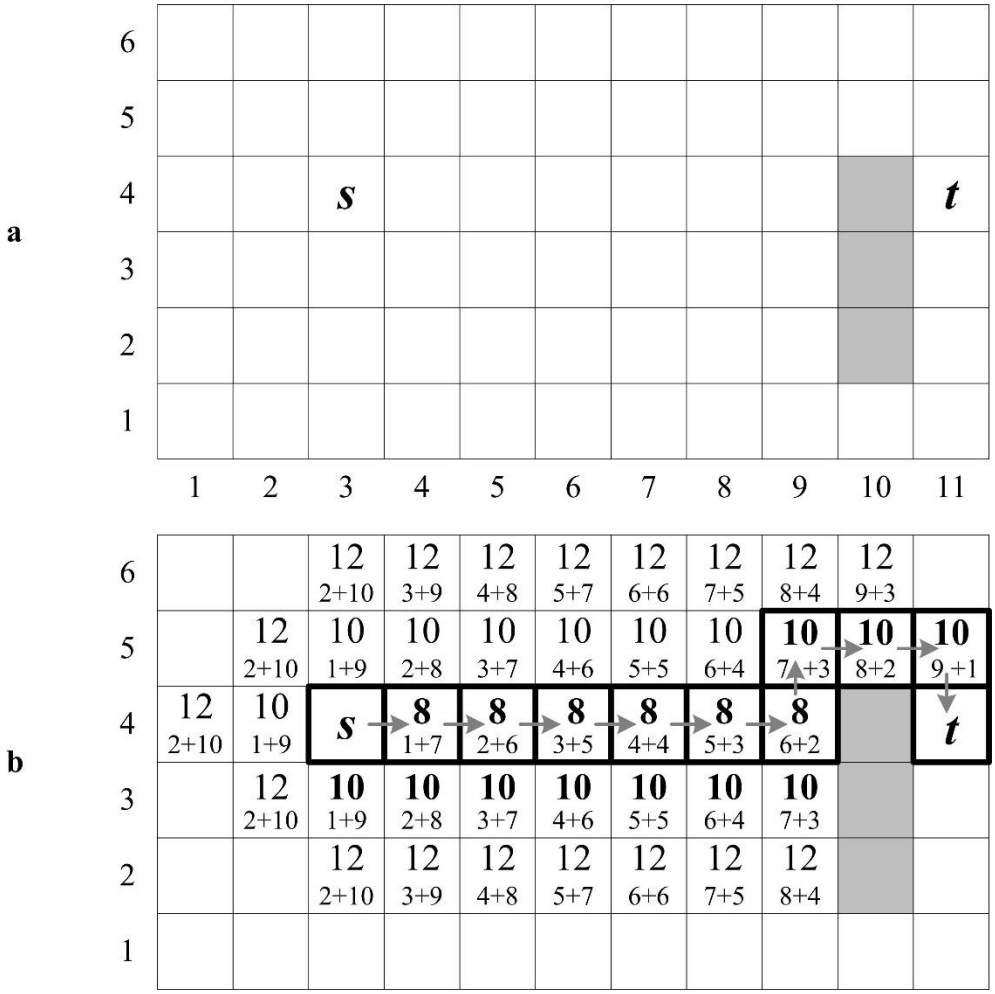


Fig. 17.2. a) The agent travels from *s* to *t*. Grey tiles denote obstacles. b) The path and the frontier

A more complicated example contains an obstacle of 5 tiles (Fig. 17.3).

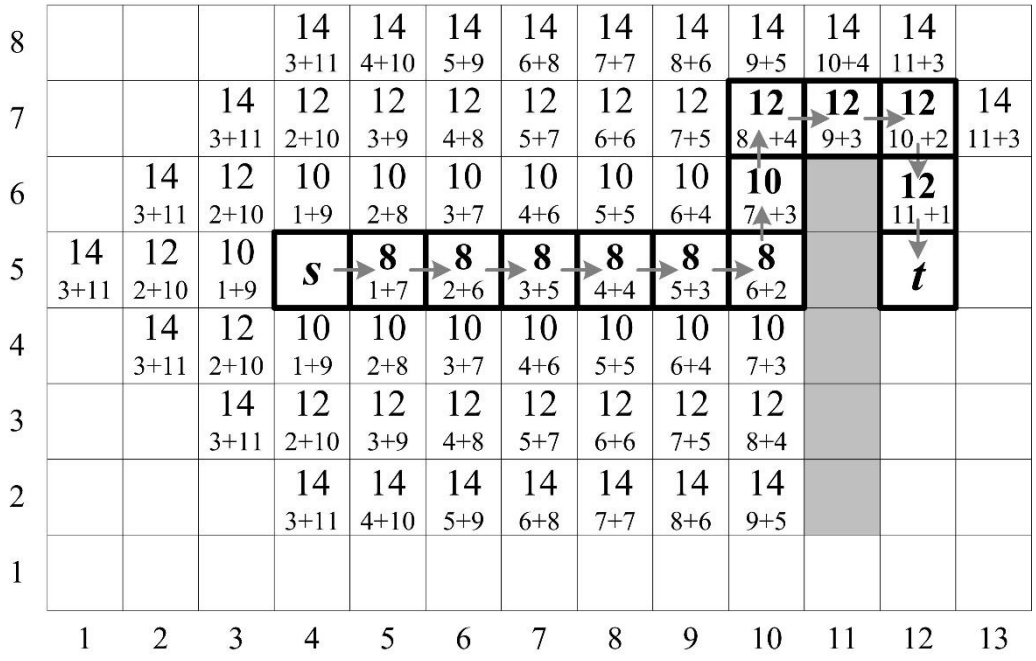


Fig. 17.3. The path from *s* to *t* and the frontier

The frontier is ellipse-shaped. This is more effective because the ellipse-shaped area contains fewer nodes than a circle-shaped one.

Next is an example with an L-shaped obstacle (Fig. 17.4).

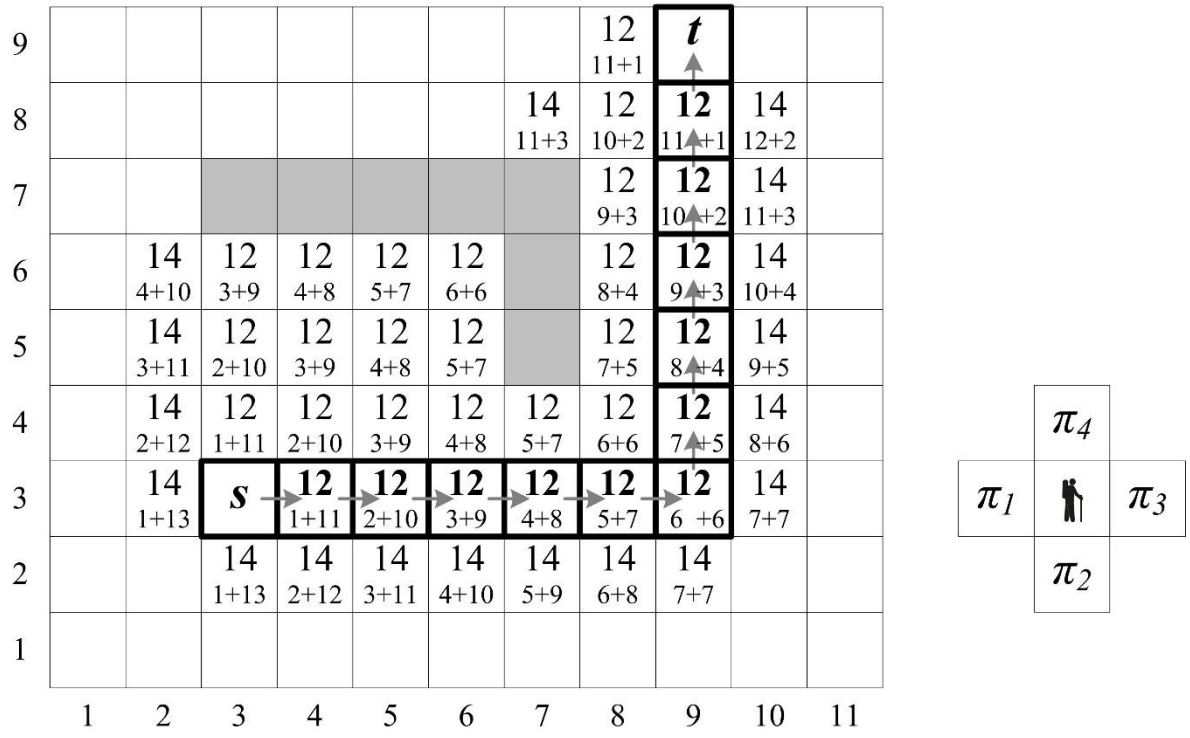


Fig. 17.4 A labyrinth with an L-shaped obstacle and a path from *s* to *t*

Next is an example with the same labyrinth, but 8 moves, in other words, the agent can move diagonally (Fig. 17.5). See also an animation in Wikipedia, https://en.wikipedia.org/wiki/A*_search_algorithm.

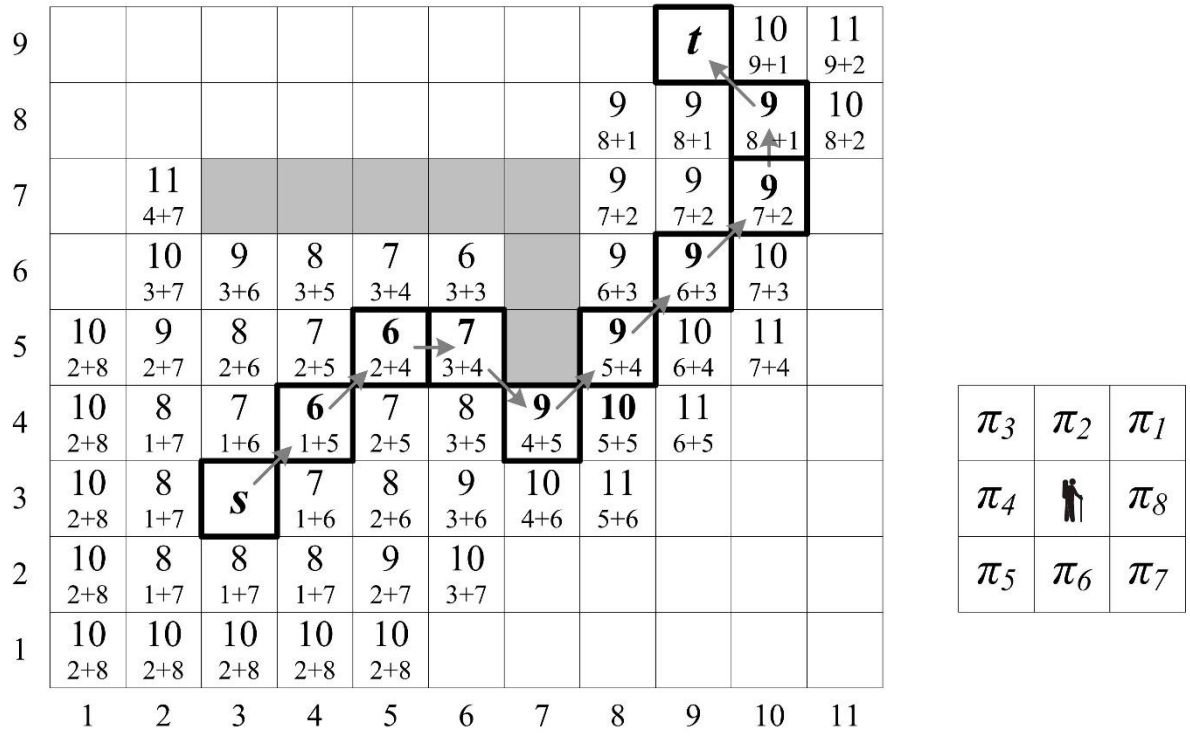


Fig. 17.5 A labyrinth with an L-shaped obstacle, but 8 moves. The path from *s* to *t* differs from the path in Fig. 17.4

The algorithm A* is optimal when the heuristic function *h(n)* is optimistic; see (Russell, Norvig 2003). A heuristic function is called *optimistic* in case the evaluation is smaller than an actual path.

17.2. An example from Russell & Norvig 2003

Further A* explanation follows (Russell, Norvig 2003, Ch. 4, p. 94–98). A path is searched from A to B in the graph which is shown in Fig. 17.6.

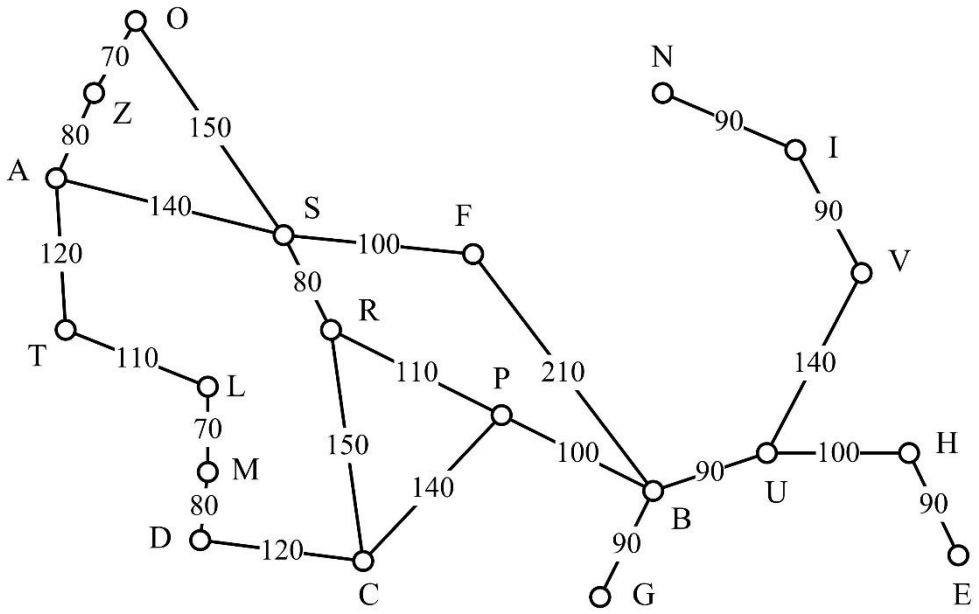


Fig. 17.6. The graph shows transport routes in Romania and is adapted from (Russell, Norvig 2003, p. 63). Path searching from A to B

Heuristic function *h(n)* is listed in Table 17.7. This heuristic evaluation presents straight distance from *n* to B.

City	*h(n)*
A	370
B	0
C	160
D	240
E	160
F	180
G	80
H	150
I	230
L	240

City	*h(n)*
M	240
N	230
O	380
P	100
R	190
S	250
T	330
U	80
V	200
Z	370

Table 17.7. Heuristic function *h(n)*. This heuristic evaluation presents straight distance from *n* to B

Search trees in each step of A* are shown in Fig. 17.8.

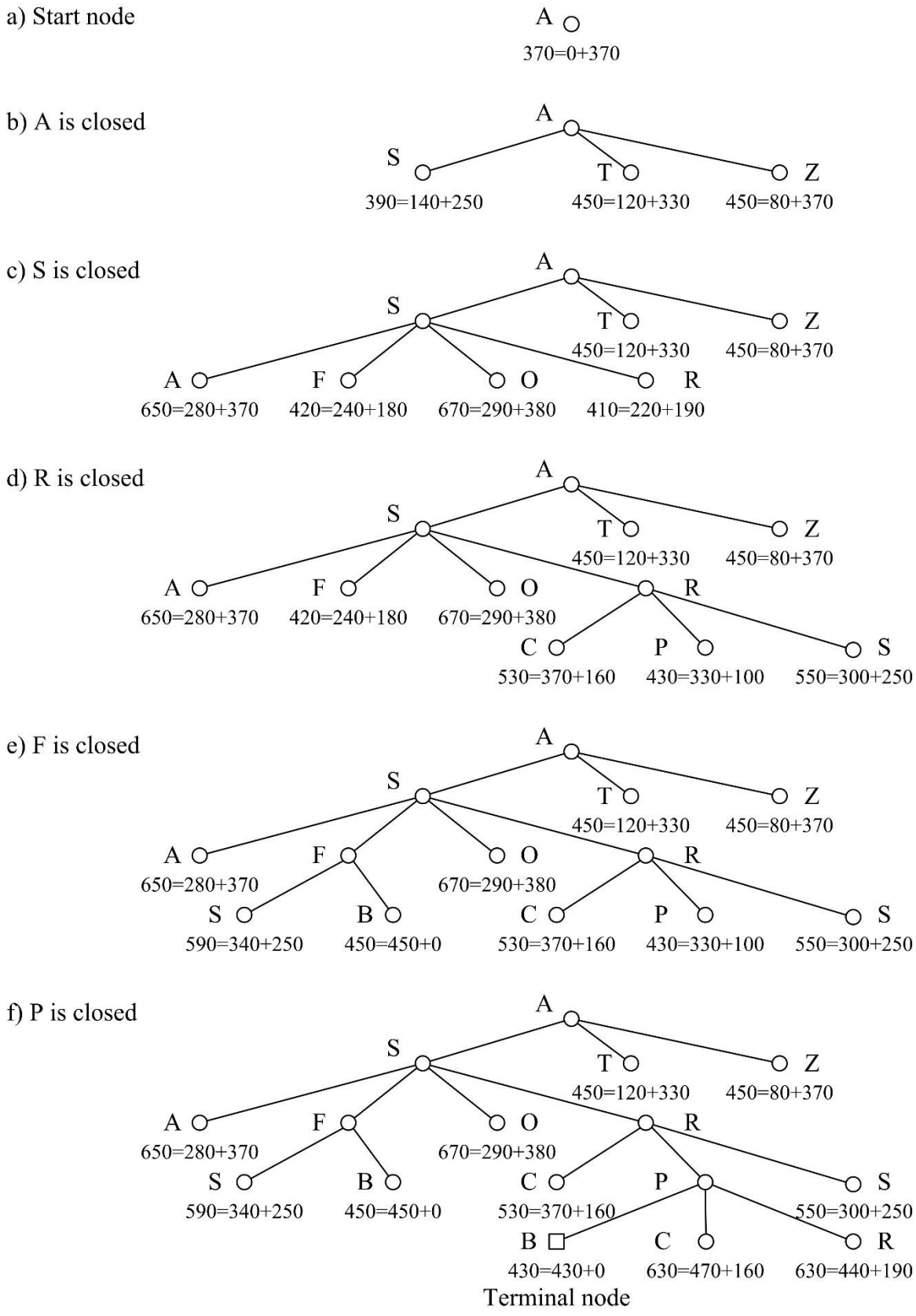


Fig. 17.8. Search trees in each step of A*. Nodes mark *f(n)=g(n)+h(n),* where *h(n)* presents a straight distance from *n* to B

18. Forward chaining and backward chaining

Suppose a sample rule system with propositional variables {A,B,C,D,F,Z}:

$$\begin{aligned} \pi_1&: \quad F, B \to Z \\ \pi_2&: \quad C, D \to F \\ \pi_3&: \quad A \quad \to D \end{aligned} \tag{18.1}$$

Each rule is of the form rule_name: antecedent → consequent. The consequent (right hand side) consists of one propositional variable and the antecedent (left hand side) of one or more. In the case the consequent would consist of several propositional variables, the rule can be replaced by several rules. For example, π_j: A, B, C → J, K is replaced with two rules: π_{j1}: A, B, C → J and π_{j1}: A, B, C → K.

The global database (GDB) consists of facts, for example, {A,B,C}. Facts are represented as propositional variables, i.e. true or false. A goal is a new fact, for example, Z.

The problem is to find a sequence of rules which produces the goal from the facts. Two algorithms – forward chaining and backward chaining – are discussed further.

Both algorithms are implemented in Web services; see the server at http://juliuschainingexample.appspot.com/. A client requires to copy-paste the JSON format result, which is produced by the server.

18.1. Forward chaining

It starts from facts and proceeds to the goal.

Suppose a rule system (18.1). Suppose an initial state of the GDB which consists of three facts {A,B,C} (Fig. 18.1).

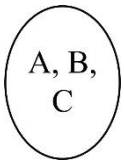


Fig. 18.1. The initial state of the GDB is {A,B,C}

Iteration 1. Rule π_3 is applied (Fig. 18.2).

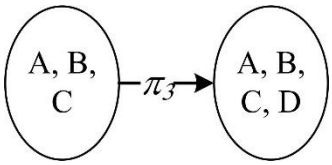


Fig. 18.2. Rule π_3 is applied. A new GDB state is {A,B,C,D}

Proceed with next iteration.

Iteration 2. Rule π_2 is applied (Fig. 18.3).

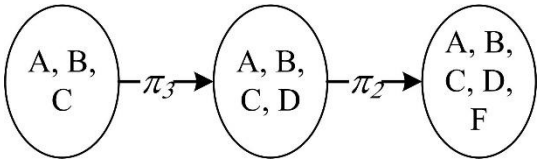


Fig. 18.3. Rule π_2 is applied. A new GDB state is {A,B,C,D,F}

Proceed with next iteration.

Iteration 3. Rule π_1 is applied (Fig. 18.4).

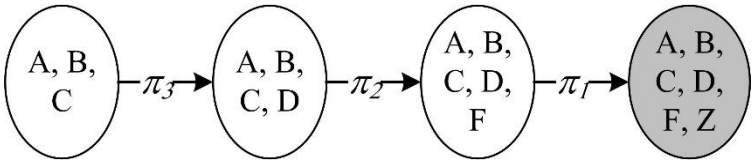


Fig. 18.4. Rule π_1 is applied. A new GDB state is {A,B,C,D,F,Z}. This is a terminal state because it contains the goal Z

The following result – a sequence of rules – is obtained. It is called a *path* (or a plan):

$$\langle \pi_3, \pi_2, \pi_1 \rangle \tag{18.2}$$

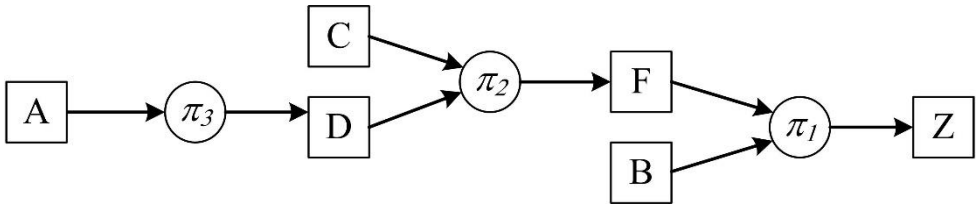


Fig. 18.5. The resulting semantic network shows a derivation of Z from {A,B,C} with the sequence $\langle \pi_3, \pi_2, \pi_1 \rangle$ in the rule system (18.1)

18.2. Backward chaining

Backward chaining starts from the goal and proceeds towards the facts. This is in contrast to forward chaining. Suppose (18.1) as an example.

Iteration 1. Rule π_1 is selected in (18.1); see Fig 18.6.

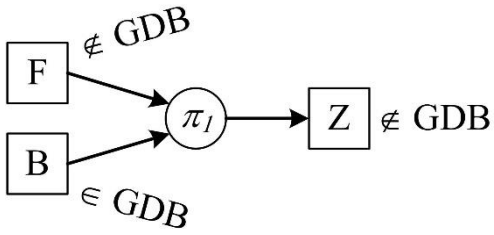


Fig. 18.6. The semantic network of the rule π_1. It shows the derivation of Z backwards. The input object B belongs to {A,B,C}, but F does not. Therefore F is taken as a new goal

Proceed with next iteration.

Iteration 2. Rule π_2 is selected in (18.1); see Fig. 18.7.

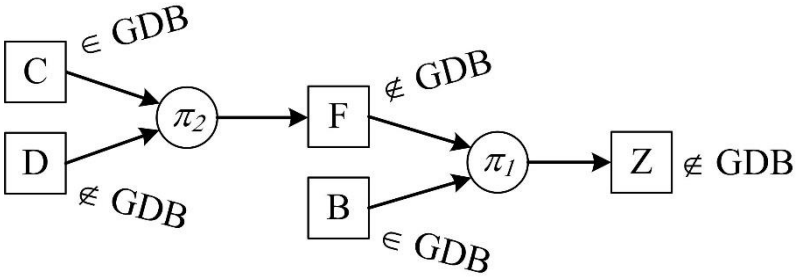


Fig. 18.7. The semantic network of the rule π_2. It shows the derivation of F backwards. The input object C belongs to {A,B,C}, but D does not. Therefore D is a new goal

Proceed with next iteration.

Iteration 3. Rule π_3 is selected in (18.1); see Fig 18.8.

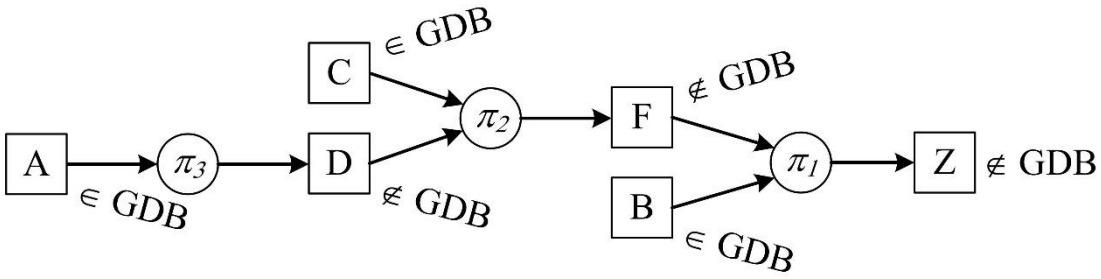


Fig. 18.8. A semantic network shows the derivation of Z from {A,B,C}. The rule sequence $\langle \pi_3, \pi_2, \pi_1 \rangle$ stands for the result

18.3. Program synthesis

Suppose procedures (18.3). Their semantics in terms of input-output pairs is represented in (18.1). The list of object names {A,B,C,D,F,Z} stands for the alphabet. The objects are of arbitrary type and need not to be Boolean variables as in the case of a rule system.

```
procedure P1;
  Z := f1(B,F)

procedure P2;
  F := f2(C,D)

procedure P3;
  D := f3(A)
```

(18.3)

Procedures `P1`, `P2`, `P3` program functional dependencies `f1`, `f2`, `f3` among the objects in a programming environment. This sort of procedures serves to represent procedural knowledge in a certain domain. Procedures normally form a software library. The semantics of procedures `P1`, `P2`, `P3` is represented analogous to rules π_1, π_2 and π_3 (see Fig. 18.9).

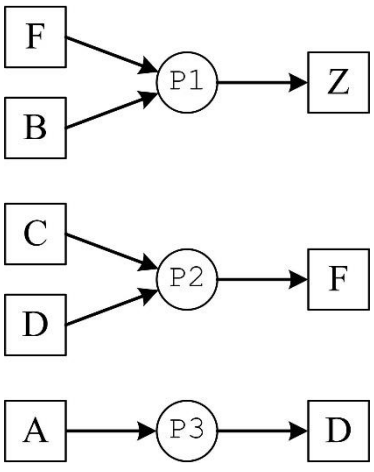


Fig. 18.9. A graphical representation of `P1`, `P2` and `P3` semantics

The sequence $\langle \pi_3, \pi_2, \pi_1 \rangle$ (18.2) corresponds to the following *synthesized* program:

```
call P3;
call P2;                                    (18.4)
call P1
```

The semantics of this program is shown in Fig. 18.10.

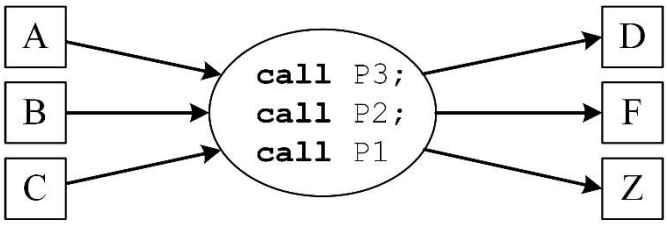


Fig. 18.10. Semantic network representation of the synthesized program

This program may be executed many times. However, it is synthesized only once. Each execution reads new values of objects A, B, C:

```
for J := 1 to 900 do   { Repeat }
begin
  readln (A, B, C);    { 1) Read A, B ir C              }

  call P3;             { 2) Invoke synthesized program }
  call P2;
  call P1;

  writeln(Z);          { 3) Print Z                     }
end
```

18.4. Superfluous rules in forward chaining

Suppose rules (18.1) and add two redundant rules π' ir π'':

$$\begin{array}{ll} \pi': & A \rightarrow L \\ \pi'': & L \rightarrow K \\ \pi_1: & F, B \rightarrow Z \\ \pi_2: & C, D \rightarrow F \\ \pi_3: & A \rightarrow D \end{array} \qquad (18.5)$$

Facts are {A,B,C} and goal Z. Forward chaining is shown in Fig. 18.11.

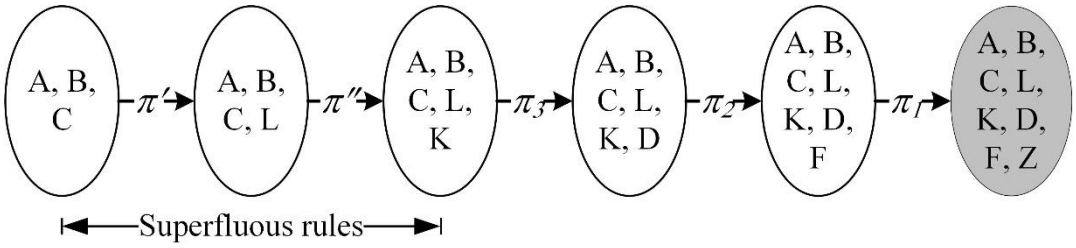


Fig. 18.11. Forward chaining produces $\langle \pi', \pi'', \pi_3, \pi_2, \pi_1 \rangle$. It contains superfluous rules π' and π'' that derive redundant objects L and K

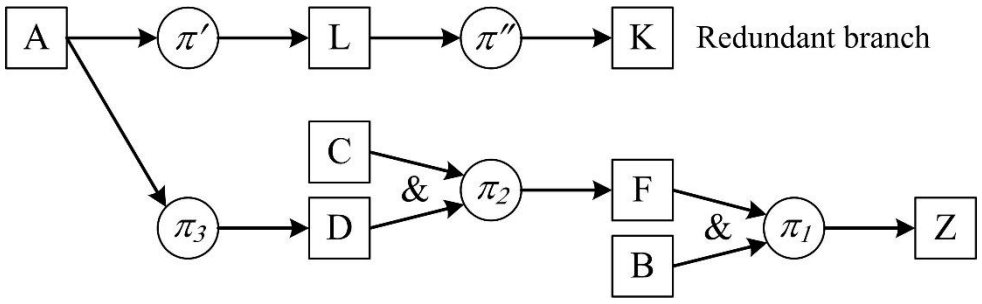


Fig. 18.12. Superfluous rules π' and π'' derive redundant objects L and K

18.5. Superfluous rules in backward chaining

Suppose rules (18.1) and add two superfluous different rules π' and π'':

$$\begin{array}{ll} \pi': & \mathrm{K} \rightarrow \mathrm{M} \\ \pi'': & \mathrm{M, N} \rightarrow \mathrm{Z} \\ \pi_1: & \mathrm{F, B} \rightarrow \mathrm{Z} \\ \pi_2: & \mathrm{C, D} \rightarrow \mathrm{F} \\ \pi_3: & \mathrm{A} \rightarrow \mathrm{D} \end{array} \qquad (18.6)$$

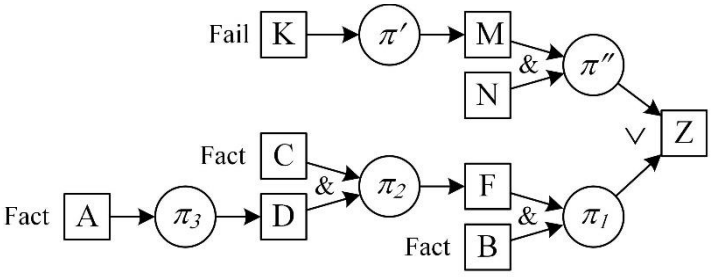


Fig. 18.13. Two superfluous rules π' and π'' in backward chaining. Their goals N and K cannot be derived. Therefore these rules are not included in the resulting path $\langle \pi_3, \pi_2, \pi_1 \rangle$

18.6. Complexity of forward chaining

Theorem. The complexity of forward chaining in a rule system of N rules (18.1) is $O(N^2)$.

Proof. The first iteration searches among N rules, the second iteration searches among $N–1$ rules, etc. In each iteration one rule is found. Thus the search space for the next iteration is reduced by one. The worst case to search is N iterations. The largest sum of iterations is:

$$N + N–1 + N–2 + \ldots + 1 = N \cdot (N–1)/2 = O(N^2)$$

18.7. Testing forward chaining

Test 1. Initial fact in right hand side
Rules see in the table.
Facts A, B, C.
Goal Z.
Path R1, R2, R7, R4, R6, R5.

R1:	A	→ L
R2:	L	→ K
R3:	D	→ A
R4:	D	→ M
R5:	F, B	→ Z
R6:	C, D	→ F
R7:	A	→ D

Input file:

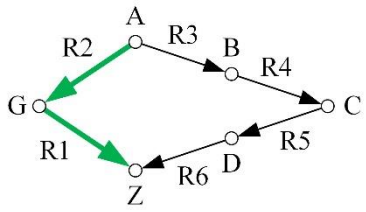
```
Student First name Last name. University, study programme.
Test 1. Initial fact in the right hand side
1) Rules
L A          // R1: A -> L. Comments.
K L          // R2: L -> K
A D          // R3: D -> A
M D          // R4: D -> M
Z F B        // R5: F, B -> Z
F C D        // R6: C, D -> F
D A          // R7: A -> D

2) Facts
A B C

3) Goal
Z
```

The trace:

```
PART 1. Data

  1) Rules
     R1: A -> L
     R2: L -> K
     R3: D -> A
     R4: D -> M
     R5: F, B -> Z
     R6: C, D -> F
     R7: A -> D
  2) Facts A, B, C.
  3) Goal  Z.

PART 2. Trace

  ITERATION 1
    R1:A->L apply. Raise flag1. Facts A, B, C and L.

  ITERATION 2
    R1:A->L skip, because flag1 raised.
    R2:L->K apply. Raise flag1. Facts A, B, C and L, K.

  ITERATION 3
    R1:A->L skip, because flag1 raised.
    R2:L->K skip, because flag1 raised.
    R3:D->A not applied, because of lacking D.
    R4:D->M not applied, because of lacking D.
    R5:F,B->Z not applied, because of lacking F.
    R6:C,D->F not applied, because of lacking D.
    R7:A->D apply. Raise flag1. Facts A, B, C and L, K, D.
```

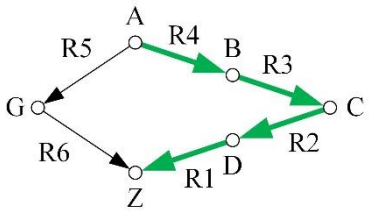
```
  ITERATION 4
    R1:A->L skip, because flag1 raised.
    R2:L->K skip, because flag1 raised.
    R3:D->A not applied, because RHS in facts. Raise flag2.
    R4:D->M apply. Raise flag1. Facts A, B, C and L, K, D, M.

  ITERATION 5
    R1:A->L skip, because flag1 raised.
    R2:L->K skip, because flag1 raised.
    R3:D->A skip, because flag2 raised.
    R4:D->M skip, because flag1 raised.
    R5:F,B->Z not applied, because of lacking F.
    R6:C,D->F apply. Raise flag1. Facts A, B, C and L, K, D, M, F.

  ITERATION 6
    R1:A->L skip, because flag1 raised.
    R2:L->K skip, because flag1 raised.
    R3:D->A skip, because flag2 raised.
    R4:D->M skip, because flag1 raised.
    R5:F,B->Z apply. Raise flag1. Facts A, B, C and L, K, D, M, F, Z.
    Goal achieved.

PART 3. Results
  1) Goal Z achieved.
  2) Path R1, R2, R7, R4, R6, R5.
```

Test 2. Čyras vs. Negnevitsky; Čyras wins
Rules see in the table.
Fact A.
Goal Z.
Path R2, R1.



R1:	G → Z
R2:	A → G
R3:	A → B
R4:	B → C
R5:	C → D
R6:	D → Z

Test 3. Čyras vs. Negnevitsky; Negnevitsky wins (however, with a redundant rule)
Rules see in the table.
Fact A.
Goal Z.
Path R4, R3, R2, R1.



R1:	D → Z
R2:	C → D
R3:	B → C
R4:	A → B
R5:	A → G
R6:	G → Z

Test 4. Goal in facts

```
PART 3. Results
  Goal A in facts. Empty path.
```

Test 5. No path
Rules R1: A → B; R2: C → Z. Fact A. Goal Z.

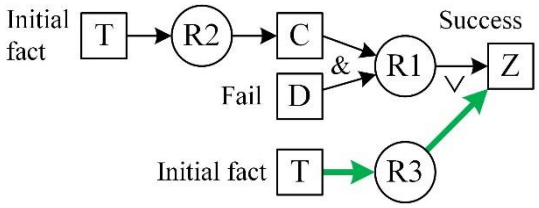
Test 6. Negnevitsky's example
Take the 5 rules example from Negnevitsky's (2011) section about forward chaining.

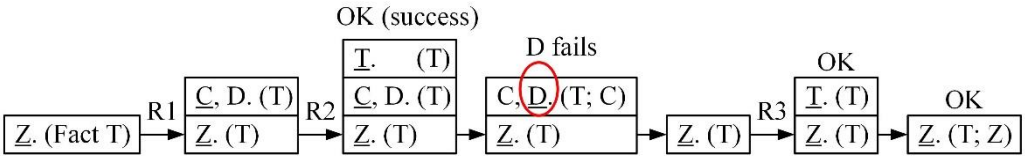
18.8. Testing backward chaining

Test 1. Failed branch

Rules: R1: C, D → Z
R2: T → C
R3: T → Z

Fact T. Goal Z. Path R3. Note that the path is not R2, R3. Facts in the end are T and Z and not T and C, Z. This happens due to backtracking and the failed branch from Z to C.



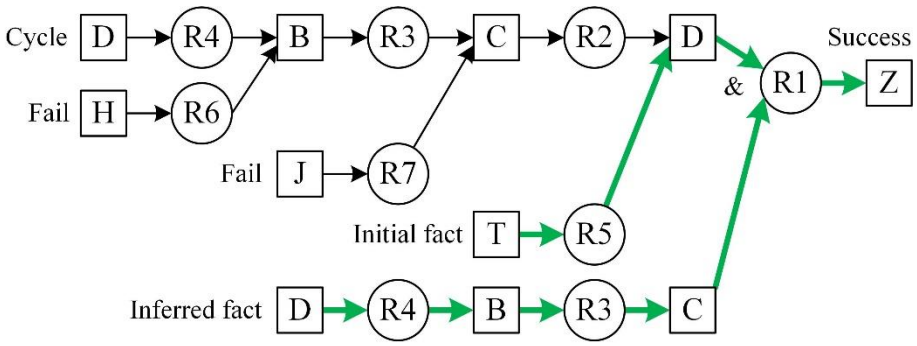



```
PART 2. Trace
 1) Goal Z. Find R1:C,D->Z. New goals C, D.
 2) -Goal C. Find R2:T->C. New goals T.
 3) --Goal T. Fact (initial), as facts are T. Back, OK.
 4) -Goal C. Fact (presently inferred). Facts T and C.
 5) -Goal D. No rules. Back, FAIL.
 6) Goal Z. Find R3:T->Z. New goals T.
 7) -Goal T. Fact (initial), as facts are T. Back, OK.
 8) Goal Z. Fact (presently inferred). Facts T and Z. OK.

PART 3. Results
  1) Goal Z achieved.
  2) Path: R3.
```

Test 2. Seven rules D, C

Rules see in the table.
Fact T.
Goal Z.
Path R5, R4, R3, R1.

R1:	D, C → Z
R2:	C → D
R3:	B → C
R4:	D → B
R5:	T → D
R6:	H → B
R7:	J → C



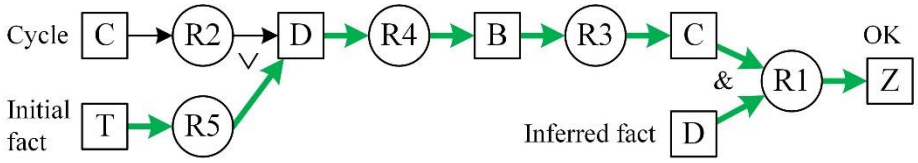

```
PART 2. Trace
 1) Goal Z. Find R1:D,C->Z. New goals D, C.
 2) -Goal D. Find R2:C->D. New goals C.
 3) --Goal C. Find R3:B->C. New goals B.
 4) ---Goal B. Find R4:D->B. New goals D.
 5) ----Goal D. Cycle. Back, FAIL.
```

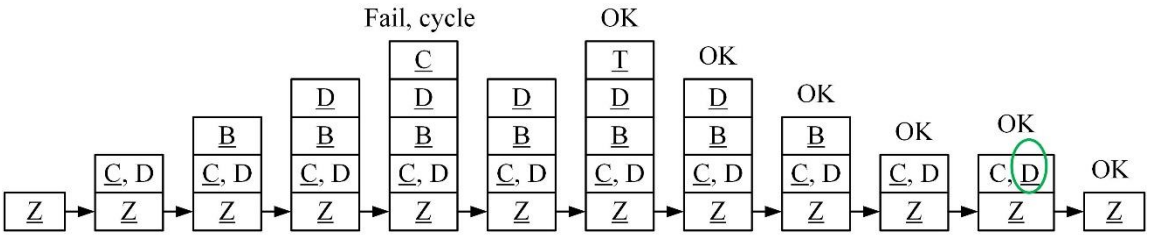
```
 6) ---Goal B. Find R6:H->B. New goals H.
 7) ----Goal H. No rules. Back, FAIL.
 8) ---Goal B. No more rules. Back, FAIL.
 9) --Goal C. Find R7:J->C. New goals J.
10) ---Goal J. No rules. Back, FAIL.
11) --Goal C. No more rules. Back, FAIL.
12) -Goal D. Find R5:T->D. New goals T.
13) --Goal T. Fact (initial), as facts T. Back, OK.
14) -Goal D. Fact (presently inferred). Facts T and D.
15) -Goal C. Find R3:B->C. New goals B.
16) --Goal B. Find R4:D->B. New goals D.
17) ---Goal D. Fact (earlier inferred), as facts T and D. Back, OK.
18) --Goal B. Fact (presently inferred). Facts T and D, B. Back, OK.
19) -Goal C. Fact (presently inferred). Facts T and D, B, C. Back, OK.
20) Goal Z. Fact (presently inferred). Facts T and D, B, C, Z. OK.

PART 3. Results
  1) Goal Z achieved.
  2) Path: R5, R4, R3, R1.
```

Test 3. Seven rules C, D

See Test 2, but R1: C, D → Z.
Fact T.
Goal Z.
The path is the same as in Test 2, namely, R5, R4, R3, R1.

R1:	C, D → Z
R2:	C → D
R3:	B → C
R4:	D → B
R5:	T → D
R6:	H → B
R7:	J → C

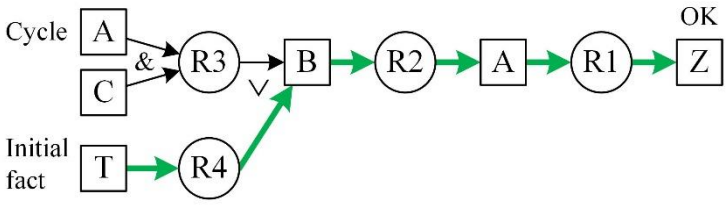




Test 4. A cycle and a failed branch

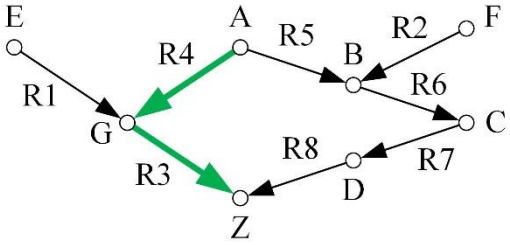
Rules:

R1: A → Z
R2: B → A
R3: A, C → B
R4: T → B
R5: T → C



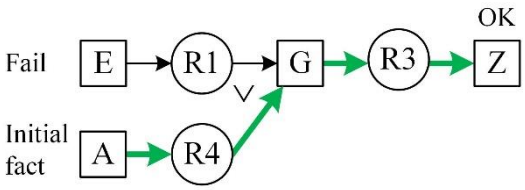
Fact T. Goal Z. Path R4, R2, R1. Note that forward chaining's path is R4, R2, R5, R3.

Test 5. Graph – short path
Rules are provided in the table.
Fact A.
Goal Z.
Path R4, R3.

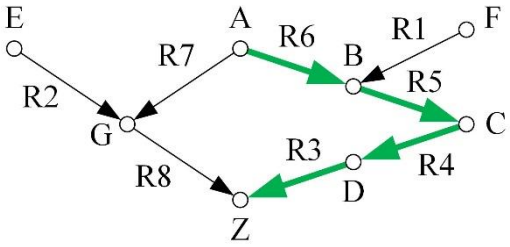


R1:	E → G
R2:	F → B
R3:	G → Z
R4:	A → G
R5:	A → B
R6:	B → C
R7:	C → D
R8:	D → Z

The search tree:

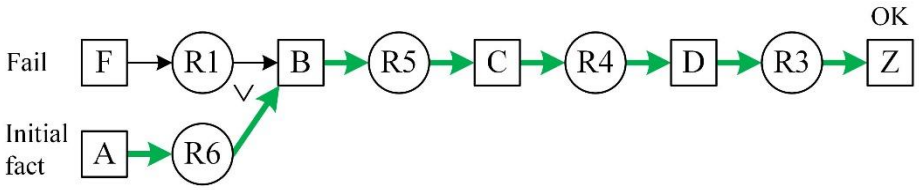


Test 6. Graph – long path
Rules are provided in the table.
Fact A.
Goal Z.
Path R6, R5, R4, R3.



R1:	F → B
R2:	E → G
R3:	D → Z
R4:	C → D
R5:	B → C
R6:	A → B
R7:	A → G
R8:	G → Z

The search tree:



Test 7. Three alternatives
Rules R1: A → Z; R2: B → Z; R3: C → Z. Fact C. Goal Z. Path R3.

Test 8. Unachievable goal
Rules R1: C, D → Z; R2: C, E → Y. Facts C, D. Goal Y. No path.

Test 9. Goal among facts – empty path

```
PART 3. Results
  Goal A among facts. Empty path.
```

Test 10. Negnevitsky's example
Take the 5 rules example from Negnevitsky's (2011) section about backward chaining.

19. Resolution

The word resolution denotes inference rule and also inference method which is explained below. See also https://en.wikipedia.org/wiki/Resolution_(logic). The method is introduced with an example from (Thayse et al. 1988). Suppose a statement "A professor can examine students of a distinct faculty". Suppose two facts:

1. James is a professor at the faculty of informatics;
2. Mary is a student at the faculty of mathematics.

The task is to prove that James is allowed to examine Mary.

Two facts are represented with predicates:

Fact F1: `Prof(Info,James)` (19.1)

Fact F2: `Stud(Mat,Mary)` (19.2)

The statement "A professor can examine students of a distinct faculty" is represented with the following rule:

R1: $\texttt{Prof(x,y)} \,\&\, \texttt{Stud(z,w)} \,\&\, \neg\texttt{Eq(x,z)} \Rightarrow \texttt{Exam(y,w)}$ (19.3)

Implication $\texttt{P} \Rightarrow \texttt{Q}$ can be replaced with negation and disjunction $\neg\texttt{P} \vee \texttt{Q}$. The reason is that these two formulas are equivalent: $\texttt{P} \Rightarrow \texttt{Q} \equiv \neg\texttt{P} \vee \texttt{Q}$. The tables of both formulas coincide (consider $\texttt{true} \vee \texttt{true} = \texttt{true}$, $\texttt{true} \vee \texttt{false} = \texttt{true}$, $\texttt{false} \vee \texttt{false} = \texttt{false}$):

P	Q	¬P	¬P ∨ Q	P ⇒ Q
true	true	false	true	true
true	false	false	false	false
false	true	true	true	true
false	false	true	true	true

Implication elimination is sound. The replacement above can also be treated as a term rewriting rule (19.4) below. Implication examples are illustrated in Fig. 19.1.

$$\frac{\texttt{P} \Rightarrow \texttt{Q}}{\neg\texttt{P} \vee \texttt{Q}}$$ Elimination of implication. (Write also $\texttt{P} \Rightarrow \texttt{Q} \vdash \neg\texttt{P} \vee \texttt{Q}$) (19.4)

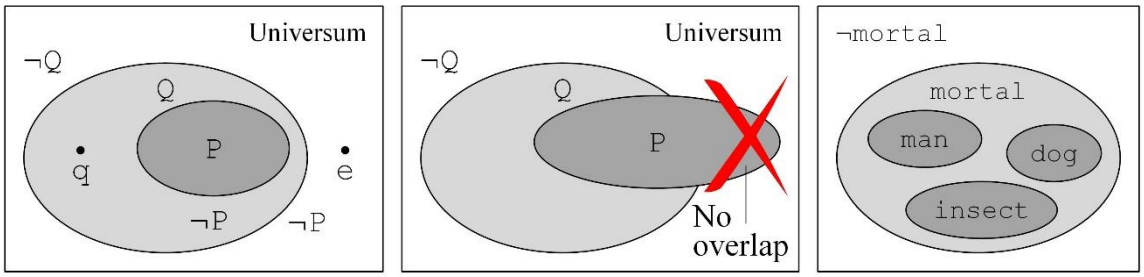


Fig. 19.1. (a) Venn diagram showing the implication $\texttt{P} \Rightarrow \texttt{Q}$. Suppose $\forall\texttt{x}\ \texttt{P(x)} \Rightarrow \texttt{Q(x)}$ and two states, `q` and `e`. First, $\texttt{P(q)} = \texttt{false}$ and $\texttt{Q(q)} = \texttt{true}$. Second, $\texttt{P(e)} = \texttt{false}$ – as in the first case – but $\texttt{Q(e)} = \texttt{false}$. **(b)** No overlap of `P` and ¬Q. **(c)** Three implications: $\texttt{man} \Rightarrow \texttt{mortal}$, $\texttt{insect} \Rightarrow \texttt{mortal}$ and $\texttt{dog} \Rightarrow \texttt{mortal}$

In the case of $P \Rightarrow Q$, a set of world states for P (where $P = true$) is a subset of states for Q (where $Q = true$): *states*(P) $\subset$ *states*(Q) (see Fig. 19.1 a). Following is an example of an implication statement: if a number is divisible by 6 then it is divisible by 3. Formally, $\forall x \in integer\ D6(x) \Rightarrow D3(x)$, where $D6(x)$ denotes the predicate "number x is divisible by 6". Note, however, that if a number **is not** divisible by 6, it can be either divisible by 3 (e.g. numbers 9, 15, 21…) or not divisible by 3 (e.g. numbers 7 or 8). Note also that the set inclusion symbol $\subset$ is directed in the opposite direction than the implication symbol $\Rightarrow$. Fig. 19.1 b illustrates that P and $\neg Q$ do not overlap. Fig. 19.1 c shows the Venn diagram with three implications $man \Rightarrow mortal$, $insect \Rightarrow mortal$, and $dog \Rightarrow mortal$.

Eliminating implication can be generalised:

$$\frac{P_1 \,\&\, P_2 \,\&\, \ldots \,\&\, P_n \;\Rightarrow\; Q}{\neg P_1 \vee \neg P_2 \vee \ldots \vee \neg P_n \vee Q} \quad \text{Elimination of implication (general)} \qquad (19.5)$$

Following is the rule to eliminate double negation. It is sound because $\neg\neg P \equiv P$:

$$\frac{\neg\neg P}{P} \quad \text{Double negation elimination} \qquad (19.6)$$

We rewrite the rule (19.3) taking into account (19.5) and obtain

$$\neg Prof(x,y) \vee \neg Stud(z,w) \vee \neg\neg Eq(x,z) \vee Exam(y,w)$$

Double negation is eliminated:

$$\text{Rule R1: } \neg Prof(x,y) \vee \neg Stud(z,w) \vee Eq(x,z) \vee Exam(y,w) \qquad (19.7)$$

To apply resolution we start explaining the *modus ponens* rule: (1) Suppose P; (2) Suppose the implication if P, then Q; (3) Therefore Q.

$$\frac{\begin{array}{l} P \quad \text{(1) Small premise} \\ P \Rightarrow Q \quad \text{(2) Great premise} \end{array}}{Q \quad \text{(3) Conclusion}} \qquad \textit{modus ponens}$$

A horizontal representation of the *modus ponens* rule (with comma between premises):

$$\frac{P, \quad P \Rightarrow Q}{Q} \qquad \textit{modus ponens}$$

An example of *modus ponens* application with formulas in predicate logic, where $\{Socrates/x\}$ is a *unifier*:

$$\frac{\begin{array}{l} \text{(1) } man(Socrates) \\ \text{(2) } \forall x\ man(x) \Rightarrow mortal(x) \end{array}}{\text{(3) } mortal(Socrates)} \qquad \{Socrates/x\}$$

The resolution rule is written as follows:

$$\frac{P, \quad \neg P \vee Q}{Q}\ \sigma \qquad \textbf{Resolution rule} \text{ (simple form)} \qquad (19.8)$$

The above rule is depicted in Fig. 19.2. Formulas $\forall x_1 \forall x_2 \ldots \forall x_n\ Q(x_1, x_2, \ldots, x_n)$ are considered; see e.g. (Norgėla 2007, p. 26, 92, 158–163; Nilsson 1998, p. 253–268).

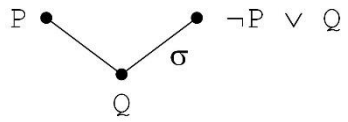


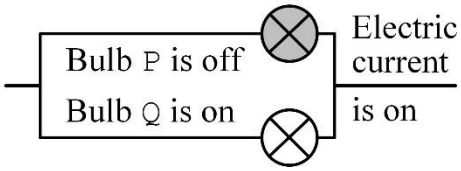
Fig. 19.2. Graphical representation of the resolution rule

Another form of representing the resolution rule is with replacing P with $\neg P$:

$$\frac{\neg P, \quad P \vee Q}{Q} \qquad (19.9)$$

The resolution rule (19.9) and deduction is illustrated with an example. Suppose a parallel circuit of two electric bulbs P and Q (Fig. 19.3). The circuit is modelled with the formula $P \vee Q$. Suppose the bulb P is broken (meaning $\neg P = \texttt{true}$) but electric current is on (meaning $P \vee Q = \texttt{true}$). Therefore we deduce that the bulb Q is light (meaning $Q = \texttt{true}$).

Fig. 19.3. Parallel circuit of two bulbs is modelled with the disjunction $P \vee Q$. The statement „Bulb P is off“ (i.e. the bulb is broken) means $\neg P$. A conclusion of these two statements is that Q is on



A general case, where $n \geq 0$:

$$\frac{P \vee H, \quad \neg P \vee Q}{H \vee Q}\ \{A_1/x_1,\ A_2/x_2, \ldots A_n/x_n\} \qquad \textbf{Resolution rule} \text{ (general)} \qquad (19.10)$$

In the unifier the variables x_i, if any, are replaced with A_i; see Fig. 9.4. The unifier σ is of the form $\{A_1/x_1, A_2/x_2, \ldots A_n/x_n\}$.

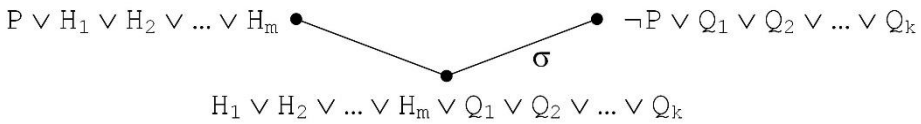


Fig. 19.4. Graphical representation of the resolution rule, where m, $k \geq 0$

19.1. Inference example

There are two directions of proof (inference) with the resolution rule: *backward proof* and *forward proof*. Backward proof goes from the goal to the facts. Forward proof goes from the facts to the goal.

Backward reasoning – from the goal to the facts. To prove `Exam(James,Mary)` suppose the opposite, i.e. negation of the goal:

$$\neg\texttt{Exam(James,Mary)} \tag{19.11}$$

Take (19.11) and the rule (19.7) and apply the resolution rule (19.9). To resolve the negative disjunct $\neg$`Exam(James,Mary)` with the positive one `Exam(y,w)`, the unifier is `{James/y, Mary/w}`.

$$\frac{\neg\texttt{Exam(James,Mary)},\ \neg\texttt{Prof(x,James)} \vee \neg\texttt{Stud(z,Mary)} \vee \texttt{Eq(x,z)} \vee \texttt{Exam(James,Mary)}}{\neg\texttt{Prof(x,James)} \vee \neg\texttt{Stud(z,Mary)} \vee \texttt{Eq(x,z)}}$$

Further take the conclusion above and the fact F1 (19.1) and apply the resolution rule (19.8). The unifier is `{Info/x}`.

$$\frac{\neg\texttt{Prof(Info,James)} \vee \neg\texttt{Stud(z,Mary)} \vee \texttt{Eq(Info,z)},\qquad \texttt{Prof(Info,James)}}{\neg\texttt{Stud(z,Mary)} \vee \texttt{Eq(Info,z)}}$$

Further take the conclusion above and fact F2 (19.2) and apply the resolution rule (19.8). The unifier is {`Mat/z`}.

$$\frac{\neg\texttt{Stud(Mat,Mary)} \vee \texttt{Eq(Info,Mat)},\qquad \texttt{Stud(Mat,Mary)}}{\texttt{Eq(Info,Mat)}}$$

Faculties of informatics and mathematics are distinct ones, formally, `Eq(Info,Mat) = false`. Therefore the conclusion above is `false`. Hence, a contradiction is obtained. Thus the goal negation leads to contradiction. Therefore the goal `Exam(James,Mary)` is true. The inference tree is shown in Fig. 19.5. **End of proof.**

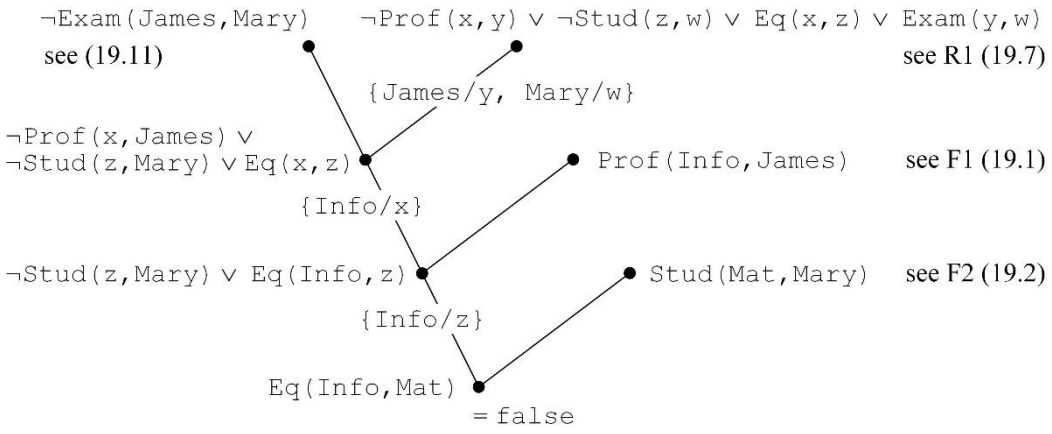


Fig. 19.5. Inference tree for the statement `Exam(James,Mary)`. Theorem's negation leads to contradiction

Forward reasoning (direct inference) – from facts to goal. Suppose two facts F1 (19.1) and F2 (19.2) and one rule R1 (19.7). We will prove `Exam(James,Mary)`.

Take fact F1 `Prof(Info,James)` and find a rule to match. Take the rule (19.7) and apply the resolution rule (19.8). The unifier is `{Info/x, James/y}`:

$$\frac{\texttt{Prof(Info,James)},\ \neg\texttt{Prof(Info,James)} \vee \neg\texttt{Stud(z,w)} \vee \texttt{Eq(Info,z)} \vee \texttt{Exam(James,w)}}{\neg\texttt{Stud(z,w)} \vee \texttt{Eq(Info,z)} \vee \texttt{Exam(James,w)}}$$

Then take fact F2 `Stud(Mat,Mary)` and match with the conclusion above. Apply the resolution rule (19.8) with the unifier `{Mat/z, Mary/w}`:

$$\frac{\texttt{Stud(Mat,Mary)},\quad \neg\texttt{Stud(Mat,Mary)} \vee \texttt{Eq(Info,Mat)} \vee \texttt{Exam(James,Mary)}}{\texttt{Eq(Info,Mat)} \vee \texttt{Exam(James,Mary)}}$$

As `Eq(Info,Mat) = false` and $\texttt{false} \vee \texttt{H} \equiv \texttt{H}$, therefore `Exam(James,Mary)`. Q.E.D. – *quod erat demonstrandum*. The inference tree is shown in Fig. 19.6. **End of proof.**

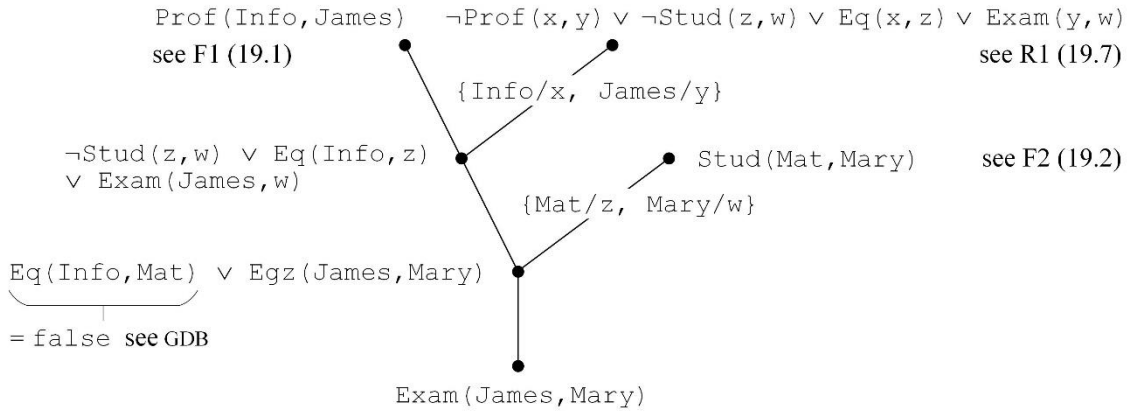


Fig. 19.6. Inference tree to prove `Exam(James,Mary)`. This is the direct proof – from the facts to the goal

Suppose one more fact: Irma is a student at the faculty of informatics. Formally, `F3: Stud(Info,Irma)`. However, the rule 19.3 fails to prove that professor James examines student Irma.

19.2. Example with three rules

Suppose the rules (18.1). Implication elimination leads to normal disjunctive form below.

	Rule	Formula	Implication eliminated
π_1:	F, B → Z	F & B ⇒ Z	¬F ∨ ¬B ∨ Z
π_2:	C, D → F	C & D ⇒ F	¬C ∨ ¬D ∨ F
π_3:	A → D	A ⇒ D	¬A ∨ D

Suppose three facts, F1: A, F2: B, and F3: C. In other words, `A=true`, `B=true`, and `C=true`. Prove `Z`.

Proof backward. Start from goal's negation ¬Z.
Find a rule to match. The rule π_1 is found. Apply the resolution rule (19.8):

$$\frac{\neg Z, \qquad \neg F \vee \neg B \vee Z}{\neg F \vee \neg B}$$

Take the above conclusion ¬F ∨ ¬B and find a rule to match. Find π_2. Then apply the resolution rule (19.10):

$$\frac{\neg F \vee \neg B, \quad \neg C \vee \neg D \vee F}{\neg B \vee \neg C \vee \neg D}$$

Take the above conclusion ¬B∨¬C∨¬D and find a rule to match. Find π_3. Then apply the resolution rule (19.10):

$$\frac{\neg B \vee \neg C \vee \neg D, \qquad \neg A \vee D}{\neg B \vee \neg C \vee \neg A}$$

The above conclusion can be rewritten because ¬B ∨ ¬C ∨ ¬A ≡ ¬(B & A & C). Next consider the facts A, B and C. Therefore the formula A & B & C is true. Apply the resolution rule (19.10) (or apply resolution three times with formulas A, B and C sequentially):

$$\frac{\neg (B \,\&\, A \,\&\, C), \;\; A \,\&\, B \,\&\, C}{\varnothing}$$

Empty resolvent ∅ means contradiction (P & ¬P ≡ false). Hence, goal's negation ¬Z implies contradiction. Therefore the goal Z is true. The proof tree is shown in Fig. 19.7. **End of proof.**

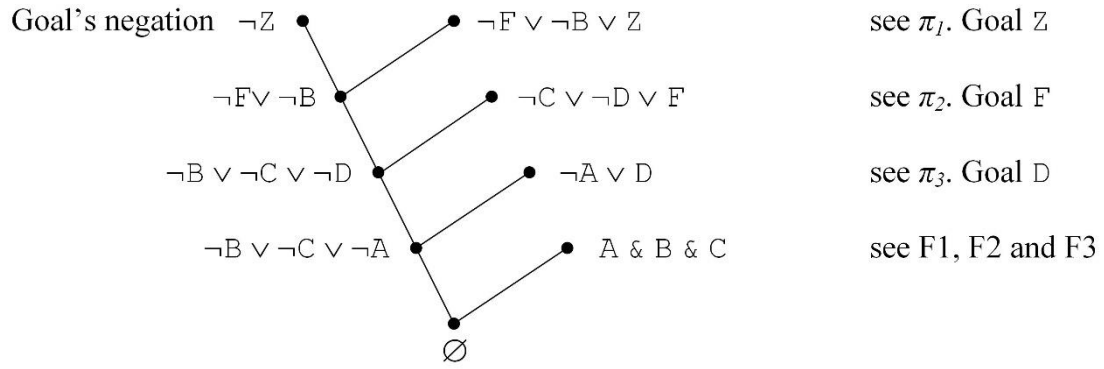


Fig. 19.7. Proof tree for Z. Inference is backward: from goal's negation to contradiction

Intelligence lies in the sequence π_1, π_2, π_3 which is inherent in the proof. This sequence is extracted from the proof and listed from its end to produce the plan $\langle \pi_3, \pi_2, \pi_1 \rangle$.

Proof forward (direct inference). Start from facts. Take fact F1: A. Search a rule to match. Find π_3, i.e. ¬A ∨ D. Apply the resolution rule (19.8):

$$\frac{A, \qquad \neg A \vee D}{D}$$

Search a rule to match. Find π_2, i.e. $\neg C \vee \neg D \vee F$. Apply the resolution rule (19.8):

$$\frac{D, \qquad \neg C \vee \neg D \vee F}{\neg C \vee F}$$

Take fact F3, i.e. C. Apply the resolution rule (19.8):

$$\frac{C \qquad \neg C \vee F}{F}$$

Search a rule to match. Find π_1, t. y. $\neg F \vee \neg B \vee Z$. Apply the resolution rule (19.8):

$$\frac{F, \qquad \neg F \vee \neg B \vee Z}{\neg B \vee Z}$$

Take fact F2, i.e. B. Apply the resolution rule (19.8):

$$\frac{B, \qquad \neg B \vee Z}{Z}$$

The goal Z is obtained. Q.E.D. Proof tree is shown in Fig. 19.8. **End of proof**.

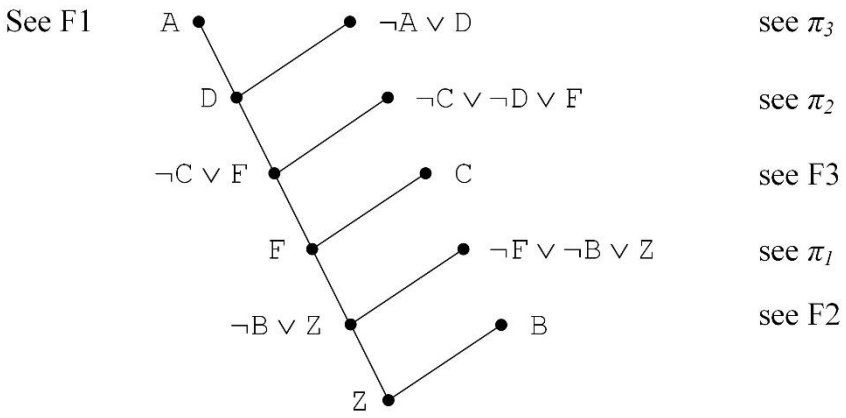


Fig. 19.8. Proof tree of Z . The proof is direct forward: from facts to the goal

19.3. Using resolution to prove theorem

This chapter follows (Nilsson 1998, 16.5, p. 260–261).

(1) $\forall x,y\ \mathtt{Package(x)} \,\&\, \mathtt{Package(y)} \,\&\, \mathtt{Inroom(x,27)} \,\&\, \mathtt{Inroom(y,28)} \Rightarrow \mathtt{Smaller(x,y)}$

Shortly, after the elimination of implication:

(2) $\neg \mathtt{P(x)} \vee \neg \mathtt{P(y)} \vee \neg \mathtt{I(x,27)} \vee \neg \mathtt{I(y,28)} \vee \mathtt{S(x,y)}$

Robot knows that A is in room 27 or 28 (but does not know in which one), formally, $\mathtt{I(A,27)} \vee \mathtt{I(A,28)}$. Robot knows that B is in room 27, $\mathtt{I(B,27)}$. Robot knows that B is not smaller than A, $\neg \mathtt{S(B,A)}$. Hence:

(3) $\mathtt{P(A)}$. Fact.

(4) $\mathtt{P(B)}$. Fact.

(5) $\mathtt{I(A,27)} \vee \mathtt{I(A,28)}$. Robot knows.

(6) $\mathtt{I(B,27)}$. Robot knows.

(7) $\neg \mathtt{S(B,A)}$. Robot knows.

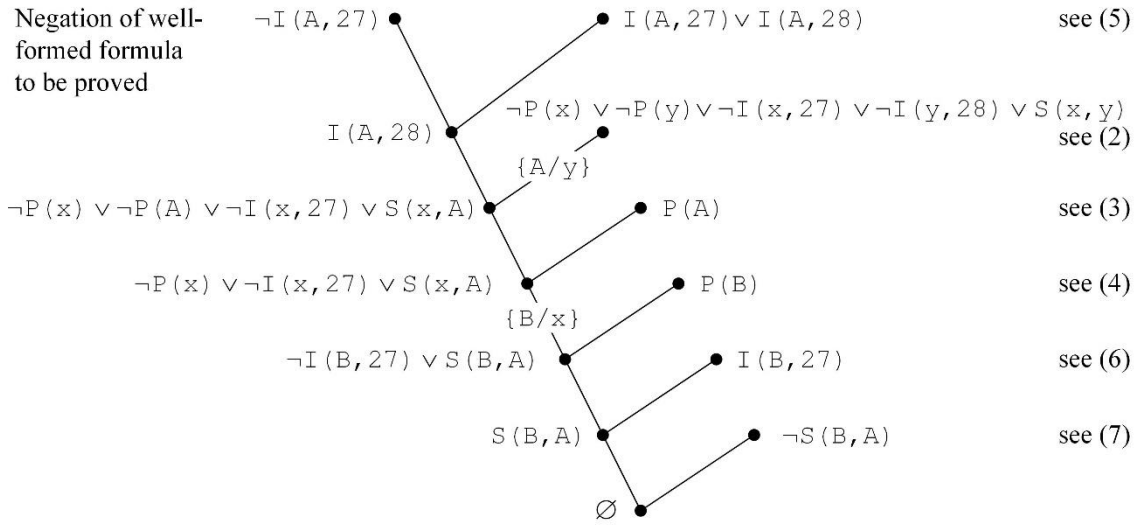


Fig. 19.9. A proof tree by resolution refutation; adapted from (Nilsson 1998, p. 261)

A possible world is shown in Fig. 19.10.

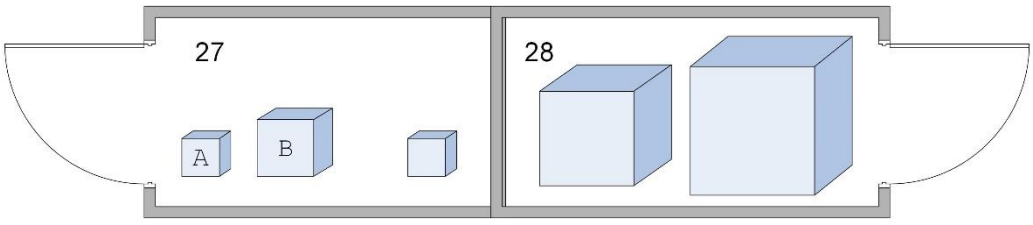


Fig. 19.10. A possible world the robot knows about

20. Expert systems

A general architecture of expert systems is shown in Fig. 20.1; see e.g. Sawyer and Foster (1986).

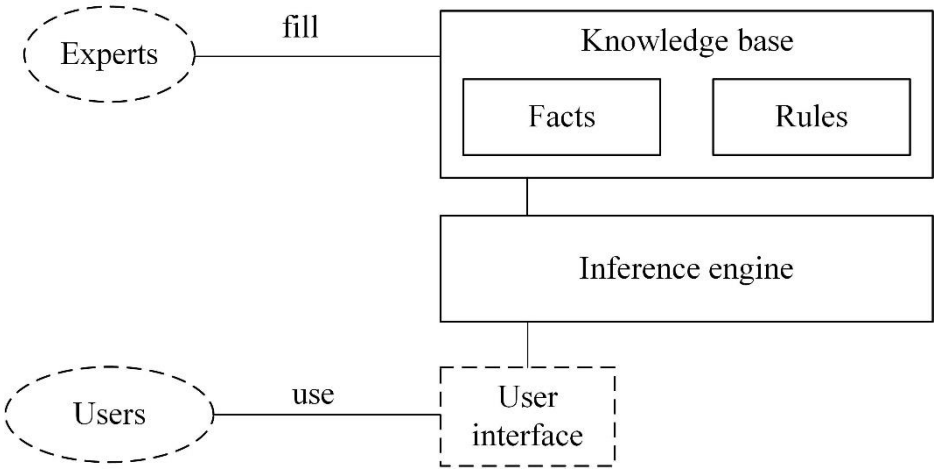


Fig. 20.1. A general architecture of an expert system

Following is an example “Health Assessment Knowledge Base” from (Sawyer, Foster 1986, p. 169–181). Facts are shown below in Fig. 20.2.

1) age= 1. '25_or_less' 2. **'25-55'** 3. '55_or_over'	2) gender= 1. 'm' 2. **'f'**	3) weight= 1. **'55_or_less'** 2. '55-85' 3. '85_or_more
4) frame= 1. **'small'** 2. 'large'	5) cholesterol= 1. **'normal'** 2. 'high'	6) salt= 1. **'normal'** 2. 'high'
7) smoker= 1. 'yes' 2. **'no'**	8) personality= 1. **'aggressive'** 2. 'docile'	9) alcohol= 1. 'none' 2. **'moderate'** 3. 'excessive'

Fig. 20.2. Facts in an expert system. Bold text shows a selection by the user

Several rules of total about 100 are shown below.

R1	If relative weight is normal then general assessment is “yes”. **if** relative_weight='normal' **then** val='yes'
R4	If heart disease risk is average then heart disease danger is low. **if** heart_disease_risk='average' **then** hddanger='low'
R8	If age is 25-55 years and gender is female then base longevity is 67 years. **if** age='25-55' & gender='f' **then** base_longevity='67'

R12	If weight is smaller than 55 kg, and frame is small and gender is female then relative weight is normal. **if** weight='55_or_less' & frame='small' & gender='f' **then** relative_weight='normal'
R27	If cholesterol is normal then heart disease risk is average. **if** cholesterol='normal' **then** heart_disease_risk='average'
R29	If salt consumption is normal then blood pressure is average. **if** salt='normal' **then** blood_pressure='average'
R37	If general evaluation is "yes" and heart disease danger is low and blood pressure is average and non-smoker then outlook is excellent. **if** val='yes' & hddanger='low' & blood_pressure='average' & smoker='no' **then** outlook ='excellent'
R43	If personality is aggressive then personality type is A. **if** personality='aggressive' **then** personality_type='type_a'
R45	If personality type is A then general risk is high. **if** personality_type='type_a' **then** risk='high'
R47	If alcohol consumption is moderate then additional factor is good. **if** alcohol='moderate' **then** add='good'
R52	If outlook is excellent and risk is high and additional factor is good then factor is none. **if** outlook='excellent' & risk='high' & add='good' **then** factor='none'
R64	If base_longevity is 67 years and factor is none then longevity prognosis is 67 years. **if** base_longevity='67' & factor='none' **then** longevity='67_yrs'

Fig. 20.3. Sample rules in an expert system

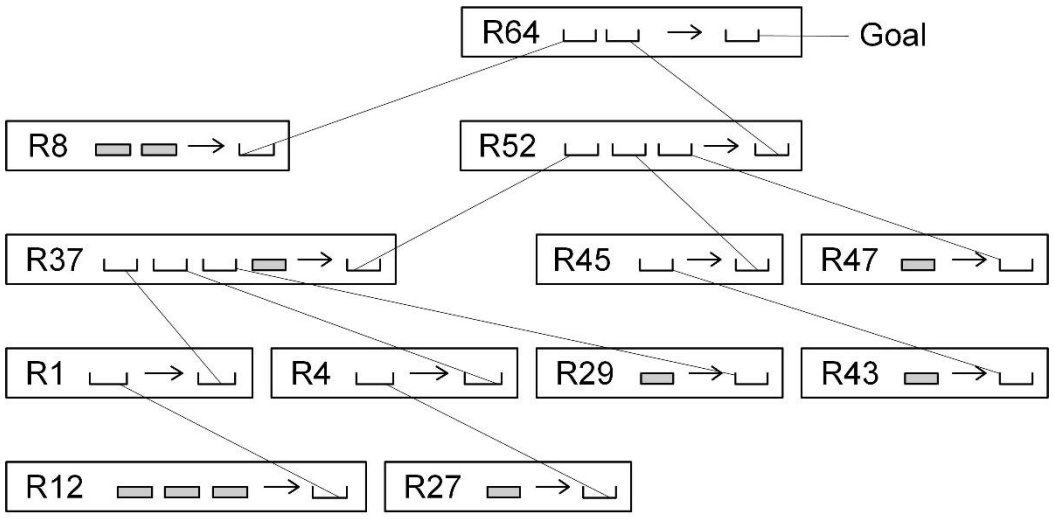


Fig. 20.4. A simplified proof tree. Facts are denoted by grey rectangles

Here rules are of the form ***if*** $o_1=v_1$ & $o_2=v_2$ & … & $o_n=v_n$ ***then*** $o_{n+1}=v_{n+1}$. Therefore the polynomial complexity of inference results. The reason is that every object o_i obtains a value from a finite enumeration set $\{v_{i1}, v_{i2,}, \ldots v_{im}\}$. This is a distinction from exponential complexity in Prolog, where rules are represented with predicates, e.g., $P_1(x,y)$ & $P_2(x)$ & $P_3(y) \rightarrow P_4(x,y)$. Therefore variables *x*, *y*, etc. require matching in infinite sets.

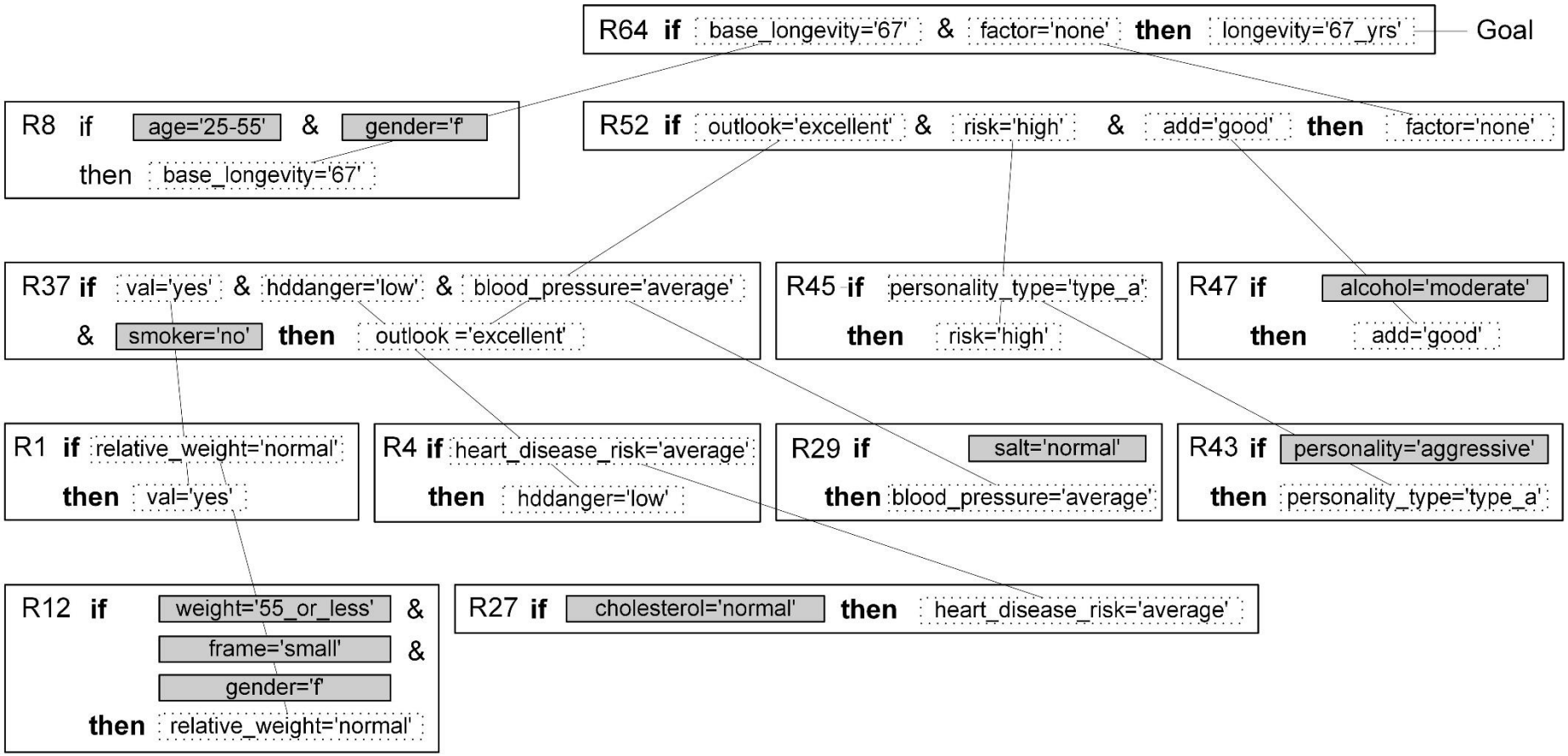


Fig. 20.5. A sample proof tree in an expert system. Facts are shown in grey rectangles. The goal longevity='67_yrs' is at the root. The longevity of 67 years is predicted for a small-framed aggressive woman (gender='f'), aged 25–55 years (age='25-55'), non-smoker (smoker='no'), consuming salt normally (salt='normal'), weight smaller than 55 kilograms (weight='55_or_less'), and normal amount of cholesterol in her blood.

21. Internet shopping

This chapter follows (Russell, Norvig 2003, p. 344–348) and adds more details.

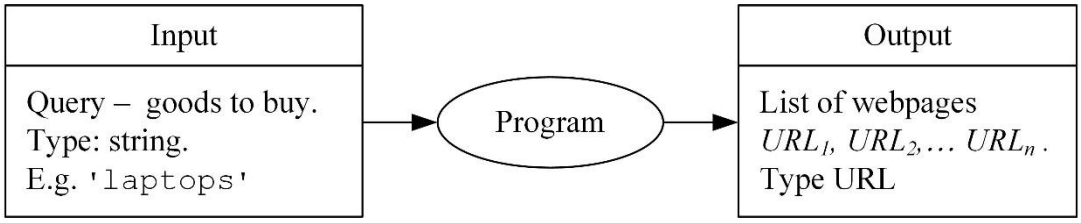


Fig. 21.1. Input-output description of an internet shopping program

The program is treated as a computer agent (Fig. 21.2); see Russell, Norvig (2003) chapter 2 Intelligent Agents.

Fig. 21.2. Input-output description of a computer agent

A typical internet shop is shown in Fig. 21.3.

Generic Online Store

Select from our fine line of products

- Computers
- Cameras
- Books
- Videos
- Music

```
<html>
<body>
<h1>Generic Online Store</h1>
<i>Select</i> from our fine line of products
<ul>
<li><a href="http://www.gen-store.com/compu">Computers</a>
<li><a href="http://www.gen-store.com/camer">Cameras</a>
<li><a href="http://www.gen-store.com/books">Books</a>
<li><a href="http://www.gen-store.com/video">Videos</a>
<li><a href="http://www.gen-store.com/music">Music</a>
</ul>
</body>
</html>
```

Fig. 21.3. A web page which is perceived in the internet and its HTML code. Suppose its address http://www.gen-store.com and named GenStore

We will use top-down refinement to formulate requirements. A computer agent is required to return URLs of internet shopping webpages.

Step 1. Predicate **RelevantOffer**

It is decomposed below. Types: page – HTML text, url – URL, query – String:

```
RelevantOffer(page,url,query) ⇔
     Relevant(page,url,query) & Offer(page)
```

Step 2. Predicate **Offer**. Either a tag "a" or "form" should contain the word 'buy' or 'price':

```
Offer(page)          ⇔    ( InTag('a',str,page) ∨
                              InTag('form',str,page) )
                        & ( In('buy',str) ∨ In('price',str) )
InTag(tag,str,page) ⇔ In('<'+tag+str+'</'+tag+'>', page)
In(sub,str)          ⇔ ∃i str[i:i+Length(sub)-1] = sub
```

For example,

```
In('KAD', 'ABRAKADABRA') = true
In('laptop', 'ABRAKADABRA') = false
'ABC' + 'DEFG' = 'ABCDEFG'
```

Step 3. Predicate **OnlineStores**(store)

The set of well-known internet shops is not empty because Amazon and eBay are the cases. Equally we would treat our GenStore. This is specified:

```
Amazon ∈ OnlineStores
           & Homepage(Amazon, 'http://www.amazon.com')
Ebay ∈ OnlineStores
           & Homepage(Ebay, 'http://www.ebay.com')
GenStore ∈ OnlineStores
           & Homepage(GenStore, 'http://www.gen-store.com')
```

Step 4. Predicate **Relevant**. It is decomposed below; see Fig. 21.4:

```
Relevant(page,url,query) ⇔ ∃ store ∃ home (
 store∈OnlineStores & Homepage(store,home)
 & ∃ url2 RelevantChain(home,url2,query)
 & Link(url2,url)& page = GetPage(url)     )
```
(21.1)

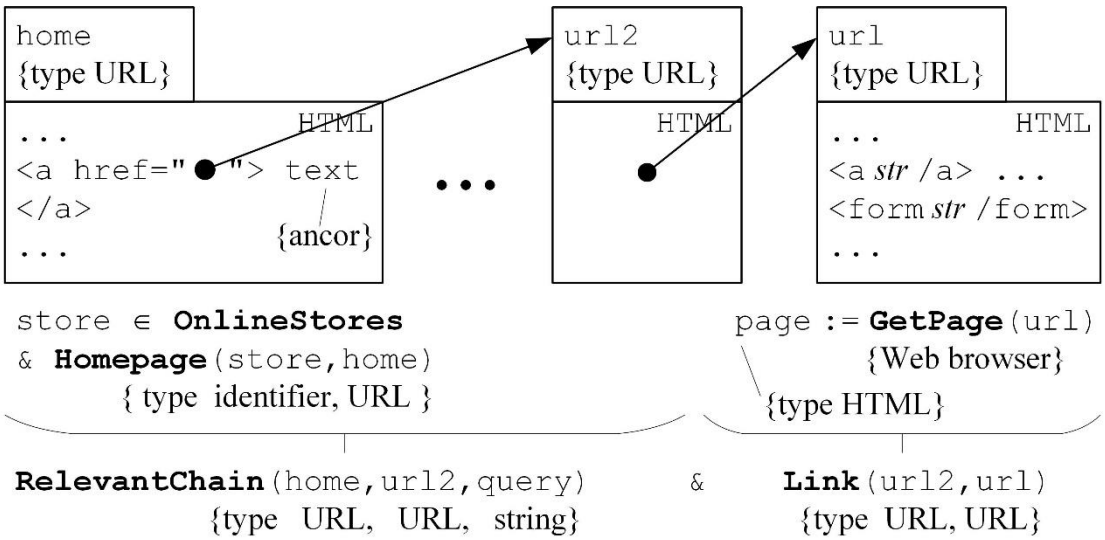


Fig. 21.4. Internet shopping specification. For example, `home = 'http://www.gen-store.com'` and `store = GenStore`

Step 5. Predicate **`RelevantChain`**.

It is decomposed below. Types: start – URL, end – URL, query – String (Fig. 21.5):

```
RelevantChain(start,end,query) ⇔ (start = end)
    ∨ ( ∃u ∃text LinkText(start,u,text)
        & RelevantCategoryName(query,text)
        & RelevantChain(u,end,query) )
```
(21.2)

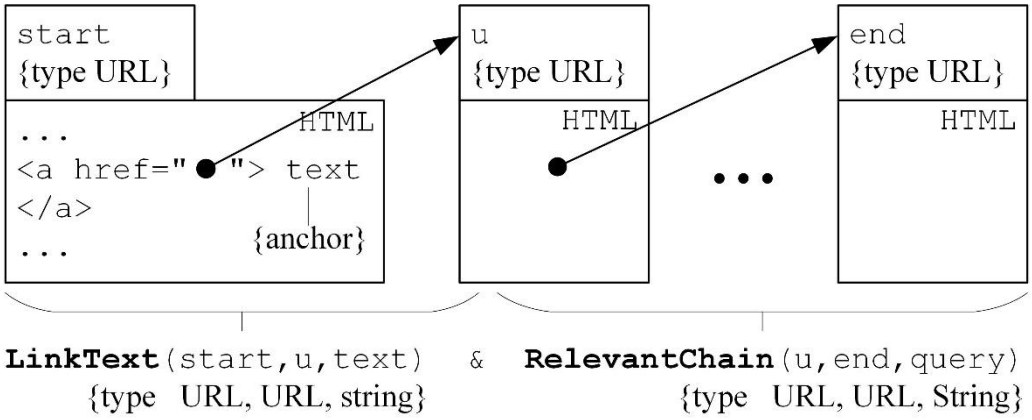


Fig. 21.5. Predicate **`RelevantChain`**`(start,end,query)`

For example, following is a specification of forward chaining:

```
ForwardChaining(s,t) ⇔ Link(s,u) & ForwardChaining(u,t)
```

Following is a specification of backward chaining:

```
BackwardChaining(s,t) ⇔ BackwardChaining(s,v) & Link(v,t)
```

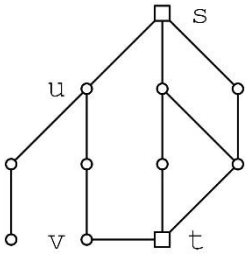


Fig. 21.6. Forward chaining and backward chaining algorithms can be used for path searching in a graph

Step 6. Assigning words to categories

Recall **`RelevantChain`**`(start,end,query)` (21.2):

`LinkText``(start,u,text) &` **`RelevantChain`**`(u,end,query)`

Here, for example, `text` obtains value `'Computers'`, and `query` – `'I need laptops'`.

A supposed is-a categorisation of products is shown in Fig. 21.7.

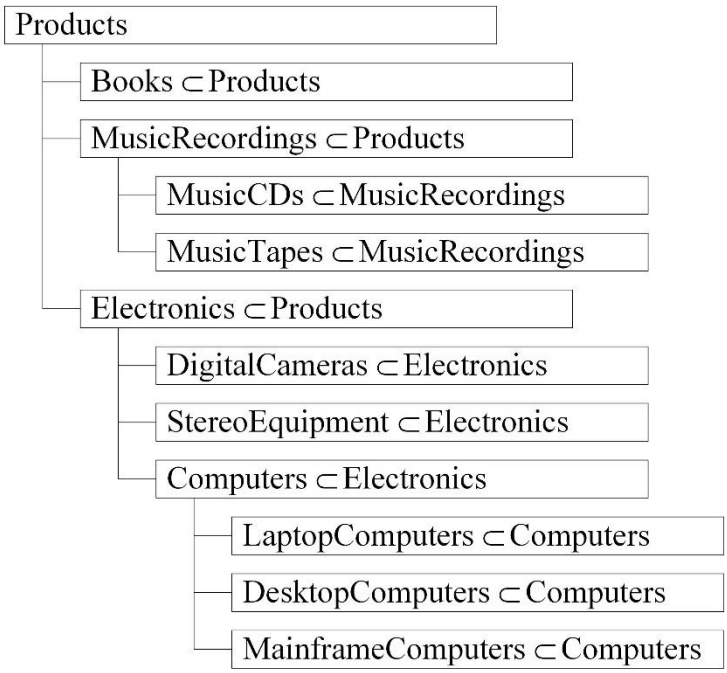


Fig. 21.7. Product hierarchy

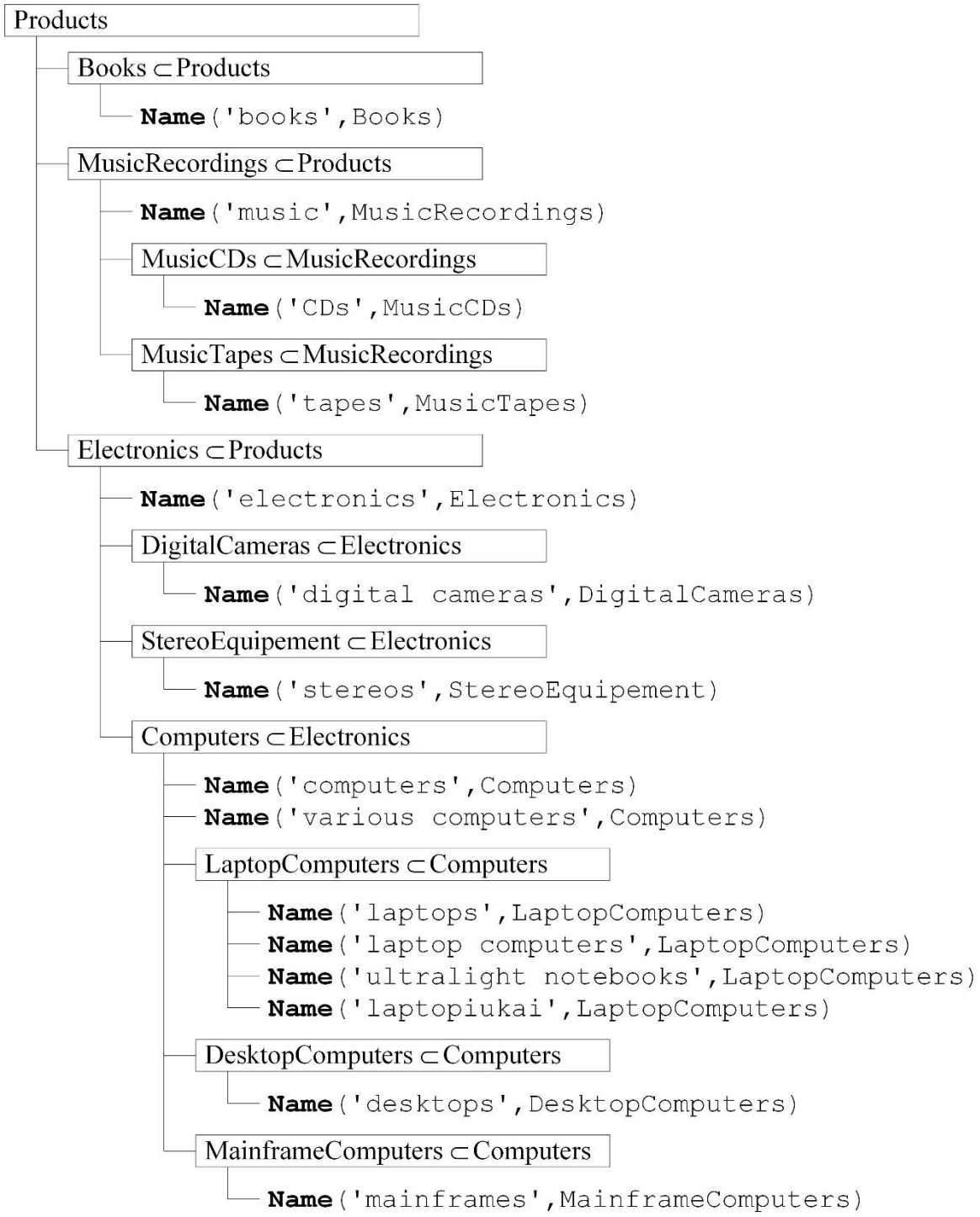


Fig. 21.8. Assigning words to categories

Step 7. Predicate **RelevantCategoryName**(query,text)

RelevantCategoryName(query,text) ⇔
  ∃c1 ∃c2 Name(query,c1) & Name(text,c2) & (c1=c2 ∨ c1⊂c2 ∨ c1⊃c2)

The case c1⊂c2 is shown in Fig. 21.9 a and the case c1⊃c2 in Fig. 21.9 b.

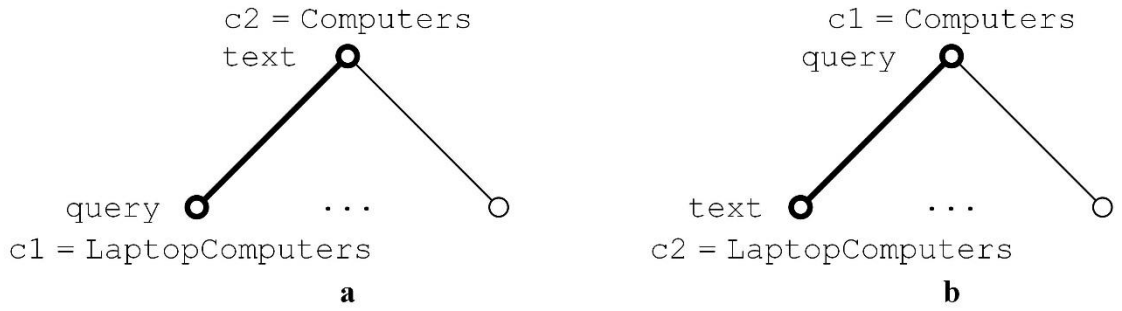


Fig. 21.9. a) The case c1⊂c2. b) The case c1⊃c2

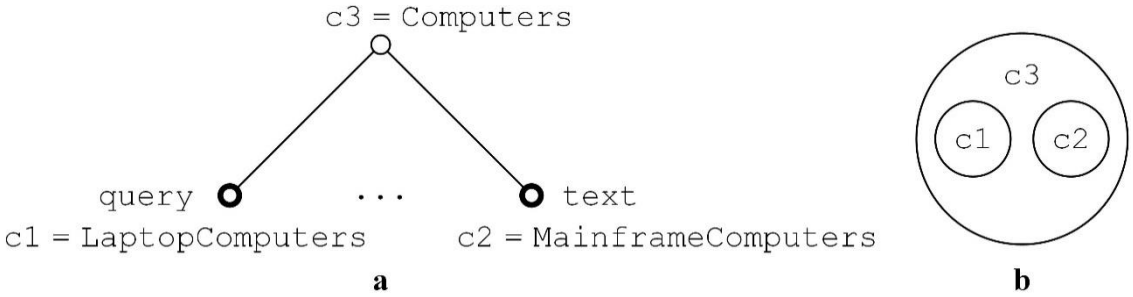


Fig. 21.10. a) The predicate is not satisfied when none of relations c1=c2, c1⊂c2 and c1⊃c2 is satisfied. b) The categories c1 are c2 not comparable

22. The Turing test

See textbooks, e.g. Luger (2005), and Wikipedia https://en.wikipedia.org/wiki/Turing_test.

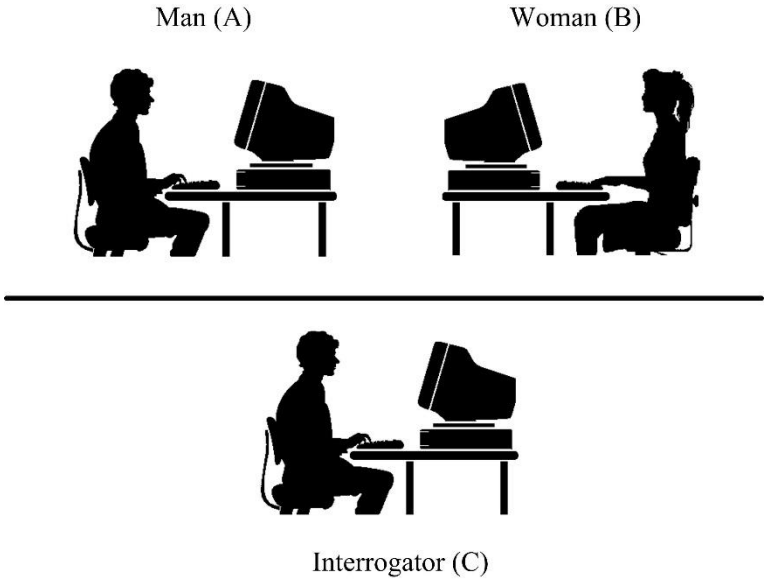


Fig. 22.1. Imitation game

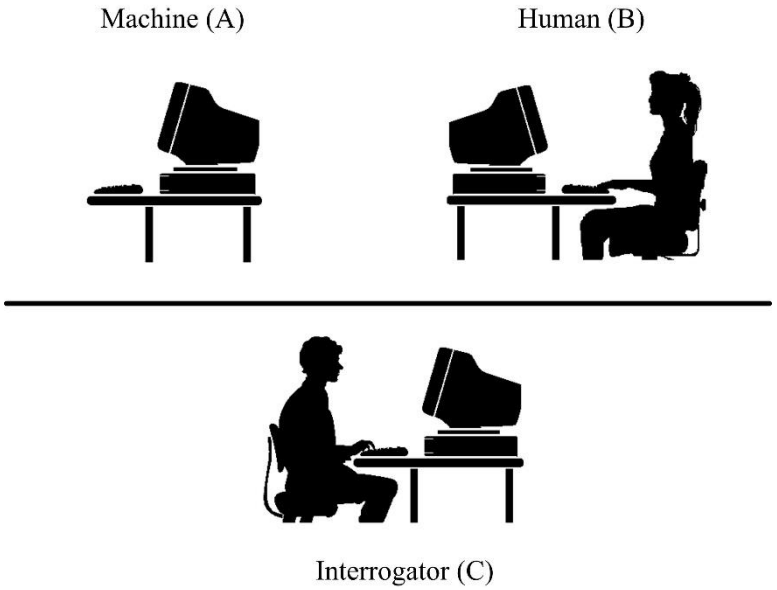


Fig. 22.2. The Turing test

A related material is John Searle's Chinese Room argument, https://en.wikipedia.org/wiki/Chinese_room); see also 1 minute film by the Open University available at https://www.youtube.com/watch?v=TryOC83PH1g. See also Weizenbaum's program Eliza, https://en.wikipedia.org/wiki/ELIZA and https://www.masswerk.at/elizabot/.

23. Intension, extension and ontology

"An intensional definition provides the meaning of an expression by specifying *necessary* and *sufficient conditions* for correct application of the expression. An intensional definition should be distinguished from an *extensional definition*, which merely provides a list of those instances in which the expression being defined is applicable. For example, we might provide an intensional definition of "bachelor" by specifying that bachelors are unmarried men. An extensional definition of bachelor, on the other hand, would consist merely of a list of those men." (Roy T. Cook. Intensional definition, in: A Dictionary of Philosophical Logic, p. 155, Edinburgh University Press, Edinburgh, 2009); see also Wikipedia, https://en.wikipedia.org/wiki/Extensional_and_intensional_definitions.

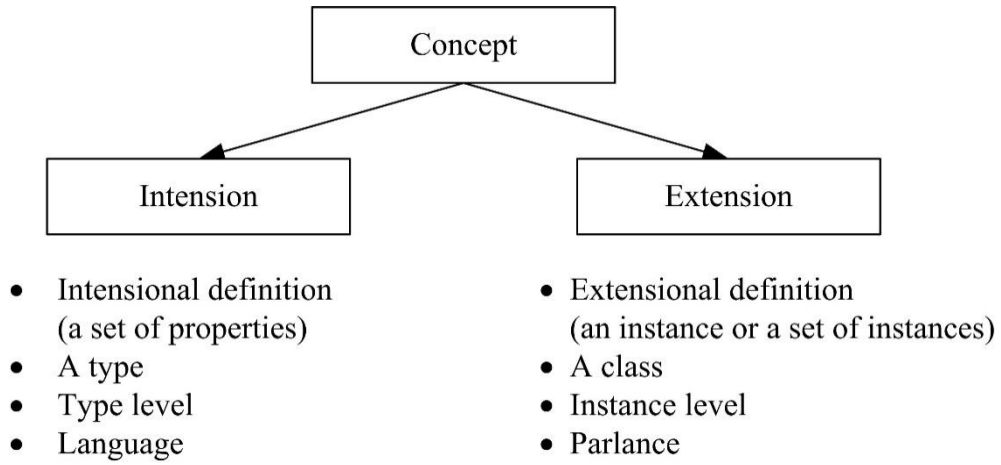


Fig. 23.1. The notion of a concept is a triad

As becomes clear, intensional definitions are best used when something has a clearly defined set of properties, and it works well for sets that are too large to list in an extensional definition. It is impossible to give an extensional definition for an infinite set, but an intensional one can often be stated concisely — there is an infinite number of even numbers, impossible to list, but they can be defined by saying that even numbers are integer multiples of two.

Definition by genus and difference, in which something is defined by first stating the broad category it belongs to and then distinguished by specific properties, is a type of intensional definition. As the name might suggest, this is the type of definition used in Linnaean taxonomy to categorize living things, but is by no means restricted to biology. Suppose we define a miniskirt as "a skirt with a hemline above the knee." We've assigned it to a genus, or larger class of items: it is a type of skirt. Then, we've described the differentia, the specific properties that make it its own sub-type: it has a hemline above the knee.

Intensional definition also applies to rules or sets of axioms that generate all members of the set being defined. For example, an intensional definition of "square number" can be "any number that can be expressed as some integer multiplied by itself." The rule—"take an integer and multiply it by itself"—always generates members of the set of square numbers, no matter which integer one chooses, and for any square number, there is an integer that was multiplied by itself to get it.

Similarly, an intensional definition of a game, such as chess, would be the rules of the game; any game played by those rules must be a game of chess, and any game properly called a game of chess must have been played by those rules; see Wikipedia, the link above.

23.1. Signs

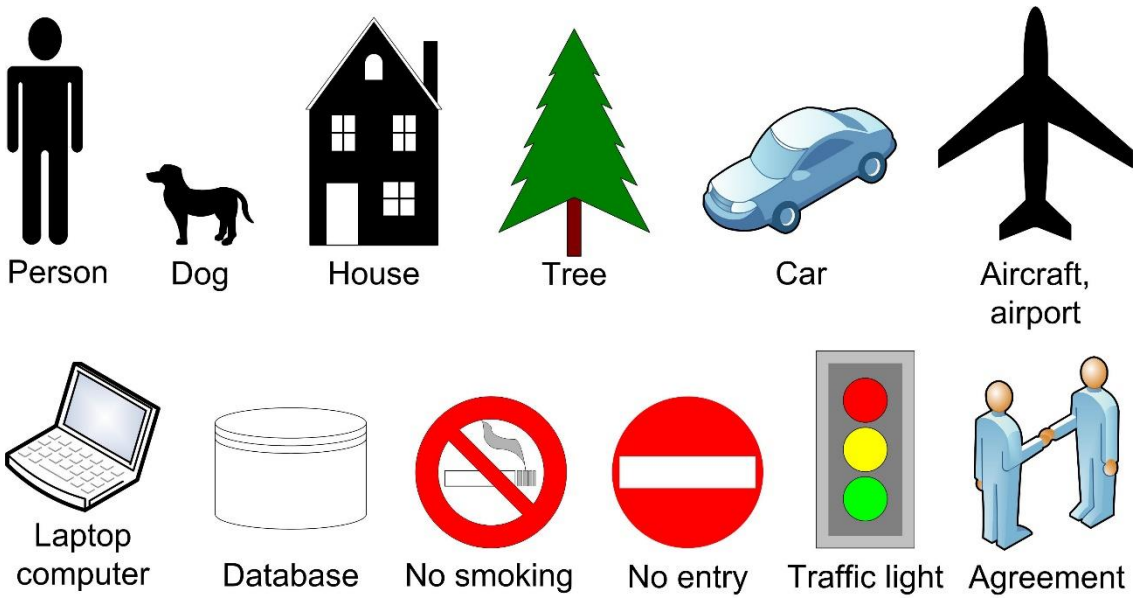


Fig. 23.2. Graphical signs

In semiotics, a *signifier* refers to a *significant*, i.e. a *meaning* (Fig. 23.3). Semiotika yra mokslas apie ženklus ir jų aiškinimą (interpretavimą).

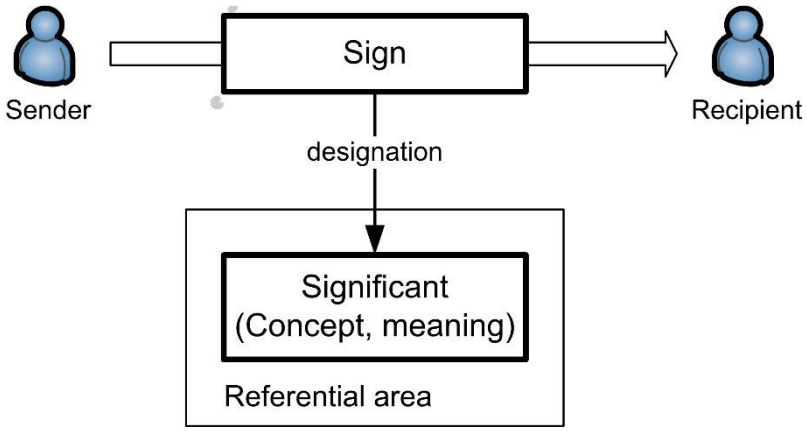


Fig. 23.3. A sign (i.e. a signifier) refers to its significant

See the problem of universals, https://en.wikipedia.org/wiki/Problem_of_universals. Example of universals: redness, cupness, homo sapiens.

Three viewpoints (Fig. 23.4):

- *universalia ante rem* (Platon)
- *universalia in re* (Aristotle)
- *universalia post rem* (nominalism, Latin *nomen* – name)

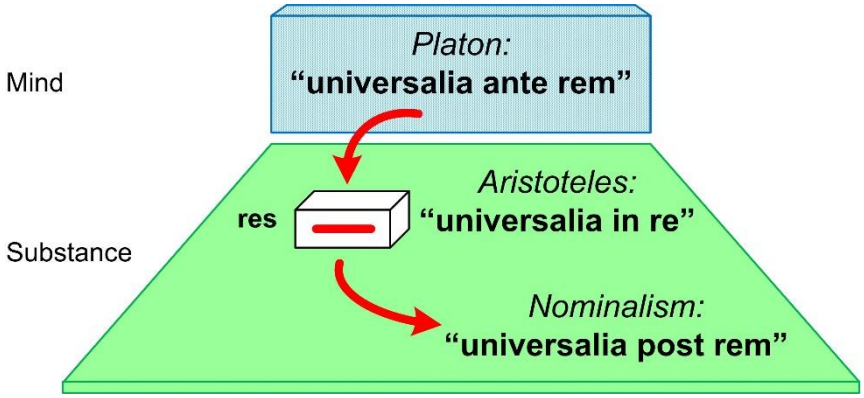


Fig. 23.4. Different viewpoints in philosophy

23.2. What is a conceptualization?

This chapter follows (Guarino et al. 2009). An ontology is a formal, explicit specification of a shared conceptualization [ibid., p. 3].

When writing about ontologies, Gruber, a predecessor of Guarino, starts with emphasis on the term "conceptualization". In his widely cited paper, Gruber (1995; see https://tomgruber.org/writing/onto-design.pdf) states: "A body of formally represented knowledge is based on a *conceptualization*: the objects, concepts, and other entities that are assumed to exist in some area of interest and the relationships that hold among them (Genesereth & Nilsson, 1987). A conceptualization is an abstract, simplified view of the world that we wish to represent for some purpose."

Definition 23.1 (Extensional relational structure $S = (D, \mathbf{R})$ **)** An extensional relational structure is a tuple (D, **R**), where

- D is a set called the universe of discourse
- **R** is a set of relations on D. (Guarino et al. 2009, definition 2.1) □

Every element of **R** is an *extensional relation*, i.e., a mathematical relation, a subset of the Cartesian product.

Example 23.2. Let us consider human resources management in a large software company with 50.000 people, each one identified by a number (e.g., the social security number, or a similar code) preceded by letter *I*. Let us assume that our universe of discourse *D* contains all these people, and that we are only interested in relations involving people. Our **R** will contain some unary relations, such as *Person*, *Manager*, and *Researcher* as well as the binary relations reports-to and cooperates-with.[5] (Guarino et al. 2009, example 2.1)

Consider an information system of the company. Let us assume that *D* contains 4 elements: $D = \{I1, I2, I3, I4\}$ (Fig. 23.5).

[5] The name of a person could also be assigned via relations, e.g., *firstname*(*I4*, '*Daniel*') and *lastname*(*I4*, '*Oberle*').

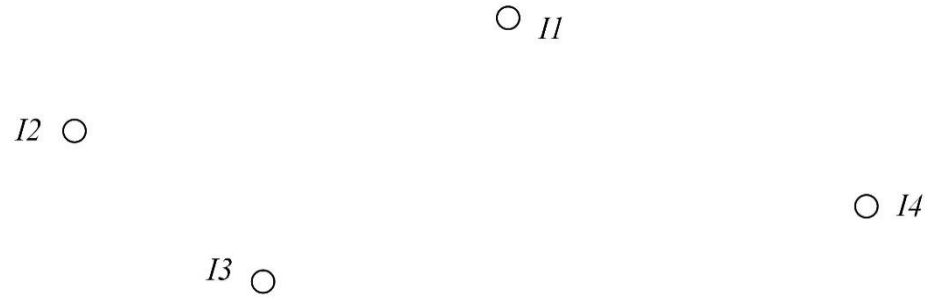


Fig. 23.5. The universe of discourse $D = \{I1, I2, I3, I4\}$

Let us assume 5 relations (5 tables):

$$\mathbf{R} = \{ \{ (I1), (I2), (I3), (I4) \},$$
$$\{ (I1) \}$$
$$\{ (I2), (I3) \}$$
$$\{ (I2, I1), (I3, I1) \}$$
$$\{ (I2, I3) \} \}$$

Let us provide names (i.e. textual symbols) to the relations:

$\mathbf{R} = \{$ *Person*, *Manager*, *Researcher*, *reports-to*, *cooperates-with* $\}$, where

Person	= { (*I1*), (*I2*), (*I3*), (*I4*) }	– unary relation
Manager	= { (*I1*) }	– unary relation
Researcher	= { (*I2*), (*I3*) }	– unary relation
reports-to	= { (*I2*, *I1*), (*I3*, *I1*) }	– binary relation
cooperates-with	= { (*I2*, *I3*) } }	– binary relation

These 5 relations can be represented graphically by 5 tables (Fig. 23.6):

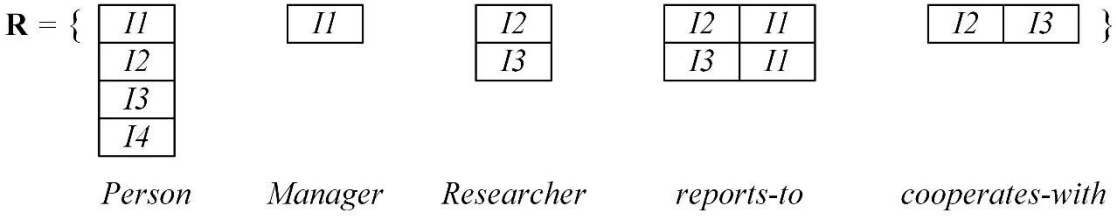


Fig. 23.6. 5 Representing 5 relations graphically by 5 tables

Example 23.3 (Guarino et al. 2009, example 2.2) Let us consider the following alteration of Example 23.2. An extensional relational structure (*D'*, **R'**) is supplemented with the edge (*I1*, *I4*):

- *D'* = *D*
- **R'** = $\{$ *Person*, *Manager*, *Researcher*, *reports-to'*, *cooperates-with* $\}$, where *reports-to'* = *reports-to* $\cup$ {(*I1*, *I4*)}

Hence (*D'*,**R'**) $\neq$ (*D*,**R**), although *reports-to'* is supplemented with one line (Fig. 36.7).

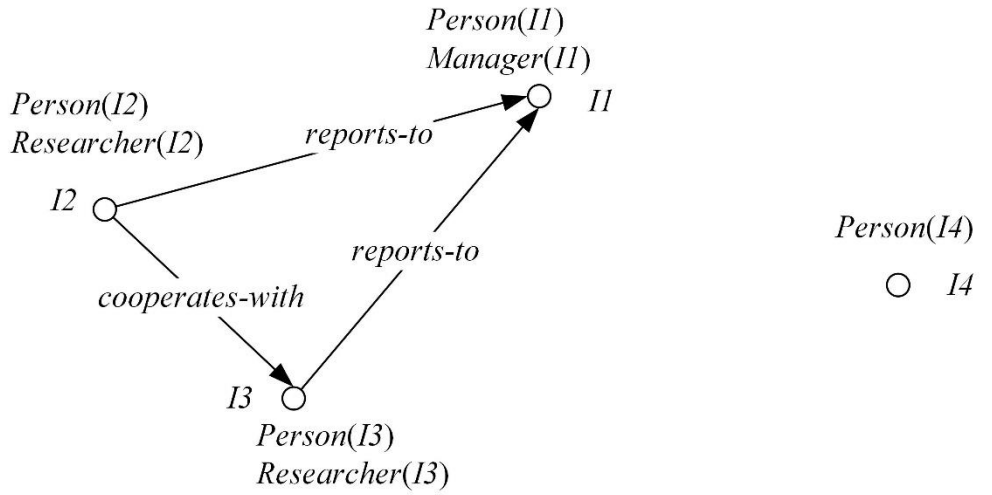


Fig. 23.7. An extensional relational structure from Example 23.2. It corresponds to a specific state of the world. This is a tiny part of a specific world with persons, managers, researchers, and their relationships in the running example of human resources in a large software company. Adapted from (Guarino et al. 2009, fig. 1)

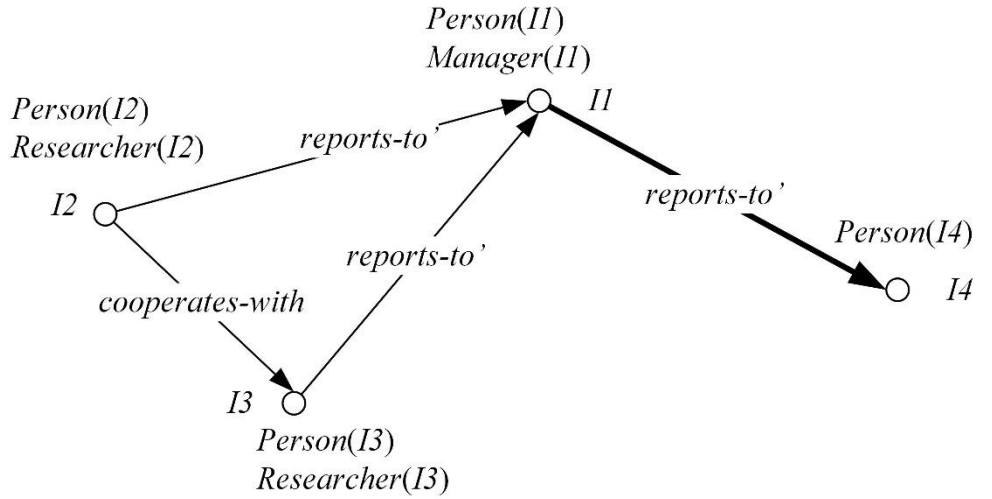


Fig. 23.8. The extensional relational structure (*D'*, **R'**) in Example 23.6. The relation *reports-to'* is supplemented with the edge (*I1*, *I4*). This corresponds to another state of the world

Definition 23.4 (World *W*). A world is a totally ordered set of world states:

$$W = \{w_1, w_2, w_3 \ldots\}$$ □

A contrast of total ordering can be illustrated with a partially ordered set. Consider a set $W = \{w_1, w_2, w_3, w_4\}$ with relations $\{w_1<w_2, w_1<w_3, w_2<w_4, w_3<w_4\}$ (Fig. 23.9). There is no relation between w_2 and w_3 – neither $w_2<w_3$ nor $w_2>w_3$ – which are called *non-comparable*.

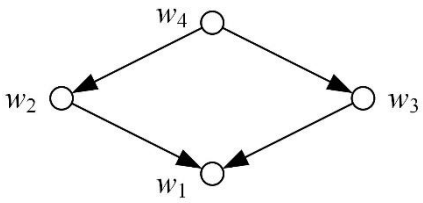


Fig. 23.9. A partially ordered set $W = \{w_1, w_2, w_3, w_4\}$

Definition 23.5 (Intensional relation, or conceptual relation $\rho^{(n)}$). An intensional relation (or conceptual relation) $\rho^{(n)}$ of arity n on (D, W) is a total function $W \rightarrow powerset(D^n)$ from the set W into the set of all n-ary extensional relations on D. In other words, $\rho^{(n)}$: $w_i \rightarrow \rho^{(n)}(w_i)$, i.e. $\rho^{(n)}$ maps a world's state w_i into $\rho^{(n)}(w_i)$, a subset of the Cartesian product D^n (w_i) – a set of tuples from D^n, in other words, an extensional relational structure. (Guarino et al. 2009, definition 2.3) □

Here *powerset*(A) denotes a set of subsets of A. Also denoted 2^A. For instance, suppose $A = \{a, b\}$. Then $powerset(A) = 2^A = \{\ \{\}, \{a\}, \{b\}, \{a, b\}\ \}$.

Definition 23.6 (Intensional relational structure C = (D, W, $\mathcal{R}$), or conceptualization) An intensional relational structure (or a conceptualization) is a triple **C** = (D, W, $\mathcal{R}$) with

- D a universe of discourse
- W a world, i.e. a set of world's states
- $\mathcal{R}$ a set of intensional relations on the domain space (D, W)

(Guarino et al. 2009, definition 2.3) □

Example 23.7 (Guarino et al. 2009, example 2.3) Coming back to Examples 23.2 and 23.3, we can see them as describing two different world's states compatible with the following intensional relational structure **C** = (D, W, $\mathcal{R}$) (Fig. 23.10):

- $D = \{I1, I2, I3, I4\}$
- $W = \{w_1, w_2, w_3 \ldots\}$
- $\mathcal{R} = \{Person^{(1)}, Manager^{(1)}, Researcher^{(1)}, reports\text{-}to^{(2)}, cooperates\text{-}with^{(2)}\}$
- $\forall i=1,2,3\ldots \quad Person^{(1)}(w_i) = \{(I1), (I2), (I3), (I4)\}$
- $\forall i=1,2,3\ldots \quad Manager^{(1)}(w_i) = \{(I1)\}$
- $\forall i=1,2,3\ldots \quad Researcher^{(1)}(w_i) = \{(I2), (I3)\}$
- $\forall i=1,2,3\ldots \ reports\text{-}to^{(2)}(w_{2i-1}) = \{(I2, I1), (I3, I1)\}$ – 2 edges in odd states
- $\forall\ i=1,2,3\ldots \ reports\text{-}to^{(2)}(w_{2i}) = \{(I2, I1), (I3, I1), (I1, I4)\}$ – 3 edges in even states
- $\forall\ i=1,2,3\ldots \ cooperates\text{-}with^{(2)}(w_i) = \{(I2, I3)\}$

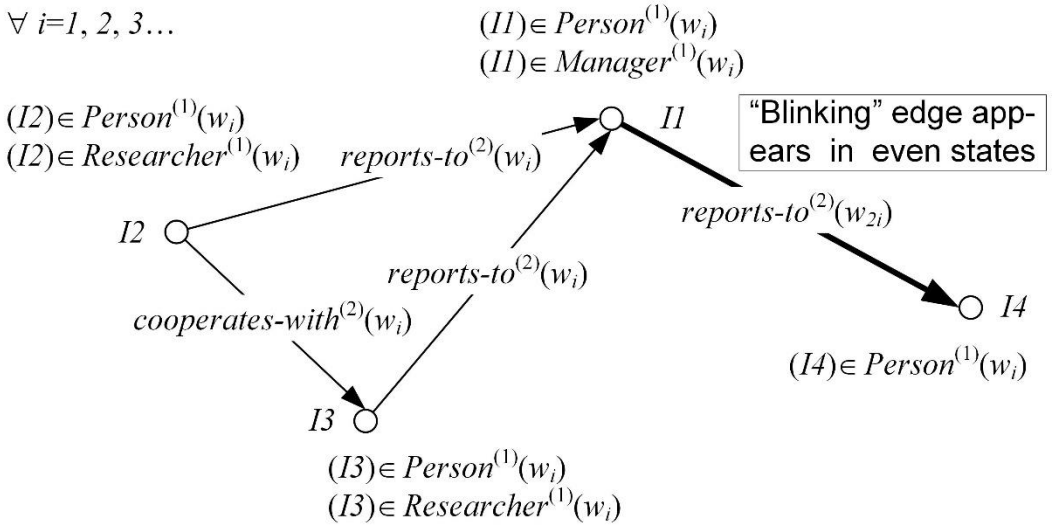


Fig. 23.10. The intensional relational structure in Example 23.7. The intensional relations *Person*$^{(1)}$, *Manager*$^{(1)}$, *Researcher*$^{(1)}$ and *cooperates-with*$^{(2)}$ do not change, but *reports-to*$^{(2)}$ changes – the edge (*I1*, *I4*) blinks

23.3. What is a proper formal, explicit specification?

In practical applications, as well as in human communication, we need to use a language to refer to the elements of a conceptualization: for instance, to express the fact that *I2* cooperates with *I3*, we have to introduce a specific symbol (formally, a predicate symbol, say '*cooperates-with*', which, in the user's intention, is intended to represent a certain conceptual relation. We say in this case that our language (let us call it **L**) *commits* to a conceptualization. Suppose now that **L** is a first-order logical language, whose non-logical symbols (i.e., its vocabulary) is V. The vocabulary is divided into two sets: 1) *constant symbols*, e.g, '*I1*', '*I2*', '*I3*', '*I4*', and 2) *predicate symbols*, e.g., '*Person*', '*Manager*', '*Researcher*', '*reports-to*', '*cooperates-with*':

$$\mathbf{V} = \textit{constant symbols} \cup \textit{predicate symbols}$$

A first-order language has the following difference from higher-order languages: predicate arguments can be formed of variables and expressions, but not of predicate names. In other words, a table field cannot be a table's name. Examples of correct first-order sentences are '*reports-to*'('*I2*', '*I1*') = '*true*' and $\forall x,y$ '*reports-to*'(x,y) $\Rightarrow$ '*Person*'(x) & '*Person*'(y), but not '*reports-to*'('*Person*', '*Person*').

Definition 23.8 (Extensional first-order structure $M = (S, I)$ **).** Let us assume

- **L** a first-order logical language with vocabulary **V**
- $S = (D, \mathbf{R})$ an extensional relational structure

An **extensional first-order structure** (also called model for **L**) is a tuple $M = (S, I)$, where I (called **extensional interpretation function**) is a total function I: $\mathbf{V} \rightarrow D \cup \mathbf{R}$ that maps each vocabulary symbol of **V** to either an element of D or an extensional relation belonging to the set **R**. In other words, I maps constant symbols to D and predicate symbols to **R**, where $S = (D, \mathbf{R})$. (Guarino et al. 2009, definition 3.1) □

Definition 23.9 (Intensional first-order structure K= (C, $\mathcal{I}$), also called **ontological commitment).** Let us assume

- **L** a first-order logical language with vocabulary **V**
- **C** = (*D, W,* $\mathcal{R}$) an intensional relational structure (conceptualization)

An **intensional first-order structure** (also called ontological commitment) for **L** is a tuple **K**= (**C**, $\mathcal{I}$), where kur $\mathcal{I}$ (called **intensional interpretation function**) is a total function $\mathcal{I}$: **V** $\rightarrow D \cup \mathcal{R}$, that maps each vocabulary symbol of **V** to either an element of *D* or an intensional relation belonging to the set $\mathcal{R}$. In other words, $\mathcal{I}$ maps constant symbols to *D* and predicate symbols to $\mathcal{R}$, where **C** = (*D, W,* $\mathcal{R}$). (Guarino et al. 2009, definition 3.2) □

Example 23.10 (Guarino et al. 2009, example 3.1) Let us come back to the extensional relational structure in our Example 23.2. An intensional first-order structure is as follows:

V = {'*I1*', '*I2*', '*I3*', '*I4*'} ∪
{'*Person*', '*Manager*', '*Researcher*', '*reports-to*', '*cooperates-with*'}

$\mathcal{I}$: '*I1*' → *I1*, '*I2*' → *I2*, '*I3*' → *I3*, '*I4*' → *I4*,
'*Person*' → *Person*$^{(1)}$, '*Manager*' → *Manager*$^{(1)}$, '*Researcher*' → *Researcher*$^{(1)}$,
'*reports-to*' → *reports-to*$^{(2)}$, '*cooperates-with*'→ *cooperates-with*$^{(2)}$

Definition 23.11 (Intended models $\mathbf{I_K}$(L)). Let us assume

- **C** = (*D, W,* $\mathcal{R}$) an intensional relational structure (conceptualization)
- **L** a first-order logical language with vocabulary **V**
- **K**= (**C**, $\mathcal{I}$) an intensional first-order structure (ontological commitment)

A model *M* = (*S, I*), with *S* = (*D,* **R**), is called an **intended model** of **L** according to **K** iff

1. For all constant symbols *c*∈**V** we have *I*(*c*) = $\mathcal{I}$(*c*)
2. There exists a world *w*∈*W* such that, for each predicate symbol *v*∈**V** there exists an intensional relation $\rho^{(n)} \in \mathcal{R}$ such that $\mathcal{I}(v) = \rho^{(n)}$ and $I(v) = \rho^{(n)}(w)$

The set $\mathbf{I_K}$(**L**) of all models of **L** that are compatible with **K** is called the **set of intended models** of **L** according to **K**. (See Guarino et al. 2009, definition 3.3) □

In Example 23.7, for instance, we have for w_1 (Fig. 23.11):

I('*Person*') = { (*I1*), (*I2*), (*I3*), (*I4*) } = *Person*$^{(1)}$ (w_1)
I('*reports-to*') = { (*I2*, *I1*), (*I3*, *I1*) } = *reports-to*$^{(2)}$ (w_1)
etc.

Definition 23.12 (Ontology $\mathbf{O_K}$). Let us assume

- **C** = (*D, W,* $\mathcal{R}$) an intensional relational structure (conceptualization)
- **L** a first-order logical language with vocabulary **V**
- **K**= (**C**, $\mathcal{I}$) an intensional first-order structure (ontological commitment)

An ontology $\mathbf{O_K}$ for **C** with vocabulary **V** and ontological commitment **K** is a logical theory consisting of a set of formulas of **L**, designed so that the set of its models approximates as well as possible the set of intended models $\mathbf{I_K}$(**L**) of **L** according to **K**.
(Guarino et al. 2009, definition 3.4) □

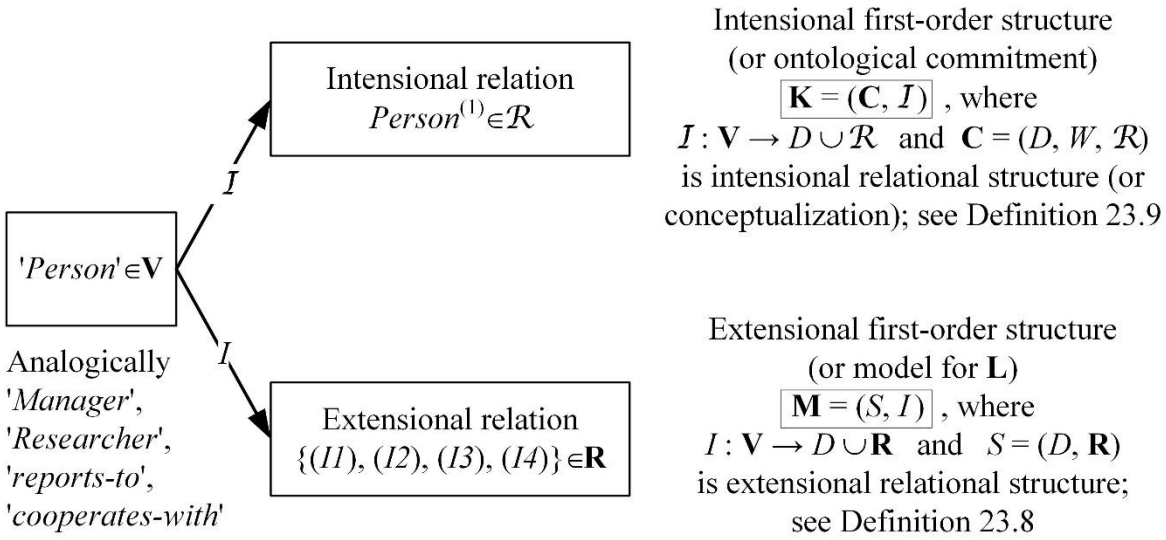


Fig. 23.11. The predicate symbol '*Person*' has both an extensional interpretation (through the usual notion of model, or extensional first-order structure) and an intensional interpretation (through the notion of ontological commitment, or intensional first-order structure). Adapted from (Guarino et al. 2009, Fig. 3)

Example 23.13. In the following we build an ontology $\mathbf{O_K}$ consisting of a set of logical formulae. Through O_1 to O_5 we specify our human resources domain (see Examples 23.2 and 23.7) with increasing precision. (Guarino et al. 2009, example 3.2)

- Taxonomic information. We start our formalization by specifying that the concepts *Researcher* and *Manager* are sub-concepts of *Person*:

$$O_1 = \{\ \text{'}Researcher\text{'}(x) \Rightarrow \text{'}Person\text{'}(x),\ \text{'}Manager\text{'}(x) \Rightarrow \text{'}Person\text{'}(x)\ \}$$

- Domains and ranges. We continue by adding formulae to O_1 which specify the domains and ranges of the binary relations:

$$O_2 = O_1 \cup \{\ \text{'}cooperates\text{-}with\text{'}(x, y) \Rightarrow \text{'}Person\text{'}(x)\ \&\ \text{'}Person\text{'}(y),\ \text{'}reports\text{-}to\text{'}(x, y) \Rightarrow \text{'}Person\text{'}(x)\ \&\ \text{'}Person\text{'}(y)\ \}$$

- Symmetry. *cooperates-with* can be considered a symmetric relation:

$$O_3 = O_2 \cup \{\ \text{'}cooperates\text{-}with\text{'}(x,y) \Leftrightarrow \text{'}cooperates\text{-}with\text{'}(y, x)\ \}$$

- Transitivity. Although arguable, we specify *reports-to* as a transitive relation:

$$O_4 = O_3 \cup \{\ \text{'}reports\text{-}to\text{'}(x, z) \Leftarrow \text{'}reports\text{-}to\text{'}(x, y)\ \&\ \text{'}reports\text{-}to\text{'}(y, z)\ \}$$

- Disjointness. There is no *Person* who is both a *Researcher* and a *Manager*:

$$O_5 = O_4 \cup \{\ \text{'}Manager\text{'}(x) \Rightarrow \neg\text{'}Researcher\text{'}(x)\ \}$$

The more formulae an ontology contains, the more constraints are concerned. Therefore, $\mathbf{I_K(L)}$ contains less intended models (Fig. 23.12). In other words, if $D' \subset D$ and $\mathcal{R}' \subset \mathcal{R}$ then $\mathbf{M}_{D'}(\mathbf{L}) \supset \mathbf{M}_{D}(\mathbf{L})$.

"We say that an agent **commits** to an ontology if its observable actions are consistent with the definitions in the ontology. […] In short, a commitment to a common ontology is a

guarantee of consistency, but not completeness, with respect to queries and assertions using the vocabulary defined in the ontology." (Gruber 1995).

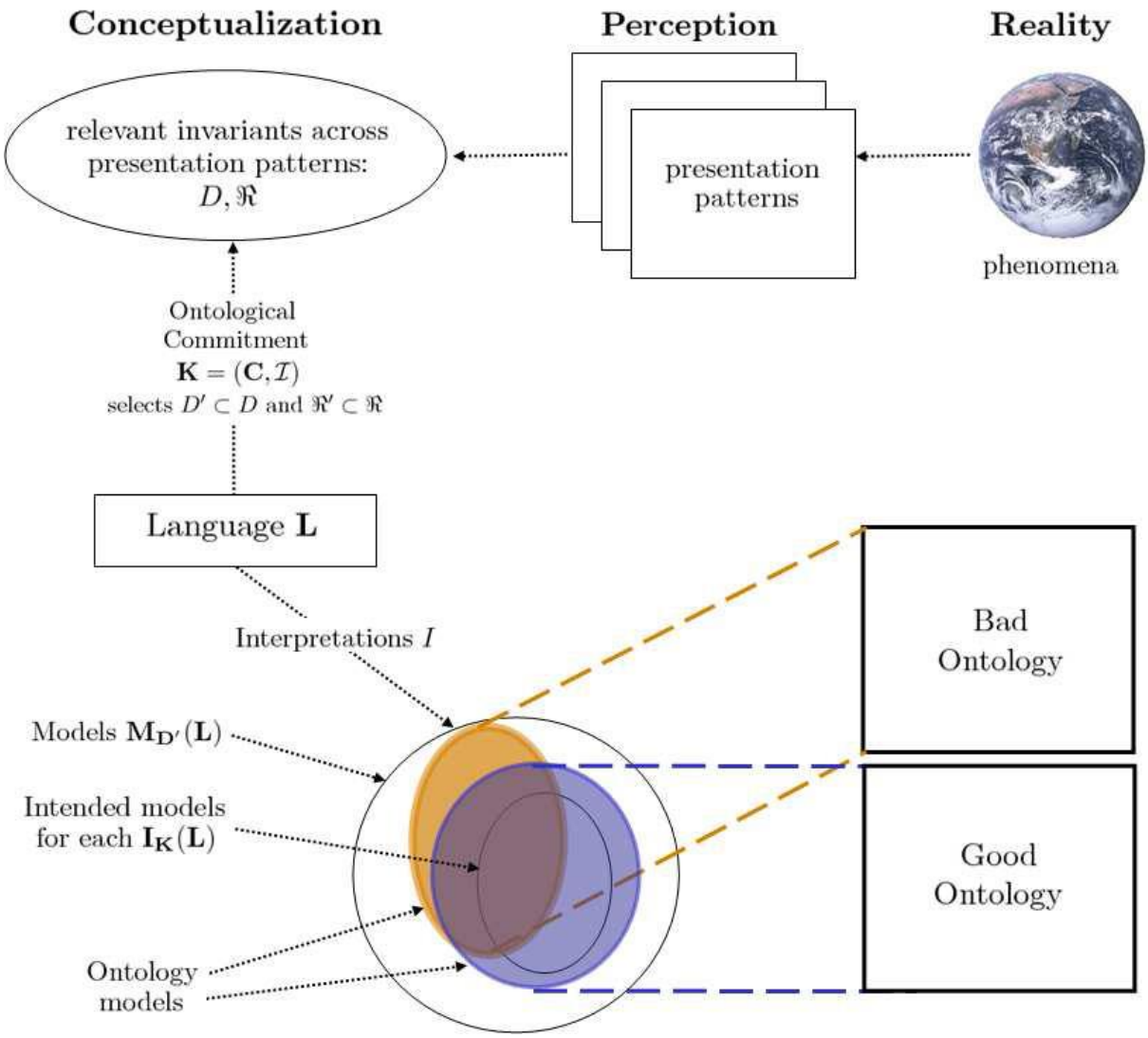


Fig. 23.12. The relationships between the phenomena occurring in reality, their perception (at different times), their abstract conceptualization, the language used to talk about such conceptualization, its intended models, and an (Guarino et al. 2009, fig. 2)

"At one extreme, we have rather informal approaches for the language **L** that may allow the definitions of terms only, with little or no specification of the meaning of the term. At the other end of the spectrum, we have formal approaches, i.e., logical languages that allow specifying rigorously formalized logical theories… Within this rightmost category one typically encounters the trade-off between expressiveness and efficiency when choosing the language **L**. On the one end, we find higher-order logic, full first-order logic, or modal logic. They are very expressive, but do often not allow for sound and complete reasoning and if they do, reasoning sometimes remains untractable. At the other end, we find less stringent subsets of first-order logic, which typically feature decidable and more efficient reasoners. They can be split in two major paradigms. First, languages from the family of *description logics (DL)*… e.g., OWL-DL", see https://www.w3.org/2001/sw/wiki/OWL for the Web Ontology Language

(OWL). “The second major paradigm comes from the tradition of *logic programming (LP)*… with one prominent representor being F-Logic” (Guarino et al. 2009, p. 12–13).

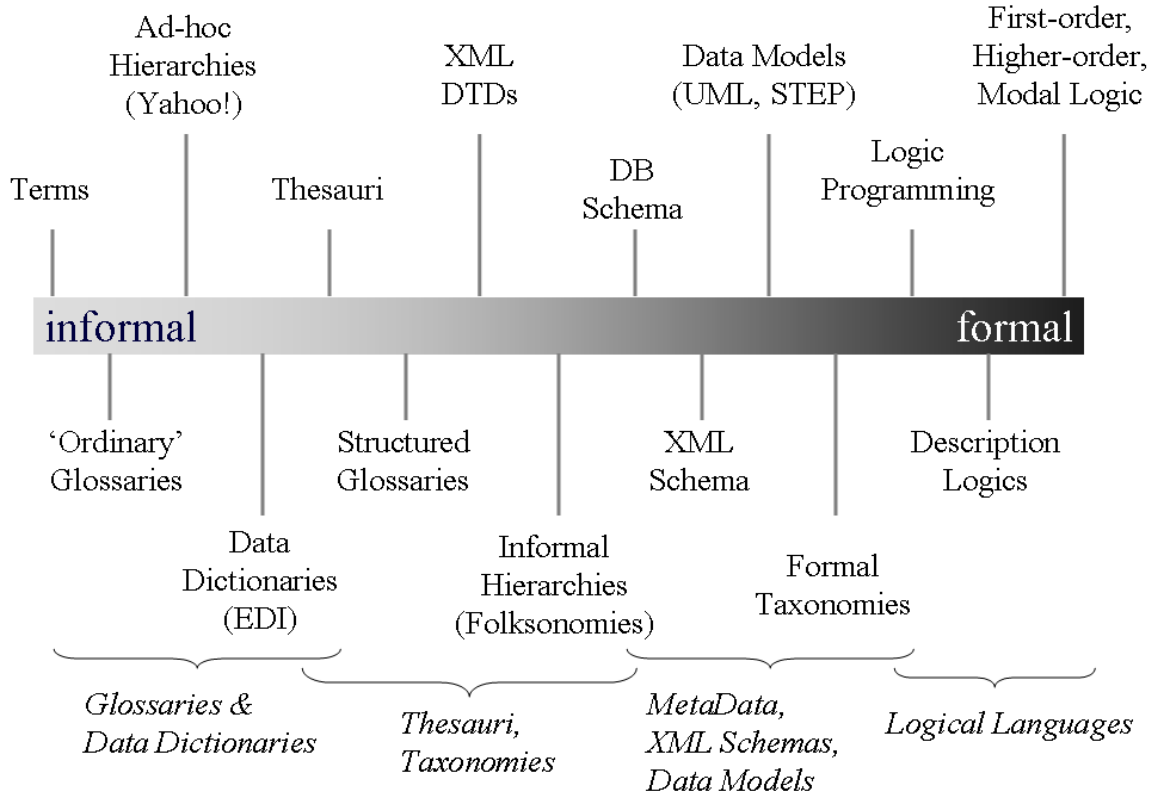


Fig. 23.13. Different approaches to the language **L**. Typically, logical languages are eligible for the formal, explicit specification, and, thus, ontologies (Guarino et al. 2009, Fig. 4)

Reference and meaning is shown in Fig. 23.14.

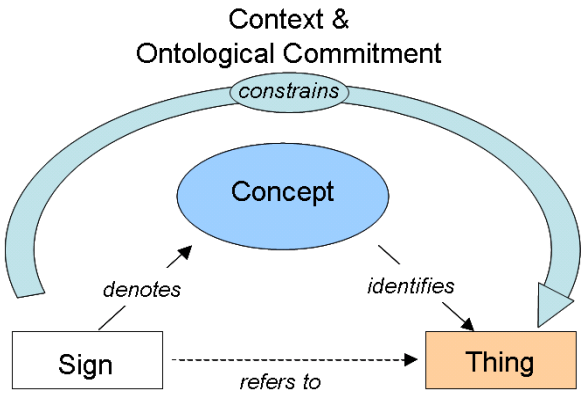


Fig. 23.14. Semiotic triangle (Guarino et al. 2009, fig. 6)

23.4. Distinct models of a specification

Following are examples of distinct models which satisfy the same set of formulas in logic.

Example 23.11 (Loeckx et al. 1996, example 2.6, p. 29). Consider a language **L** whose alphabet **V** consists of one constant symbol '*0*' and one functional symbol '*Succ*'. The latter is the name of arity *1* (one argument) operation '*Succ*': _ → _. Both are syntactic objects (i.e. names); no semantics is assigned to them yet.

$$\mathbf{V} = \{\text{'}0\text{'}\} \cup \{\text{'}Succ\text{'}\}$$

An example of a term in **L** is '*Succ*'('*Succ*'(*x*)). A set of ground terms (terms without free variables) of **L** is

$$\{\text{'}0\text{'},\ \text{'}Succ\text{'}(\text{'}0\text{'}),\ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'})),\ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'}))),\ldots\}$$

The notion of a term is defined inductively. Constant symbols and variables are terms. If 'f' is an operation of arity k, $k \geq 1$, and $t_1,\ldots,t_k$ are terms then 'f'($t_1,\ldots, t_k$) is a term. A term can be represented as a tree. Branches occur at the vertices of operations of arity *2* or greater.

Let an ontology be an empty set of formulas $O = \varnothing$. Three models of **L** are described below.

Model *Nat*. The "classical" model is a set of natural numbers $D = Nat = \{0, 1, 2, 3,\ldots\}$. Here the successor function is interpreted as adding *1* to its argument:

$$\mathcal{I}_{Nat}(\text{'}Succ\text{'})(n) = n + 1$$

The interpretation $\mathcal{I}_{Nat}$ maps terms (syntactic objects) to semantic objects – natural numbers:

$$\mathcal{I}_{Nat}:\ \text{'}0\text{'} \to 0,\ \ \text{'}Succ\text{'}(\text{'}0\text{'}) \to 1,\ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'})) \to 2,\ \ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'}))) \to 3,\ldots$$
$$\text{'}Succ\text{'} \to \_ + 1$$

Model *Bool*. Next model is a Boolean set $D = Bool = \{true, false\}$. The constant '*0*' is interpreted as *false*. The successor function is interpreted as "flipping" the argument:

$$\mathcal{I}_{Bool}(\text{'}Succ\text{'})(x) = \begin{cases} false & if\ x = true \\ true & if\ x = false \end{cases}$$

The interpretation function $\mathcal{I}_{Bool}$ maps:

$$\mathcal{I}_{Bool}:\ \text{'}0\text{'} \to false,\ \ \text{'}Succ\text{'}(\text{'}0\text{'}) \to true,\ \ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'})) \to false,$$
$$\text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'}))) \to true,\ldots$$

Model *C*. Next model is a set of one element, $D = D_C = \{\#\}$. (This character is called hash.) The constant '*0*' is interpreted as #. The successor function maps # to #:

$$\mathcal{I}_C(\text{'}Succ\text{'})(\#) = \#$$

The interpretation function $\mathcal{I}_C$ maps:

$$\mathcal{I}_C:\ \text{'}0\text{'} \to \#,\ \ \text{'}Succ\text{'}(\text{'}0\text{'}) \to \#,\ \ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'})) \to \#,\ \ \text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}Succ\text{'}(\text{'}0\text{'}))) \to \#,\ldots$$

Example 23.12 (Loeckx et al. 1996, example 5.24 iv, p. 87).). Consider a language **L** whose alphabet **V** consists of one constant symbol '*0*' and one functional symbol '+':

$$\mathbf{V} = \{\text{'}0\text{'}\} \cup \{\text{'+'}\}$$

Let an ontology *O* consist of four formulas:

$$\begin{aligned} O = \{ \quad & x \text{ '+' } (y \text{ '+' } z) = (x \text{ '+' } y) \text{ '+' } z, \\ & x \text{ '+' } \text{'}0\text{'} = x, \\ & \text{'}0\text{'} \text{ '+' } x = x, \\ & \forall x \, \exists y \, (\, x \text{ '+' } y = \text{'}0\text{'} \, \& \, y \text{ '+' } x = \text{'}0\text{'} \,) \, \} \end{aligned} \tag{23.1}$$

The first formula expresses a property known as associativity. The second and third formula are about the neutral element '*0*' for addition operation '+'. The forth formula is about the inverse element *y*. All formulas in an ontology serve as axioms that specify a model.

Any group (an algebraic structure in abstract algebra) is a model for the ontology (23.1). Below we distinguish between infinite groups and finite groups.

Model *Z*. The "classical" model is a set of integer numbers $D = Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. Here the addition operation '+' is interpreted as addition of integers:

$$\mathcal{I}_Z (\text{'+'}) (n,m) = n + m$$

The existence of the inverse element *y* in the fourth formula in (23.1) can be replaced with a Skolem term operation '*Inv*'(*x*). This replacement is called Skolemization. The interpretation $\mathcal{I}_Z$ maps '*Inv*' terms to invocations of an arity *1* function *minus* (and denoted –) for integers:

$$\mathcal{I}_Z (\text{'}Inv\text{'}) (n) = -n$$

Integer numbers satisfy all four formulas in the ontology *O* (23.1) above. For all $x, y \in Z$ it is true that $x + (y + z) = (x + y) + z$, $x + 0 = x$, $0 + x = x$, and the inverse element of *x* is $-x$, where $-(-x) = x$.

Model Z_n. A model is set of integers $D = Z_n = \{0, 1, 2, \ldots, n-1\}$. They form a finite group called remainder group. Here the addition operation '+' is interpreted as addition modulo *n*:

$$\mathcal{I}_{Zn} (\text{'+'}) (l,m) = (l + m) \; modulo \; n \qquad \text{or simply } l \oplus_n m$$

In the case $n=3$ we have $Z_3 = \{0, 1, 2\}$ and

$$\begin{aligned} &(0 + 0) \; modulo \; 3 = 0 \qquad \text{or } 0 \oplus_3 0 = 0 \\ &(0 + 1) \; modulo \; 3 = 1 \qquad \text{or } 0 \oplus_3 1 = 1 \\ &(0 + 2) \; modulo \; 3 = 2 \qquad \text{or } 0 \oplus_3 2 = 2 \\ &(1 + 2) \; modulo \; 3 = 0 \qquad \text{or } 1 \oplus_3 2 = 0 \\ &(2 + 2) \; modulo \; 3 = 1 \qquad \text{or } 2 \oplus_3 2 = 1 \end{aligned}$$

In the case $n=2$ we have $Z_2 = \{0, 1\}$ and write $0 \oplus 0 = 0$, $0 \oplus 1 = 1$, $1 \oplus 1 = 0$.

In modular arithmetic, two integers are added and then the sum is divided by a positive integer called the *modulus*. The result of modular addition is the remainder of that division. For any modulus, *n*, the set of integers from *0* to $n - 1$ forms a group under modular addition: the inverse of any element *a* is $n - a$, and *0* is the identity element. This is familiar from the addition

of hours on the face of a clock: if the hour hand is on *9* and is advanced *4* hours, it ends up on *1*. This is expressed by saying that *9 + 4* equals *1* "modulo 12" or, in symbols, *9 + 4 ≡ 1 modulo 12*. The group of integers modulo *n* is written Z_n; see https://en.wikipedia.org/wiki/Group_(mathematics).

Example 23.13 (Loeckx et al. 1996, example 5.24 iv, p. 87). Consider a language **L** whose alphabet **V** consists of one constant symbol '*0*' and two functional symbols, '+' and '*Succ*':

$$\mathbf{V} = \{'0'\} \cup \{'+', 'Succ'\}$$

Let an ontology consist of two formulas:

$$\begin{aligned} O = \{\ & x\ '+'\ '0' = x, \\ & 'Succ'(x\ '+'\ y)\ =\ x\ '+'\ 'Succ'(y)\} \end{aligned} \tag{23.2}$$

The first formula is about the neutral element '*0*' for addition operation '+'. The second formula is about a flowdown of '*Succ*'.

A model is constituted by natural numbers $D = \{0, 1, 2, 3, \ldots\}$. Here the successor function is interpreted as adding *1* to its argument. Arithmetical addition, +, is the second function on *D*. The interpretation function $\mathcal{I}$ maps:

$$\begin{aligned} \mathcal{I}: \quad & '0' \to 0,\ \ 'Succ'('0') \to 1,\ 'Succ'('Succ'('0')) \to 2,\ \ 'Succ'('Succ'('Succ'('0'))) \to 3, \ldots \\ & '+' \to +, \\ & 'Succ' \to \_ + 1 \end{aligned}$$

The meaning of the last line, '*Succ*' → _ + *1*, is as follows. The functional symbol '*Succ*' is mapped to arity *1* (one argument) function which adds *1* to its argument. Natural numbers satisfy both formulae in the ontology *O* (23.2): for all $x, y \in D$ it is true that $x + 0 = x$ and $(x + y) + 1 = x + (y + 1)$.

Next model is constituted by even numbers, $D = \{0, 2, 4, 6, \ldots\}$. Here the successor operation '*Succ*' is interpreted as adding *2* to its argument. The interpretation function $\mathcal{I}$ maps:

$$\begin{aligned} \mathcal{I}: \quad & '0' \to 0,\ \ 'Succ'('0') \to 2,\ 'Succ'('Succ'('0')) \to 4,\ \ 'Succ'('Succ'('Succ'('0'))) \to 6, \ldots \\ & '+' \to +, \\ & 'Succ' \to \_ + 2 \end{aligned}$$

The last line above, '*Succ*' → _ + *2*, means that the functional symbol '*Succ*' is mapped to arity *1* function which adds *2* to its argument. The set of even numbers is isomorphic to the set of natural numbers. Therefore even numbers are not considered as a model.

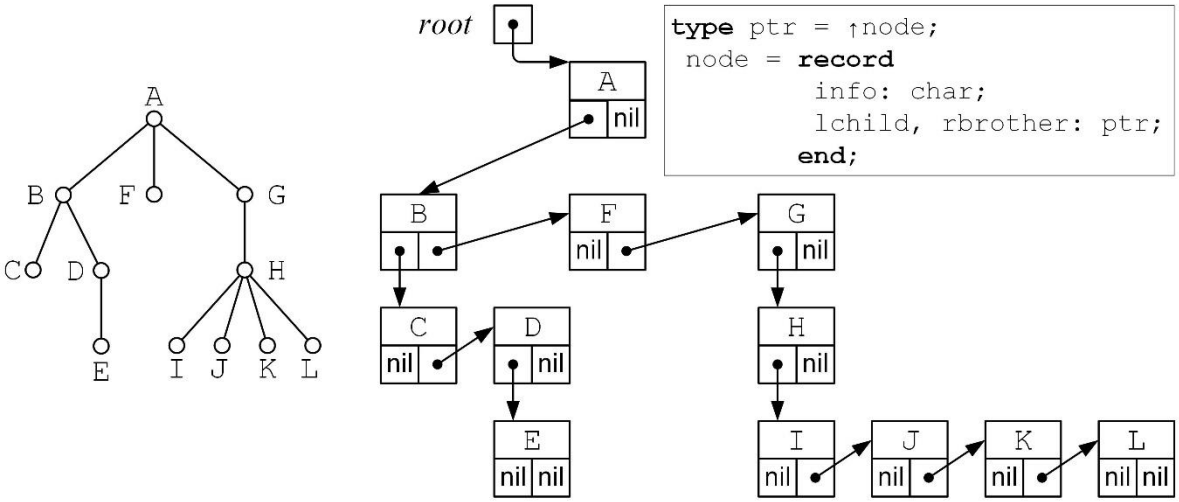
24. Examination questions

1. Artificial intelligence production system, a formalisation, PRODUCTION algorithm, examples. The concept of artificial intelligence (according to your reading).

2. BACKTRACK and BACKTRACK1 search algorithms. The concept of backtracking. Is an infinite loop possible? Examples. The concept of heuristic.

3. Depth-first search and breadth-first search in a labyrinth (write programs in pseudocode).

4. Prefix and postfix traversal of a tree. Binary trees and general trees. Write pseudocode for general trees: 1) enter a general tree, 2) prefix order traversal, and 3) postfix order traversal. Tree representations: `ABC.DE...F.GHI.J.K.L` or `A(B(CD(E))FG(H(IJKL)))`



5. Depth-first search and breadth-first search in a tree. The shortest path problem for non-weighted graphs. Provide an algorithm.

6. The shortest path problem for weighted graphs. Provide an algorithm.

7. Depth-first search in a graph. The concepts of solver and planner.

8. Procedure GRAPHSEARCH. Uniform search, heuristic search. A difference between BACKTRACK1 and GRAPHSEARH-DEPTH-FIRST. A counterexample.

9. A* search algorithm.

10. Forward chaining and backward chaining. Examples, semantic graphs, program synthesis and the complexity of inference.

11. The resolution rule. Inference examples. Forward chaining and backward chaining strategies in theorem proving. Examples of inference trees. The example with rooms 27 and 28.

12. Expert systems as artificial intelligence systems. An architecture. An example.

13. Internet shopping specification according to Russell & Norvig. Product tree as an ontology.

14. The Turing test and the philosophy of artificial intelligence (according to your reading).

15. The infeasibility of achieving several goals. The punishment problem as an example.

16. Extensional relational structure, world, intensional relation, intensional relational structure, extensional first-order structure (a model for a language), intensional first-order structure (ontological commitment), intended models, and ontology. Examples.

17. Distinct models of a specification. Examples.

25. References

Main list

1. Luger, George (2009) *Artificial intelligence: structures and strategies for complex problem solving* (6th ed.). Addison-Wesley. http://www.cs.unm.edu/~luger/. VU MIF: 004.8/Lu-59.
2. Michael Negnevitsky (2011) *Artificial intelligence: a guide to intelligent systems* (3rd ed.). Addison-Wesley, 2011. VU MIF: 004.8/Ne-44.
3. Russell, Stuart; Norvig, Peter (2010) *Artificial intelligence: a modern approach* (3rd ed.). Prentice Hall. http://aima.cs.berkeley.edu. VU MIF: 004.8/Ru-151.

Supplementary list

4. Antoniou, G.; Groth, P.; van Harmelen, F.; Hoekstra, R. (2012) *A semantic web primer* (3rd ed.). The MIT Press. VU MIF 004.8/An-154.
5. Nilsson, Nils (2010) *The quest for artificial intelligence: a history of ideas and achievements*. Cambridge University Press. VU MIF: 004.8/Ni-133.
6. Nilsson, Nils (1998) *Artificial intelligence: a new synthesis*. Morgan Kaufmann Publishers. VU MIF: 004.8/Ni-133.
7. Nilsson, Nils (1982) *Principles of artificial intelligence*. Springer, Berlin Heidelberg.
8. Sowa, John F. (2000) *Knowledge representation: logical philosophical, and computational foundations*. Brooks/Cole, Pacific Grove.

Other references

9. Bench-Capon, T. J. M.; Prakken, H. (2006) Justifying actions by accruing arguments. In: *Computational Models of Argument – Proceedings of COMMA 2006*, IOS Press, pp. 247–258. IOS Press. http://www.cs.uu.nl/groups/IS/archive/henry/action.pdf, slides http://www.cs.uu.nl/deon2006/tbc+hp.pdf.
10. E. D. Falkenberg, W. Hesse, P. Lindgreen, B. E. Nilsson, J. L. H. Oei, C. Rolland, R. K. Stamper, F. J. M. Van Assche, A. A. Verrijn-Stuart, K. Voss (1998) *A framework of information system concepts, The FRISCO Report (Web edition)*, IFIP, the Netherlands. http://www.mathematik.uni-marburg.de/~hesse/papers/fri-full.pdf.
11. Gruber, T. R. (1995) Toward principles for the design of ontologies used for knowledge sharing. International Journal of Human-Computer Studies, 43(5-6):907–928.
12. Guarino, N.; Oberle, D.; Staab, S. (2009) What is an Ontology? In: *Handbook on Ontologies*, S. Staab, R. Studer (eds.) pp. 1–17, Springer. http://iaoa.org/isc2012/docs/Guarino2009_What_is_an_Ontology.pdf.
13. Hesse, W.; A. Verrijn-Stuart, A. (2001) Towards a theory of information systems: the FRISCO approach. In: H. Jaakkola et al. (eds.*) Information Modelling and Knowledge Bases XII*, p. 81–91. IOS Press, Amsterdam.
14. Loeckx, Jacques; Ehrich, Hans-Dieter; Wolf, Markus (1996) Specification of abstract data types. Wiley-Teubner, Chichester.
15. McCarthy, John (2007) What is artificial intelligence? http://www-formal.stanford.edu/jmc/whatisai.pdf.
16. Sawyer, Brian; Foster, Dennis (1986) *Programming expert systems in Pascal*. John Wiley, New York.
17. Thayse, André; Gribomont, Pascal; Louis, Georges et al. (1988) *Approche logique de l'intelligence artificielle*. Dunot, Paris.
18. Turing, Alan M. (1950) Computing machinery and intelligence. *Mind*, 1950, vol. 59. no. 236, pp. 433–460. http://mind.oxfordjournals.org/content/LIX/236/433.