

Oh, Buoy!
**Optimal Design of a Vibration Energy Harvester
for Wave Power**

MTHE 493 Final Thesis — GROUP G3

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Executive Summary

Vibration energy harvesting is the phenomenon of converting mechanical energy extracted from vibrations into electrical energy using a method of transduction. The goal of this project is to design a VEH system that extracts a maximum of power for a relevant and meaningful application. The research was conducted regarding the topic of vibration energy harvesters (VEHs), the possibility of transduction materials, and various past works with related designs to the solution generation results.

The general idea proposed as a design solution to the wasted energy problem is a system of interconnected buoys that will harvest energy from ocean waves. Each free-floating buoy will be surrounded by a cage-like shell, where each buoy will have a VEH that first transforms the up-and-down movement of the waves into mechanical energy and then transforms that mechanical energy into electrical energy. Variations of this general idea were explored and evaluated based on environmental, social, and economic considerations, and the finalized design was decided on using an evaluation matrix. The proposed design with the best ranking in the evaluation consists of an electromagnetic transducer on each buoy, where the buoy's are in a parallel circuit with one central capacitor that all the energy outputs get filtered into.

In terms of stakeholders, the team used an in-depth Triple Bottom Line analysis to evaluate the design thus far. Social considerations include the need to locate the system away from fishing, tourism, and recreation areas, and the creation of jobs. Economic considerations include the cost of transducers, energy converter systems, and the battery and cable. Environmental considerations include the impact on marine ecosystems and the use of non-toxic materials. Ethical considerations include the evaluation of impacts on stakeholders and the need to make ethical decisions during the decision-making process. These criteria explored affect decisions regarding the proposed design relating to implementation location and choice of materials. The implementation location was decided to be in the town of Ucluelet near Vancouver Island in British Columbia, Canada based on expected power outputs and ease of implementation. This text discusses the design choices made for the different components of a buoy system that converts ocean wave energy into electrical energy. The design choices were made based on research, industry standards, specific design criteria, and optimization. Different components such as resistors, capacitors, springs, and wiring coils were discussed, and their respective criteria for selection were outlined. The chosen materials for each component were justified based on their ability to meet the specific criteria and their availability and cost-effectiveness.

An in-depth mathematical model was produced for the proposed of wave energy harvesting system using a buoy equipped with an electromagnetic transducer. The transducer is made up of a ferromagnetic mass, a conductive coil, a spring, and a damper. Lenz's Law is used to relate the mechanical energy of the waves to the electrical energy stored in the capacitors. A differential equation is derived to describe the mass-spring-damper system, taking into account an external force function determined by the oscillating buoyant force of the ocean waves. The equation is used to find the system's natural frequency, which helps in maximizing power output. The harvested energy can be outputted to a battery or another electrical circuit. The design was then simulated using software tools chosen to be MATLAB and Simulink after evaluation. The system produces a realistic and ideal power output after various design iterations and the optimization of chosen values in the system.

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1 Introduction

A vibration energy harvester (VEH) is a device that takes the free vibrational mechanical energy within a system and turns it into electrical energy [1]. In theory, any vibrating object or system can be used to power a VEH, from using ambient sound within an environment to the induced rumblings of cars driving over a bridge.

The principal mechanics behind a VEH are straightforward in theory, but can become quite convoluted in practice. Several parts are required for the VEH system. Since it is a vibrational energy that is harvested, the system requires some application that itself vibrates, such as a mass on a spring that corresponds to the applied vibrations. Next, it needs some mechanism to translate the mechanical energy of the mass on the spring to electrical energy. This can be done via one of the several methods of transduction, such as: electromagnetic, piezoelectric, electrostatic, magnetostrictive, and triboelectric [2]. Each transducer has its own benefits depending on what system it is being equipped to. Thus, there also must be a circuit system to allow the current to form, and a resistor for the power to be drawn out of, such as a lightbulb.

It should be noted that there is a difference between vibration energy harvesting and vibration energy conversion (VEC). For VEH, the source of the energy is typically ambient to the system and otherwise untapped as a source; consider the example of wind turbines. For VEC, the energy being converted is already being used as a source for a system, such as a VEC being placed on the wheels of a bike [1]. Therefore, the VEH is being placed on a source of free energy, as such a source is typically untapped and/or sustainable.

The objective of this project is to design a VEH for a chosen application, with the added goal of extracting the maximum power from the energy source. While the output will be designed to be as efficient as possible, in order to approach maximum power extraction from the system, it is desirable for the system to be operating at resonance.

Resonance is the phenomenon where the frequency of an input wave closely matches (or is) the natural frequency of the system. The natural frequency of a system is the frequency at which the system can oscillate freely about equilibrium without gaining amplitude or collapsing to the system's origin. When resonance occurs, it allows the system to achieve a greater amplitude than normal. A greater amplitude results in a larger displacement at the wave peaks, which in turn results in a greater energy extraction.

2 Research

All research was done using published scientific or engineering articles and so holds a high degree of authority, currency, credibility, and objectivity.

2.1 Transduction Materials

VEHs require a method of transduction that allows the mechanical energy generated by the vibrations to be converted to electrical energy. There are various options for said transduction methods (referred to as the generator of the VEH) including but not limited to piezoelectric, electromagnetic, electrostatic, magnetoelectric, triboelectric, and dielectric transducers.

The piezoelectric transducer contains an electric dipole that creates charge separations such that an electric charge can be produced from the deformation of the piezoelectric materials [23]. Piezoelectric transduction from mechanical energy to electrical energy is called the direct piezoelectric effect. This generator can alternatively convert electrical energy to mechanical energy; this is called the converse piezoelectric effect. Moreover, the strain of this transducer is comprised of the mechanical strain and the actuation strain, resulting from the mechanical stress and applied electric voltage respectively [24]. Furthermore, the cantilever beam is the best internal structure for this generator since it has a resonance frequency that optimizes the electrical output. Equations for the mechanical properties of the piezoelectric systems can be derived as the following:

$$\delta = \frac{\sigma}{Y} + dE \quad (1)$$

$$D = \epsilon E + d\sigma \quad (2)$$

Where δ and σ are the mechanical strain and stress respectively, Y is Young's Modulus, d is the piezoelectric strain coefficient, E electric field, and D electric displacement [23]. These equations help visualize how the materials and the system interact with the external vibrations. One can see that electric displacement D is dependent on the various strains of the system; given the use of a cantilever beam design, the produced strain is usually above average strain. In addition, the frequency of the device depends directly on the device's measurements and composition, in which the respective resonance frequency is always equal to the highest possible electrical output [23].

The electromagnetic generator is centered around Faraday's Law of Electromagnetic induction and consists of a wire coil, that enables the flow of current, connected to a mass-spring system. The transducer works as such: in an induced magnetic field, vibrations cause a relative displacement between the coil and the base magnet of the system, which in turn generates a voltage above said coil [23]. Similarly to the piezoelectric transducer, the electromagnetic generator only operates when the vibrational frequency is equal to the resonance frequency; tuning techniques are used to obtain said resonance frequencies. Furthermore, electromagnetic generators have a higher power output relative to piezoelectric generators and are easier to implement in harsher and more complicated applications [24]. In modelling the electromagnetic system, equations for the voltage produced and the maximum harvested power can be defined using Faraday's Law:

$$\epsilon = \beta lv \quad (3)$$

Where ϵ is the EMF voltage, β is the strength of the magnetic field, l is the length of the wire and v is the relative velocity between the magnet and the wire.

$$P_{max} = \frac{mY_0^2\omega^2}{4\zeta} \quad (4)$$

Where P_{max} is the maximum harvested power, Y_0 and ω are the vibrational amplitudes/frequencies, and ζ is the transducer damping constant [25]. As directly seen from Equations 3 and 4, maximizing harvested power requires the transducer damping factor to be small and the frequencies squared to be proportionally larger than 4 times the transducer damping factor. In terms of generated voltage, all variables are directly proportional to it, so increasing these variables maximizes the EMF voltage [25].

2.2 Transducer Limitations

Energy harvesting requires a small yet intricate system and the materials involved are expensive. High-power output is difficult to obtain with very small systems; to obtain maximal power output, the size of the VEH must be considered [25].

Both the piezoelectric and electromagnetic transducers have their own constraints. For instance, the piezoelectric transducer is only when used for single-direction motion, given its use of cantilever-based geometry [24]. On the other hand, electromagnetic generators need various vibrations at different frequencies to increase the bandwidth of energy harvesting. Moreover, as previously mentioned, each generator needs to be tuned to exactly match the system's resonance frequency. In addition, electromagnetic transducers are typically used for systems with larger amplitudes while piezoelectric transducers are more applicable for harvesting larger power outputs [25].

There are also considerations to take in terms of the optimization of systems. For instance, when the system is dependent on various moving parts, all must be considered to have relative limitations. This is the case for mass-spring systems, in which the relative displacement is constrained by the mass and the spring stiffness. Another consideration of note is that piezoelectric transducers should only be used for applications with vibrations of small amplitude, given the needed deformation of materials [27].

2.3 Past Work

The first past work studied involves a patent that was developed for the harvesting of electricity from ocean wave energy using a piezoelectric cantilever placed on buoys [28]. The patented system is comprised of a cantilever member attached to a free-floating buoy. The cantilever radiates outwards to generate electrical power from the motion induced by the rotation of the object and the motion of the buoy. The buoy has various forces applied to it, as outlined in the Free-Body Diagram:

$$F(t) = F_{float}(t) + F_{ac1}(t) + F_{dr1}(t) + F_{ac2}(t) + F_{dr2}(t) \quad (5)$$

Where $F_{float}(t)$ is the buoyant force, $F_{ac1}(t)$ and $F_{dr1}(t)$ are the initiate and drag force respectively by the water on the buoy, and $F_{ac2}(t)$ and $F_{dr2}(t)$ are the same forces but for the water on the piezoelectric cantilevers [28]. And then Newtown's Second Law of Motion can be applied using the total forces to obtain the displacement of the of the free-floating buoy, where this equation is in terms of the double derivative of that displacement. This then results in a non-homogenous system of differential equations with initial conditions assumed to be 0:

$$M_B \frac{d^2w(t)}{dt^2} = F(t) \quad (6)$$

Where M_B is the mass of the buoy and $w(t)$ is the displacement of the floating buoys, and $F(t)$ is the total sum of forces as seen in Equation 5. 6 defines the system of equations is proven to be quite difficult to solve as seen in the patents. Mitigations can include making assumptions and design criteria that remove some of the forces from the equation to make the system easier to solve. The research shows that the design of the slender floater and large sinker structure is a smart design for harvesting maximal energy since the buoys will experience transverse ocean waves that need to be accounted for. The coupled piezoelectric cantilevers increase the overall efficiency in the energy harvesting due to the larger wave force applied. In addition, the choice to have a larger floater result in a deeper location of the sinker, which leads to reductions in the ocean drag force and vibrational amplitude of the buoy [28].

The second past work studied is an article from 2012 that suggests a restricted floating buoy system to harvest wave energy [22]. The design has an oscillating buoy that is restricted by a frame that keeps it from moving non-vertically. The buoy internally uses an electromagnetic transducer with a dampened mass spring system.

The study derived a system of equations for the relation of motion between the movement of the magnet to the movement of the buoy. The following equations, 7 , represent the motion of the two-body system:

$$\begin{aligned}(m_1 + \mu)\ddot{z} + m_2\ddot{y} + B\dot{z} + \rho g S z &= F_D \\ m_2\ddot{y} + c(\dot{y} - \dot{z}) + k(y - z) &= 0\end{aligned}\tag{7}$$

where m_1 is the buoy mass, m_2 is the magnet mass, z and y are the coordinates of the heave motion of the buoy and magnet, respectively, \dot{z} and \dot{y} are the velocity of the buoy and magnet, \ddot{z} and \ddot{y} are the acceleration of the buoy and magnet, B is the total-damping coefficient, S is the water-plane area, μ is the added mass, k is the spring stiffness, and c is the damping coefficient. From these equations, the mechanical power is extracted which is then used with the buoy amplitude to find the electrical power [22].

2.4 Location

In a technical report conducted by the Canadian Hydraulics Centre, multiple different locations were analyzed to quantify and map Canada's renewable marine energy resources from waves [20]. Areas off the Pacific and Atlantic coasts were monitored. On the west coast, the Pacific Ocean provides a significantly larger potential wave power, with a mean annual wave energy flux of around 25 kW/m at the coast of Vancouver Island and up to 55 kW/m in deeper waters further up the coast of British-Columbia. On the east coast, it was found that off the coast of Newfoundland wave energy varied between 20-45 kW/m and around 10-20kW/m off the southern shore of Nova Scotia. It was also determined that in the Northwest Atlantic, the mean wave power is 4-5x larger in the winter than summer, whereas off the B.C. coast it is 6-7x larger in winter [20]. This research was used to help narrow down a specific location for the implementation of the project.

Another research study was developed to model and assess the feasibility of wave energy harvesting on the west coast of Vancouver Island in B.C. [18]. Three wave buoys were implemented in areas off the shore of Tofino and Ucluelet, B.C. with ocean depths ranging from 40 to 2040 meters. The monthly average significant wave height and peak wave period were recorded for each buoy system. For the buoy closest to the shore, wave heights reached around 2.5 to 3 meters in the winter months and around 1.5 meters in the summer months [18]. It was found that the wave power and harvestable energy increased further offshore as wave conditions become more extreme. The study also conducted a spatial overlap analysis with ecological characteristics and categories impacting human lifestyle

and businesses. It was determined that the buoy system could produce enough energy to support 100 to 167 households [18]. This study was used to validate the feasibility and benefits of the proposed location.

2.5 Modelling Techniques

The final work studied is a conference paper that presents a design for a vibration energy harvester that converts mechanical vibrations into electrical energy using a dampened mass-spring system. For this introduced design, an electromagnetic transducer that creates a magnetic field, connecting to a rectifier, converts AC voltage to DC voltage [29].

This studied work was used primarily to aid in the modeling process for this particular design. Because the overall structure of the energy conversion relates to the general solution structure in question, this paper provided sufficient insight into how to model the system using Simulink.

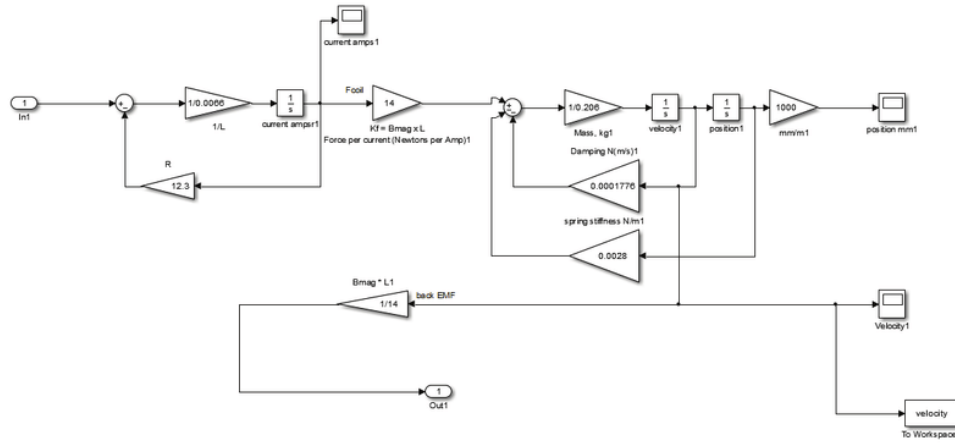


Figure 1: Simulink model from a conference paper discussing low-frequency energy harvesting using a dampened-mass-spring system [29].

The Simulink model provided as seen in the figure above consists of a dampened mass-spring system that replicates the mechanical behavior of the system design, and the magnet coil system represents the electrical behavior of the energy conversion. Figure 1 depicts the internal circuit of each circuit, and the paper presents a four-assembly vibration energy harvester. The system is based on provided equations that represent the physics behind the system. Under various input frequencies and amplitudes, the simulation results show that assembling four VEHs together produces a substantially higher power output than a single-assembly [29].

3 Problem Overview

3.1 Problem Definition

The overarching goal of this project is to harvest wasted energy and render it useful. This is to be done by designing a VEH system that extracts a maximum of power for a relevant and meaningful application. This project involves understanding the basics of vibration energy harvesting, researching past work done in this field, selecting a significant application, designing an efficient solution, modelling the proposed VEH using complex mathematical models and simulating technologies. Economic, environmental, societal, and safety factors, as well as various constraints to ensure feasibility, will all be considered in the solution design.

3.2 Problem Considerations

The source of free energy studied is the ocean - specifically, the oscillations of ocean waves. A buoy paired with a VEH can use the bobbing motion of the waves as the vibrational input. This buoy can then be directly connected to the mainland via power cables to transmit the electrical energy, or the buoy can be equipped with batteries for energy storage. Using the oscillatory behaviour of ocean waves proves to be impactful and optimal, since it can generate large amounts of power, and can have various meaningful applications for this direct power source. Such free energy choice mitigates the stability issue, since these waves can be modelled to adjust the system accordingly based on other factors.

The implementation of the VEH system requires the authorization from the province for the use of Crown land resources for ocean energy projects. The application includes a crown land application, ocean energy development plan and an investigative plan [19]. The crown land application is used to request permission for the use of provincial crown land in British Columbia. The ocean energy application requires the submission of at least two digital maps: a general location map and a site plan. It also requires an investigative plan that includes the location and type of monitoring equipment proposed for placement on crown land. The third requirement for this application is a development plan containing information on the project's construction, operation, monitoring and decommissioning. The investigative plan requires background information about the project, including land management plans and regional growth strategies, seasonal expectations and engagement with First Nations. It also requires information about the location, including a general description of the area, a justification for use and historical use. Finally, it needs information about the infrastructure [19].

For ocean energy project proposals generating below 50 megawatts, the application is reviewed by the ministry of Land, Water and Resource Stewardship and for projects exceeding 50 megawatts, the review is led by the Environmental Assessment Office [19]. At the initial investigative stage of a project, an investigative Crown land license is issued for up to 10 years. At the initiation phase of the project a multi tenure is applied for that is issued for up to 40+ years with the exact time period corresponding to the electricity purchase agreement period. Without an electricity purchase agreement, the project is limited to a 10-year approval. Gaining access to the land results in a rental fee and application fees that is determined from the type of rights being transferred [19].

Health and safety constraints must also be considered. The system must be made of non-toxic materials to prevent health issues that may arise from people swimming in the surrounding water or fish consuming toxic materials. As well, the capacitors and wires connecting the buoys must be encased to prevent any safety issues that may arise if the system were to be damaged. The buoy

system must also be visible enough to avoid accidents with boats and must be in a location easy to install and easy to reach if maintenance is needed.

4 Solution Idea Generation

4.1 General Solution Idea

The general idea proposed as a design solution to the wasted energy problem is a system of interconnected buoys that will harvest energy from ocean waves. See Figures 2 and 3 for visuals of the general solution idea.

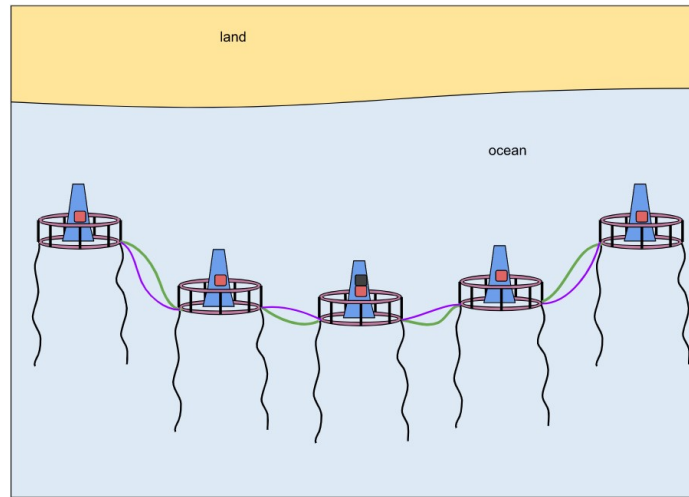


Figure 2: General solution idea to harvest wasted vibrational energy from ocean waves.

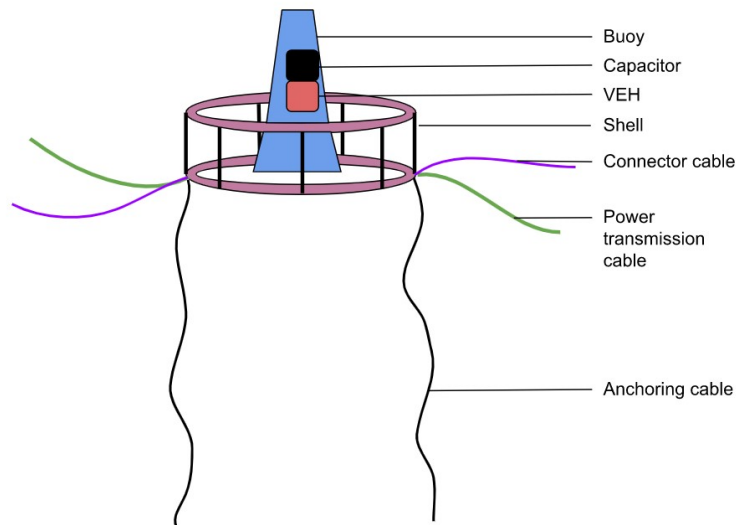


Figure 3: Components of the general solution idea.

Each free-floating buoy will be surrounded by a cage-like shell, which will be anchored to the ocean floor. The shell will be designed such that it does not impede ocean waves from moving the buoy, constrains the motion of the buoy to the vertical or z-direction, and minimizes friction between the shell and the buoy. Connector cables will connect the shells to each other; this serves mainly as a risk management mechanism in the sense that if a grounding cable happens to break, the connector cables will be able to provide some sort of stability to the system until repairs can be made. It will be ensured that connector cables have enough slack such that they do not impede motion of the shell or buoy during normal operation (i.e. when grounding cables are in place).

On each buoy will be a VEH that first transforms the up-and-down movement of the waves (and so of the buoys) into mechanical energy, and then transforms that mechanical energy into electrical energy. The electrical energy is then ported via power transmission cables from each VEH to a central capacitor located on a central buoy, where it is stored. Once the capacitor is filled, it needs to be manually recuperated, and then the harvested energy can be put to use.

The main item that differentiates the proposed solution from past work is the collection – rather than the immediate use – of the harvested energy. By using a capacitor as the system’s output, the harvested energy can be used in innumerable different ways.

The idea generation for specific design details will consider a number of factors that will render this design superior to existing solutions. These factors include adaptability and scalability, cost, and environmental friendliness.

- Adaptability and scalability of the system will be considered in terms of ocean temperature, climate, anchor length, battery size, and more. This means the designed system will be able to be used in a number of different oceans, at different distances from the shoreline, and for varying lengths of shoreline.
- Cost will be an important factor when considering types of shells, types of buoys, types of connectors, materials, etc. in order to deliver the most efficient and least costly solution.
- Principles of environmental friendliness will be incorporated both in the solution design and the material choices to reduce any potential negative impact on oceans, ocean species, fishermen, adjoining cities, etc.

Potential design solutions, as well as their benefits and drawbacks, will be fully detailed in the next presentation.

4.2 Possible Solution Configurations

Having established the general idea solution, the team then brainstormed and developed a few different system configurations to be considered. See Figure 4 for a graphical representation of each of the possible solution configurations and see Table 1 for a comparison of configuration components.

- Option 1 involves having a transducer and a capacitor on each buoy. At a given time interval, the energy stored in each buoy’s capacitor will be discharged to a central, larger capacitor. Once the central capacitor is full, it will be collected, and the extracted energy can be used.
- Option 2 involves all parts of option 1, except for the central capacitor. This means that once each buoy’s capacitor is full, it will have to be collected individually. This option avoids any energy loss that would result from routing the energy to a larger capacitor but involves more human effort.

- Option 3 involves having a transducer on each buoy, but only one, central, capacitor. This means that as soon as the electrical energy is generated by the transducer, it will be routed to and stored in the central capacitor. Once the central capacitor is full, it will be collected.

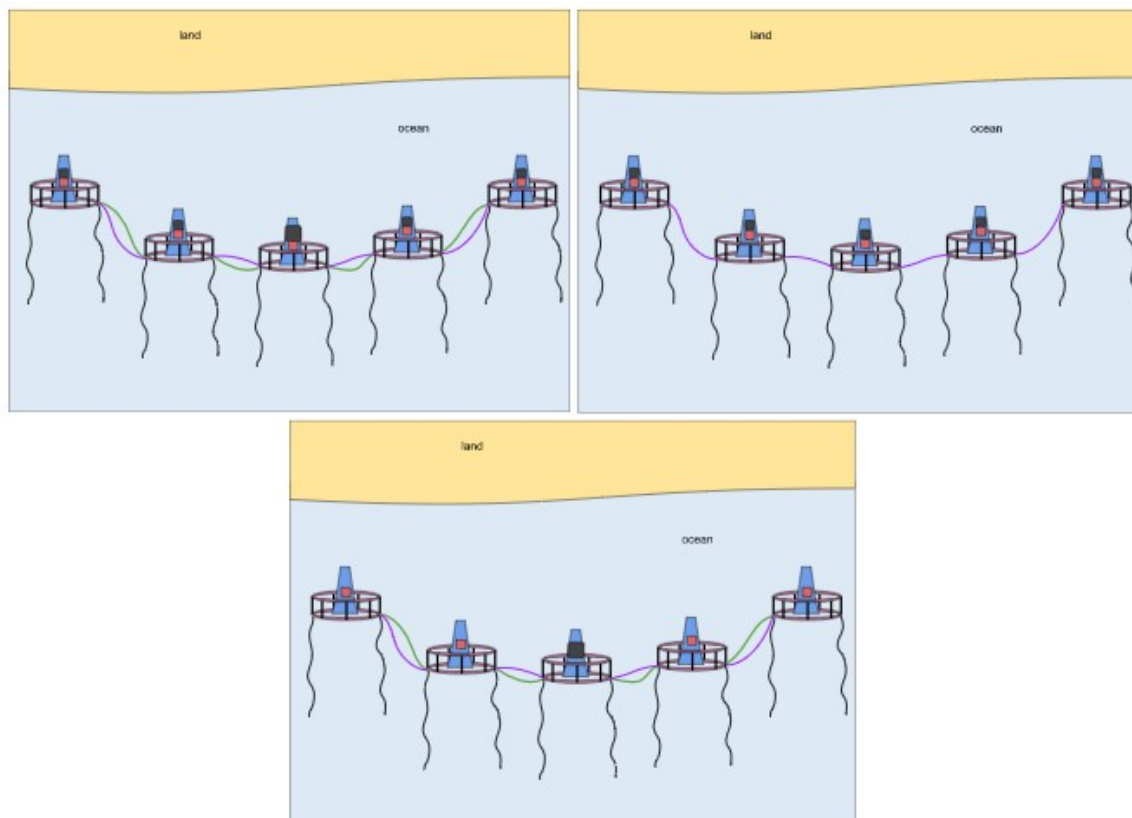


Figure 4: Layout of possible design solutions: Option 1 (top left), Option 2 (top right), Option 3 (bottom).

Table 1: Alternate representation of components per option.

	Option 1	Option 2	Option 3
Transducer on each buoy			
Capacitor on each buoy			
Central capacitor			
Circuit switches			

*For the sake of redundancy, components that are found in all potential configurations, namely the shell and grounding cables, are not included in the above table. Also, power transmission cables were not included in the table as they are directly tied to the incorporation of a central capacitor.

4.3 Evaluation of General Solution Idea

Table 2: Evaluation of three potential solution configurations via evaluation matrix.

Evaluation Criteria	Option 1 Transducer on each buoy Capacitor on each buoy Central capacitor	Option 2 Transducer on each buoy Capacitor on each buoy	Option 3 Transducer on each buoy Central capacitor
Capital Cost	High capital cost (all components plus added cost of switches)	Low capital cost (all components minus cost of power transmission cables)	Low capital cost (all components minus cost of individual capacitors)
Circuit Complexity	High complexity (circuit with switches)	Low complexity (series circuits)	Medium complexity (parallel circuit)
Human Effort	Low effort required	High effort required	Low effort required
Energy Loss	Potential energy loss	No potential energy loss	Potential energy loss
Risk to System	Low potential risk (if one circuit fails, the rest are unaffected)	Low potential risk (if one circuit fails, the rest are unaffected)	Medium potential risk (parallel circuit)
Total	10	12	13

*Here, orange represents 1 point, yellow represents 2 points, and green represents 3 points. A breakdown of the point system used can be seen in Table 3.

Table 3: Breakdown of points for each of the evaluation criteria used in evaluating potential solution configurations.

Evaluation Criteria	1 point	2 points	3 points
Capital Cost	High capital cost	Medium capital cost	Low capital cost
Circuit Complexity	High complexity	Low complexity	Medium complexity
Human Effort	High effort required	Medium effort required	Low effort required
Energy Loss	Definite energy loss	Potential energy loss	No energy loss
Risk to system	High potential risk	Medium potential risk	Low potential risk

*Medium complexity equates 3 points because, while the team does not want the circuit to be too complex, some level of challenge is wanted.

The three configurations previously described were then evaluated on a number of different fronts; these evaluation criteria are outlined in the table above. (The table only lists criteria on which the configurations differ; criteria for which there is no significant difference between configurations, namely environmental impact, feasibility, and adaptability are not included in the above table for the sake of redundancy.) As one can see from Table 2, option 3 scored the highest with a total of 13 points; the team will move forward with this solution configuration.

5 Stakeholder Considerations

The idea of transforming wasted, free energy into usable energy has a great number of social, economic, environmental and ethical considerations.

5.1 Social Considerations

The implementation of a VEH in an ocean buoy have several stakeholders that must be considered. The first stakeholder involved are the surrounding residents. The system must be located away from recreation and tourism areas. It must also be in a location away from fishing areas to prevent disturbances in fish habitats that would result in loss of business for fisherman. As well, building and installing the buoys would create short-term jobs and since the system stores the generated energy into capacitors they will need to be collected, thus producing future long-term jobs. The energy generated can be sold to produce a profit, which can be used to expand the system to different locations or saved for future maintenance costs.

A buoy system is a clean and renewable way to collect and supply electrical power to residents. Its consistency and sustainability would help reduce emissions from other power sources, thus benefiting the environment. Another stakeholder is the government since it would help encourage the transition to a clean energy source and help Canada reach its goal to cut greenhouse gas emissions by 2030 [12]. As well, the created system must abide by any national laws and policies put in place by the government. It was estimated, from the Electric Power Research Institute, that wave energy could meet 10% of total worldwide electric demand as it is widely available and near highly populated areas and industries [13]. However, it has not been adopted due to economic and logistical factors, thus presenting the opportunity to create a low-cost system.

5.2 Economic Considerations

Next, the cost of the transducers must be considered. In current studies, most piezoelectric energy harvesters are costly as they are designed to be fixed on the seabed and are mostly applicable in shallow water. Electromagnetic and hydraulic pump-based generators are able to produce extremely large amounts of energy; however, they are also costly, require a large space and have a long construction time [14]. Dielectric elastomers are another option that are low-cost; however, are more challenging to create a long lifetime for large areas. Additionally, there are different energy convertor systems for the transduction mechanisms. When comparing an acceleration-based convertor to a deformation-based convertor the costs are around \$1000 and \$500, respectively; however, the power output from the acceleration-based convertor was around 122 times higher [15].

For wave energy harvesting it is desirable to use material with a high permittivity, high dielectric breakdown strength, and low leakage to minimize electrical losses [13]. Common materials used to protect the power generating unit are natural rubber, acrylics, and high density polyethylene (HDPE) [13]. Natural rubber has a much lower cost, however, a poor weather resistance [13][16]. The use of an acrylic or HDPE casing will increase the durability of the system against harsh environmental conditions of the sea, such as strong salinity, high humidity, strong ultraviolet and hot/cold weather [17] [38]. This strong durability will minimize the amount of maintenance and repairs of the buoys, thus saving costs. As well, selecting a transducer with a long construction and installation time should be avoided to reduce labor costs. The battery and cable cost must also be considered; however, the size will not be able to be chosen until future tests are completed to determine the amount of energy generated from the system and distance between buoys.

5.3 Environmental Considerations

Wave energy harvesting provides continuous and consistent power generation without producing greenhouse gases or contaminating the surrounding environment. However, it can still have a negative effect on marine ecosystems as large wave converter systems make noise that could disturb surrounding sea life [15]. As well, the systems built near the shoreline may disturb the seafloor and change the direction of water flow, negatively impacting the fish and marine life habitats [15]. The system must also be built from non-toxic materials and the battery used to collect the power must be secured and durable to prevent it from polluting the environment. The wires used must be encased in a water resistant casing to avoid damaged wires polluting the ocean.

5.4 Ethical Considerations

During the decision process of the ideal buoy system, the team must make ethical decisions to evaluate impacts to stakeholders and pick an optimal solution.

In a previous study, an assessment of the human overlap with the buoy system was undergone for projects in Ucluelet and Tofino, B.C., categories were given one of the following rankings: very low, low, moderate, or high overlap [18]. It was found that at the chosen location, at depths of around 40m, there was a very low overlap with tenures and offshore energy, indicating that the chosen location would have minimal impact on aquaculture, log handling, and offshore petroleum. There was also a very low overlap with tourism and recreation, illustrating the low impact it would have on coastal campsites, kayaking and boating routes, scuba diving sites, marine and coastal facilities, and anchorages. As well, there was a low overlap with commercial fisheries, indicating that there would be minimal impact to fishermen in the area. However, there was a moderate overlap with shipping and transport, including ferry routes and terminals, tow boat reserves, tug vessels. Therefore, this aspect will need to be closely monitored during the implementation of the system. It was also found that at the chosen location, there was very low overlap with ecological characteristics and a low overlap with commercial fisheries, indicating that the chosen location would have minimal impact on marine ecosystems and fishermen in the area [18]. This past research study validates that the chosen solution will have minimal negative impacts on the stakeholders and thus would be beneficial to the area. However, before the implementation, the team will need to ensure that there is no concerning overlap with First Nation communities or government activities.

6 Design Overview

6.1 Approach to Design

The following paragraphs present an overview of the team’s approach to the design, modelling, and iteration of our solution.

The team first finalized the math model, which consists of two parts, the mechanical sub-model and the electrical sub-model. This done, a list of all the variables involved was established. Through brainstorming and discussion, the team then determined which variables most drastically alter the output, in this case energy, relative to other variables. These variables will henceforth be referred to as “mathematically-significant”.

It was then decided that values for the less mathematically-significant variables would be selected via research into past works, design criteria, industry norms, while values for the mathematically-significant variables would be determined through optimization.

Having established the above, it was then time to begin modelling the math model. Different tools were explored but MATLAB was ultimately chosen for its familiarity as well as for its Simulink function, which allows individuals to model their system using graphics and visuals in a more user-friendly interface.

The MATLAB/Simulink model was designed to replicate the flow of the math model, and so is also separated into mechanical and electrical sub-models. (In the lines below, the phrases “MATLAB model” and “Simulink model” will be used interchangeably.) The Simulink model was also designed to mimic the circuit and buoy set up of the physical system, which allowed the team to better visualize how all the system components came together. This was an extremely iterative process as multiple versions of the model with needed to be created and tested for functionality for resemblance to the physical system.

Once the Simulink model was designed, optimization of the power output via selection of the mathematically-significant variables was undertaken. This too was an extremely iterative step of the design process as based on the power output, small changes were made to the variable values, over and over again, until satisfactory outputs were obtained.

6.2 Implementation Location

It was decided to implement the design off Vancouver Island, B.C., in a town called Ucluelet. Although both the Atlantic and Pacific coasts of Canada provide a large amount of energy, the Pacific Ocean was chosen as a better fit since it produces more energy year-round. This location was also chosen because of its consistent weather. Since in B.C. the summers are colder and the winters are warmer than the east coast of Canada, it was decided that the energy generated would have a longer and more significant impact on the west coast.

The chosen location allows the generation of a meaningful amount of wave energy without being too far off the coast. Our system will be located around 5 km off the shore at a depth of around 40 m. Although the implementation of a system further off the shore would produce more energy, it was decided that an area closer to shore would be more beneficial due to the continuous collection of energy and the shorter distance needed to travel for maintenance and repair costs. The average wave height was found to be around 2.5 m in the winter and 1.5 m in the summer [18]. The average energy period is around 8.5 seconds in the summer and 9.5 s in the winter [21]. Since past research demonstrates that buoy VEHs in B.C. located at least 5km off-shore produced a significant and useful amount of harvested energy, it was decided that the project no longer serves as a good replacement for beach-side buoys.

The chosen area is home to around 40,000 people with around 60% of total energy demand on the island coming from the mainland [18]. Therefore, the use of the VEH buoy system could help the area become more self-sufficient regarding energy consumption.

6.3 Choice of Materials

Design choices were made based on research, past work, specific design criteria, and optimization.

For components that are isolated and less significant mathematically, i.e. that have less influence in the mathematical model established, design choices were made based on research and industry standards. For components that heavily affect the energy output, material options were researched and compared. Finally, for components that are interdependent or whose mathematical significance is unsure, optimization of parameters via modelling was performed. Such components include magnet size/mass, resistance of resistor, capacitance of capacitor, length of coils, spring constant, damping coefficient. The numerical values of these components are not included here below but can be found in the Optimization subsection of the Modelling section.

The circuit system and larger buoy system are each comprised of many small items which each contribute an important aspect to the overall system. Thus, it is important to compare and evaluate each item and its qualities to ensure that the system is properly optimized. Additionally, since each item is unique to the other, each item will have its own unique criteria.

For resistors, lower price options are more desirable. The resistor material should have minimal capacitance to ensure minimal energy is lost to it, and should also have minimal inductance so as to not interfere with the capacitor or the conducting coil of the mass-spring system.

6.3.1 Spring Material

The spring material for the mass-spring system should be cheap and not easily deform-able. Resistance to deformity is important to keep the location of static equilibrium constant. Otherwise, the spring may deform over time, raising or lowering the position of static equilibrium, which ultimately will change the resonant frequency.

The choice for the material for the spring is under the same criteria for the wiring material, with the additional criterion of resistance to deformation. It is pertinent that the spring retain its shape for as long as possible to ensure the mass-spring-damper system does not change over time, thus, longevity is the most important criterion for this choice.

While nickel alloys "offer a high level of strength and durability" [7], stainless steel is a more common and accessible material to use for the spring material, and is the dominant material for springs while still offering "high tensile strength and great resistance to fatigue". Thus, stainless steel was chosen as the spring material.

6.3.2 Capacitor Material

The capacitor is the object that will store the charge until it is ready to discharge. Thus, the higher capacitance a capacitor has, the better. There are many standardized options for capacitors in circuits, and which capacitor is best for which circuit is dependent on the input voltage and the desired range of capacitance. For the chosen solution, this input voltage is in the range of 0V - 10V, and the desired range of capacitance is simply the highest capacitance available to maximize the energy stored.

Aside from the cost, the other important factor in choosing the capacitor type is the ability to store a suitable amount of energy and do so while withstanding power surges and having a minimal

inductance.

Film capacitors become the immediate choice in this regard, meeting both of the desired criteria [9]. While other capacitors offer a high range of voltages, such as polymer capacitors, or usability at more extreme temperatures, such as class 2 ceramic capacitors, they also have drawbacks of being typically more expensive or having more apt use in other applications such as audio or circuit control as opposed to energy storage.

6.3.3 Resistor Material

For the resistor material, it is desirable to have a resistor that will have minimal impact on the system outside of the resistance it provides. Further, since the circuit is equipped with a capacitor and no inductor, the resistor should have minimal to no capacitance and minimal to no inductance. Also, the resistor should work over a wide range of voltages.

Foil resistors are thus the best choice for this aspect, as they have "very low capacitance and no inductance at all" [8], and can be designed for very specific resistance values. Other resistor types fail to meet the criteria based on having high inductance values, such as wire wound resistors, or by being easily damaged by voltage surges, such as metal film resistors [8].

6.3.4 Wiring & Coil Material

It is crucial to select appropriate wiring for the system as it will be transmitting the energy from the mechanical state to the electrical state. When selecting the wiring material, one of the most important parameters is cost. The cost of the material is also reflective of its availability. Another important parameter is the efficiency of the material, as it is most desirable to lose the least amount of energy possible while transferring it to the capacitor(s).

The wiring material of the design is a critical aspect of the solution as it essentially acts as the skeleton of the circuit system. The chosen material should, ideally, be cheap to acquire, be effective at transmitting a current, and have minimal impact on the local environment.

Several metals were considered for which would be best to use for the wiring. Among copper, gold, aluminum, and lead, copper was determined to be the best option. While gold is the most efficient conductor among the four metals, it is also the most significant by a large margin, costing \$22,372 USD per pound [5], compared to around \$1.30 - \$3.335 USD per pound for copper [4].

Aluminum stands as a good second choice given it is cheap, costing about \$.70 USD per pound [4], but it is not as efficient at transmitting current as copper is.

Lastly, although lead wiring would be the cheapest option at \$.44 USD per pound [6], it is also the most inefficient material among the metals, is the heaviest, and has middling availability. Further, lead is the most toxic metal among the four, meaning it has the most potential to harm local wildlife.

6.3.5 Magnet Type

The magnet chosen for the solution is another crucial component to the system, as the induced electromotive force from the electromagnetic transducer is proportional to the speed of the magnet and the magnitude of its magnetic field. Therefore, the size of the magnetic field of the magnetic material is an important parameter. Another important parameter is cost, as the solution must be

economically viable.

Neodymium magnets are the best choice for the magnetic material. They are the industry standard for applications which require magnets, and are the "strongest permanent magnets commercially available" [10]. They are readily available for both commercial and private use, while also being the cheapest option among other magnet choices.

6.3.6 Buoy

The buoy used will be a cylindrical shape with a diameter of around 3 meters. This size was determined as it is a commonly used size for systems in deep waters in the ocean. The exterior is made of polyethylene as it has good weather resistance and impact resistance. It also can resist against ultraviolet rays, frost, ocean chemicals and corrosion [38].

6.3.7 Shell

The shape of the shell needs to be carefully considered such that the shell satisfies all of its design requirements, which are: the shell must constrain the motion of the buoy to the z-direction; the shell shape must minimize friction between the shell and the buoy; and the shell must not impede ocean waves from moving the buoy. The shell will be made of a high density polyethylene sheet as it is a durable and strong material. Polyethylene sheets are resistant to impact, moisture and chemicals making it an ideal choice to encase the buoy system [37].

Through brainstorming, it was suggested that shell shapes that satisfy these design requirements are circular and square shells (both illustrated in Figure 5). In order to reduce frictional forces, small protrusions would need to be added to the circular shell; the square shell requires no such modification.

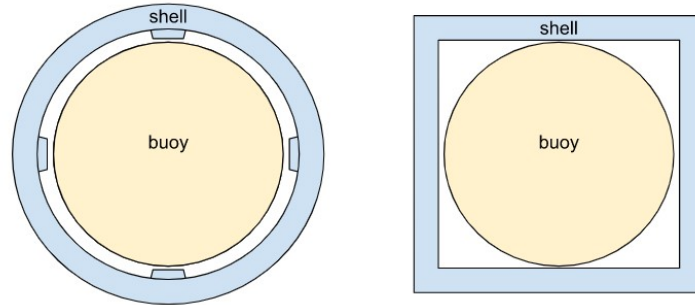


Figure 5: Top view comparison between circular and square shell shapes.

The shell, no matter the shape, will also be cage-like in structure in the sense that water will be able to move through it, as illustrated in Figure 6 for the circular shell.

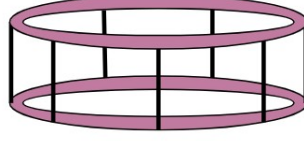


Figure 6: Side view of a circular shell design.

6.3.8 Grounding Cables

It was decided that the grounding cables should be fixed to the shell rather than to the buoy, so as to not impede the movement of the buoy. The project assumes that this component configuration also results in negligible force interactions between the grounding cables and the buoy.

7 System & Mathematical Model

7.1 System Overview

The system input will be the oscillatory motion of the ocean - specifically the up/down motion of waves. The VEH will be placed on a buoy; buoys are excellent contenders as the vehicle for a VEH system given their ease of implementation and maintenance, and their consistent oscillatory nature. Additionally, given that most water-based locations involving human activity (such as trading ports, public beaches, marinas, etc) already use buoys, there is an existing infrastructure for buoys upon which the VEH system could be built. The VEH system is comprised of an electrical circuit including a transducer, resistor, and capacitor. The harvested energy can either be outputted to a battery or another electrical circuit. In sum, the system input are the vibrations from ocean waves (wasted mechanical energy) and the output is electrical energy.

The equations which describe the circuit and consequent power output of a VEH will depend on the type of transducer chosen. However, since the design of this project's solution is using an electromagnetic transducer, the construction of the mathematical system is done specifically with an electromagnetic transducer equipped. An electromagnetic transducer includes a ferromagnetic mass, a conductive coil, a spring, and a damper.

7.2 Lenz' Law

The electromagnetic transducer is the component of the system that transforms the mechanical energy of the waves to the electrical energy stored in the capacitors. To relate these two quantities, Lenz' Law is applied. Lenz' Law is a special law of conservation of energy in relation to changes in magnetic flux. As a magnet moves through a conductive coil, the magnetic flux in the area of the coil changes. In response, a current is induced in the coil, which creates a magnet field around the coil and a counteractive flux which conserves the energy of the system.

This induced current can then be related to an induced voltage, which is the electromotive force of the design solution. This electromotive force is described by Equation 8 below, where V is the induced voltage, B is the magnetic field strength of the magnet, l is the axial length of the coil, and $x \dot{}$ is the velocity of the magnet as it oscillates on its mass-spring-damper system.

$$V = Bl\dot{x} \quad (8)$$

7.3 Spring System

A magnet attached to a spring-damper system is described by the following differential equation [3], where m is mass, \ddot{x} is acceleration, c is the damping coefficient, \dot{x} is the velocity, k is the spring constant, and x is the position.

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (9)$$

However, since the mass-spring-damper system is enclosed within the buoys, there will be an external force applied to the system, denoted as $F(t)$. The force function here is determined by the oscillating buoyant force of the ocean waves.

7.4 External Force Function

For the purposes of finding the force function, it is assumed that ocean waves move in a sinusoidal pattern. From past works, values for wave height and wave period can be found for specific locations. This means that for an equation describing the motion of the ocean waves, where $x(t)$ is the position of the wave, A is the amplitude, ω_0 is the frequency, and t is the time:

$$x(t) = A\sin(\omega_0 t) \quad (10)$$

However, Equation 9 above provides a value in the units of meters, not Newtons. Therefore, in order to change the wave position function, the oscillatory motion of the waves is treated similar to that of a mass-spring system, treating the water itself as a spring with its own spring constant.

Considering Hooke's law,

$$k = \frac{F}{x} \quad (11)$$

the force in Equation 11 above can be related to the buoyant force of the ocean wave F_b , and x its amplitude. Thus, the equation becomes

$$k = \frac{F_b}{A} \quad (12)$$

Then, recombining Equation 12 with Equation 10:

$$F(t) = kx(t) = \frac{F_b}{A}A\sin(\omega_0 t) = F_b\sin(\omega_0 t) \quad (13)$$

Thus, Equation 13 is the external force function of the mass-spring-damper system.

7.5 Resonance and Natural Frequency

In order to maximize the power output of the system, it is desirable to have the system operate at resonance. Resonance is the phenomenon where an oscillatory system acting under no external forces indefinitely oscillates around its equilibrium point under ideal conditions. The frequency at which the system operates during resonance is called the natural frequency.

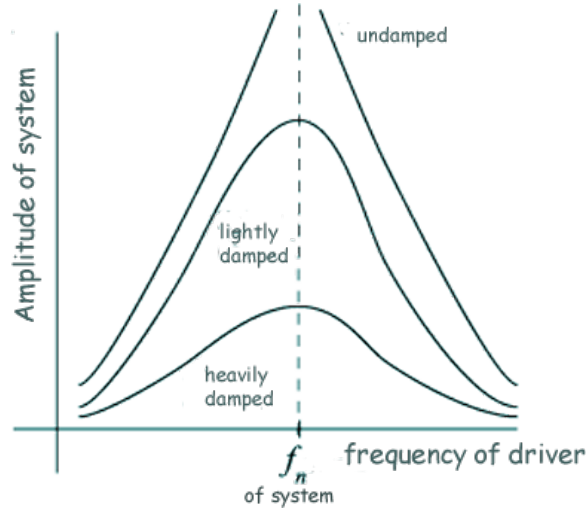


Figure 7: Simple diagram demonstrating the effect of a system's natural frequency affecting resonance.

Mass-spring-damper systems do not have natural frequencies as the damper causes the system to tend towards resting at equilibrium as time increases. To include a damper, and also ensure a natural frequency can be found, the damper was chosen to have a coefficient extremely close to 0. Then, the system can be analyzed by treating the damping coefficient as equal to zero and then finding a natural frequency.

With the force function found and the damping coefficient set to zero, the differential equation

$$m\ddot{x} + kx = A\sin(\omega_0 t) \quad (14)$$

is found to have a natural frequency of $\omega_0 = \sqrt{\frac{k}{m}}$ [3], which provides an additional constraint to the design of the solution.

7.6 Circuit System

The second half of the math model involves analyzing the circuit system equipped with the electromagnetic transducer. In Figure 8 below, R represents the resistance of the resistors, V is the voltage induced by the electromagnetic transducers, and C is the capacitance of the capacitor.

The circuit allows for the structure that transports the transformed energy to be stored in the buoy's capacitors. The circuit is equipped with several electromagnetic transducers which act as the voltage generators for the circuit analysis, resistors to help control the system, and a capacitor to store the energy. Each loop of the circuit functionally represents a buoy being connected to the overall buoy system.

Kirchoff's loop rules can be applied to each individual loop to find the contributing terms of voltage gain/loss. The equations describing each loop are shown in Equation 16 below, where V is the voltage produced from the electromagnetic transducer, R is the resistance of the resistor, \dot{Q} is the current, Q_c is the charge on the capacitor, and C is the capacitance of the capacitor. i indexes the relevant value, denoting which term relates to which loop.

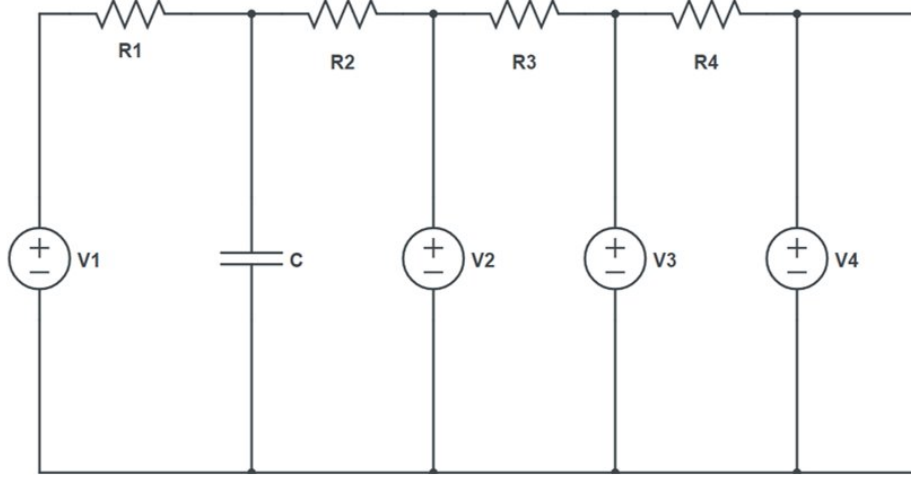


Figure 8: Circuit diagram representing VEH equipped buoy system.

$$\begin{aligned}
 V_1 - R_1 \dot{Q}_1 - \frac{Q_c}{C} &= 0 \\
 V_2 - R_2 \dot{Q}_2 - \frac{Q_c}{C} &= 0 \\
 V_n - R_n \dot{Q}_n - V_{n-1} &= 0
 \end{aligned} \tag{15}$$

Equation 16 can be substituted in to put the differential equations in terms of the velocity of the magnet:

$$\begin{aligned}
 Bl\dot{x}_1 - R_1 \dot{Q}_1 - \frac{Q_c}{C} &= 0 \\
 Bl\dot{x}_2 - R_2 \dot{Q}_1 - \frac{Q_c}{C} &= 0 \\
 Bl\dot{x}_n - R_n \dot{Q}_n - V_{n-1} &= 0
 \end{aligned} \tag{16}$$

The system of differential equations above is then simulated in MATLAB to find the total energy stored. The energy stored in a capacitor is described in the equation below, where Q_c is the total charge on the capacitor, and V_c is the voltage across the capacitor:

$$E = \frac{1}{2} Q_c V \tag{17}$$

7.7 Math Model Assumptions

The accuracy of the math model is restricted by some of the assumptions made.

Values for parameters such as wave height amplitude, wave period (natural frequency), and water density are taken at their average value at specific time frames. However, such values will fluctuate throughout the year as the seasons change and the water at the location of the buoy becomes colder/hotter. Additionally, since weather is inherently chaotic, taking average values of such parameters will exclude outliers from the analysis. Further, long-term effects such as climate change

may drastically change these parameters.

The inefficiency of a physical system has also not been analyzed for the system. This includes the loss of energy from sound, friction, heat loss via the intrinsic resistivity of wires, and energy leakage of the capacitor.

8 Modelling

8.1 Potential Simulation Tools

A software was required to properly model the buoy-VEH system. The goal was to model the behavior and mathematics of the system given the optimized parameters. The goal was not only wanted to solve the ordinary differential equations but also to obtain the resulting power output. An evaluation matrix was created to explore three main options for the modeling of the finalized design. Each option is evaluated based on a specific criterion and a respective weighting to obtain the best choice for a modeling technique. Each rating is out of ten, where ten is the best score and two is the worst. The following criteria are weighted based on importance in the development process.

1. **Ease of Learning & Development:** Some software options may not be familiar to all team members, so it is imperative that the team research and learn how to use the program option before approaching the modeling design in that respective program.
2. **Math Customization:** The system's mathematics is quite complex, with tons of overlap and added mathematics that properly models the system. The options are evaluated on how difficult the customization of the embedded mathematics is implemented in each of the options.
3. **Data Visualization:** The options are evaluated based on the software's ability to visualize the outputs in an effective and organized manner.
4. **Runtime:** The run-time defines how long each simulation would take to run before outputting results. This component is not a major component but does affect the speed of the team's development.
5. **Research Conducted:** Papers written on Vibration Energy Harvesting using ocean movements are limited, so access to research that relates to the defined topic is essential to aid in accurately modeling the system.

The main options explored were Python, MATLAB exclusively, or a combination of both MATLAB and Simulink. The evaluation matrix for the decision of which tool can be seen below.

Table 4: Evaluation matrix to decide on one of three potential modeling approaches using various software.

		Option 1 Python Script to Solve Differential Equation System, New Script to Visualize		Option 2 MatLab Script to Solve Differential Equation System, Same Script to Visualize		Option 3 Simulink Model of System with Data Output, Visualization Using MatLab Script	
Evaluation Criteria	Weighting	Rating	Weighted Score	Rating	Weighted Score	Rating	Weighted Score
<i>Software Development Ease</i>	20%	6	1.2	4	0.8	10	2
<i>Math Customization</i>	30%	6	1.8	4	1.2	8	2.4
<i>Data Visualization</i>	25%	10	2.5	6	1.5	6	1.5
<i>Runtime</i>	10%	8	0.8	2	0.2	2	0.2
<i>Research Conducted</i>	15%	4	0.6	2	0.3	8	1.2
TOTAL	100%	6.9		4		7.3	

Based on ease of use and desired flexibility in design, Option 3 proves to be the best tool to use for the chosen solution having the highest weighted score out of ten comparatively.

8.2 Option 3: MATLAB & Simulink

This option entails modeling the external and internal system using Simulink, outputting total potential energy across the capacitor, and visualizing it using a simple MATLAB code. As seen in Figure 4 to its high rankings in Software Development, Math Customization, and Research Conducted.

To begin, both MATLAB and Simulink have embedded functions to solve and model differential equations. Learning how to model a differential equation in Simulink proved to be quite efficient and easy to implement. Because of the interactions between the two systems, it is clear that customizing the model is imperative to obtaining accurate results. This is because compared to MATLAB and Python scripts, the differential equations are hard-coded and leave more room for customization in the model. Therefore, using this option guarantees the system can be properly customized with proper specifications in a much simpler way proven than the other options.

Finally, based on academic papers regarding Vibration Energy Harvesting with ocean waves, very few use Python or MATLAB scripts alone to fully model the system, whereas most use a combination of various software. However, multiple sources use Simulink to model the EMF specifically to obtain accurate results for the system. The research that used Python for modeling purposes had very different project designs mostly using different transduction materials with very different designs aside from ocean energy harvesting. Based on all these criteria and access to materials, it is clear that Option 3 is the most desired for the finalized design of this system.

8.3 Overview of Simulation

In the Simulink model, there are two major components: the external system and the internal system. The goal was to model, given the optimized parameters, how much electrical energy and power the system would output altogether. Altogether, the Simulink model is based on five analyzed equations, two of which are the primary differential equations seen in 18 and 19. From there, a Simulink system was developed to model the mathematics and behavior of the buoy system. A MATLAB code was then developed to obtain the results accurately.

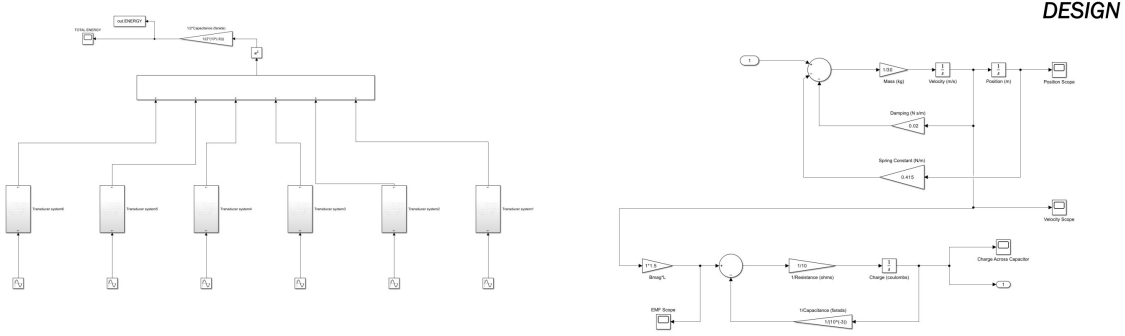


Figure 9: An image of all the simulation components, with the external system being on the left, and the internal system being on the right. More detailed diagrams are seen in Figures 10 and 12, 15 respectively.

From Figure 9, the external system on the left depicts the coupled buoys in a parallel circuit with a single capacitor that has to be discharged. Each buoy has a sinusoidal function (the buoyant force function) as an input and outputs its respective charge across the capacitor. The charges are then coupled and converted into total energy stored in the capacitor.

Each buoy has its own internal circuit, which is where the internal system comes into play. These two differential equations are the basis of the individual modeled circuits:

$$m\ddot{x} + \beta\dot{x} + kx = F_b * \sin(\omega t) \quad (18)$$

$$Bl\dot{x} - R\dot{Q} - \frac{Q}{C} = 0 \quad (19)$$

They must be modeled together because the output of the mass-spring system, the velocity of the magnet (\dot{x}), is used to obtain the generated EMF and used as an input for the charging system.

8.4 Simulink: External System

The system in Simulink is modeled using six buoys in total. Because the team projects the buoys to be around 3 meters in diameter, the power obtained from a six-buoy system proved to be more than sufficient. Each buoy has its own sinusoidal wave as an input to its internal system, and the output of each's circuitry is its respective charge across the capacitor. The charges for all the buoys are then combined to obtain the total charge and converted into electrical energy across the capacitor.

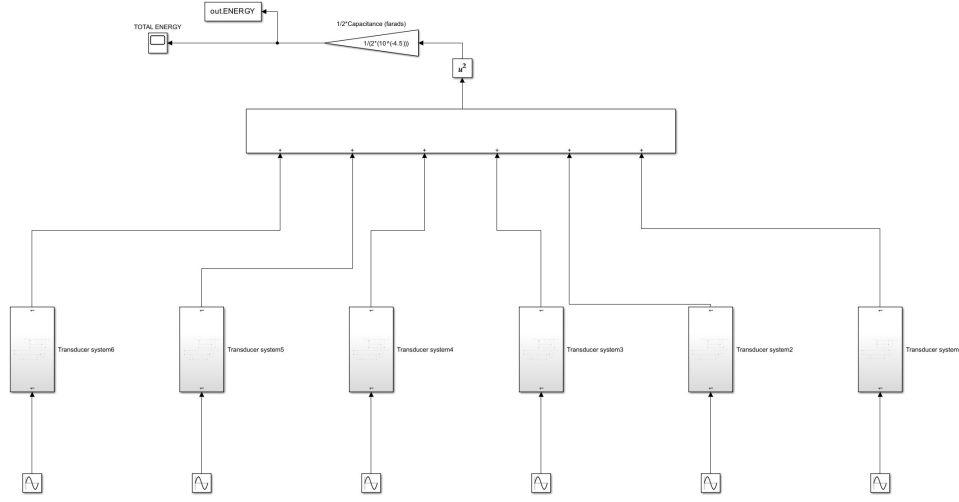


Figure 10: Annotated and enlarged image of the mass-spring modeled equation within the buoy's internal system.

Since the internal circuits are modeled based on Equations 18 and 19, the mass-spring system requires the buoyant force function, $F(t)$, as an input. Therefore, each buoy has a sine wave input accounting for its buoyant force amplitude and the respective resonance frequency. The calculations for this function are seen in Section 7.5, and the function input for each is as follows.

$$F(t) = 18670 * \sin(0.1176t) \quad (20)$$

Although each input function is the same in terms of the buoyant force amplitude and the resonance frequency, the phase of each function has been adjusted to account for the chaos of the waves to allow for more accurate outputs. However, the system itself has not accounted for the entirety of the potential chaos of the system.

The individual systems then output their respective charge across the capacitors which are then put through a sum block to obtain the signal for the total charge across the capacitor. Through research, it was deduced that the power in this particular system is additive, and therefore so is the charge. From the mathematical model, Equation 17 and the fundamental equation $V = \frac{Q_C}{C}$ to convert the total charge across the capacitor, Q_C , to the electrical potential energy outputted by the entire system, E .

$$E = \frac{1}{2} * \frac{Q_C^2}{C} \quad (21)$$

Looking at Figure 10, this equation is where the gain after the sum block comes from. It can be seen that the total charge goes through a square block where the function is squared and then multiplied by $1/C$ to obtain the energy. From this modeled energy, the team is then able to determine power since the outputted power is in units of Joules per second. Therefore, based on the time the energy is discharged, the power can be calculated using a simple fraction.

8.5 Simulink: Internal System

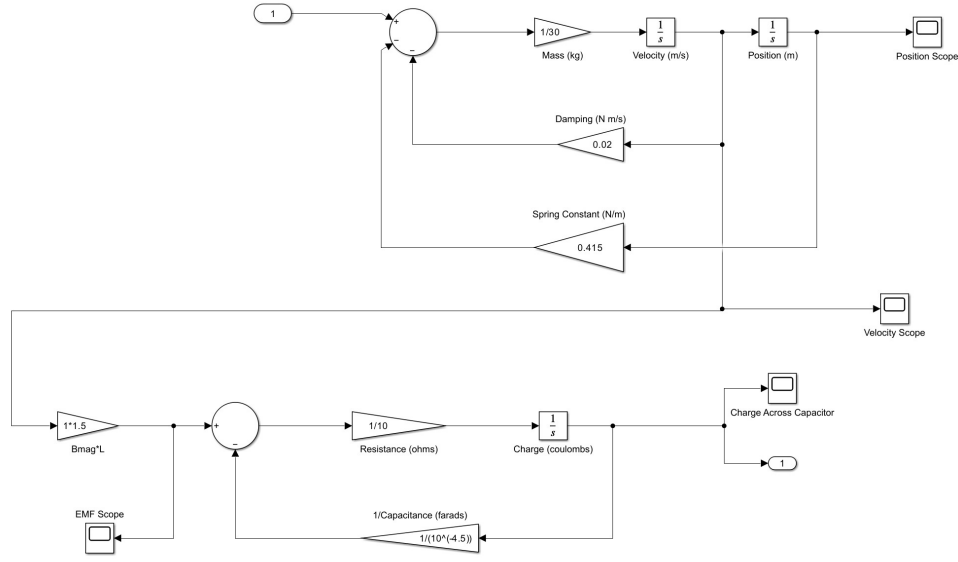


Figure 11: The individual circuit inside each buoy.

As mentioned, each buoy has its own modeled circuit system. Figure 11 shows that the system has two major components: the first being the mass-spring system in the top right, and the charge across the capacitor system in the bottom left. These two differential equations had to be modeled together because the goal was to use , the velocity of the magnet, as an input for the charging system.

8.5.1 First Differential Equation: Mass-Spring System

Let's begin with the first differential equation, the mass-spring system. This is modeled using Equation 18 where $F(t) = F_b * \sin(\omega t)$ is the buoyant force function as seen in Equation 20.

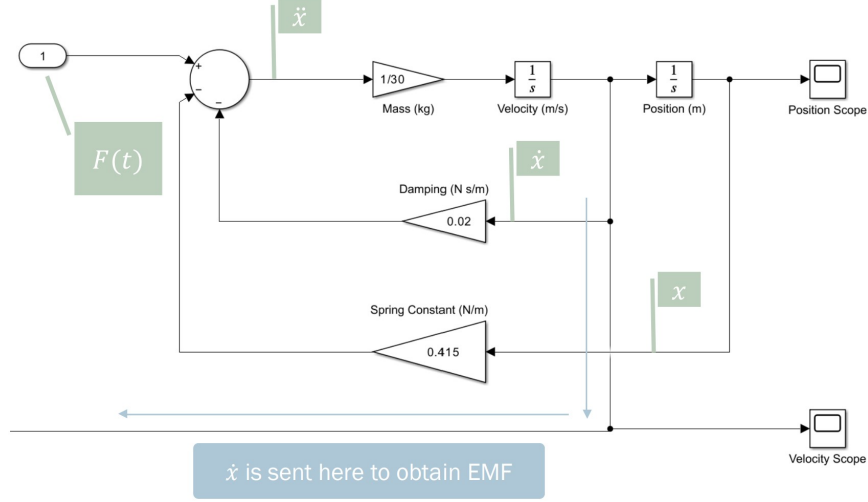


Figure 12: Annotated and enlarged image of the mass-spring modeled equation as seen in Equation 18 within the buoy's internal system.

Notice how the equation has various gains and integrators. That is because, for the model, the highest order derivative has to be isolated using the equation and becomes the following equation.

$$\ddot{x} = \frac{1}{m}(F_b * \sin(\omega t) - \beta \dot{x} - kx)$$

The highest order derivative then becomes the output of the sum block. As seen, once the \ddot{x} goes through the mass gain and then through the integrator, we then obtain \dot{x} which then gets multiplied by the damping constant, and so on. From here, we can connect a wire right after the first integrator to obtain \dot{x} which we can use in the second differential equation to obtain the EMF.

8.5.2 Second Differential Equation: Charge Across the Capacitor

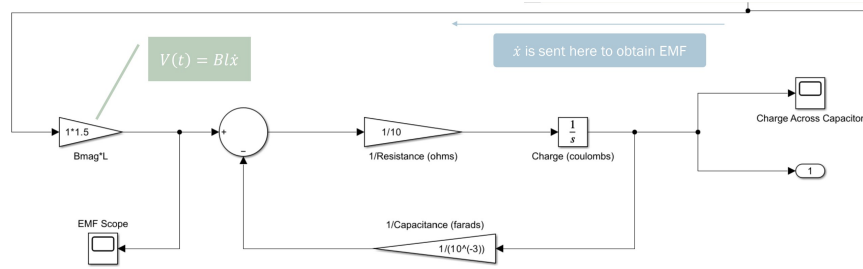


Figure 13: Annotated and enlarged image of the mass-spring modeled equation as seen in Equation 19 within the buoy's internal system.

\dot{x} is then sent to the EMF gain where it is multiplied by the magnetic field B and the axial length of the coil l and is used in the second differential equation where the goal is to obtain the charge across the capacitor for that respective buoy. The same structure as the mass-spring system for modeling

the equation, meaning the highest order derivative needs to be isolated as follows.

$$\dot{Q} = \frac{1}{R}(Bl\dot{x} - \frac{Q}{C})$$

In the case of this equation, Q is the charge across the capacitor, which would be after \dot{Q} , the current as a function of time, gets integrated into charge. This obtained value is then what is sent as an output to the external system and summed together with the other outputs.

8.6 MATLAB: Code Description

A script was developed in MATLAB using the data outputs from the Simulink model to properly visualize the behavior of the system. Although using a script may be deemed unnecessary based on the scopes placed in the Simulink model, the scopes do not visualize the outputs of the system in an organized and readable fashion the way that a well-written MATLAB script can. The MATLAB script can input the matrices saved from the Simulink model, plot it on a specific graph, assign appropriate labels, and have a more accurate and efficient visualization of the energy outputs for the simulated system.

8.7 Results & Outputs

After the simulation in Simulink went through countless design iterations and optimization techniques (see Section 8.8.2 for optimization process) the results were captured using the MATLAB script as designed after 10 minutes and around 3.33 hours respectively.

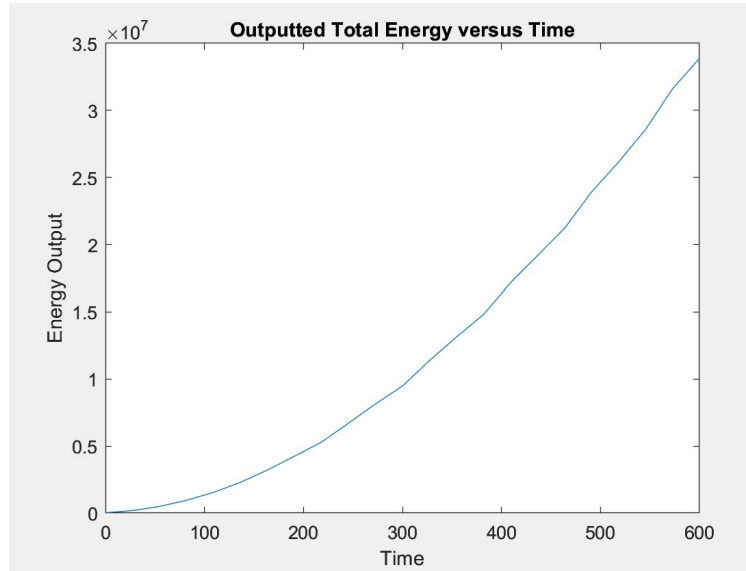


Figure 14: Output of MATLAB script where the energy output is in joules and the time is in seconds. This is the energy output graph after around 10 minutes.

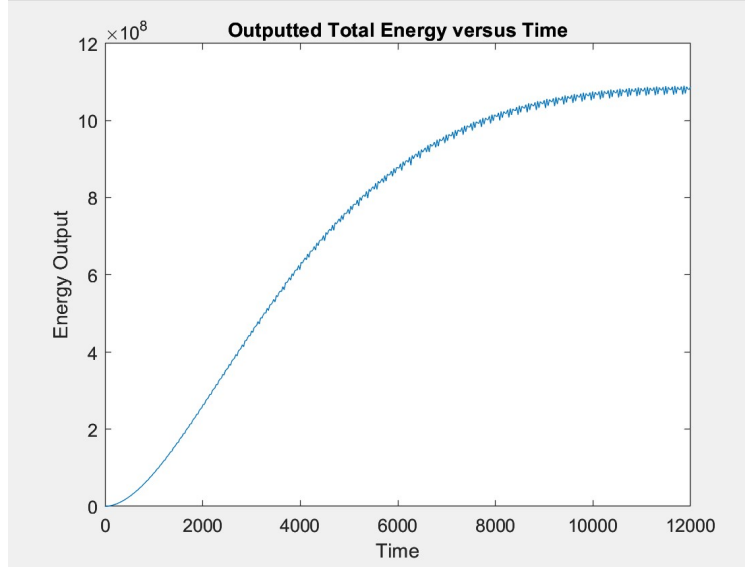


Figure 15: Output of MATLAB script where the energy output is in joules and the time is in seconds. This is the energy output graph after 3.33 hours.

Note that these are the outputs of the system with the optimized values as inputs. Therefore the energy output obtained for the designed system is expected to be $12 * 10^8$ joules.

8.8 Variable Specifications

The goal of this finalized design is to extract as much electrical potential energy as possible while being realistic and using appropriate values in simulations. In the optimization process, the team decided on specific values to optimize while deciding on preset values that, in testing, did not seem to affect the overall output by much. Therefore, three specific values were optimized, while the rest of the values were based on accuracy from research, and access and cost of resources.

8.8.1 Assumptions

The mass of the magnet and the spring stiffness could be predetermined based on access to materials. Using the phenomenon of resonance frequency, we use the equation $\omega_o = \sqrt{\frac{k}{m}}$, where research at this particular location indicates that the expected resonance frequency is $0.1176Hz$, and because from Section 6.3.5 the expected material tells us the expected mass of the magnet is 30 kg, we can solve for the spring stiffness.

$$M = 30kg$$

$$K = (0.1176Hz)^2(30kg) = 0.415N/m$$

Note that the resonance frequency $0.1176Hz$ is used as an input for the buoyant force function as seen in Equation 20, where the amplitude is determined using the following equations.

$$V = \pi r^2 h$$

$$F_b = \rho V g$$

Plugging in the mass of the magnet, the radius of the buoy which is chosen to be $1.5m$, and the expected density of the water, and the expected height of the buoy-water displacement $0.3m$, we can solve for the amplitude as follows.

$$\rho = 0.8978 \frac{g}{cm^3} \left(\frac{\frac{1kg}{1000g}}{\frac{1cm^3}{100^3m^3}} \right) = 897.5 \frac{kg}{m^3}$$

$$V = \pi(1.5m)^2(0.3m) = 2.12m^3$$

$$F_b = (897.5 \frac{kg}{m^3})(2.12m^3)(9.81 \frac{N}{kg}) = 18,670N$$

Where the amplitude of the buoyant force function is $18,670N$ which is plugged into the buoyant force function which is how we obtain Equation 20.

When testing the model, the back EMF was determined to be negligible since it was extremely small compared to the actual EMF output. In addition, the resistance when varying for optimization purposes turned out to not affect the output at all, so a standard resistance of 10Ω was chosen.

8.8.2 Optimization

Damping Constant & Magnetic Field Strength

From Section 7.5, it was determined that the spring system had to be dampened in order to fit the specifications for an electromagnetic transducer. Because the damping coefficient would be incredibly minor, in solving for the resonance frequency as seen in this section, it can be assumed to be zero to obtain an easier and more accurate result. However, the system has to be dampened, so the constant is not negligible entirely. Therefore, a small damping constant would optimize the resonance frequency and the overall system. Based on this logic and optimal damping constants, the value is set as:

$$d = 0.02N * \frac{s}{m}$$

The next optimized component regards the magnet. As discussed in Section 6.3, the material for the magnet was decided to be Neodymium, and the mass was found to be $30kg$ based on material density and average size. The maximum possible magnetic field strength for this material is 1.4 Teslas. It was determined that the higher the magnetic field strength the larger the output proportionally, but 1.4 Teslas is not a realistic expectation for this system. Therefore, by validating with sources, B can be accurately set between the range of $0.3T$ and $1T$, so an optimal choice for the magnetic field strength is:

$$B = 1T$$

Selection of Capacitance for Optimal Energy Output

The BC Hydro website states that households in BC use 29.1918 kWh/day on average [40]. Using the unit conversion below, we can calculate how much energy would be required to power one household for a week. (We are using weeks as our timeframe since the collection of capacitors will occur once a week.)

We here assume that energy can be discharged at an extremely fast rate, i.e. that no conversion needs to be made between the amount of energy stored and the amount of energy needed; these will be viewed as the same.

$$\frac{22.1918kWh}{day} = \frac{22.1918kJ * h}{s * day} * \frac{7days}{1week} * \frac{1GJ}{1000000kJ} = \frac{0.5592GJ}{week} \quad (22)$$

In other words, 0.5592 GJ is the average amount of energy consumed by one household in any given week. To determine how much energy is consumed by 100 households in any given year, we simply multiply this number by 100 and obtain 55.92 GJ. This means that the designed solution must generate 55.92 GJ per week in order to power 100 households per week.

The capacitor chosen for this project has a fill-time of 12000s (or 3.33 hrs); i.e. after this time, no further energy can be stored in the capacitor, even if more energy is harvested. As such, in order to determine how many central capacitors are required of the system, we divide 1 week by 12000 seconds, which gives ≈ 51 capacitors. In order to determine how much energy each capacitor must store, we divide 55.92 GJ by 51, which gives ≈ 1.1 GJ per capacitor.

Given the above, a capacitance between $1 * 10^{-3}$ and $1 * 10^{-9}$ that produces these results must be chosen. Using a capacitance of $1 * 10^{-4.45}$ results in each capacitor storing $1.2 * 10^9$ GJ in 12000 seconds. Capacitors will be set up in series such that once one capacitor is full (i.e. once 12000s have passed), energy will start to be stored in the 2nd capacitor, and so on for the entire week until all 51 capacitors are full. Note: while the modeled system output is $1.2 * 10^9$ GJ per capacitor per 12000s, the following math will presume that $1.1 * 10^9$ GJ is stored per capacitor to account for variability between the model and reality, this is due to assumptions, wave behavior, energy losses, etc..

$$\text{energy output rate per capacitor} = \frac{1.1GJ}{12000s} = \frac{1.1GJ}{1 \text{ week}} \quad (23)$$

The energy output per 12000s per capacitor is equal to the energy output per week per capacitor since once a capacitor is full it cannot store more energy.

$$\text{fill-times per week} = \frac{604800 \text{ seconds}}{12000s} = 50.4 \approx 51 \quad (24)$$

$$\text{weekly energy output} = \frac{1.1GJ}{1 \text{ week} * 1 \text{ capacitor}} * 51 \text{ capacitors} = \frac{55.92GJ}{\text{week}} \quad (25)$$

9 Final Design

9.1 Final Variable Values

Table 5 outline the final values for variables used in the MATLAB model.

Table 5: Values and their units for all variables used in the MATLAB model.

Variable	Value & Units
Magnet weight	30 kg
Spring constant	0.415 N/m
Resistance	10 ohms
Axial coil length	1.5 m
Damping coefficient	0.02 Ns/m
Buoy diameter at largest point	3 m
Capacitance	$1 * 10^{-4.45}$ farads
Magnetic field strength	1 tesla

9.2 Physical System

Through research into past works and the chosen implementation location, it was decided that 6 buoys would be the minimum number of buoys that would render the system useful, in terms of energy output. In an effort to minimize capital cost, no further buoys will be added to the system.

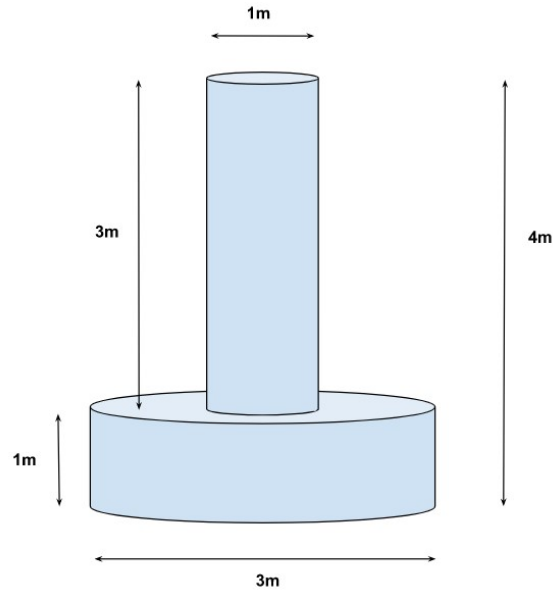


Figure 16: Chosen buoy dimensions.

Figure 16 illustrates the chosen shape of the buoy. The buoy will be bottom-heavy and have a cylindrical base that is 1m high and 3m wide. Fixed to this base will be another cylinder, this one 3m high and 1m wide. In other words, each buoy will be 3m in diameter at its widest point and 1m

in diameter at its narrowest point, and 4m in height. Around 0.333m of the buoy will be submerged. Buoys will be made of polyethylene and will be connected to each other via power transmission cables. Each buoy will be enclosed by a square polyethylene shell. Each shell will be anchored to the ocean floor by steel grounding cables and a steel anchor; the shell will also be connected to the other shells by steel connector cables.

Each buoy has a its own VEH. Each VEH is an encased circuit involving a resistor, an electromagnetic transducer (magnet, spring, long coiled rod, and damper), and a capacitor. The shell and the buoy were designed such that when the motion of the waves causes the shell and buoy to move/ , the shape of the shell restricts the motion of the buoy to the z-direction, which maximizes the acceleration of the magnet inside the transducer. The shell is grounded to provide more strength to the anchorage, which improves range of the system and allows it to withstand stronger waves. Materials and components used for the VEH were chosen for their efficiency, safety, and cost.

9.3 Results

The project's desired output is energy. The current system outputs 55.92 GJ per week, through the use of 51 capacitors organized in series in one of the buoys. As 55.92 GJ is the average amount of energy used per week by 100 households in BC, this means that the current system can power 100 households.

10 Evaluation

In terms of evaluation of our solution, multiple forms of analysis were undertaken.

10.1 Model Validation

We performed a model validation of the mechanical subsystem. Using data from past works as our inputs, we tested the accuracy of our model structure (there are various ways to organize items in Simulink). Similar results were achieved, indicating that that section of the MATLAB model is accurate.

The team evaluated and iterated the MATLAB model by discussing and iterating the model structure, comparing our energy outputs to those of past works, comparing the math model to the MATLAB model.

10.2 TBL Analysis

10.2.1 Economic Analysis

After the final solution was chosen, an economic analysis was conducted to determine the overall cost of the final design. In Table 6 , a cost breakdown of the system parts for one buoy is shown. The total price for one buoy will be \$7,186.47, and therefore for the chosen final design of a 6 buoy system, the final cost will be \$43,118.82.

Table 6: Cost breakdown of system parts for a single buoy.

System Part	Material	Cost
Wiring [30]	Copper	\$11.50
Magnet [31]	Neodymium	\$8.66
Spring [32]	Stainless steel	\$8.99
Resistor [33]	Foil resistor	\$33.81
Mooring lines [18]	Steel	\$900
Buoy [34]	Polyethylene shell	\$5,000
Capacitor [35]	Aluminum electrolytic capacitor	\$3.51
Connecting lines [18]	Steel	\$120
Anchor [36]	Steel	\$100
Shell [37]	High density polyethylene sheet	\$1,000
Total for 1 buoy		\$7,186.47

10.2.2 Social & Environmental Analysis

The implementation of the system will benefit surrounding residents as it would create jobs for locals and allow residents to use power from local sources. Since electricity generation becomes more efficient each year due to continuous development, the amount of energy needed to power homes will decrease over time. Therefore, the energy generated from the buoy system will be able to power more homes each year and thus would generate more profit over time. The ability to power more homes from clean energy sources will benefit the environmental due to the continuous transition away from fossil fuel powered energy.

10.3 Breakeven Analysis

The team also performed a breakeven analysis using cost data from BC Hydro. The current price for energy as stated by BC Hydro states is 14.22 cents per kWh. [40] Using this and the solution's energy output rate, we can determine the project's profit per year.

$$\frac{14.22cents}{kWh} = \frac{14.22cents * seconds}{kJ * h} * \frac{1\$}{100cents} * \frac{1hr}{3600s} = \frac{0.0000395\$}{kJ} \quad (26)$$

$$profit = \frac{0.0000395\$}{kJ} * \frac{55.92GJ}{week} * \frac{1000000kJ}{1GJ} * \frac{52weeks}{1yr} \approx \frac{114860\$}{year} \quad (27)$$

The project capital cost can be calculated by multiplying the cost per buoy (\$7,186.47) by 6 buoys; this equals \approx \$43119 per 6-buoy system. Using the project's profit rate and capital cost, we can determine the time period in which the running profit of the project equals its capital cost. This point is referred to as the breakeven point.

$$breakevenpoint = \frac{\frac{43119\$}{system}}{\frac{114860\$}{year * system}} = \frac{43119\$}{system} * \frac{year * system}{114860\$} = 0.375year \quad (28)$$

The project's breakeven point is 0.375 year or 4.5 months. This is an extremely fast time period in which to breakeven.

10.4 Graphical Analysis

Numerous graphs comparing the number of buoys per system to other results were created and are analyzed here below. Note that a fixed capacitance of $1 * 10^{-4.45}$ farads was used for all evaluations. Also note that there will be slight variation in the values for a 6-buoy system presented in the graphs below and reported in the previous sections. This is due the decision to use 1.1GJ/12000s per capacitor as opposed to 1.2GJ/12000s in calculations of profit and breakeven point to partially compensate for the inevitable difference between modelled and real energy outputs.

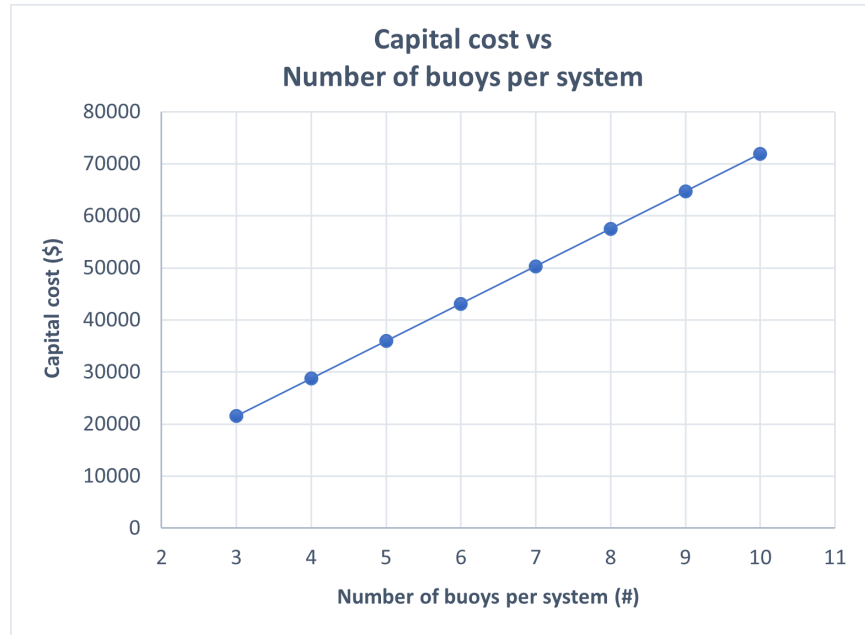


Figure 17: Graph illustrating the increasing linear relationship between the capital cost of a system and the number of buoys per system.

As expected, the capital cost increases linearly as the number of buoys per system increases. This makes sense given that the capital cost of each buoy is fixed.

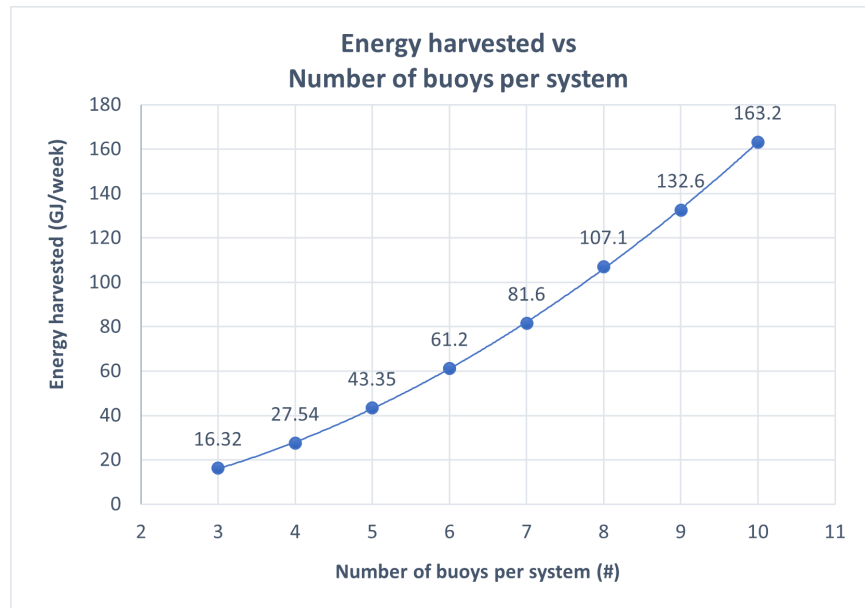


Figure 18: Graph illustrating the increasing non-linear relationship between the amount of energy harvested per week and the number of buoys per system.

Figure 18 compares the amount of energy harvested weekly to the number of buoys per system. As expected, a higher number of buoys results in a higher total energy output. As one can see, the trendline is not linear, indicating that efficiency of the system increases with the number of buoys.

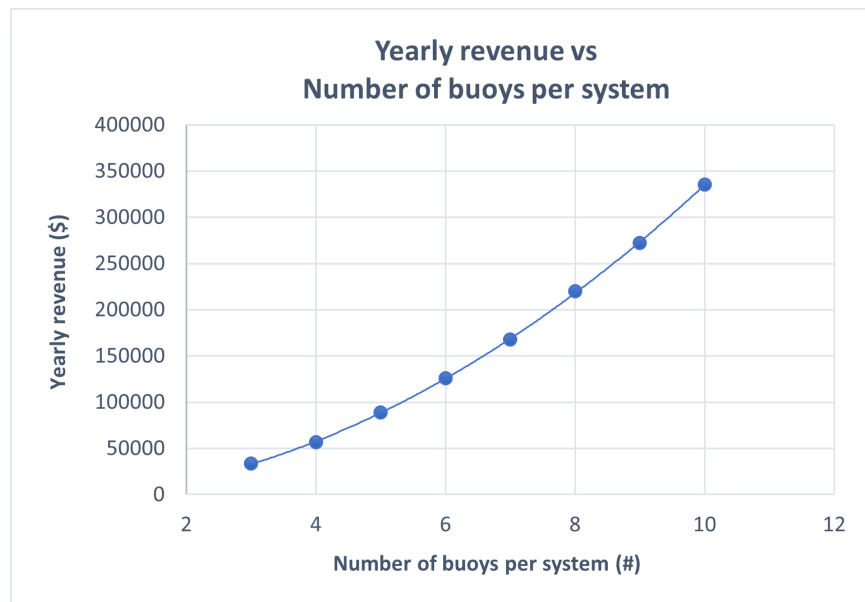


Figure 19: Graph illustrating the increasing non-linear relationship between the system's yearly revenue and the number of buoys per system.

Note, this graph illustrates yearly revenue, not yearly profit (where profit is defined as the absolute

value of the difference between capital cost and revenue). I.e. the graph solely outlines the revenue incoming due to sales of harvested energy.

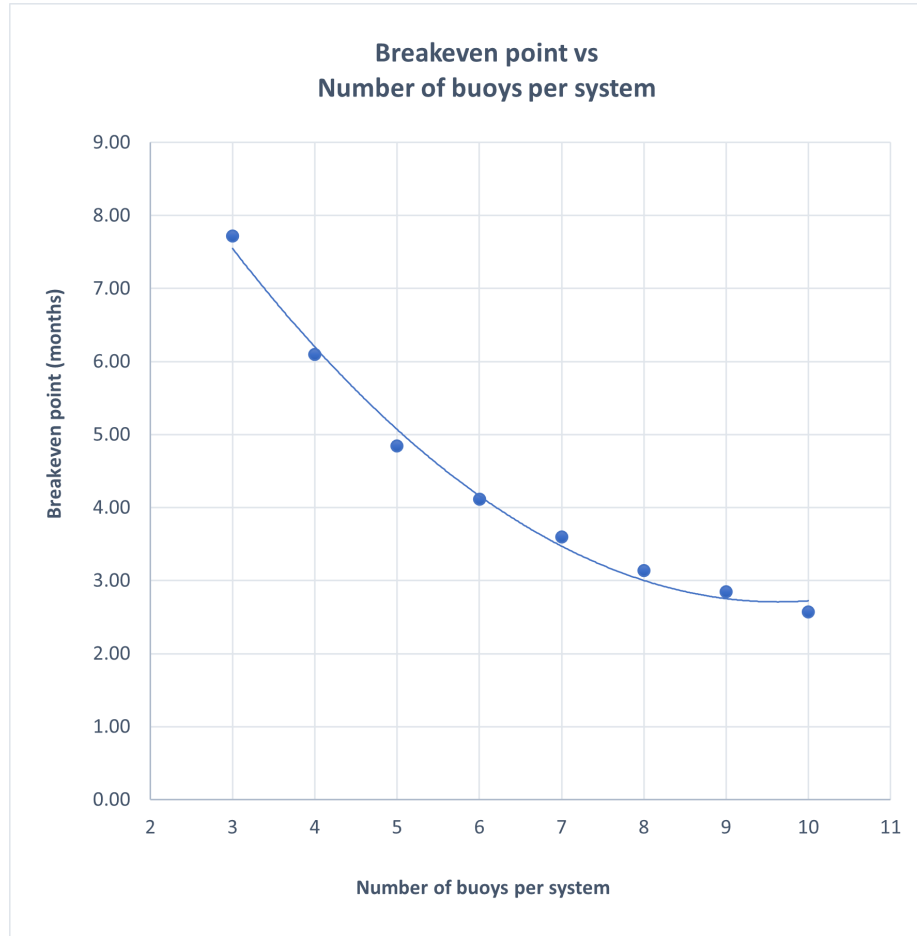


Figure 20: Graph illustrating the decreasing non-linear relationship between the system's breakeven point and the number of buoys per system. The graph is stretched vertically to emphasize the difference between breakeven points.

Using the calculated values for capital cost and yearly revenue, breakeven points were determined for all *i*-buoy systems, where *i* ranges from 3 to 10. The non-linearity of the trendline observed in Figure 20 indicates that breakeven points and number of buoys per system are not proportional. We can see that as the number of buoys increase, the trendline becomes flatter; this indicates that the breakeven point decreases by a lesser amount for every buoy added on. In other words, at a certain point, adding on buoys no longer decreases the system's breakeven point by an important amount.

Additionally, from Figure 20, we can see that the jumps in breakeven points from 3 to 4 buoys, 4 to 5 buoys, and 5 to 6 buoys are relatively large compared to the following jumps. In other words, jumping from 3 to 4 buoys, from 4 to 5 buoys, or from 5 to 6 buoys is much more efficient (breakeven point wise) than jumping from 6 to 7, 7 to 8, 8 to 9, or 9 to 10 buoys. This supports the finding that 6 buoys is the optimal number of buoys per system.

In short, based on all the above, 6 is the ideal number of buoys for the designed system, as a 6-buoy system was found to suitably balances energy output, revenue, capital cost, and breakeven point.

11 Implementation & Next Steps

This section outlines the obstacles and considerations that surround implementation of the solution, and additionally addresses next steps to be taken to further refine the system.

11.1 Implementation

For one, there will be a delay between the implementation date of the solution and when the harvested energy will be able to be used. This is simply due to the fact that the system must complete one cycle (or one week) before extracted energy can be used to power 100 standard households for the following week. In other words, energy harvested in week 1 will be used in week 2, energy harvested in week 2 will be used in week 3, and so on. In the last week of the year, households will be powered but no energy will be harvested.

Next, it is expected that there be some variation between the model and reality. For one, while the current model partially accounts for chaotic wave behavior through the phasing of sine waves, it does not fully address the complexity of fluid dynamics.

Further, the energy use data from BC Hydro is an average – that is, the model currently assumes that households do not fluctuate in energy use seasonally or monthly, or daily which is known to be untrue. This data was used to calculate the breakeven point, the number of households powered, the revenue, and the profit, and so it is acknowledged that all of these values will be slightly inaccurate due to differences in home size, family size, number of devices used, etc..

According to data gathered from BC Hydro, average energy use per household decreases yearly, mainly due to increased efficiency of lightbulbs, powerlines, and power systems.[39] This means that this system, without any modification, will be able to power more and more households every year.

Finally, the implementation of this design requires an employee to be hired to boat to and from the buoy system once a week to discharge the central capacitors.

11.2 Next Steps

The main improvement that could be made to the proposed solution is refining the force function. The force function presently makes use of many assumptions that could result in a less accurate model. For one, the model currently uses phased sine waves as its input; future work could include designing an input function that varies in amplitude, period, and force to better mimic real wave behavior. Based on additional research, the input function could also consider how wave behavior fluctuates depending on location and season. Also, the model currently assumes that both frictional and normal forces between the shell and the buoy are negligible; the next steps could entail exploring how these influence the force function. Another assumption made is that placing the grounding cables on the shell would negate the effects of the grounding cables on the buoy; in the future, the team could explore how the math model would change if this assumption was not made. Lastly, the current force function is one-dimensional, i.e. we are only considering forces in the z-direction. This is due to two assumptions, that the shell would constrain the motion of the buoy to the z-direction,

and two that constraining the motion of the buoy to the z-direction would output the most energy (as no lateral forces are involved). Future work could involve creating a three-dimensional force function to verify these assumptions.

In terms of the energy output, the model currently assumes that any energy loss (from heat dispersion during transport across the power transmission cables) is negligible. The next steps could include calculating energy loss and challenging this assumption.

With the aim of powering more households and harvesting more energy, the system could also be expanded to include more buoys and be implemented in more locations, where appropriate. Bodies of water other than oceans could also be considered.

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