The Rise and Fall of Ideas' Popularity*

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Abstract

In the dynamic landscape of contemporary society, the popularity of ideas, opinions, and interests fluctuates rapidly. Traditional dynamical models in social sciences often fail to capture this inherent volatility, attributing changes to exogenous shocks rather than intrinsic features of the system. This paper introduces a novel, tractable model that simulates the natural rise and fall of ideas' popularity, offering a more accurate representation of real-world dynamics. Building upon the SIRS (Susceptible, Infectious, Recovered, Susceptible) epidemiological model, we incorporate a feedback mechanism that allows the recovery rate to vary dynamically based on the current state of the system. This modification reflects the cyclical nature of idea adoption and abandonment, driven by social saturation and renewed interest. Our model successfully captures the rapid and recurrent shifts in popularity, providing valuable insights into the mechanisms behind these fluctuations. This approach offers a robust framework for studying the diffusion dynamics of popular ideas, with potential applications across various fields such as marketing, technology adoption, and political movements.

Keywords: Idea diffusion; SIRS model; Popularity cycles; Google Trends analysis.

Jel Classification codes: **C61** Dynamic Analysis – **D83** Learning, Communication.

1 Introduction

The SIRS model, traditionally used in epidemiology to describe the spread of infectious diseases, can be adapted to study the diffusion of popular ideas within a population. Below is an interpretation of the Susceptible, Infectious, and Recovered states in the context of idea diffusion:

(S): **Susceptible** – Individuals in this state are not currently aware of or influenced by the popular idea. However, they are open to adopting the idea if exposed to it. These individuals can be considered potential followers or adopters.

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(I): **Infectious** – Individuals in this state have adopted the popular idea and are actively promoting it to others. They influence those in the susceptible state, encouraging them to adopt the idea. These individuals can be likened to enthusiastic supporters or advocates of the idea.

(R): **Recovered** – Individuals in this state were previously influenced by the idea but are no longer actively promoting it. They might have lost interest, become disillusioned, or moved on to other ideas. Over time, these individuals can return to the susceptible state, making them open to adopting the idea again or being influenced by new ideas.

(S): **Susceptible again** – After a period of being recovered, individuals return to the susceptible state. This reflects the cyclical nature of idea adoption, where individuals can re-engage with the same idea or become receptive to new ones over time.

The standard SIRS model converges asymptotically to a steady state. However, in reality, we observe that the popularity of ideas and fads is often subject to significant fluctuations. For example, social media trends can spike rapidly, reaching wide adoption in a short period, only to diminish just as quickly as public interest wanes (Kwak *et al.*, 2010). This phenomenon is not only observed in social media but also in fashion (Aspers and Godart, 2013), technology adoption (Rogers *et al.*, 2014), political movements (Tufekci, 2014), and other areas where ideas compete for public attention. The dynamic nature of idea popularity underscores the need for models that can capture these rapid shifts and cyclical patterns.

So, contrary to the standard model, we study a case where the parameter governing the transition from the infected to the recovered state is not constant. Instead, it depends linearly on both the number of infected individuals (positively) and the number of susceptible individuals (negatively). This interpretation allows us to capture the dynamic and recurring nature of how ideas spread and fade within a population, accounting for the repeated cycles of interest and disinterest that characterize popular ideas.

Positive dependence on the number of infected individuals implies that the more people are actively promoting the idea (infected), the faster individuals lose interest in the idea and transition to the recovered state. This could be due to oversaturation or the idea becoming less novel and interesting as more people adopt and promote it. In other words, widespread adoption and promotion can lead to a quicker decline in enthusiasm and a faster rate of disillusionment. We call this tendency *interest saturation* (see Jetten and Hornsey, 2014 for a survey of the literature in social psychology that describe deviant behaviors with respect to conformism).

Negative dependence on the number of susceptible individuals suggests that the more people are open to adopting the idea (susceptible), the slower individuals currently promoting the idea (infected) transition to the recovered state. This could be interpreted as a social dynamics effect where the presence of many potential adopters makes current promoters increase their focus, as if they had an objective function that increases in the number of successful transmissions. This is also in line with the assumptions at the basis of the economic analysis of *cultural transmission*, started by Bisin and Verdier (2001). We call this tendency *influencing enthusiasm*.

Remarkably, this model shows steady states that are fluctuating and can represent situations where the popularity of ideas is volatile, with fast cycles of adoption and abandonment influenced by social saturation and the shifting focus of the population's attention.

1.1 Relation to the Literature

The diffusion of ideas, akin to the spread of infectious diseases, has been extensively studied using epidemiological models. These models provide a framework for understanding how ideas propagate within a population, mirroring the mechanisms of infection, recovery, and reinfection.

Continuous-Time Models

Several studies have utilized continuous-time models to explore the dynamics of idea diffusion. Notably, Young (2006, 2009) employed epidemiological approaches to model the spread of innovations and behaviors within social networks. These works laid the foundation for understanding how social influence and network structures impact the adoption of ideas over time. More recently, Badr *et al.* (2021) and Chen *et al.* (2020) extended these ideas by specifically applying the SIRS (Susceptible, Infectious, Recovered, Susceptible) model to capture the cyclical nature of idea popularity. As we already discussed, the SIRS model, traditionally used to study the dynamics of infectious diseases, is well-suited to modeling phenomena where individuals can lose and later regain interest in an idea.

Discrete-Time Models

While continuous-time models provide valuable insights, most models in this domain operate in discrete time, reflecting the periodic nature of data collection and decision-making in real-world scenarios. These models often involve a discrete number of agents and typically converge to a steady-state equilibrium, where the spread of the idea stabilizes. However, several studies highlight exceptions to this pattern. Acemoğlu *et al.* (2013) demonstrated that the presence of stubborn agents—individuals resistant to changing their opinions—can prevent convergence, leading to persistent fluctuations in opinion dynamics. Similarly, the recent work by Danenberg and Fudenberg (2024) introduces a stochastic component into the model, reflecting the random nature of attention and information spread in digital environments. This addition helps to capture the unpredictable nature of idea diffusion in the context of modern social media platforms.

Conceptual Parallels

A concept akin to the “recovered” or “uninterested” individuals in the SIRS model is present in the influence campaigns studied by Sadler (2023). In this model, individuals who lose interest in a particular idea may still be susceptible to future influence, particularly as new or modified ideas emerge. This cyclical susceptibility is a key feature in understanding how ideas wax and wane in popularity, similar to the patterns observed in our extended SIRS framework.

Simulation-Based Studies

In addition to analytical models, simulation-based studies provide insights into the dynamics of idea diffusion in more complex settings. Hethcote *et al.* (1981) explored nonlinear oscillations in epidemic models, demonstrating how small changes in parameters can lead to significant fluctuations in outcomes. Khalifi and Britton (2022) extended the traditional SIRS model by allowing for gradual waning of immunity, which parallels the gradual loss of interest in an idea. These studies, though focused primarily on disease dynamics, offer valuable analogies for understanding the nonlinear and often unpredictable patterns of idea spread in social systems.

2 The model

The SIRS model is represented by a system of ordinary differential equations (ODEs) that describe the rates of change for the three states over time:

(S): **Susceptible** - The rate of change in the susceptible population (dS/dt) is positively influenced by the rate at which individuals become *open* to adopt the idea and become susceptible (ξR with $\xi > 0$), and is decreased by the rate at which susceptible individuals *adopt* actually it ($-\beta SI$ with $\beta > 0$).

$$\frac{dS}{dt} = -\beta SI + \xi R, \tag{1}$$

(I): **Infectious** - The rate of change in the population "infected" by the idea (dI/dt) is increased by the rate at which susceptible individuals adopt the idea ($+\beta SI$) and decreased by the rate at which individuals that previously adopted the idea change their mind ($-\gamma I$ with $\gamma > 0$).

$$\frac{dI}{dt} = \beta SI - \gamma I, \tag{2}$$

(R): **Recovered** - The rate of change in the population that changed their mind dR/dt is determined by the rate at which individuals infected by the idea change their mind (γI) and the rate at which individuals become open to adopt the idea again ($-\xi R$).

$$\frac{dR}{dt} = \gamma I - \xi R \tag{3}$$

The set of Equations (1), (2), and (3) define a dynamical system, which is characterized by an invariant of the dynamics, namely the total number of individuals $S + I + R = 1$ (normalized to one for convenience), allowing to reduce by one the number of equations by constraining one dependent variable, e.g. $R = 1 - S - I$. In these equations, (i) β is the transmission rate, representing the likelihood that the idea spreads from "infected" population to individuals open to change their mind; (ii) ξ is the rate at which individuals who were closed to adopt the new idea becomes open to it; finally, (iii) γ is the 'recovery' rate at which individuals change their mind and abandon the idea previously adopted. The intuition on these transition rates from one state to the other mediated by constant parameters β, ξ, and γ is crucial: the period of stay in a given state is the inverse of the rate. For example, all individuals in I at time t transition to R after a period $1/\gamma$.

The standard SIRS model predicts a steady state for the long-run dynamics, see, e.g., Anderson and May (1991). However, many real-world epidemiological examples exhibit dynamics that deviate significantly from the predictions of the SIRS model, displaying oscillations, mean reversion, and often periodic evolutions. Attempts to include such patterns in simple epidemic models in continuous time date back to 70's and proposed solutions revolve around a few key intuitions: (i) exogenous time-varying patterns, possibly seasonal, for the transmission rates as in Hethcote (1973); (ii) delay effects for the transition rates resulting in integro-differential equations as in Cooke and Kaplan (1976); (iii) more complex interactions like a nonlinear incidence for the transmission rate ($\beta S^p I^q$) as in Hethcote *et al.* (1989).

We model the coupled mechanism of *interest saturation* and *influencing enthusiasm* within the general context of the SIRS framework in a straightforward manner. Specifically, we make the recovery rate γ a dependent variable, denoted as $\Gamma(t)$, and we model a linear dependence of $\log \Gamma(t)$ on the states $I(t)$ and $S(t)$ as[1]

$$\frac{d\Gamma}{dt} = \Gamma(\alpha I - \delta S), \tag{4}$$

with $\alpha, \delta > 0$. This approach describes a recovery transition process, where the transition from infected (I) to recovered (R) happens more quickly when the number of infected individuals is high (indicating that many infected individuals lose interest in a commonly adopted idea) and more slowly when the number of susceptible individuals is high (indicating that a few infected individuals retain the idea longer to promote its spread). In other words, the state R can be considered as a reservoir that introduces a delay in the transition from I to S, while the recovery transition process acts as a feedback mechanism, which favors the less represented compartment between I and S when the other one has increased its numbers.

We demonstrate that this straightforward feedback mechanism disrupts the steady-state equilibrium, breaking the stability of the fixed-point dynamics. In a restricted version of the model ($\alpha = \delta$), we prove this happens through a Hopf bifurcation, which leads to the emergence

[1]Modeling $\log \Gamma$ in the place of Γ is convenient to ensure that the recovery rate remains positive.

of stable limit cycles. Furthermore, from a methodological standpoint, it is noteworthy that cycles can arise endogenously within one of the simplest systems of interactions. This occurs without the need for external drivers or complex delay effects, providing a very simple framework for studying periodic behavior in opinion dynamics.

2.1 Stability analysis

The model reads as the following system of ODEs

$$\frac{dS}{dt} = -\beta SI + \xi(1 - S - I), \tag{5}$$

$$\frac{dI}{dt} = \beta SI - \Gamma I, \tag{6}$$

$$\frac{d\Gamma}{dt} = \Gamma(\alpha I - \delta S), \tag{7}$$

where $\boldsymbol{X} \equiv (S, I, \Gamma) \in [0,1]^2 \times \mathbb{R}_{>0}$ and $\beta, \xi, \alpha, \delta \in \mathbb{R}_{>0}$, with the flow operator defined by

$$\Phi(\boldsymbol{X}) = \begin{pmatrix} -\beta SI + \xi(1 - S - I) \\ \beta SI - \Gamma I \\ \Gamma(\alpha I - \delta S) \end{pmatrix}. \tag{8}$$

The solutions of $\Phi(\boldsymbol{X}) = \mathbf{0}$ define the fixed points of the system. In the domain of the state vector $\boldsymbol{X}$, there exists only one solution[2]

$$\boldsymbol{X}^* = \begin{cases} S^* = \dfrac{-(\delta+\alpha) + \sqrt{(\alpha+\delta)^2 + 4\alpha\delta\frac{\beta}{\xi}}}{2\delta\frac{\beta}{\xi}}, \\ I^* = \dfrac{\delta}{\alpha} S^*, \\ \Gamma^* = \beta S^*. \end{cases} \tag{9}$$

The stability of $\boldsymbol{X}^*$ can be studied in perturbation analysis via the Jacobian of the flow operator, see, e.g., Guckenheimer and Holmes (2013).

Proposition 1. *Solution $\boldsymbol{X}^*$ for ODEs (5)–(7) is locally unstable if and only if:*

$$\begin{cases} \alpha > \beta + \xi \\ \delta \geqslant \dfrac{\alpha^2 \xi}{(\alpha-\beta)(\alpha-\beta-\xi)}. \end{cases} \tag{10}$$

.

Proof. See Appendix A.1. □

When $\boldsymbol{X}^*$ is an unstable fixed point, also known as a repeller, small perturbations away from the fixed point lead to trajectories that diverge from it over time. This instability means that the fixed point cannot act as an attractor for the system; instead, it causes nearby trajectories to be expelled in its vicinity. As a result, the system tends to evolve away from the unstable fixed point, potentially leading to either periodic, complex, or divergent dynamics. This can manifest in various ways, such as chaotic behavior, bifurcations, or transitions to other regions of the state space, significantly influencing the overall long-term behavior of the system. Figure 1 shows an example decomposition of the parameter space into stable and unstable regions. Extensive numerical simulations of the model in the region of instability support the conclusion that stable limit cycles appear, leading to a periodic evolution of the system.

We demonstrate below that a restricted version ($\alpha = \delta$) of the model undergoes a Hopf bifurcation at the crossing point to the unstable region, under a condition on parameters β and ξ.

[2] There are two further solutions $X^*_{10} = (1, 0, 0)$ and $X^*_{01} = (0, 1, 0)$ of $\Phi(X) = 0$, but outside the domain (indeed at the boundary) because of $\Gamma^*_{10} = \Gamma^*_{01} = 0$.

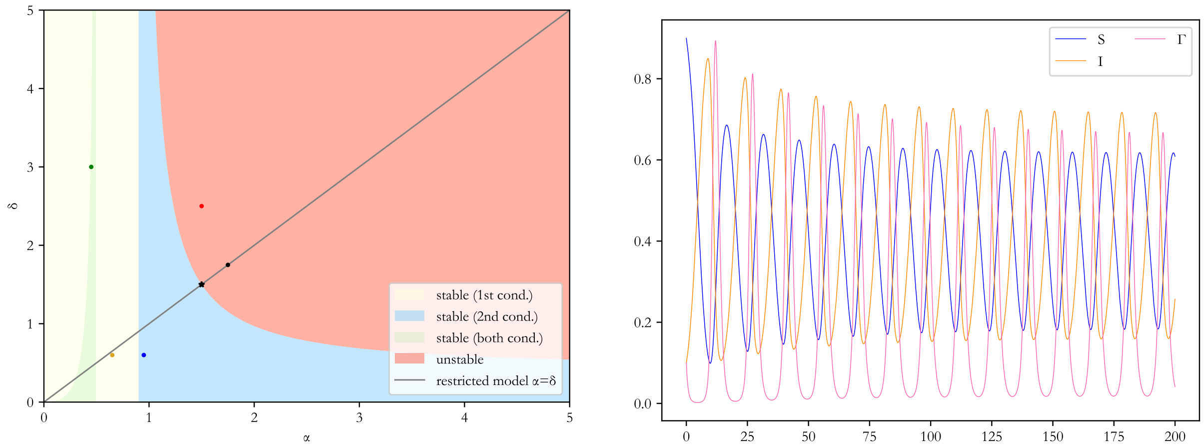

Figure 1: Left: layout in the (α, δ) parameter space, for $\beta = 0.5$ and $\xi = 0.4$, of the region of instability (red), and of stability due to the first condition (yellow), to the second (blue) or to both conditions (green) of Eq. (14). The five dots represent the parameters used to produce the graph in the right panel (black dot), and Figures 2 (yellow dot), 3 (blue dot), 4 (green dot), and 5 (red dot) in the supplementary material. The black star highlights the Hopf bifurcation point of the restricted model with $\alpha = \delta$ (see Section 2.2). Right: Solution of the model for $\beta = 0.5$, $\xi = 0.4$, and $\alpha = \delta = 1.75$ (black dot on the left graph), with $S(0) = 0.9$, $I(0) = \Gamma(0) = 0.1$.

2.2 Bifurcation analysis

A Hopf bifurcation is a critical phenomenon in dynamical systems where a fixed point of a system undergoes a qualitative change as a parameter is varied, leading to the emergence of a periodic orbit. Specifically, it occurs when a pair of complex conjugate eigenvalues of the Jacobian matrix of the flow operator crosses the imaginary axis from the left half-plane to the right half-plane while all the other eigenvalues have negative real part, see, e.g., Marsden and McCracken (2012). This transition indicates a shift from stability to instability of the fixed point, resulting in oscillatory behavior.

Proposition 2. *Solution $\boldsymbol{X}^*$ of ODEs (5)–(7) with $\alpha = \delta$ is a Hopf bifurcation at*

$$\alpha = \beta + \xi + \sqrt{\xi(\beta + \xi)}$$

if

$$\beta > \frac{11}{25}\xi.$$

Proof. See Appendix A.2. □

This sufficient condition provides a rigorous example of the model displaying periodic orbits when the fixed-point equilibrium is broken, confirming the intuition that the combined mechanism of interest saturation and influencing enthusiasm could give rise to repeated cycles of interest and disinterest in opinion dynamics as described within the SIRS modeling framework. For example Figure 1 shows a simulation of the restricted model in the region of the instability, displaying periodic orbits for the three state variables.

3 Empirical application

Digitalization allows us to track ideas' popularity in many ways nowadays, from low-level measures based on granular social network data to high-level indicators aggregating all the information. Google Trends represent aggregated indicators of the popularity of *research queries* in Google Search across various regions and languages, measured as the relative weekly search volumes normalized to the highest value in the investigated period, which is set to 100. We show

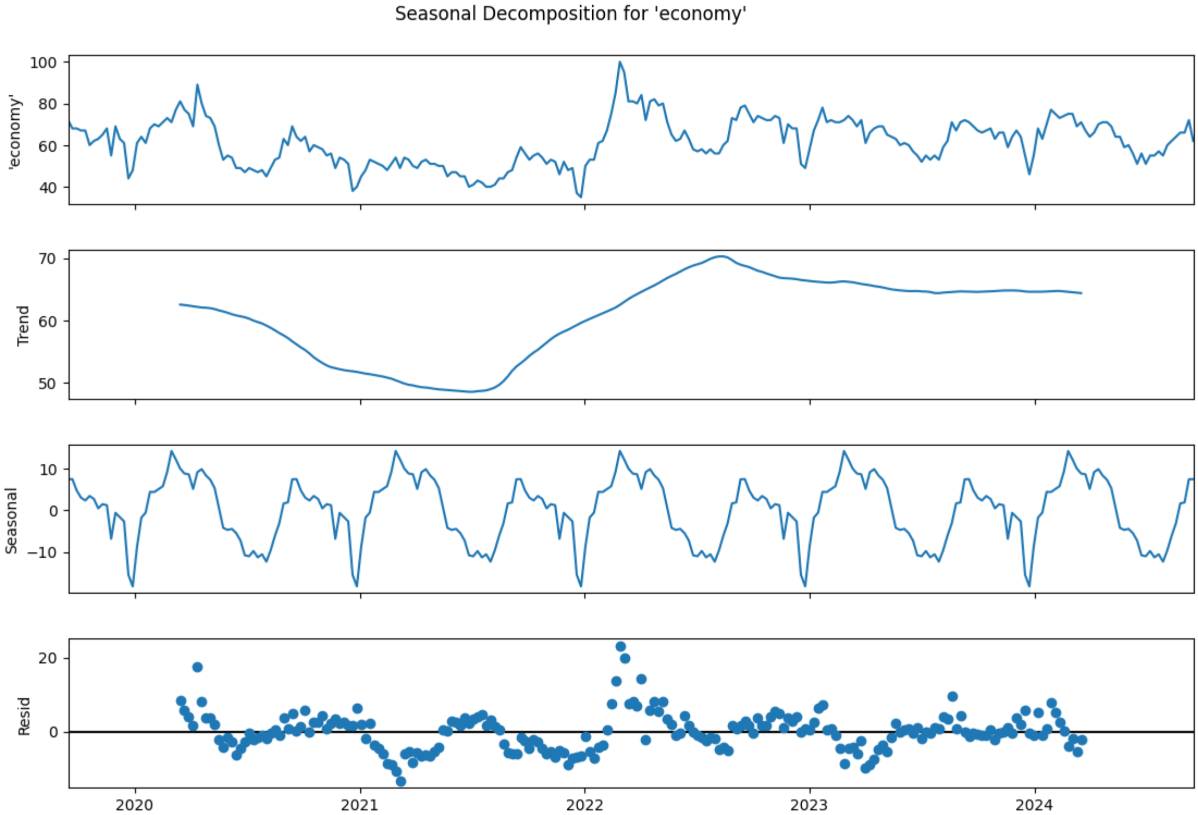

Figure 2: Seasonal decomposition for the word '*economy*' searched in Google Trends. After removing the trend and annual seasonality, we are left with the residuals (bottom panel). See supplementary material for additional information.

the research query for '*economy*' in a 5-year period from September 15th, 2019 to September 15th, 2024 in the top panel of Figure 2. Observed search volumes can depend on several effects, either exogenous or endogenous.

Media coverage of a particular topic can generate significant activity in Google searches, leading to trends in the search volume over time. Seasonal factors naturally create periodic patterns in Google searches. For example, queries like '*swimsuit*' or '*skiing*' are largely influenced by the time of year. Generally, any event, whether regional or global, can capture people's attention, causing random shifts in search volume dynamics. Meanwhile, conversations, discussions, and other forms of communication can also draw people's attention to specific topics. As such, social interactions can drive search volumes over time. There may also exist many other social mechanisms, all of them contributing to the aggregated dynamics of Google trends.

This section aims to assess whether Google Trends, as a proxy for the popularity of ideas, exhibits dynamical patterns that can be consistently explained by the endogenous effects outlined in our model. After accounting for known influences like trending and seasonality, endogenous effects should manifest as cyclical patterns instead of purely random evolution.

We collected a dataset of the 1000 most searched queries in the United States in September 2024 from Data For SEO website.[3] For each word, the time series of search volume observed on any week from September 15th, 2019 to September 15th, 2024 is a proxy of the word's popularity over time, providing insight on how interest in each topic fluctuates. For a genuine assessment, we first remove both trend and annual seasonality: (i) trend is estimated as a local average of observations over a rolling time window of 52 weeks, while (ii) annual seasonality is estimated starting from the detrended time series of observed search volume as the average of the same weeks over every year. An example of the two patterns for the word '*economy*' is

[3]Detailed information on data, methods, and code are available in the supplementary material.

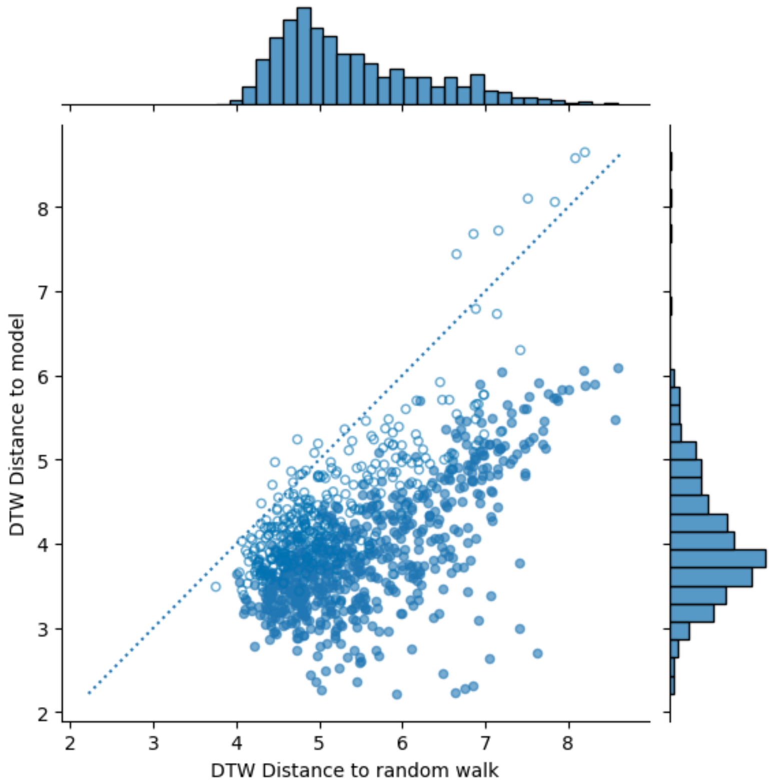

Figure 3: Scatter plot showing the relationship between the Dynamic Time Warping (DTW) distance of residuals to a random walk (x-axis) and the DTW distance to the model (y-axis). (See supplementary material for additional information.) Each circle represents a word from the dataset. Marginal histograms indicate the distribution of DTW distances for both reference processes. The dotted diagonal line represents the case when the average DTW distance to the random walk equals the DTW distance to the model. A circle below the diagonal suggests that the residuals are more similar to our model than to the random walk. The circle is filled with blue when the DTW distance associated with the model is smaller than the first quartile of the empirical distribution built with 500 simulations of the random walk, indicating that the model better captures the underlying oscillations, in a statistical sense.

shown in the middle panels of Figure 2. After removing trend and seasonality, the time series of residuals (see the bottom panel of Figure 2) is standardized by rescaling data in a range $[-1, 1]$ and compared with two reference processes: the proposed model of endogenous oscillations – a system simulating regular, internally driven cycles of 'infected' - and a random walk - a model describing the contagion of an idea as a purely random variation of 'infected' at any time. Here, we connect the concept of 'infected' with the popularity of an idea, based on the intuition that the larger the number of 'infected,' the larger the observed search volume of the corresponding word. For a fair comparison, the same standardization scheme applied to the time series of residuals is also applied to the simulations of both reference processes.

The comparison uses Dynamic Time Warping (DTW) as distance, see, e.g., Hastie *et al.* (1991). DTW is the generalization of the Euclidean distance between two time series, allowing for non-linear stretching or shrinking of the time axis to find the optimal alignment between the two. DTW is particularly useful when comparing time series that may not be perfectly synchronized but display similar patterns.

We first measure the DTW distance between the time series of standardized residuals and our model with the best value for β (i.e. the transmission rate) in terms of DWT.[4] Then, the DTW distance is computed for 500 simulations of a random walk, whose average is finally compared to our model.

[4] A detailed explanation of the implementation is in the supplementary material and the code is available at this link.

Figure 3 shows the scatter plot between the DWT distances associated with our model and the random walks for the 1000 most searched queries. The time series of residuals systematically show a closer match to the dynamic behavior predicted by our model. To ensure robustness, a circle in the scatter plot is filled with blue when the DTW distance associated with our model is smaller than the first quartile of the empirical distribution built with 500 simulations of the random walk. Our analysis shows that the endogenous cyclical effects capture the underlying patterns of Google searches statistically better than what would be expected from random fluctuations in the 67% of cases.

These results provide evidence that the temporal dynamics of online search behavior are not purely random but exhibit structured oscillations that can be explained, at least in part, by endogenous processes. Our model, therefore, offers a meaningful framework for understanding how certain search queries experience periodic surges in interest, reflecting broader patterns in human behavior.

4 Conclusion

Our analysis demonstrates the existence of complex dynamics, including cycles and potential bifurcations, providing insights into the underlying mechanisms that govern the rise and fall of ideas. This framework can be extended to explore additional feedback mechanisms and their broader implications in the diffusion of innovations and cultural trends. The feedback mechanism introduced in our modified SIRS model captures the dynamic interaction between interest saturation and influencing enthusiasm, leading to fluctuating steady states. This cyclical behavior mirrors real-world observations where ideas periodically gain and lose popularity.

Moreover, our study highlights the importance of considering endogenous factors in modeling idea diffusion. The rate at which individuals lose interest (interest saturation) and the influence exerted by current promoters (influencing enthusiasm) are both crucial in understanding how ideas spread and fade. By incorporating these factors, our model provides a more comprehensive view of the diffusion process, which can be applied to various fields, including marketing, public health campaigns, and the spread of innovations.

Future research could expand on this framework by incorporating stochastic elements to account for random external influences and by exploring the impact of network structures on the diffusion dynamics. Additionally, empirical validation of the model using real-world data on idea diffusion could further enhance its applicability and robustness.

In conclusion, our modified SIRS model with feedback provides a valuable tool for understanding the complex and dynamic nature of idea popularity. It offers a theoretical foundation that can be built upon to explore various aspects of social influence and cultural evolution, enabling the development of more effective strategies in managing and promoting ideas.

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A Proofs

A.1 Proof of Proposition 1

Proof. First, we compute the eigenvalues of the first-order derivatives of Φ (the Jacobian).

At $\boldsymbol{X} = (S, I, \Gamma)$, the Jacobian of Φ is

$$J(\boldsymbol{x}) = \begin{pmatrix} -(\beta I + \xi) & -(\beta S + \xi) & 0 \\ \beta I & \beta S - \Gamma & -I \\ -\delta\Gamma & \alpha\Gamma & \alpha I - \delta S \end{pmatrix} ,$$

having characteristic polynomial

$$\begin{aligned} p(\lambda) &= \det[J(\boldsymbol{X}) - \lambda\mathbb{I}] \\ &= -\lambda^3 \\ &\quad - [(\beta-\alpha)I + (\delta-\beta)S + \Gamma + \xi]\lambda^2 \\ &\quad - (\alpha\Gamma + \beta^2 S + \beta\xi)I\lambda \\ &\quad - [\beta(2\delta\Gamma - \beta\delta S + \alpha\beta I)SI + \beta\xi(\alpha I - \delta S)S + \delta\xi(S+I)\Gamma] \ . \end{aligned} \tag{11}$$

Recall that $p(\lambda)$ is said to be *stable* if all roots have negative real part. Ignoring uninteresting cases, assume all parameters to be strictly positive and $\boldsymbol{X} = (I, S, \Gamma) \in [0,1]^2 \times \mathbb{R}_{>0}$. Under these assumptions, the odd-order coefficients of $p(\lambda)$ are strictly negative. The Routh–Hurwitz stability criterion (Rahman and Schmeisser, 2002, sec. 11.4) for $p(\lambda)$ becomes then:

$$\begin{cases} (\beta-\alpha)I + (\delta-\beta)S + \Gamma + \xi > 0 \\ \beta(2\delta\Gamma - \beta\delta S + \alpha\beta I)SI + \beta\xi(\alpha S - \delta S)S + \delta\xi(S+I)\gamma > 0 \\ (\alpha\Gamma + \beta^2 S + \beta\xi)[(\beta-\alpha)I + (\delta-\beta)S + \Gamma + \xi] - \beta(2\delta\Gamma - \beta\delta S + \alpha\beta I)SI \\ \qquad -\beta\xi(\alpha I - \delta S)S - \delta\xi(S+I)\Gamma > 0 \end{cases} \tag{12}$$

As model (5)–(7) is non-linear, condition (12) is a necessary and sufficient condition for *local* stability at $\boldsymbol{X}$ of the model.

At the fixed point $\boldsymbol{X}^* = (I^*, S^*, \Gamma^*)$ in Eq. (9), the characteristic polynomial (11) simplifies as

$$p(\lambda) = -\lambda^3 - (\beta I^* + \xi)\lambda^2 - (\alpha\Gamma^* + \beta^2 S^* + \beta\xi)I^*\lambda - (\beta\delta S^* + \delta\xi + \alpha\beta I^* + \alpha\xi)\Gamma^* I^* \ , \tag{13}$$

and has strictly negative coefficients, hence the first two inequalities of (12) are satisfied. Therefore, the local stability condition at $\boldsymbol{X}^*$ is the third inequality only, which with some algebra can be stated as

$$\alpha \le \beta + \xi \qquad \text{or} \qquad \delta < \frac{\alpha^2\xi}{(\alpha-\beta)(\alpha-\beta-\xi)} \ . \tag{14}$$

Therefore, its complement, i.e. the statement, provides the local instability condition.

This concludes the proof. □

A.2 Proof of Proposition 2

Proof. When $\alpha = \delta$, the fixed point $\boldsymbol{X}^*$ in Eq. (9) reads as

$$\boldsymbol{X}^* = \begin{cases} S^* = \dfrac{\left(-1+\sqrt{1+\frac{\beta}{\xi}}\right)}{\frac{\beta}{\xi}} = \dfrac{1}{1+\sqrt{\frac{\beta+\xi}{\xi}}}, \\ I^* = S^*, \\ \Gamma^* = \beta S^*. \end{cases} \tag{15}$$

and the characteristic polynomial in Eq. (11) at $\boldsymbol{X}^*$ simplifies as

$$\begin{aligned} p(\lambda) &= -\lambda^3 - (\beta S^* + \xi)\lambda^2 - (\alpha\beta S^* + \beta^2 S^* + \beta\xi)S^*\lambda - [2\alpha\beta^2(S^*)^3 + 2\alpha\beta\xi(S^*)^2] \\ &= a\lambda^3 + b\lambda^2 + c\lambda + d \end{aligned} \tag{16}$$

where

$$a = -1, \qquad b = -\sqrt{\xi(\beta+\xi)}, \qquad c = -\frac{\alpha+\beta}{\beta}(\xi+b)^2 + \xi(\xi+b), \qquad d = 2\frac{\alpha}{\beta}b(\xi+b)^2.$$

The solution of the cubic equation $a\lambda^3 + b\lambda^2 + c\lambda + d = 0$ is well-known in Algebra; see, e.g., the pioneering work by Cardano *et al.* (2007). First, a change of variable is applied, i.e. $\lambda = y - \frac{b}{3a}$. Then, a reduced form $y^3 + py + q = 0$ is obtained, with (after some algebra)

$$\begin{aligned} p &= \frac{c}{a} - \frac{b^2}{3a^2} = \frac{\alpha\xi + (\alpha + \frac{2}{3}\beta)(\beta+\xi) + (2\alpha+\beta)b}{\beta}\xi, \\ q &= \frac{d}{a} - \frac{bc}{3a^2} + \frac{2b^3}{27a^3} = \left[-\frac{5}{3}\frac{\alpha}{\beta}(\xi+b)^2 + \frac{1}{3}\xi b + \frac{7}{27}b^2\right] b. \end{aligned}$$

The type of solution of Eq. (16) depends on

$$\Delta = \frac{q^2}{4} + \frac{p^3}{27}; \tag{17}$$

in particular, the cubic equation has one real root and two complex conjugate roots if and only if $\Delta > 0$.

A sufficient condition for $\Delta > 0$ is $p > 0$. The condition $p > 0$ is equivalent to

$$\alpha\xi + (\alpha + \frac{2}{3}\beta)(\beta+\xi) - (2\alpha+\beta)\sqrt{\xi(\beta+\xi)} > 0,$$

which becomes after some algebra

$$\alpha\left[\left(\sqrt{\xi} - \sqrt{\beta+\xi}\right)^2\right] > \left(\sqrt{\xi} - \frac{2}{3}\sqrt{\beta+\xi}\right)\beta\sqrt{\beta+\xi}.$$

As $\beta, \xi > 0$, the left-hand side of this last inequality is positive. Therefore, the sufficient condition for $\Delta > 0$ can be stated as

$$\alpha > \frac{(\sqrt{\xi} - \frac{2}{3}\sqrt{\beta+\xi})}{(\sqrt{\xi} - \sqrt{\beta+\xi})^2}\beta\sqrt{\beta+\xi}. \tag{18}$$

When Eq. (18) is satisfied, the characteristic polynomial in Eq. (16) has one real root and two complex conjugate roots. Moreover, the real root is always negative because of Descartes' rule of signs applied to Eq. (16), but with a sign change for the variable ($\lambda \to -\lambda$).

In the domain of parameters, the crossing point to the unstable region is given by the second inequality in Eq. (10) with equality sign for $\alpha = \delta$, provided that the first inequality is satisfied. It is

$$\alpha = \beta + \xi + \sqrt{\beta\xi + \xi^2}. \tag{19}$$

It remains to prove that the crossing point from stable (all roots have negative real part) to unstable (the two complex conjugate roots have positive real part) regions is an interior point of the parameter subspace defined by Eq. (18). In this case, Proposition 1 guarantees that the real part of the two complex conjugate roots is zero at the crossing point because of the continuity of the solution of the cubic equation.

The crossing point is an interior point of the open subspace defined by Eq. (18) when

$$\beta + \xi + \sqrt{\beta\xi + \xi^2} > \frac{\left(\sqrt{\xi} - \frac{2}{3}\sqrt{\beta + \xi}\right)}{\left(\sqrt{\xi} - \sqrt{\beta + \xi}\right)^2}\beta\sqrt{\beta + \xi}.$$

The last inequality can be stated as

$$\beta\frac{5(\beta + \xi) - 6\sqrt{\xi(\beta + \xi)}}{3(\beta + 2\xi) - 6\sqrt{\xi(\beta + \xi)}} > 0. \tag{20}$$

Eq. (20) is satisfied when both the numerator and denominator on the left-hand side are either positive or negative. As $\beta, \xi > 0$, the latter condition must be discarded. The former one results in a system of inequalities whose solution is

$$\beta > \frac{11}{25}\xi.$$

This concludes the proof. □