

# **Introduction to statistics**

## **(Day 1)**

# Housekeeping

- **Who:**
  - Paola Dalmasso  
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  - Alessia Visconti  
[alessia.visconti@unito.it](mailto:alessia.visconti@unito.it)
- **Exam:**
  - Multiple-choice questions  
(*via* Moodle)

# Introduction



# **Why are we here?**

# Will you buy this mouthwash?



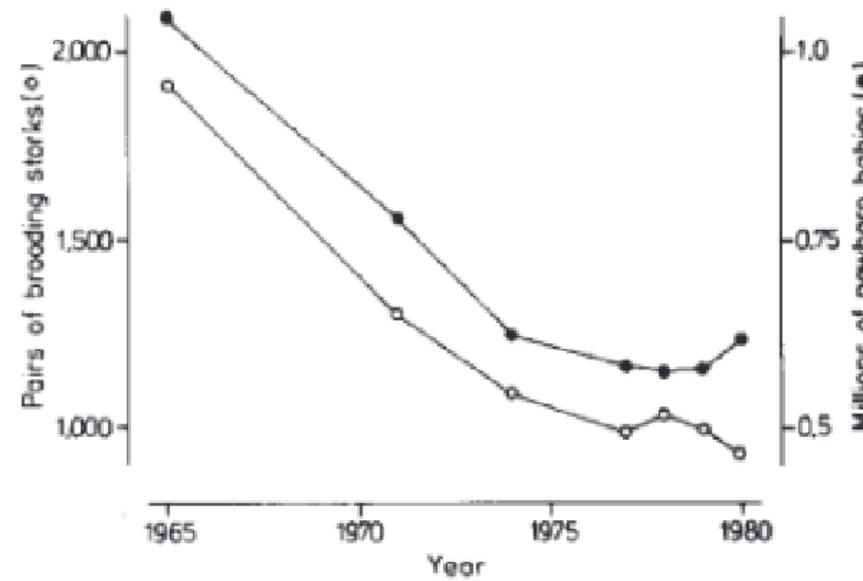
Elimina fino al  
99% dei batteri  
residui dopo il  
lavaggio



Raggiunge fino al  
100% della bocca

# Storks and babies

SIR—There is concern in West Germany over the falling birth rate. The accompanying graph<sup>1,2</sup> might suggest a solution that every child knows makes sense.

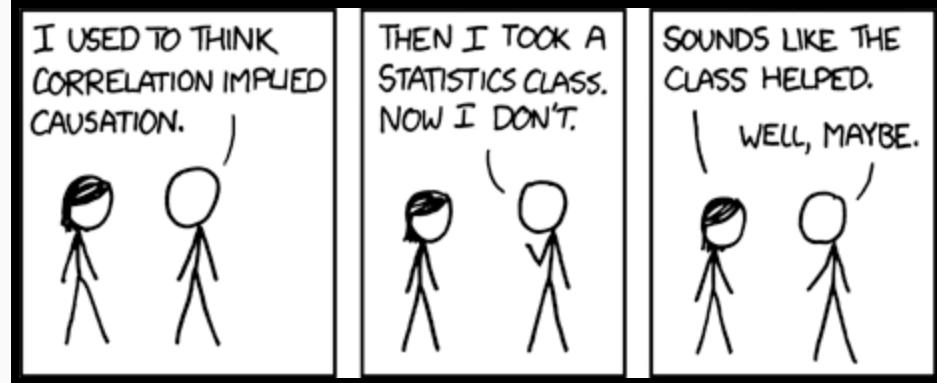


# Milk and tumours

- English women, who consume large quantities of milk, develop some types of tumours 18 times more frequently than Japanese women, who rarely drink it
- Tumours usually appear late in life.
- British women live, on average, 12 years more than Japanese women

# Why are we here?

- Because "numbers" (or rather, the way they are presented) are sometimes deceiving



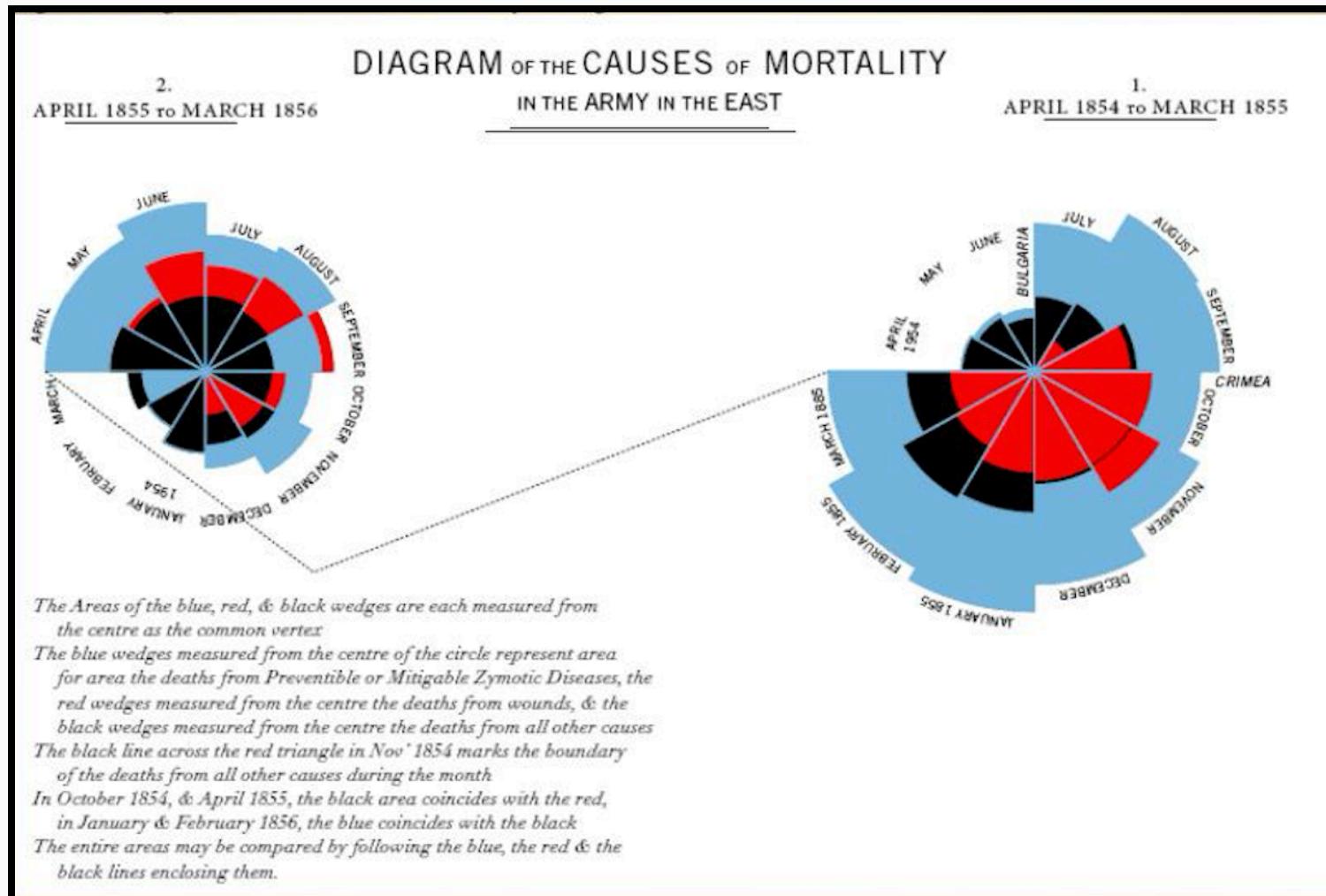
# **Why are we here?**

- Why do soldiers die?

# Who is this person?



# Why do soldiers die?



# Why are we here?

- Because "numbers" (or rather, the way they are presented) are sometimes deceiving
- Because “numbers” (and the way they are presented) help us describe, understand, and change the world

# **Why are we here?**

- To learn how to read, understand, and critically analyse scientific papers
- To be able to carry out research involving the acquisition, processing, and analysis of data

# **Scientific research**

# **Problem**

- Why do soldiers die?

# Plan

- How do we answer?

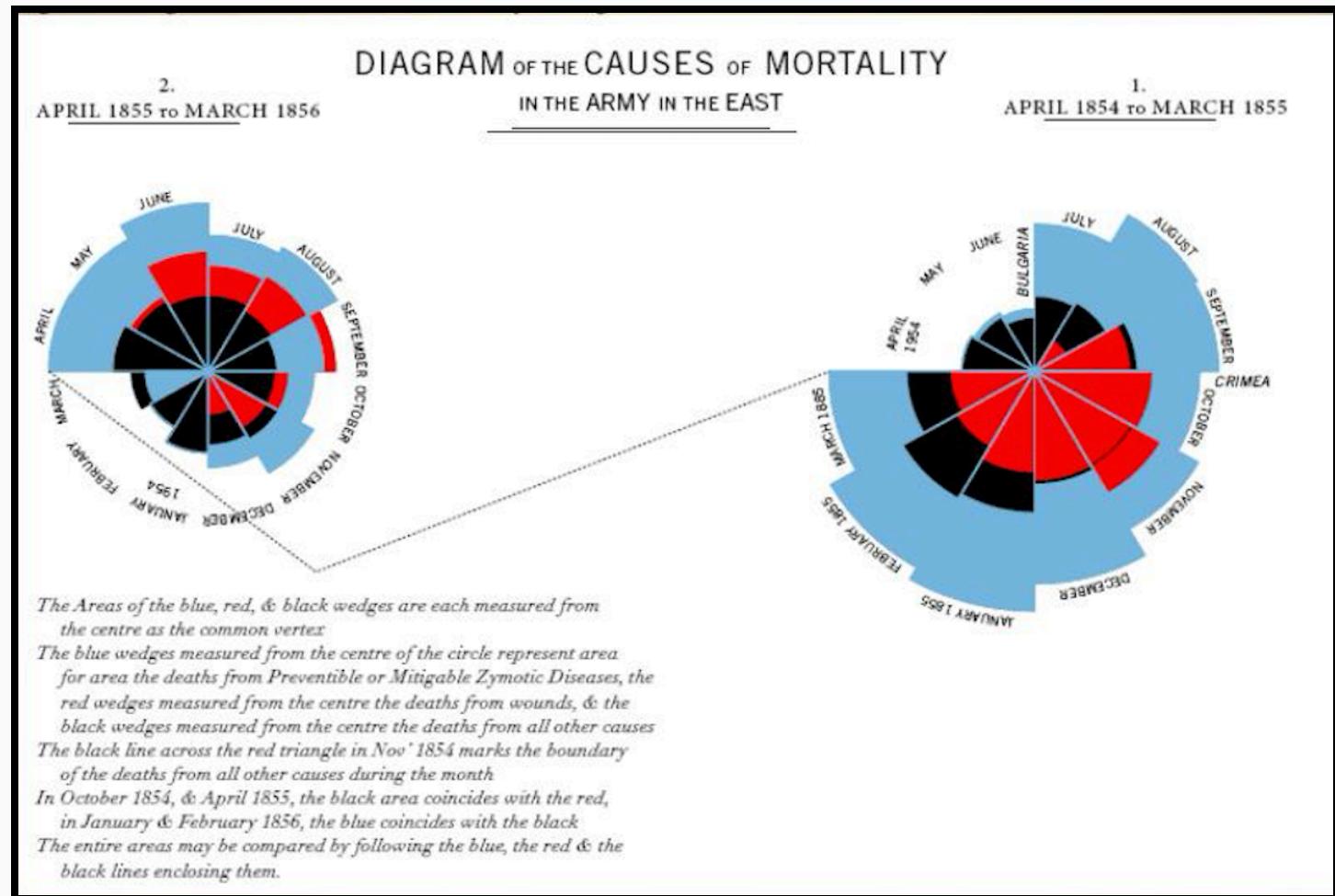


# Data



Scutari Hospital. J.A. Benwell. about 1856

# Analysis

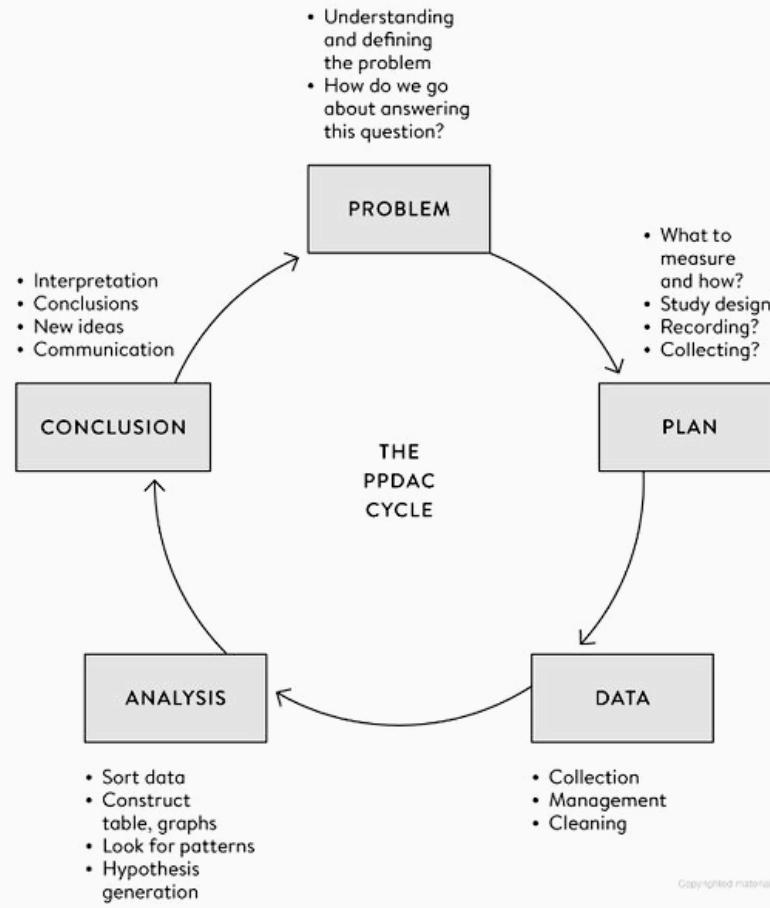


# Conclusions



Florence Nightingale (1820 - 1910) at Scutari Hospital in Turkey around 1855, unknown artist

# The data problem-solving cycle



# **Inductive reasoning**

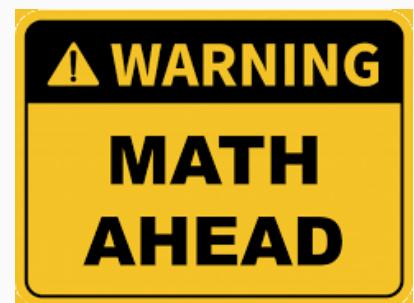
Derive a general rule from observations

# What is Statistics?

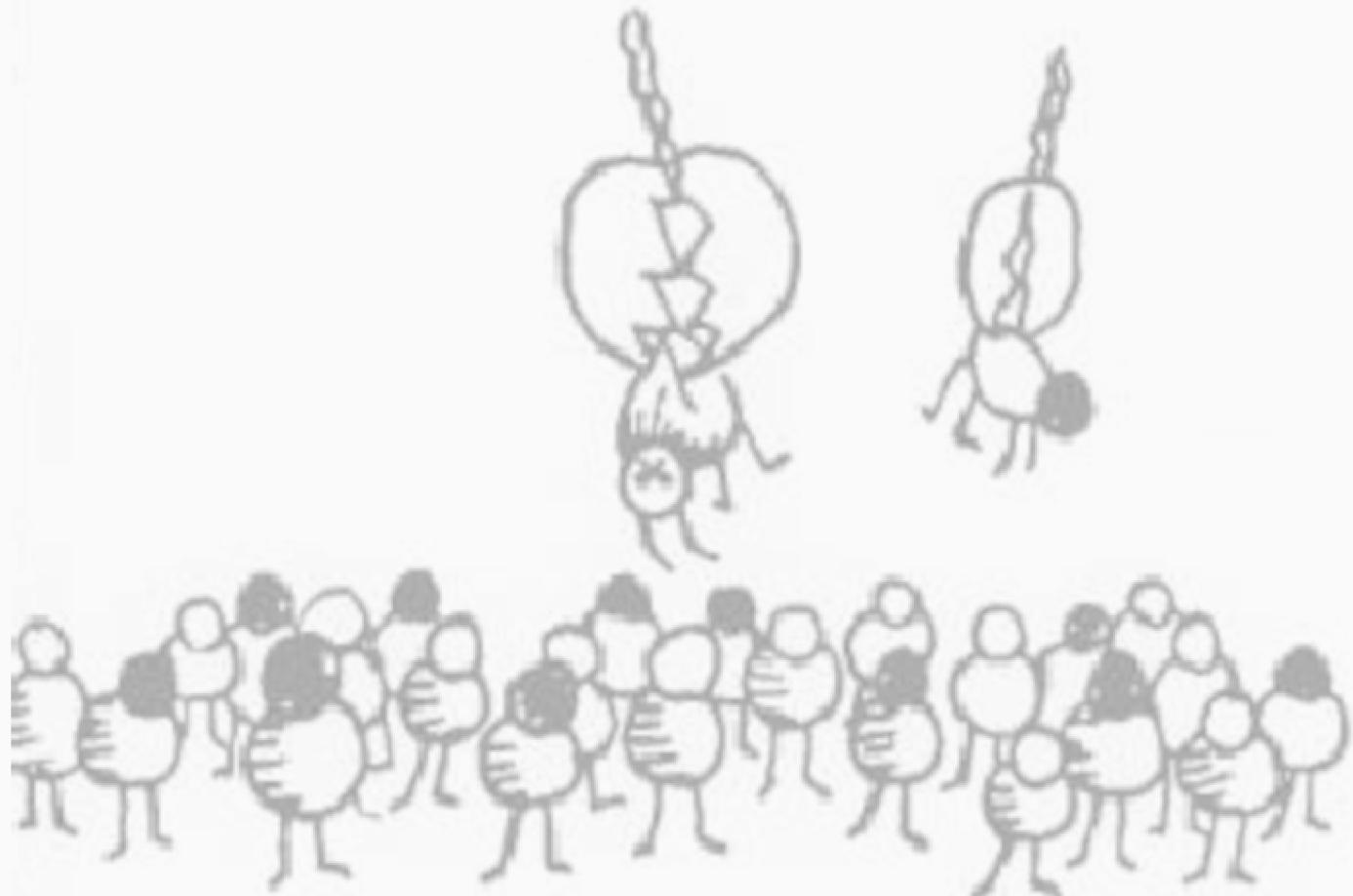
- The collection, organisation, summarisation, and analysis of data  
→ *Descriptive* statistics
- The drawing of inferences about a body of data when only a part of the data is observed  
→ *Inferential* statistics

# What will we learn?

- How to collect data
- How to summarise data
- How to make decision with data



# **Sampling**



# Learning objectives

- Understand the difference between population and sample
- Understand the difference between sampling strategies
- Understand sampling error and bias



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# Population vs sample



Istat | Istituto Nazionale  
di Statistica

## POPULATIONS AND SAMPLES

### PERMANENT CENSUS OF POPULATION AND HOUSING

The permanent census of the population and housing begins in October 2018. For the first time ISTAT conducts not a ten-yearly but an annual survey of the main characteristics of the country's resident population and its social and economic conditions at national, regional and local levels.

The new permanent census of population and housing do not involve all Italian families, but a sample of them each year: about 1,400,000 families resident in 2,800 Italian municipalities.

Moreover, only a percentage of the municipalities (about 1,100 of them) will take part by census operations every year; the remainder will be called to participate once every four years. In this way, all municipalities will be surveyed at least once by 2021.

CENSIMENTI PERMANENTI



PUBLIC INSTITUTIONS

NONPROFIT INSTITUTIONS

AGRICULTURE

AND HOUSING

# Population vs sample (in the clinic)

## Delirium as a Predictor of Mortality in Mechanically Ventilated Patients in the Intensive Care Unit

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E. Wesley Ely, MD, MPH

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Ayumi Shintani, PhD, MPH

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Brenda Truman, RN, MSN

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Theodore Speroff, PhD

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Sharon M. Gordon, PsyD

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Frank E. Harrell, Jr, PhD

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Sharon K. Inouye, MD, MPH

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Gordon R. Bernard, MD

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Robert S. Dittus, MD, MPH

**Context** In the intensive care unit (ICU), delirium is a common yet underdiagnosed form of organ dysfunction, and its contribution to patient outcomes is unclear.

**Objective** To determine if delirium is an independent predictor of clinical outcomes, including 6-month mortality and length of stay among ICU patients receiving mechanical ventilation.

**Design, Setting, and Participants** Prospective cohort study enrolling 275 consecutive mechanically ventilated patients admitted to adult medical and coronary ICUs of a US university-based medical center between February 2000 and May 2001. Patients were followed up for development of delirium over 2158 ICU days using the Confusion Assessment Method for the ICU and the Richmond Agitation-Sedation Scale.

# Exercise #1

**Objective** To determine whether intravenous dexamethasone increases the number of ventilator-free days among patients with COVID-19-associated ARDS.

**Design, Setting, and Participants** Multicenter, randomized, open-label, clinical trial conducted in 41 intensive care units (ICUs) in Brazil. Patients with COVID-19 and moderate to severe ARDS, according to the Berlin definition, were enrolled from April 17 to June 23, 2020. Final follow-up was completed on July 21, 2020. The trial was stopped early following publication of a related study before reaching the planned sample size of 350 patients.

? Which is the population of this study?

- a) Patients with Acute respiratory distress syndrome (ARDS)
- b) Patients with COVID-19-associated ARDS
- c) Brazilian patients with COVID-19-associated ARDS
- d) Patients without COVID-19-associated ARDS

Tomazini, B.M., et al., Effect of dexamethasone on days alive and ventilator-free in patients with moderate or severe acute respiratory distress syndrome and COVID-19: the CoDEX randomized clinical trial.", JAMA, 2020, doi:10.1001/jama.2020.17021

## Exercise #2

**Objective** To determine whether intravenous dexamethasone increases the number of ventilator-free days among patients with COVID-19-associated ARDS.

**Design, Setting, and Participants** Multicenter, randomized, open-label, clinical trial conducted in 41 intensive care units (ICUs) in Brazil. Patients with COVID-19 and moderate to severe ARDS, according to the Berlin definition, were enrolled from April 17 to June 23, 2020. Final follow-up was completed on July 21, 2020. The trial was stopped early following publication of a related study before reaching the planned sample size of 350 patients.

? Which is the sample used in this study?

- a) Patients with Acute respiratory distress syndrome (ARDS)
- b) Patients with COVID-19-associated ARDS
- c) Brazilian patients with COVID-19-associated ARDS
- d) Patients without COVID-19-associated ARDS

Tomazini, B.M., et al., Effect of dexamethasone on days alive and ventilator-free in patients with moderate or severe acute respiratory distress syndrome and COVID-19: the CoDEX randomized clinical trial.", JAMA, 2020, doi:10.1001/jama.2020.17021

# Opportunity vs random sample

- 🎯 An **opportunity** sample is the sample drawn from the part of the population that is close to hand (and which may not represent the whole population)
- 📌 All the patients presenting to a given clinic in a given period of time are enrolled

# Opportunity vs random sample

- 🎯 A **random** sample is the sample in which the probability of getting any particular sample may be calculated (and which should represent the whole population)
- 📌 A randomly selected set of patients with the disease is enrolled

# Strategy 1: Simple random sampling

- 🎯 A sample of size  $n$  drawn from a population of size  $N$  ensuring that every possible sample of size  $n$  is equally likely

# Strategy 1: Simple random sampling



$N = 90$

$n = 10$

La Tombola di PianetaBambini.it TABELLONE									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90

# Strategy 1: Simple random sampling



$$N = 90$$

$$n = 10$$

49, 65, 25, 74, 18

90, 47, 24, 71, 37

La Tombola di PianetaBambini.it <b>TABELLONE</b>									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
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31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90

# Strategy 2: Systematic Sampling



$$N = 90$$

$$n = 10$$

$$x = 42$$

$$step = N/n = 90/10 = 9$$

La Tombola di PianetaBambini.it TABELLONE									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
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# Strategy 2: Systematic Sampling



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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90

# **Strategy 3: Stratified Random Sampling**

- 🎯 The population is divided into homogenous group (strata) and a simple random sample is drawn from each stratum
  - Variation #1: stratified systematic sample
  - Variation #2: stratified sampling proportional to size

# Strategy 3: Stratified Random Sampling



$$N = 90$$

$$N_{female} = 60$$

$$N_{male} = 30$$

$$n = 9$$

$$n_{female} = 6$$

$$n_{male} = 3$$

*Females* : 46, 20, 26,  
50, 47, 3

*Males* : 69, 85, 87

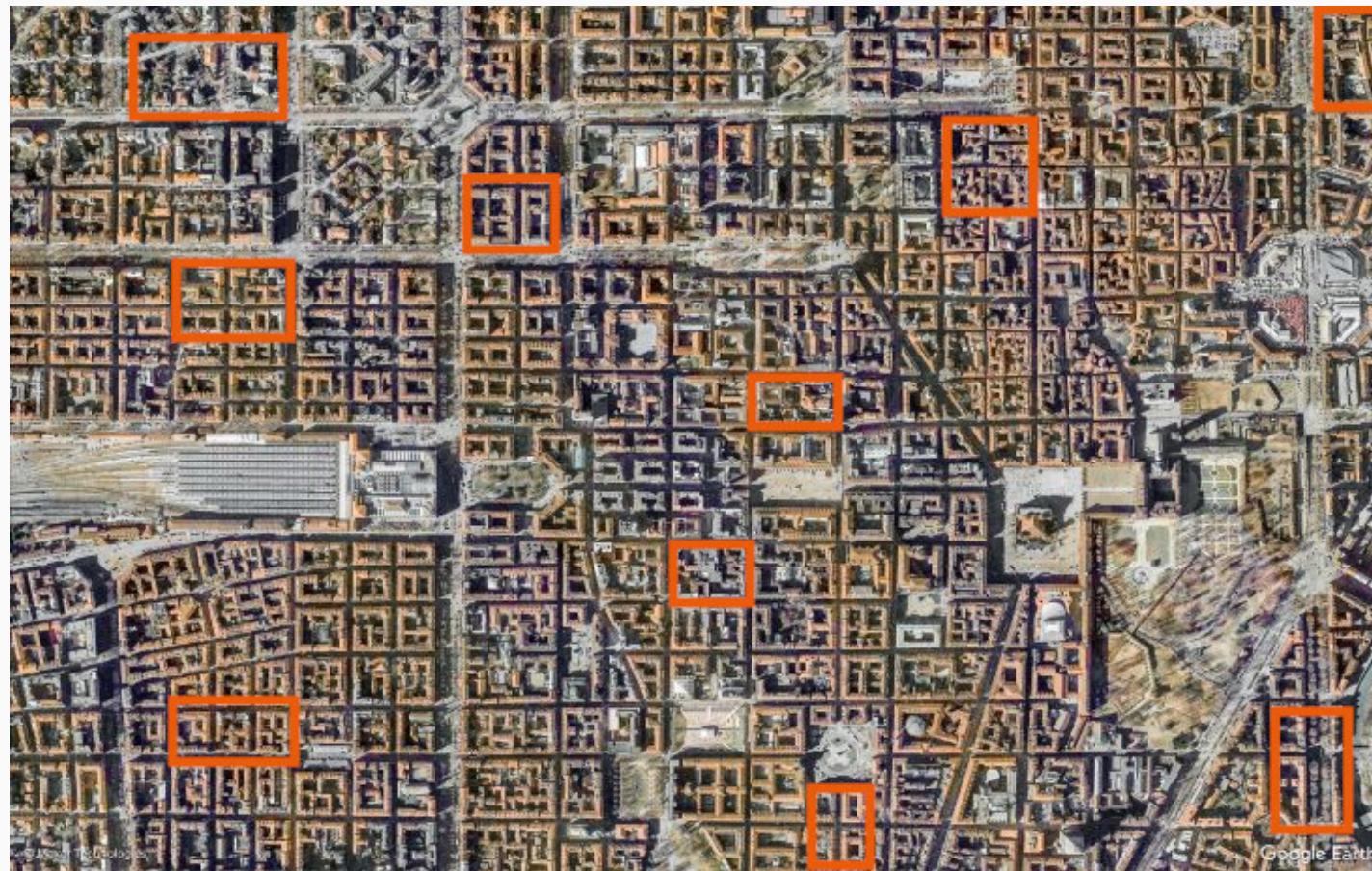
La Tombola di PianetaBambini.it <b>TABELLONE</b>									
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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90

# **Strategy 4: Cluster sampling**

- 🎯 The population is divided into clusters, and a simple random sample is drawn

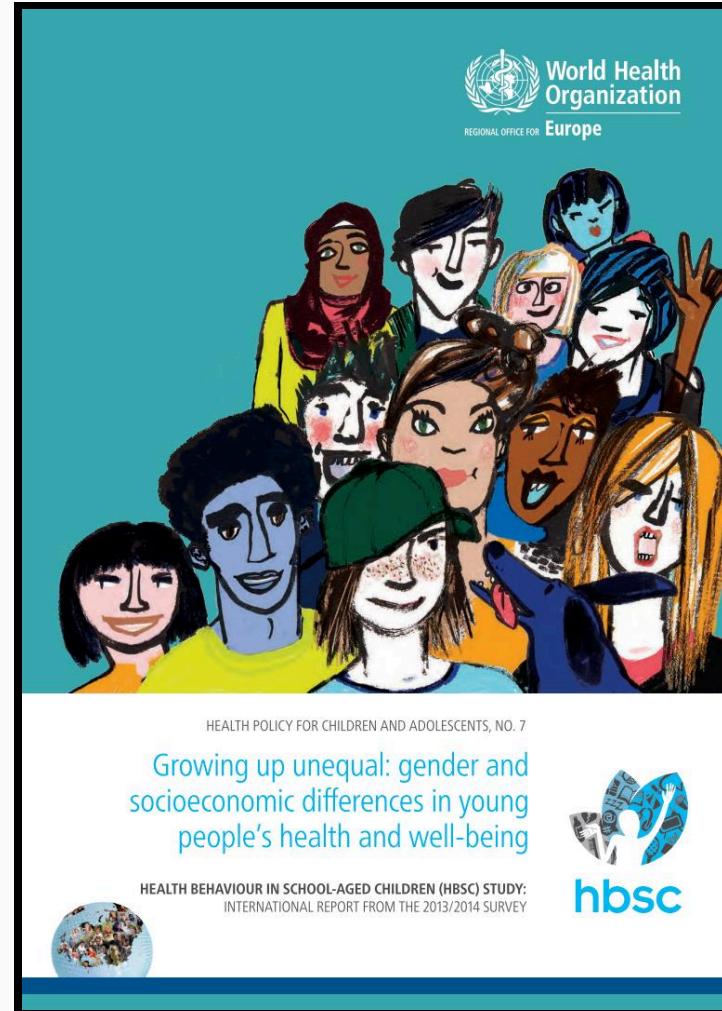
Variation: one stage (observing everything) vs  
two stage (sampling within clusters)

# Strategy 4: Cluster sampling



# Sampling in the wild

<https://hbsc.org>



## Exercise #3

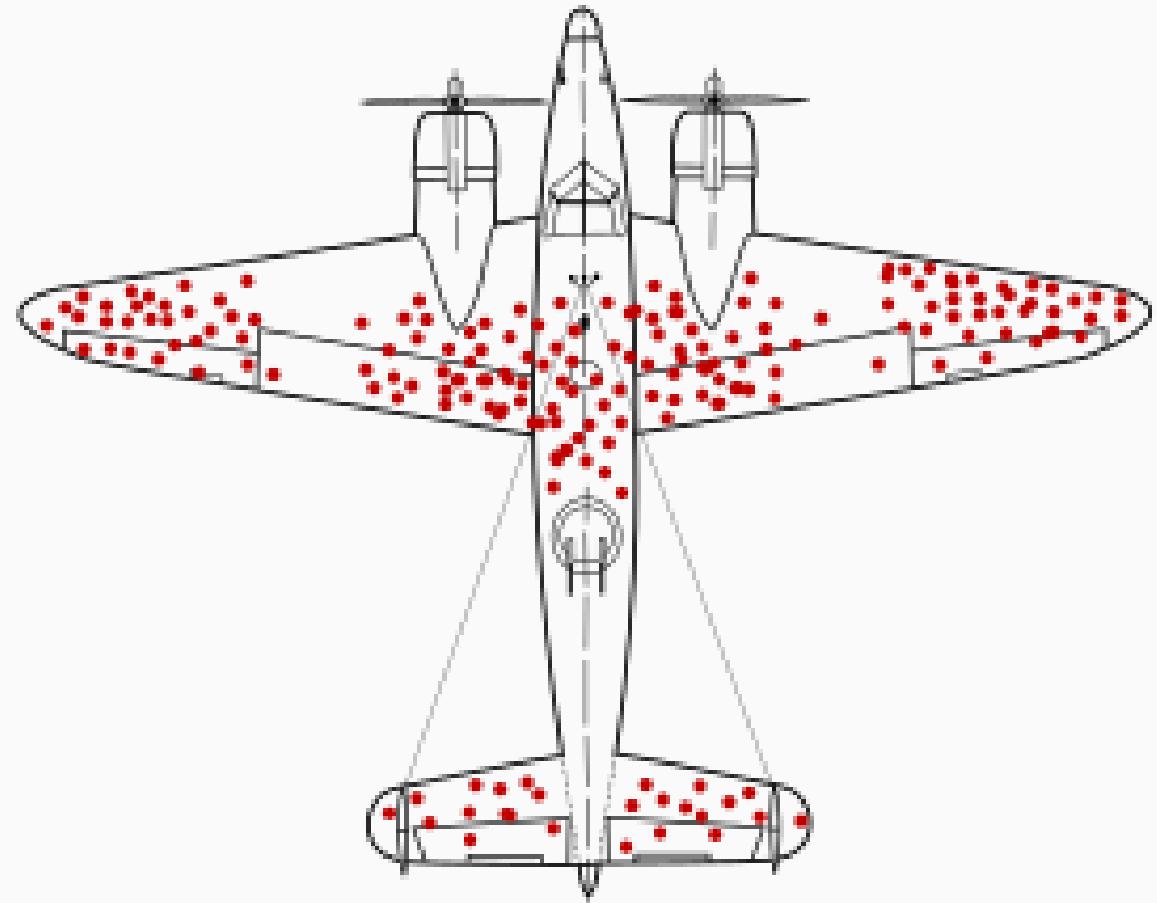
- ? A representative of a cheese factory is asking questions on cheese consumption to every 5th customer entering the supermarket

Which kind of sampling strategy are they using?

- a) simple random sampling
- b) systematic sampling
- c) stratified sampling
- d) none of the above

# Selection bias

- Survivor bias



# Selection bias

- Survivor bias

*"Buildings used to be more beautiful/longer lasting"*

# Selection bias

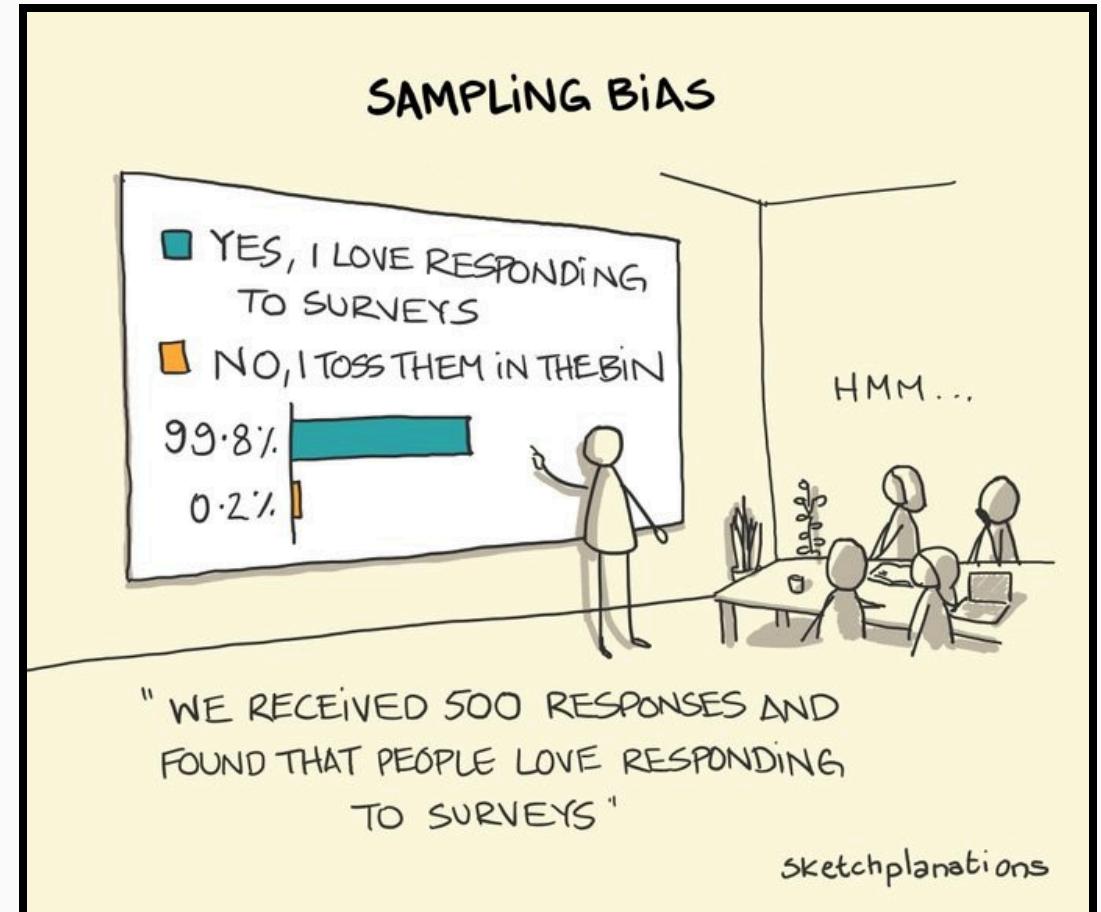
- Survivor bias

*"Buildings used to be more beautiful/longer lasting"*

*"I don't understand why, nowadays, one should do X,  
I never did it, and I'm still here to tell it"*

# Selection bias

- Survivor bias
- Volunteer bias



# Selection bias

- Survivor bias
- Volunteer bias

A teacher wonders if extra lessons improve exam performance.  
She prepares them, inviting interested students to sign up.

# **Selection bias**

- Survivor bias
- Volunteer bias
- Lost to follow up bias

# Selection bias

- Survivor bias
- Volunteer bias
- Lost to follow up bias

A pharma company is testing a new drug on a court of 100 cancer patients recruited in a center of excellence, 30 of whom did not show up at the follow up. What do we conclude about this new drug knowing that these 30 people...

# Selection bias

- Survivor bias
- Volunteer bias
- Lost to follow up bias

A pharma company is testing a new drug on a court of 100 cancer patients recruited in a center of excellence, 30 of whom did not show up at the follow up. What do we conclude about this new drug knowing that these 30 people...

- died?

# Selection bias

- Survivor bias
- Volunteer bias
- Lost to follow up bias

A pharma company is testing a new drug on a court of 100 cancer patients recruited in a center of excellence, 30 of whom did not show up at the follow up. What do we conclude about this new drug knowing that these 30 people...

- died?
- stopped the drug?

# Selection bias

- Survivor bias
- Volunteer bias
- Lost to follow up bias

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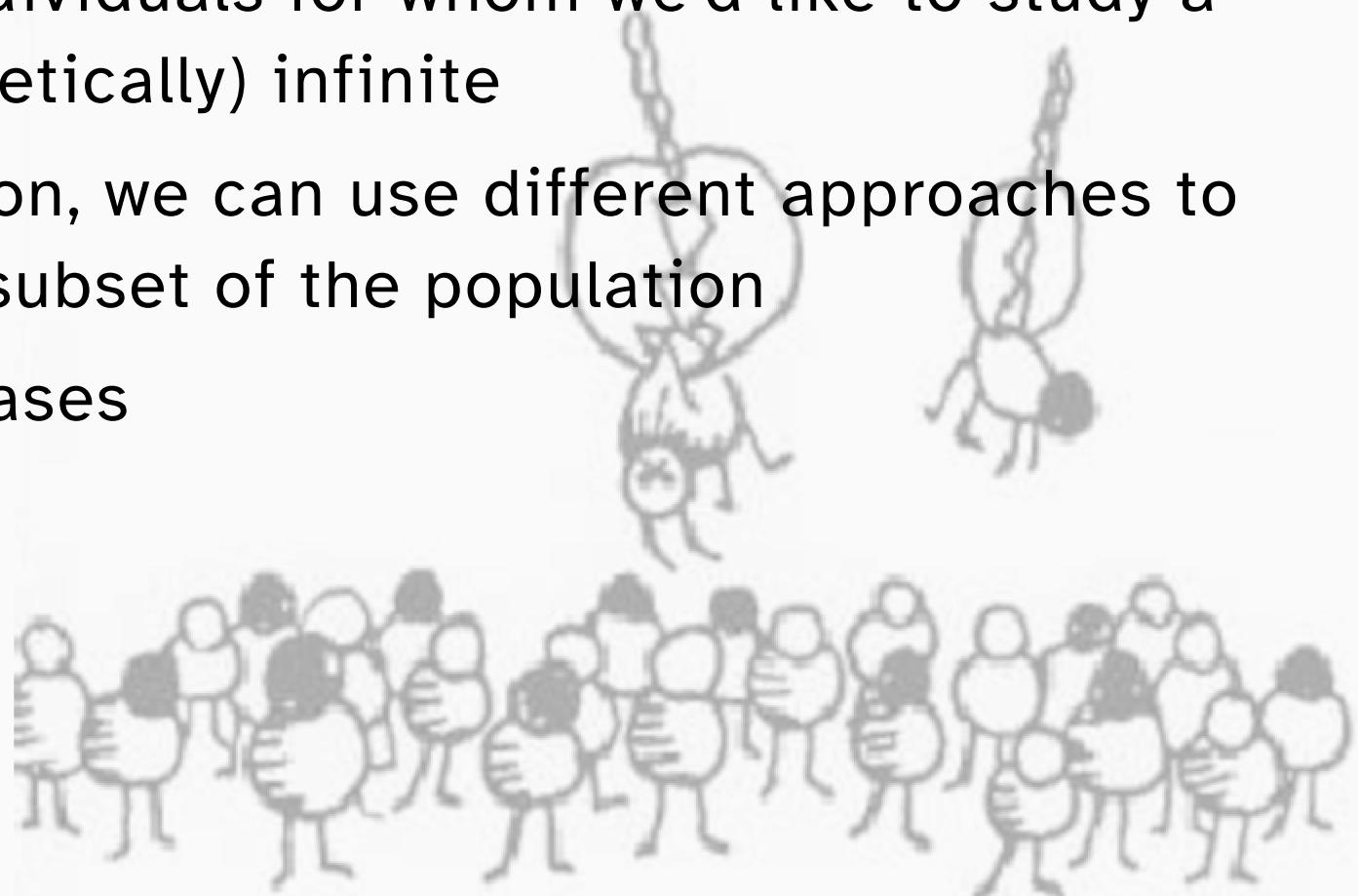
- died?
- stopped the drug?
- went back home?

# **Selection bias**

- Survivor bias
- Volunteer bias
- Lost to follow up bias
- ...

# Summary

- A population includes all individuals for whom we'd like to study a phenomenon, and it's (theoretically) infinite
- When can't study a population, we can use different approaches to sample a (representative?) subset of the population
- Samples may suffer from biases

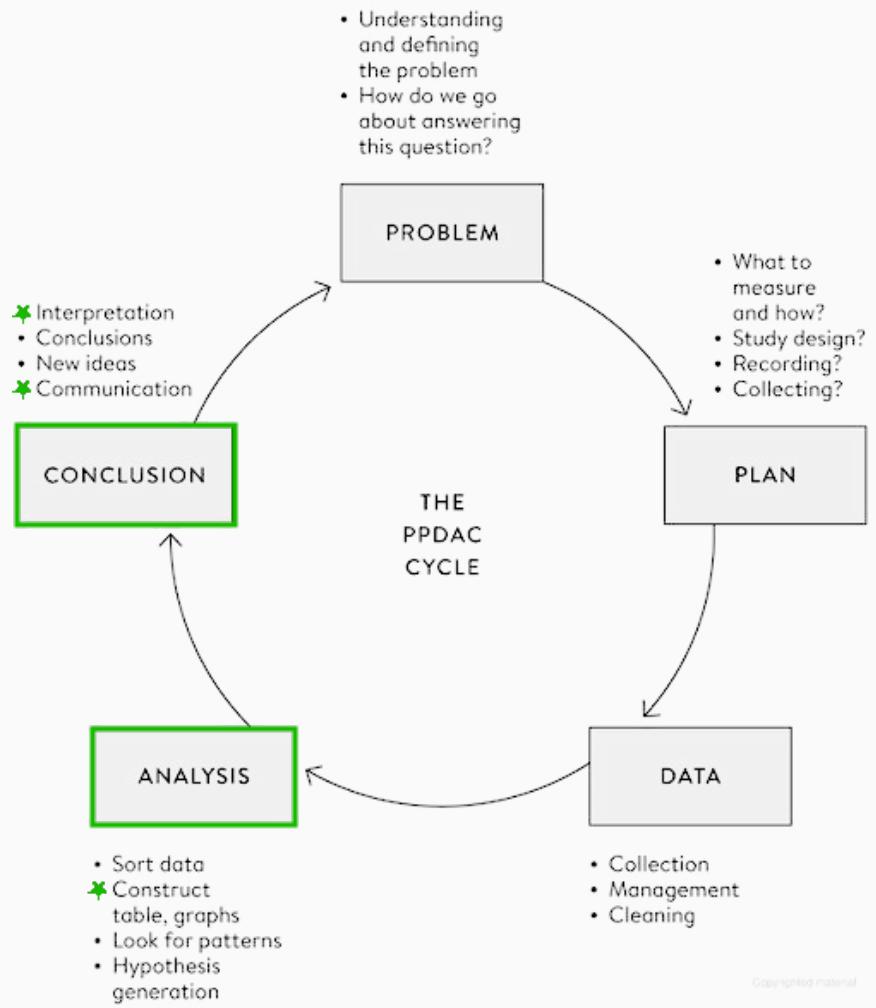


# Summarise data



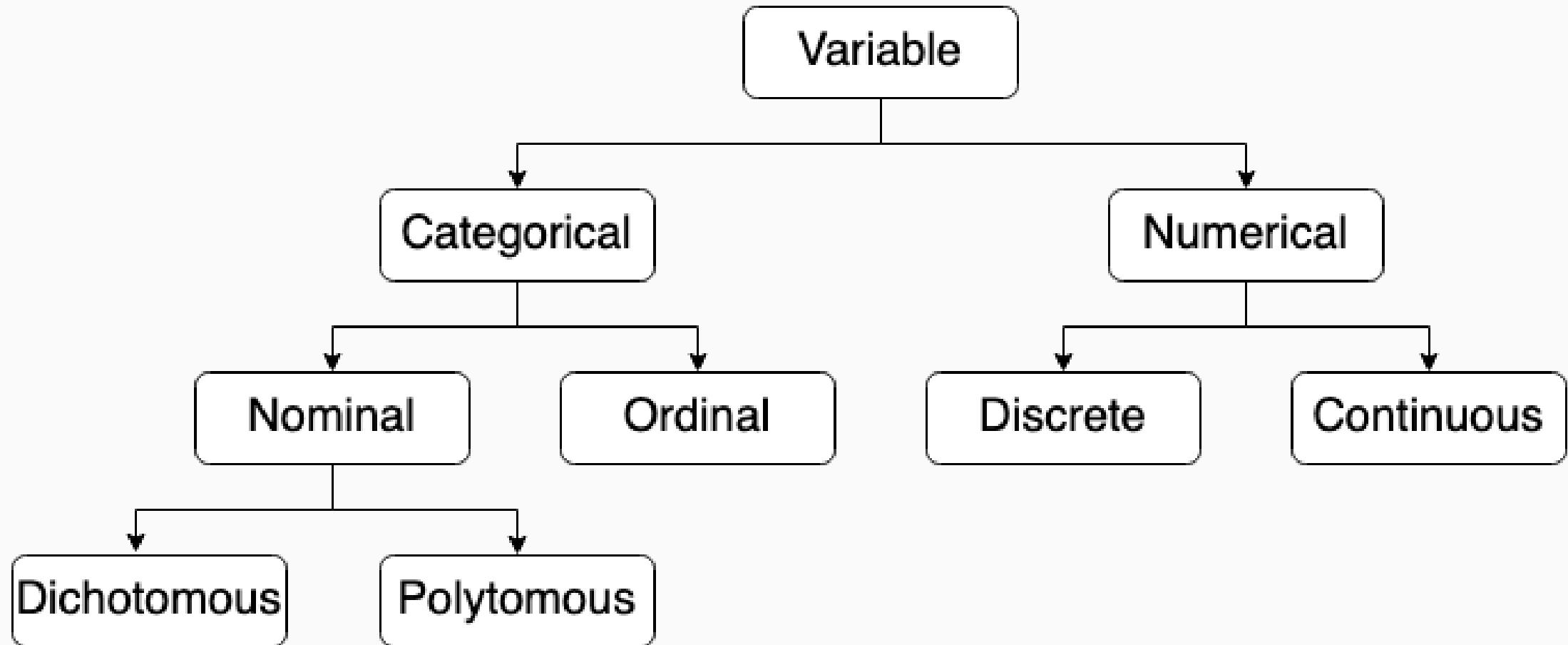
# Learning objectives

- Understand the differences between data types
- Be able to summarise each data type
- Understand the difference between parameters and statistics
- Understand why visualise your data is important

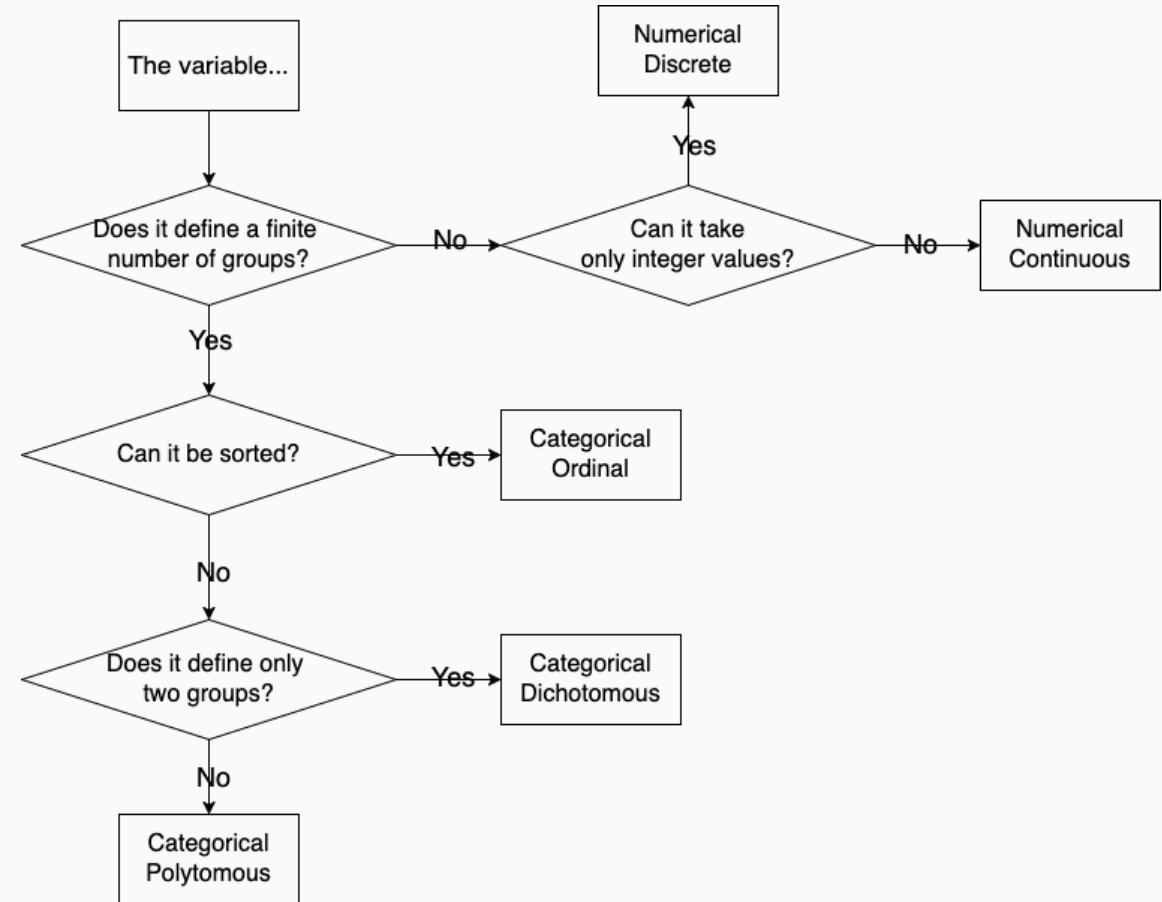


Spiegelhalter, D., *The Art of Statistics: Learning From Data*, Pelican, 2019

# Type of data

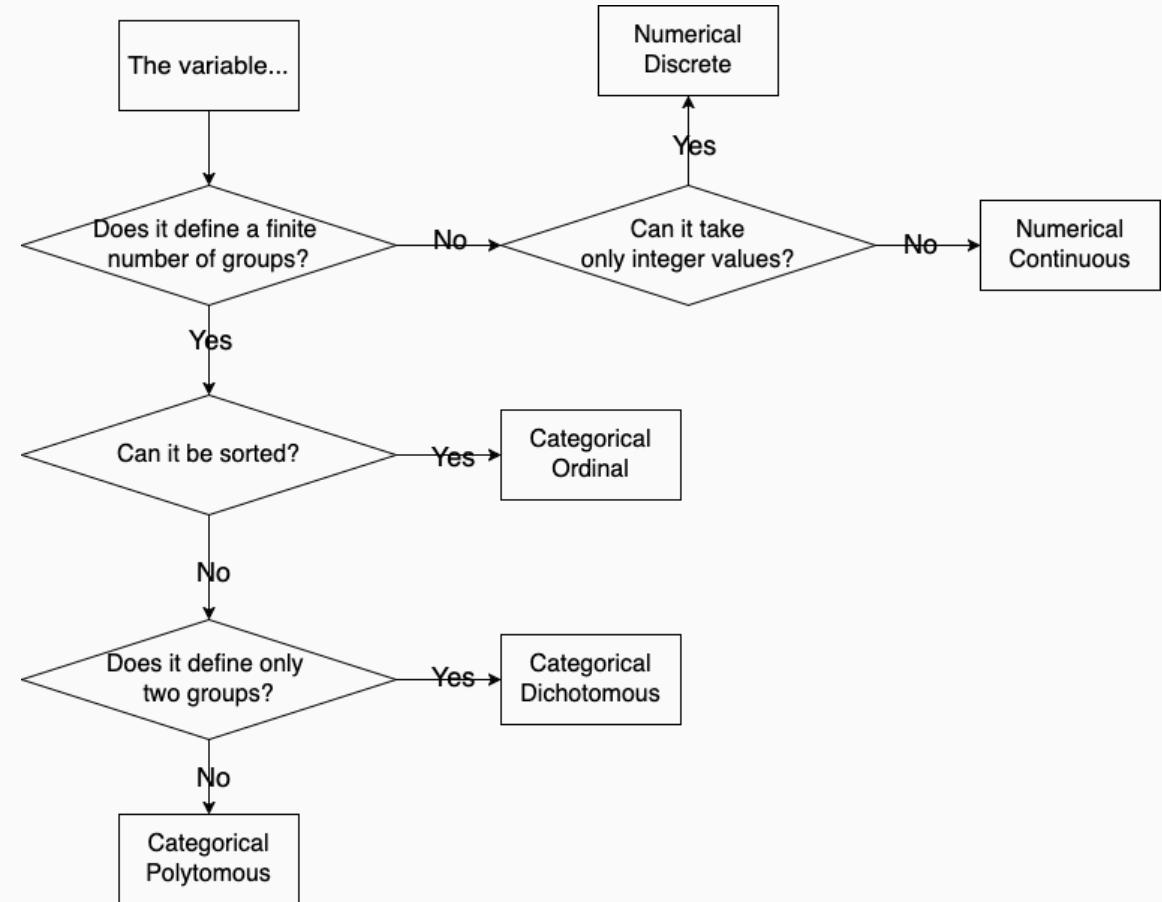


# What type of data is this?



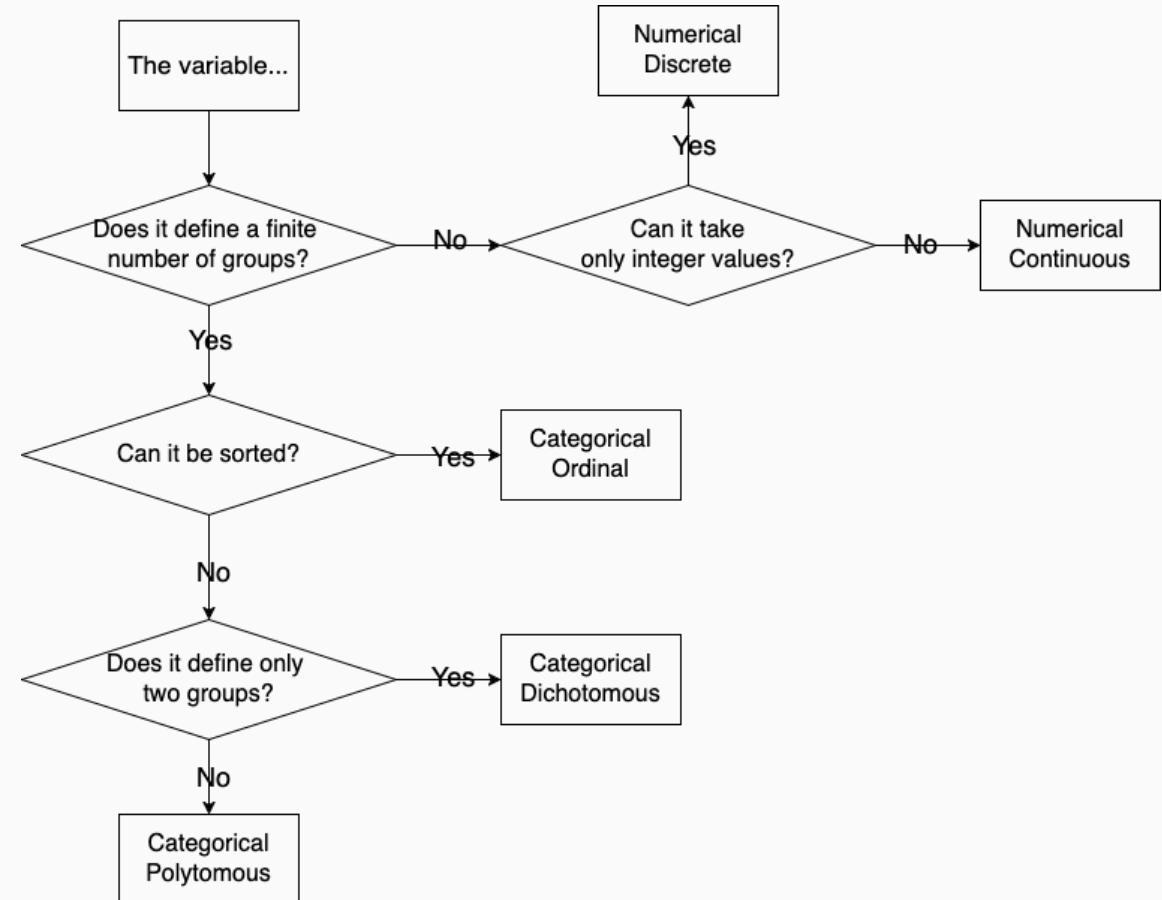
# What type of data is this?

? The number of death in a hospital



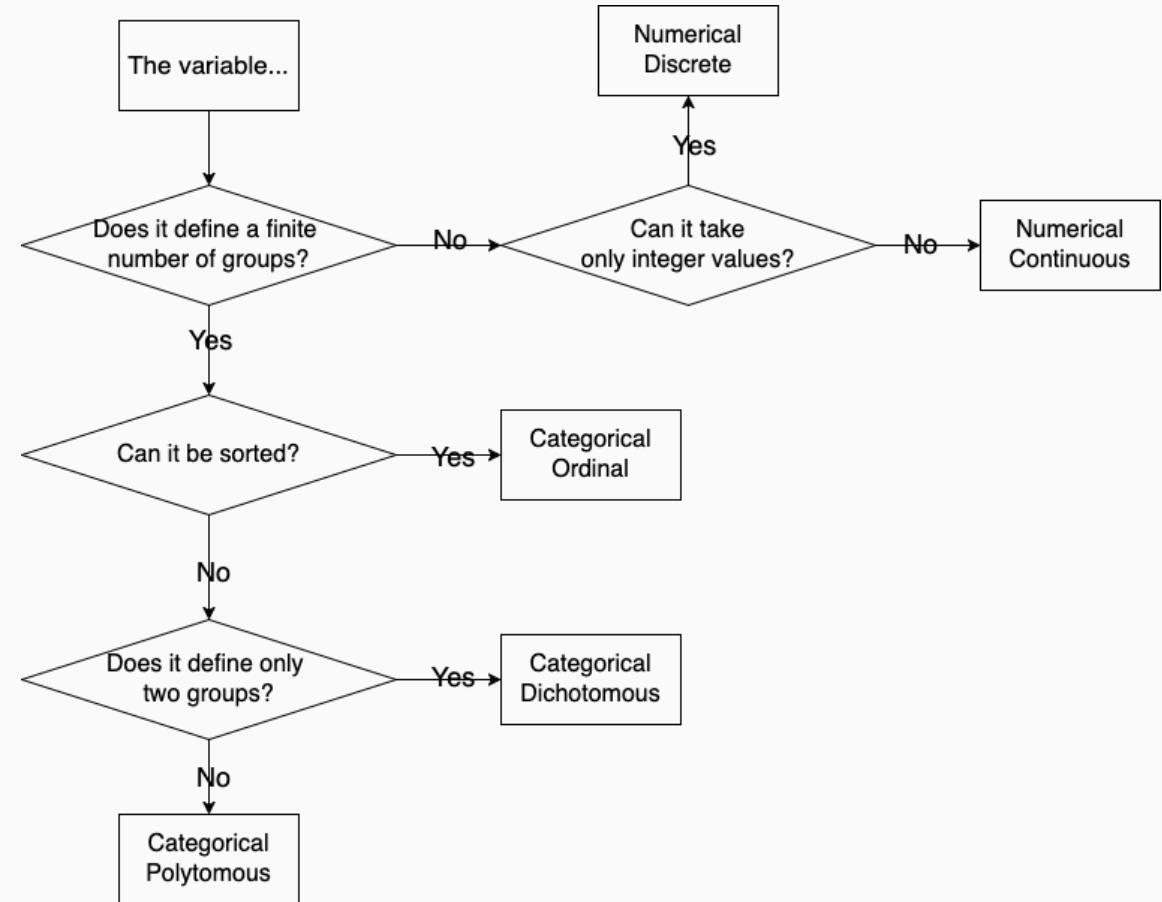
# What type of data is this?

? The size of a T-shirt



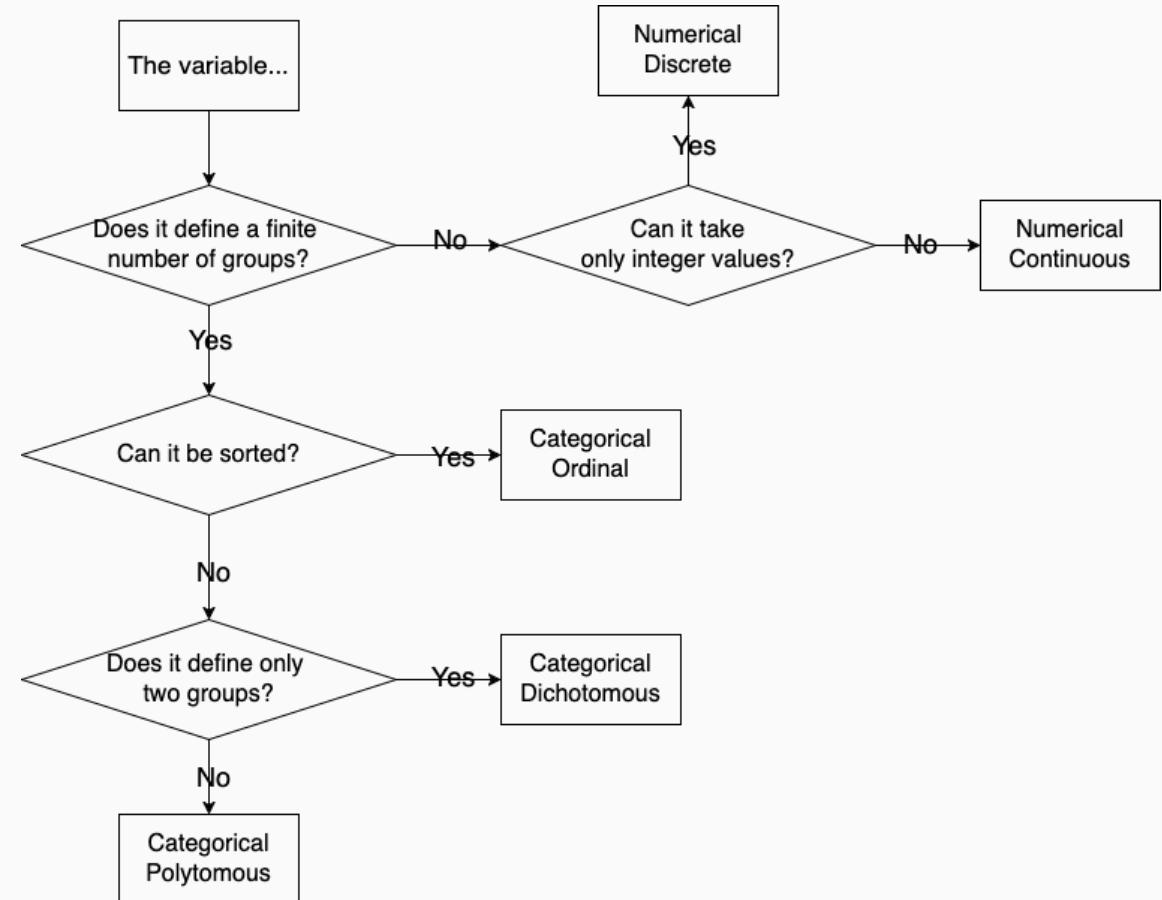
# What type of data is this?

? One's nationality



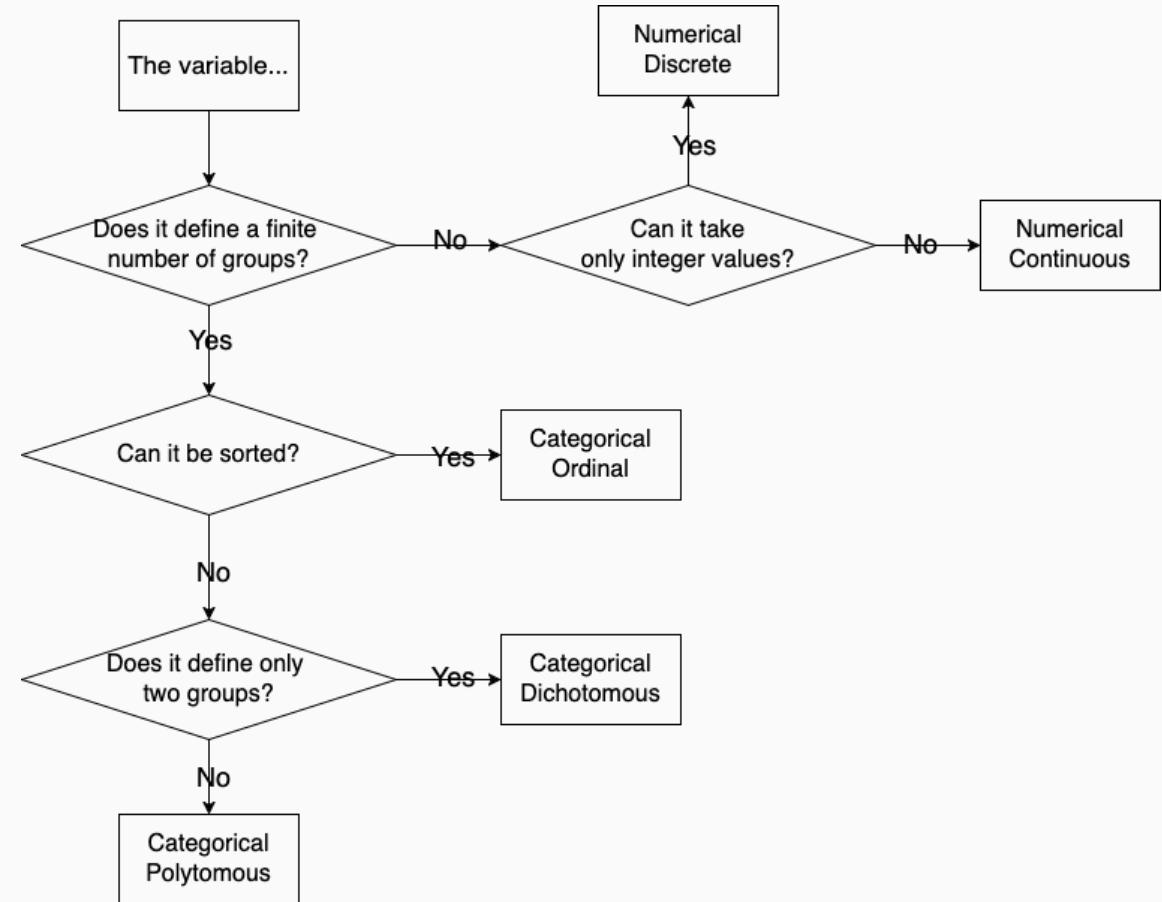
# What type of data is this?

? Fasting blood sugar levels



# What type of data is this?

? Passing the driving licence test



# Exercise #4

? Which types of data are included in this table?

Visconti A., et al., Total serum *N*-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3

**Table 1** Patient characteristics.

	All cohorts
<b>N (pre-treatment)</b>	88
<b>N (follow-up)</b>	66
<b>Sex</b>	
<i>Male</i>	57 (64.8%)
<i>Female</i>	31 (35.2%)
<b>Age (years)</b>	60.5 ± 15.0
<b>BMI (kg/m<sup>2</sup>)</b>	28.0 ± 5.4
<b>BRAF mutant</b>	40 (45.5%)
<b>LDH (<math>\leq</math>ULN)</b>	58 (65.9%)
<b>Metastatic stage</b>	
<i>Stage III unresectable</i>	2 (2.3%)
<i>M1a</i>	14 (15.9%)
<i>M1b</i>	17 (19.3%)
<i>M1c</i>	32 (36.4%)
<i>M1d</i>	23 (26.1%)
<b>ECOG performance status</b>	
0	47 (53.4%)
1	31 (35.2%)
2	8 (9.1%)
3	2 (2.3%)
<b>ICI therapy</b>	
<i>Ipilimumab</i>	1 (1.1%)
<i>Pembrolizumab</i>	20 (22.7%)
<i>Nivolumab</i>	30 (34.1%)
<i>Ipilimumab + Nivolumab</i>	37 (42.0%)

# Why is it important?

**Table 1** Patient characteristics. Categorical variables are presented as number (percentage). Continuous variables are presented as mean $\pm$ standard deviation.

Visconti A., et al., Total serum *N*-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3

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# Categorical variables

## Frequency table

- absolute frequency (the number)
- relative frequency (the percentage)

Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3

**Table 1** Patient characteristics.

	All cohorts
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Female	31 (35.2%)
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ICI therapy	
Ipilimumab	1 (1.1%)
Pembrolizumab	20 (22.7%)
Nivolumab	30 (34.1%)
Ipilimumab + Nivolumab	37 (42.0%)

# Exercise #5

? Which sex is more frequent?

**Table 1** Patient characteristics.

	All cohorts
N (pre-treatment)	88
<b>Sex</b>	
Male	57 (64.8%)
Female	31 (35.2%)

Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3

# Contingency table

**Study Objectives:** Persistent insomnia, although very common in general practice, often proves problematic to manage. This study investigates the clinical effectiveness and the feasibility of applying cognitive behavior therapy (CBT) methods for insomnia in primary care.

**Design:** Pragmatic randomized controlled trial of CBT versus treatment as usual.

**Setting:** General medical practice.

**Participants:** Two hundred one adults (mean age, 54 years) randomly assigned to receive CBT ( $n = 107$ ; 72 women) or treatment as usual ( $n = 94$ ; 65 women).

# Exercise #6

- Complete the table with the correct absolute and relative frequencies using the information contained in the abstract

**Participants:** Two hundred one adults (mean age, 54 years) randomly assigned to receive CBT ( $n = 107$ ; 72 women) or treatment as usual ( $n = 94$ ; 65 women).

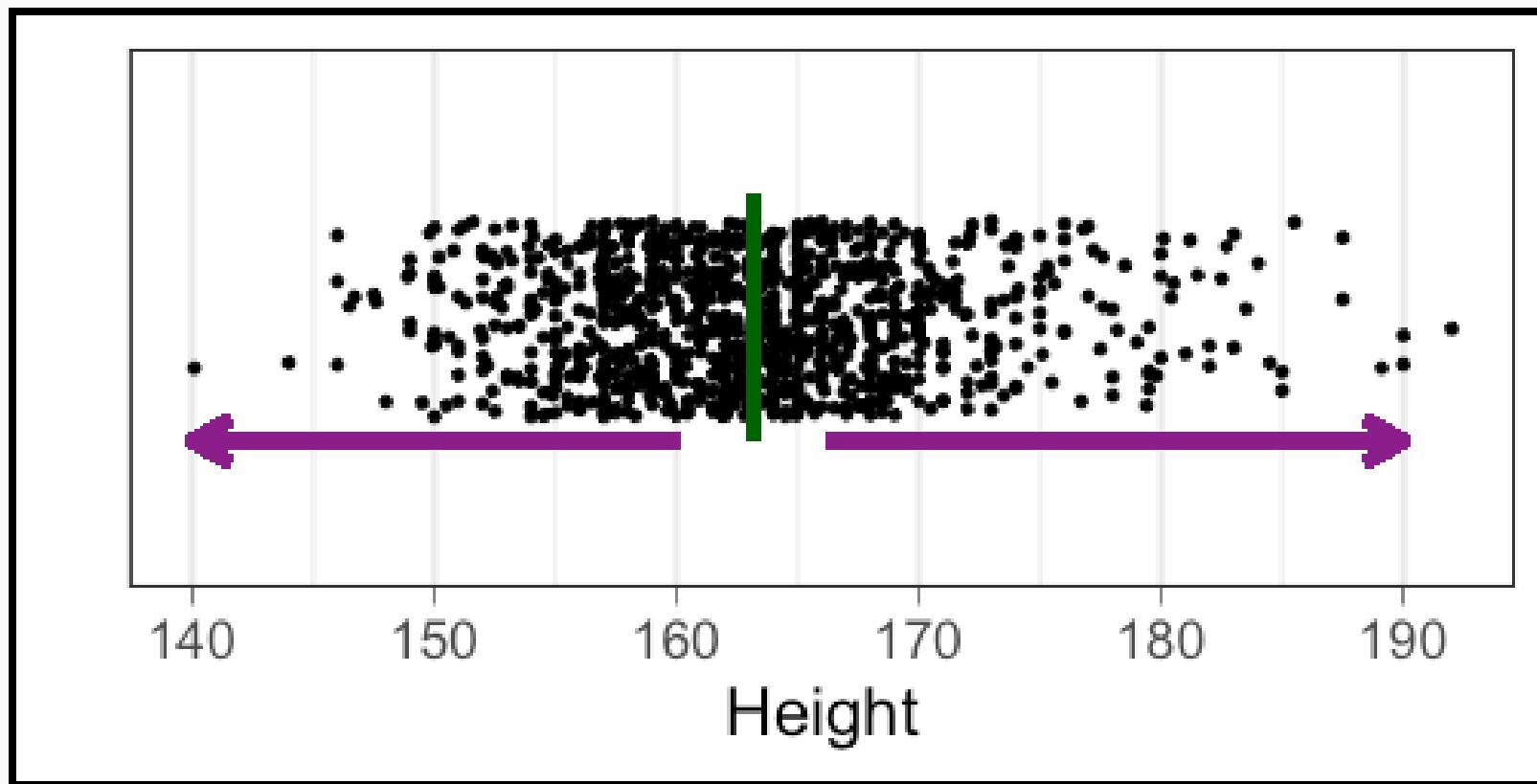
	CBT	Standard	Total
Males			
Females			
Total			

# Exercise #7

- ? Is the proportion of men and women similar?
  - a) True
  - b) False
  - c) I need more elements to decide
  
- ? Comparing the two arms, is the proportion of men and women similar?
  - a) True
  - b) False
  - c) I need more elements to decide

	CBT	Standard	Total
Males	35 (32.7%)	29 (30.9%)	64 (31.8%)
Females	72 (67.3%)	65 (69.1%)	137 (68.2%)
Total	107	94	201

# Measures of central tendency and dispersion



# Measure of central tendency: mode

1



The most frequent item



$$x = \{1, 1, 1, 3, 4, 4, 7, 8, 8, 9, 9\}$$

$$\text{mode}(x) = 1$$

## Exercise #8

? Calculate the mode of the following data sets

$$y = \{1, 1, 1, 3, 4, 4, 4, 7, 8, 8, 9, 9\}$$

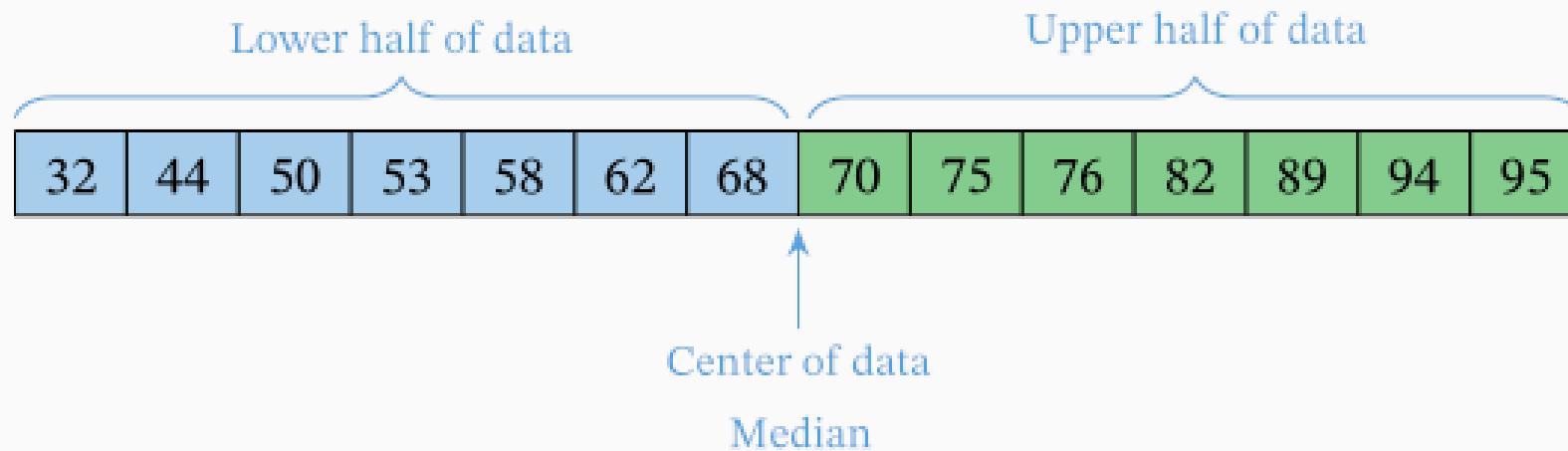
$$\text{mode}(y) = ?$$

$$z = \{1, 3, 4, 7, 8, 9, 11, 17, 21, 42\}$$

$$\text{mode}(z) = ?$$

# Measure of central tendency: median

🎯 The "middle" value



⚠️ Data should be sorted!

# Measure of central tendency: median

 The "middle" value

  $n = 7, x = \{1, 3, 3, 6, 7, 8, 9\}$

$$\text{median}(x) = x_{(n+1)/2} = x_{(7+1)/2} = x_4 = 6$$

  $n = 8, x = \{1, 2, 3, 4, 5, 6, 8, 9\}$

$$\begin{aligned}\text{median}(x) &= \frac{x_{(n/2)} + x_{((n/2)+1)}}{2} = \frac{x_{(8/2)} + x_{((8/2)+1)}}{2} \\ &= \frac{x_4 + x_5}{2} = \frac{4+5}{2} = 4.5\end{aligned}$$

 Data should be sorted!

## Exercise #9

? Calculate the median of the following data sets

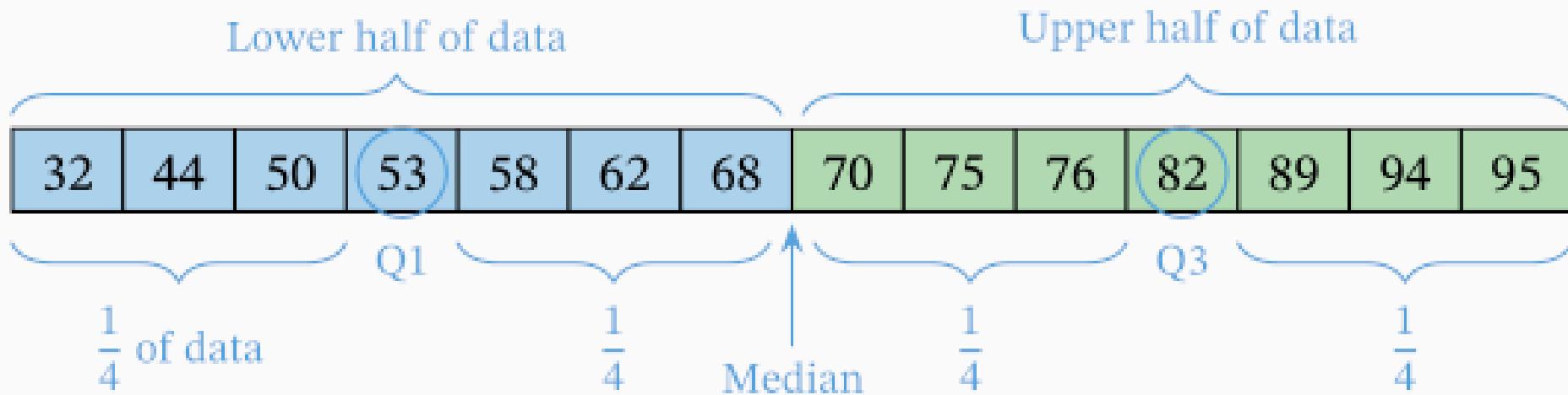
$$y = \{6, 34, 40, 55, 75\}$$

$$\text{median}(y) = ?$$

$$z = \{6, 34, 40, 55, 175\}$$

$$\text{median}(z) = ?$$

# Quartiles



⚠ Data should be sorted!

# Measure of central tendency: mean



Arithmetic mean

$$\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$



$x = \{4, 36, 45, 50, 75\}$

$$\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right) = \frac{4+36+45+50+75}{5} = 42$$

## Exercise #10

? Calculate the mean of the following data sets

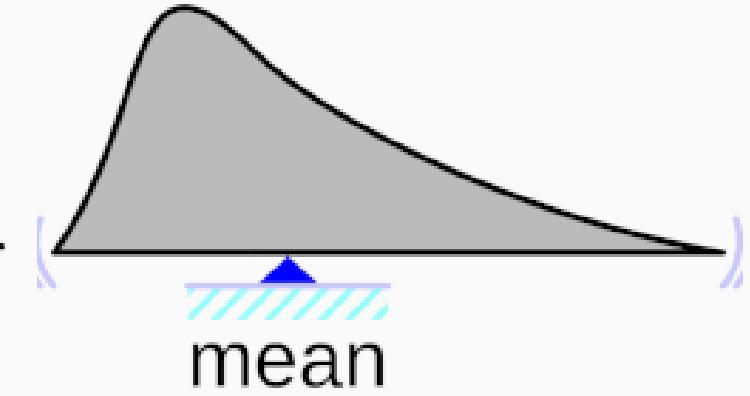
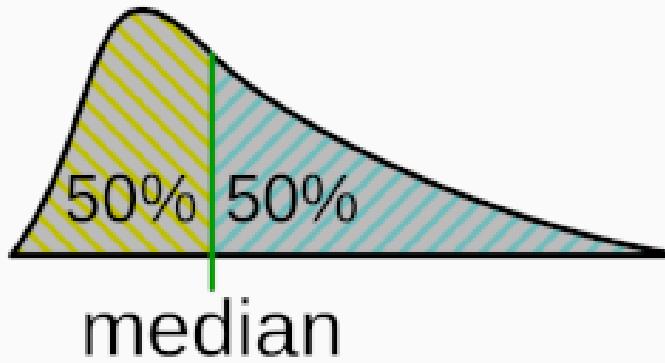
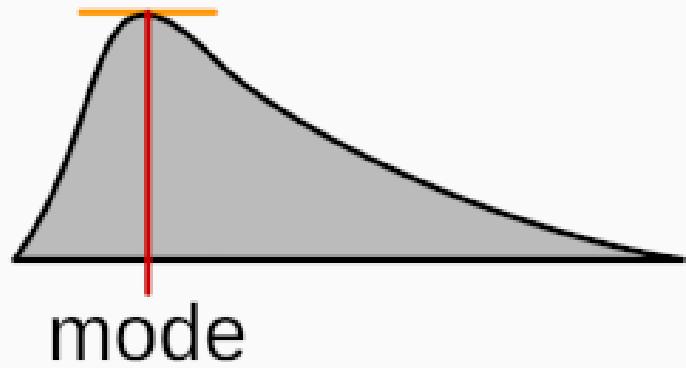
$$y = \{6, 34, 40, 55, 75\}$$

$$\bar{y} = ?$$

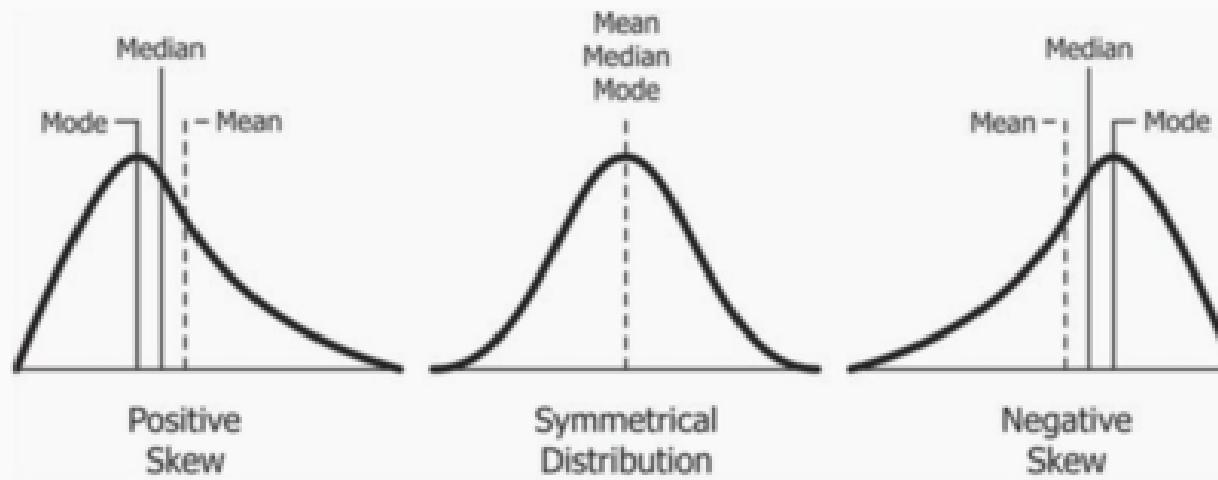
$$z = \{6, 34, 40, 55, 175\}$$

$$\bar{z} = ?$$

# Mode vs median vs mean



# The shape of a distribution



# The shape of a distribution



## Exercise #11

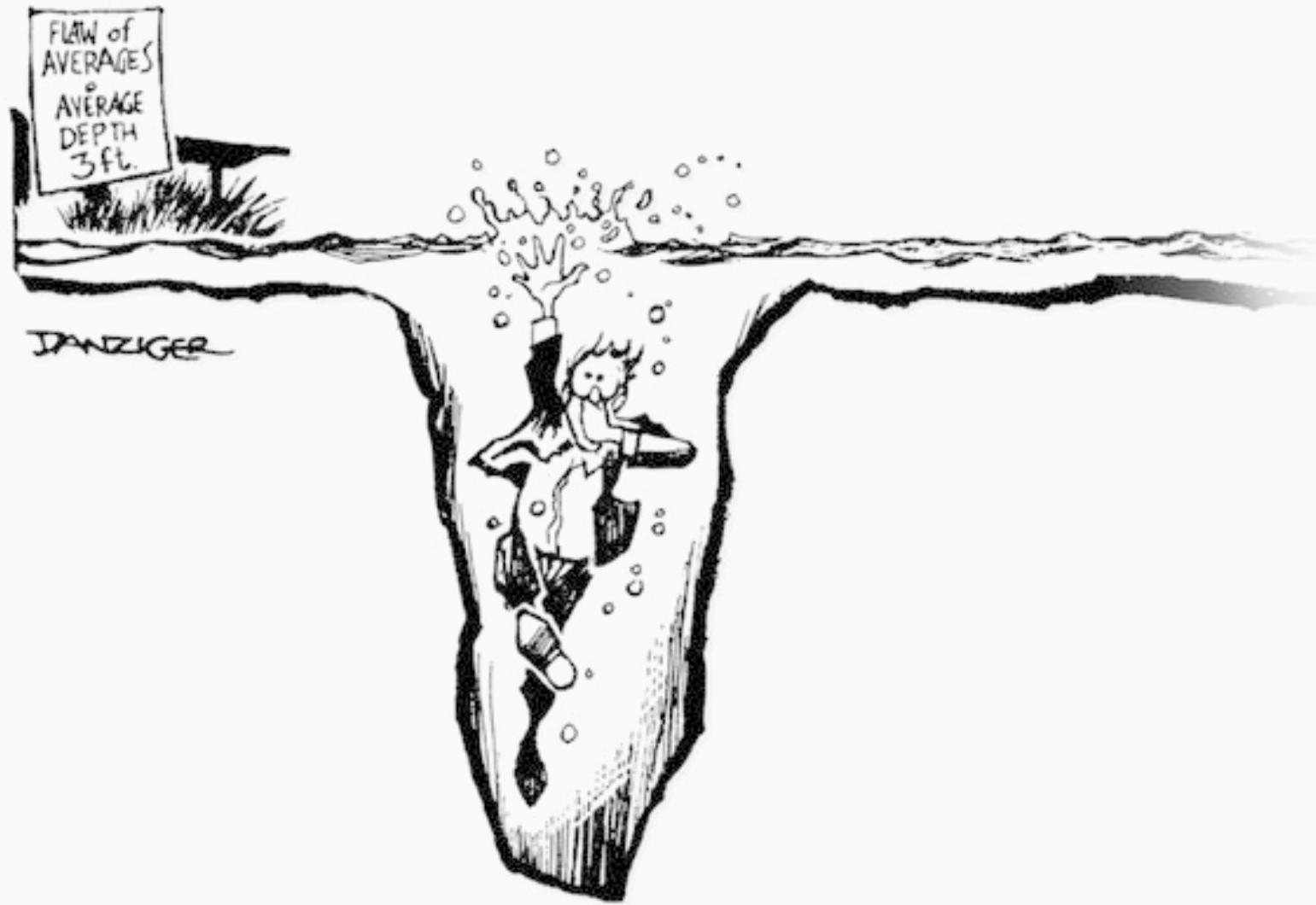
- ? In the results section, the authors reported the following

*The mean length of stay was 22.4 days (median: 14 days).*

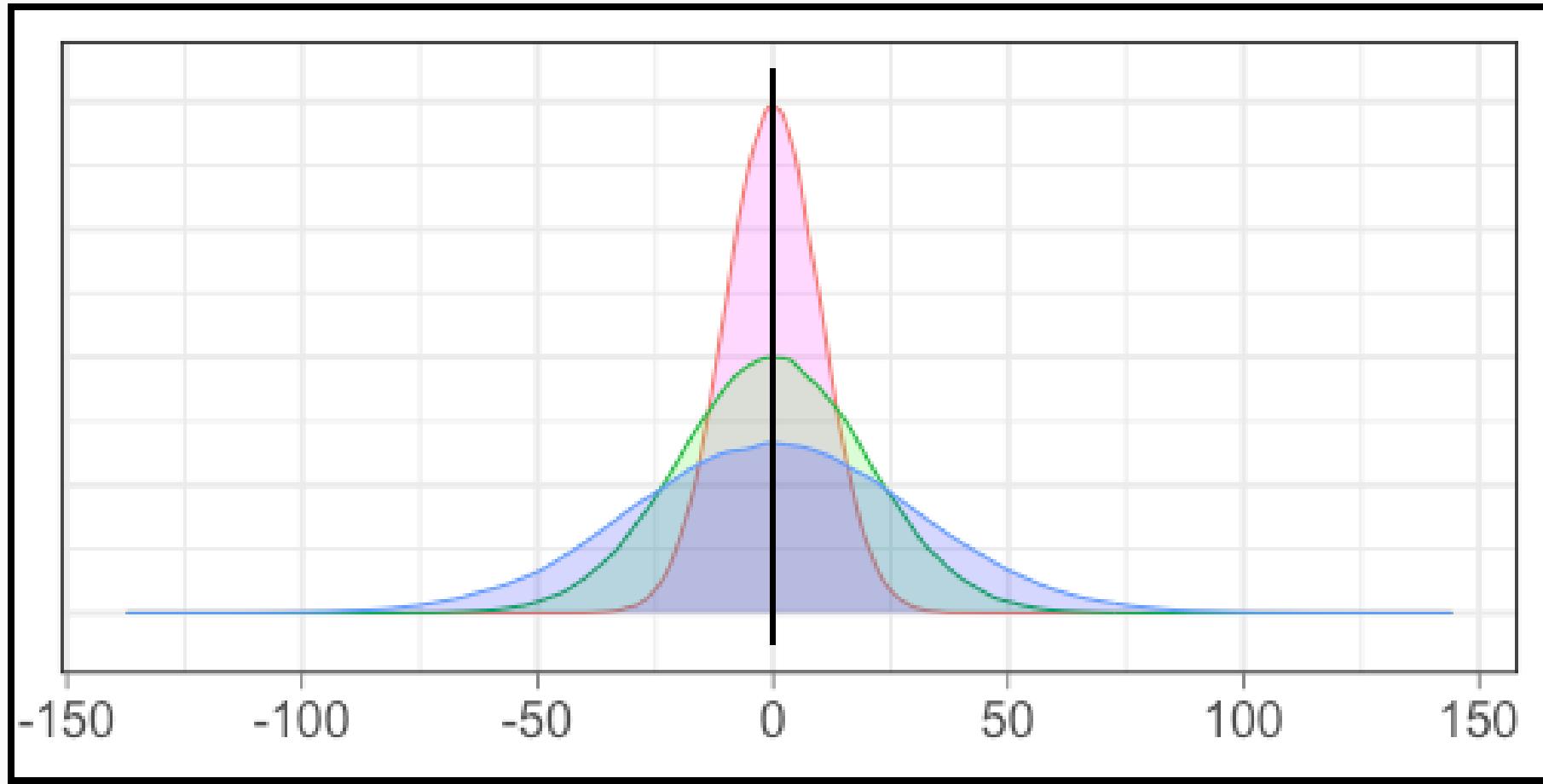
The shape of the distribution is...

- a) symmetric
- b) positive skewed
- c) negative skewed
- d) I need more information to answer

# Measures of dispersion



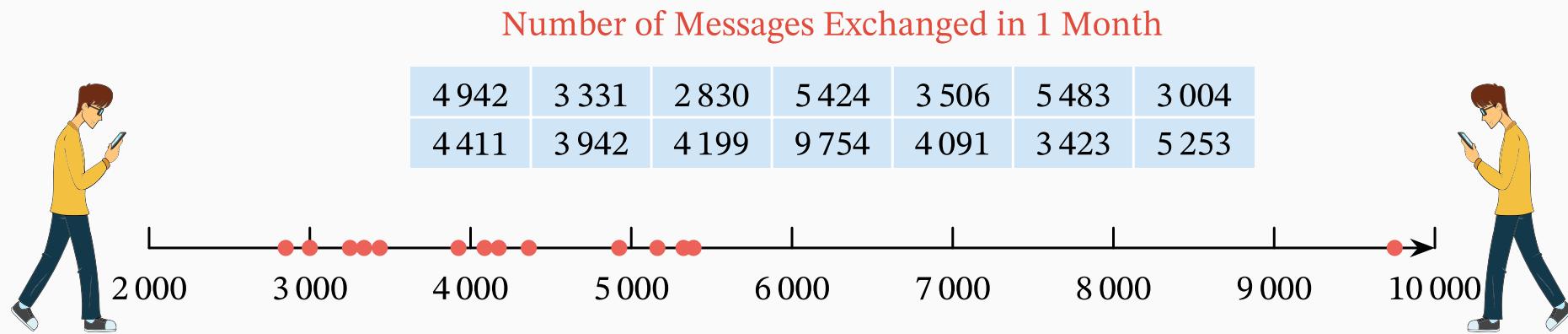
# Measures of dispersion



# Measure of dispersion: range



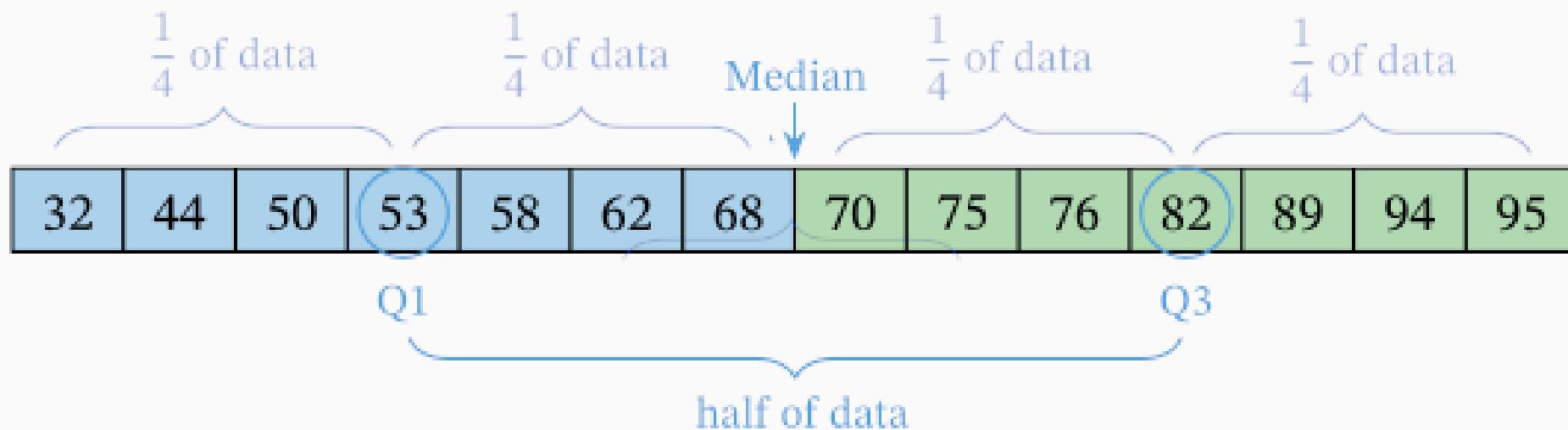
$$\text{range}(x) = \max(x) - \min(x)$$



$$\text{range}(x) = \max(x) - \min(x) = 9,754 - 2,830 = 6,924$$

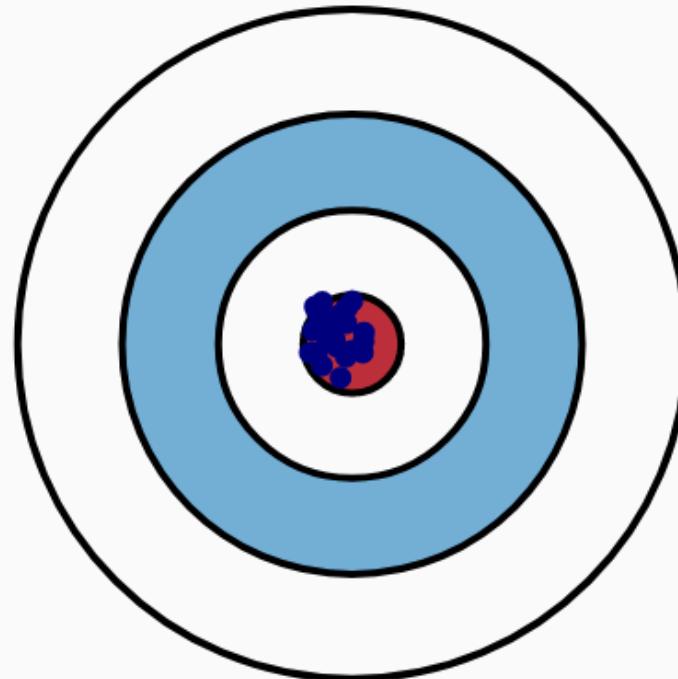
# Measure of dispersion: interquartile range

  $\text{IQR}(x) = \text{Q3}(x) - \text{Q1}(x)$

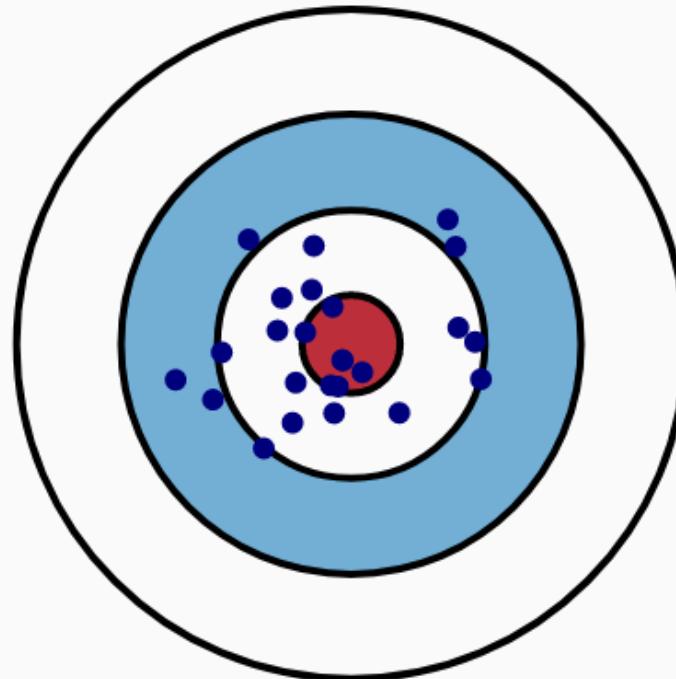


# Measure of dispersion: variance

Low Variance



High Variance



# Measure of dispersion: variance


$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

where  $\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right)$


$$x = \{1, 2, 3\} \quad \bar{x} = \frac{1+2+3}{3} = 2$$

$$\begin{aligned} s &= \frac{1}{3-1} \times [(1-2)^2 + (2-2)^2 + (3-2)^2] = \\ &= \frac{1}{2} \times [1^2 + 0^2 + 1^2] = \frac{1}{2} \times 2 = 1 \end{aligned}$$

# Measure of dispersion: variance


$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

why  $\frac{1}{n-1}$  ?  $\rightarrow \sum_{i=1}^n (x_i - \bar{x}) = 0$


$$x = \{1, 2, 3\} \quad \bar{x} = \frac{1+2+3}{3} = 2$$

$$\begin{aligned} \sum_{i=1}^n (x_i - \bar{x}) &= (1 - 2) + (2 - 2) + (3 - 2) = \\ &= -1 + 0 + 1 = 0 \end{aligned}$$

# Measure of dispersion: standard deviation


$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

where  $\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right)$


$$x = \{1, 2, 3\} \quad \bar{x} = \frac{1+2+3}{3} = 2$$

$$\begin{aligned} s &= \sqrt{\frac{1}{3-1} \times [(1-2)^2 + (2-2)^2 + (3-2)^2]} = \\ &= \sqrt{\frac{1}{2} \times [1^2 + 0^2 + 1^2]} = \sqrt{\frac{1}{2} \times 2} = \sqrt{1} = 1 \end{aligned}$$

# Centrality, dispersion, and data types

Data type	Centrality Measure	Dispersion Measure
Nominal	Mode	-
Ordinal	Mode, Median	Range, IQR
Numeric	Mode, Median, Mean	Range, IQR, standard deviation

## Exercise #12

- ? In the results section, the authors reported the following

*Coronary-artery calcium scores averaged  $68.9 \pm 244.2$  (range 0 to 1526) in patients and  $8.8 \pm 41.8$  (range 0 to 243.4) in controls.*

Which measures do we use to describe this variable?

- a) mean and standard deviation
- b) median and interquartile range
- c) median and standard deviation
- d) I need more information to answer

# Outliers in the wild

TABLE 3. Length of In-Patient Stay, by Surgical Procedure

Procedure	No. of procedures	Length of stay, d	
		Mean $\pm$ SD	Median (IQR)
Breast surgery	1,338	3.3 $\pm$ 4.4	3 (0-5)
Coronary artery bypass graft	570	9.6 $\pm$ 15.2	8 (7-9)
Cesarean section	4,831	4.9 $\pm$ 6.4	4 (3-5)
Repair of fractured neck of femur	2,303	13.8 $\pm$ 12.2	10 (7-17)
Hip replacement	6,432	8.7 $\pm$ 5.9	7 (6-9)
Abdominal hysterectomy	1,484	5.4 $\pm$ 4.0	5 (4-6)
Knee replacement	4,483	8.2 $\pm$ 5.0	7 (6-9)
Major vascular surgery	269	22.4 $\pm$ 23.1	14 (8-30)
Overall	21,710	7.8 $\pm$ 8.0	6 (4- 9)

The mean length of stay was 7.8 days but was greatly influenced by 2 patients with lengths of stay of almost 1 year. The median length of stay was 6 days, with 90% of patients discharged within 14 days after the procedure. Table 3 displays measures of central tendency (mean and median values) and dispersion (SDs and interquartile ranges) for the length of stay for each type of surgical procedure.

# Parameters vs statistics

- Parameters: calculated on the population
- Statistics: calculated on the sample

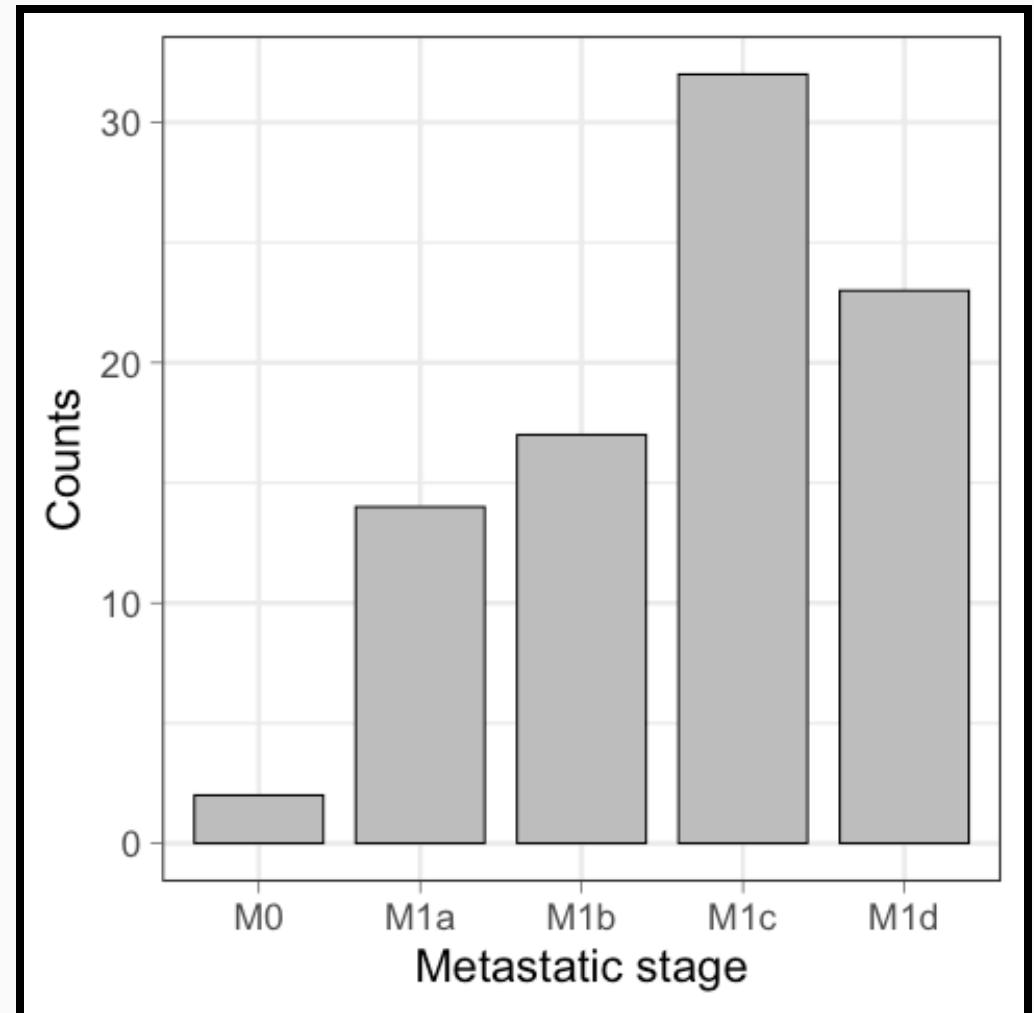
	Parameter	Statistic
Size	$N$	$n$
Mean	$\mu$	$\bar{x}$
Standard deviation	$\sigma$	$s$
Proportion	$\pi$	$p$

# **Notes on data visualization**

# Bar chart

- categorical data
  - absolute frequency
  - relative frequency

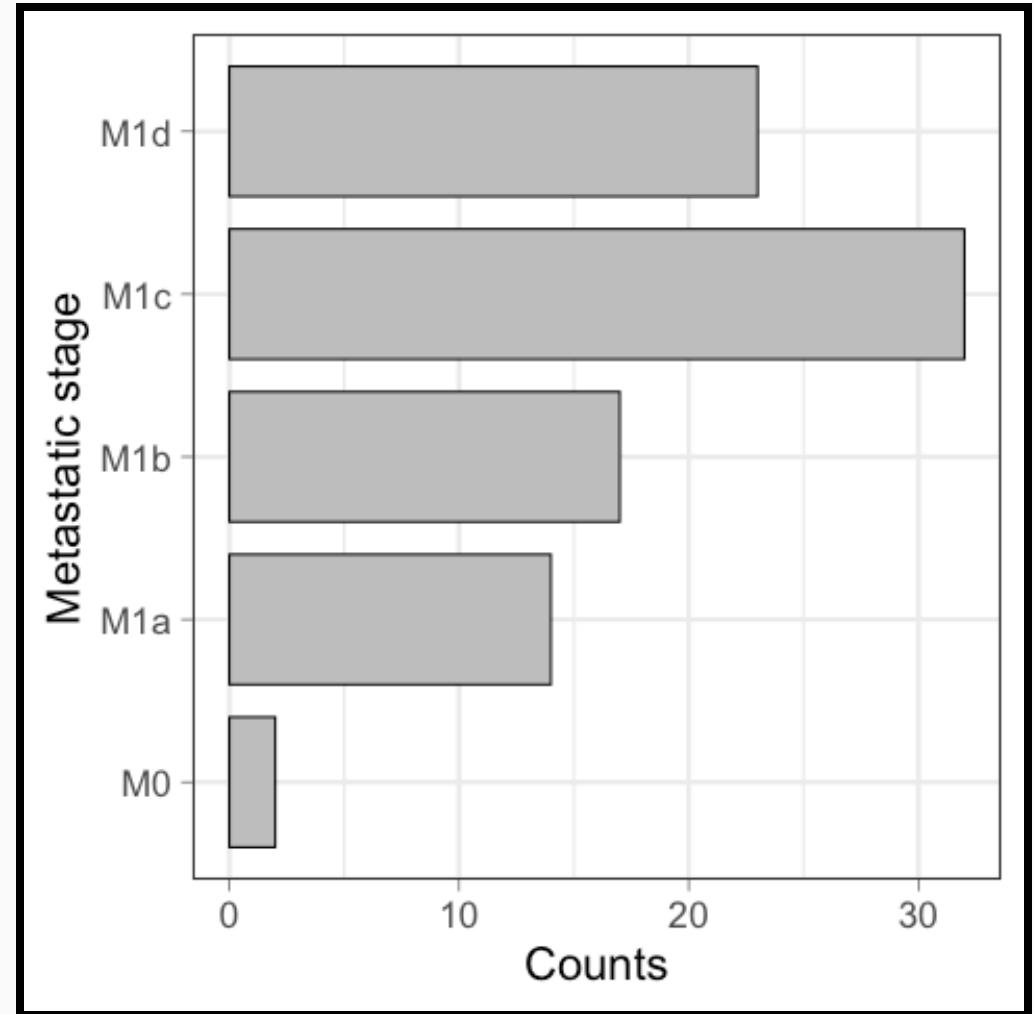
Visconti A., et al., Total serum *N*-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Horizontal bar chart

- categorical data
  - absolute frequency
  - relative frequency

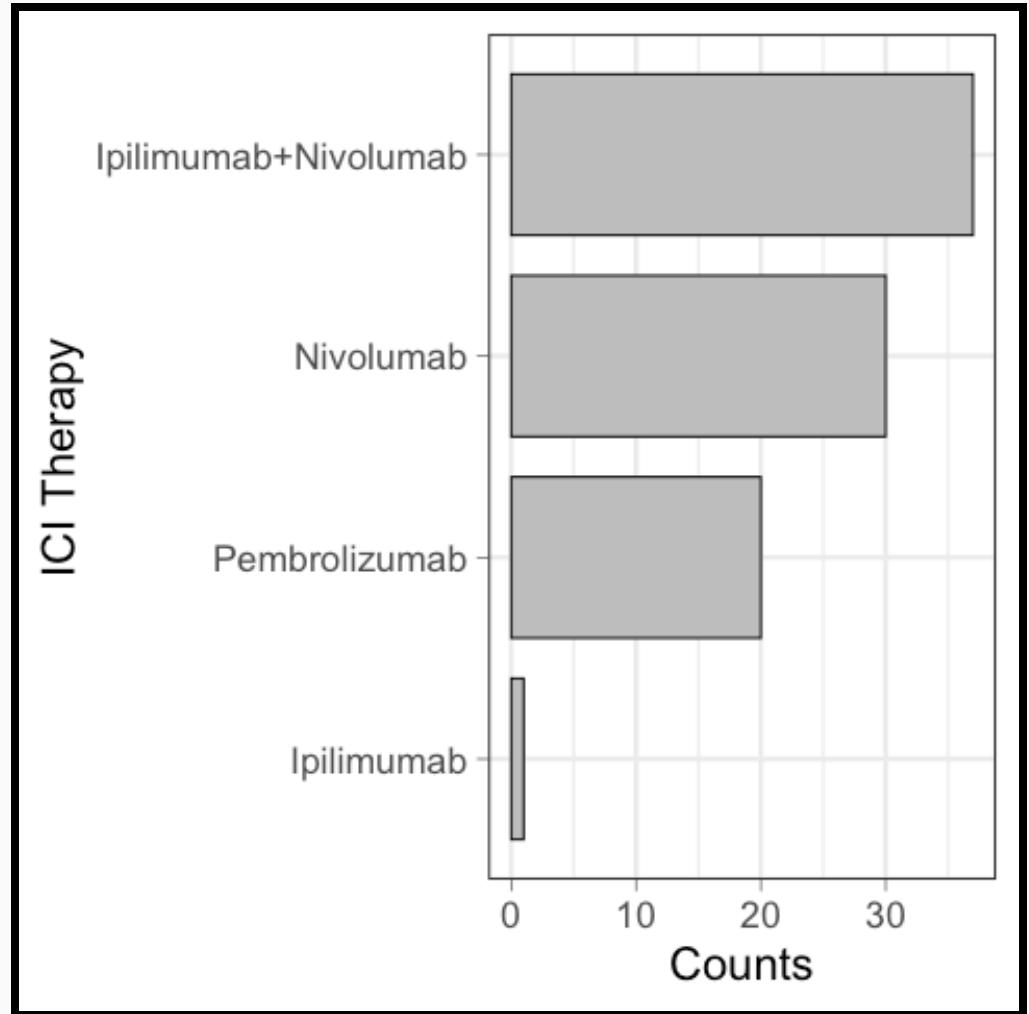
Visconti A., et al., *Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma*, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Horizontal bar chart

- categorical data
  - absolute frequency
  - relative frequency

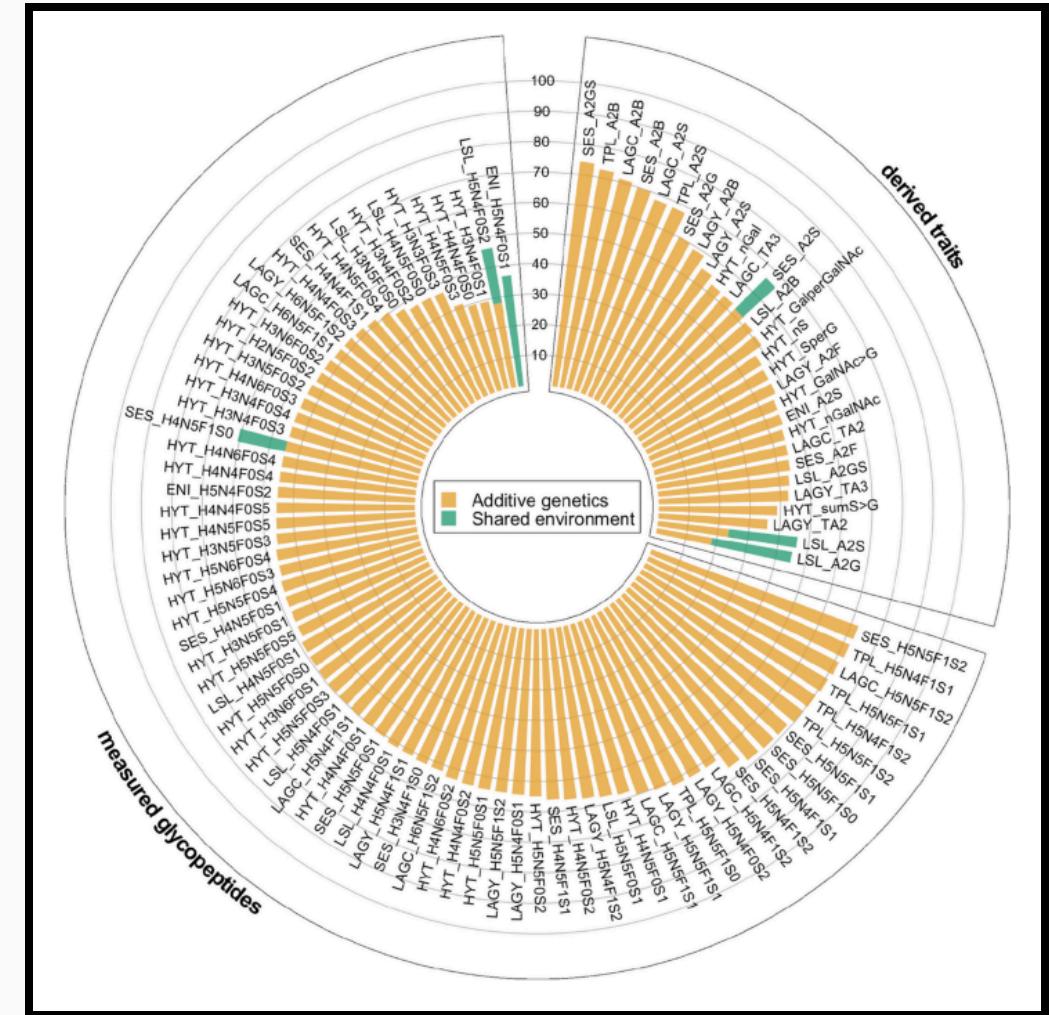
Visconti A., et al., *Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma*, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Circular bar chart

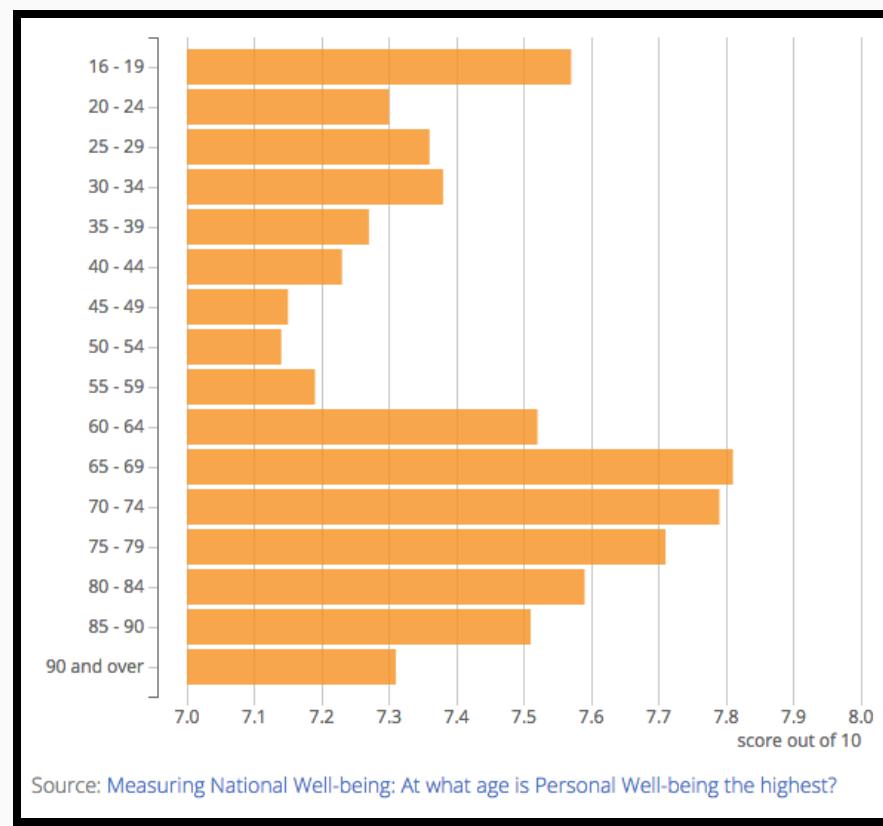
- categorical data
    - absolute frequency
    - relative frequency

Visconti, A., et al.. *The genetics and epidemiology of N-and O-immunoglobulin A glycomics.*, 2024, doi:10.1186/s13073-024-01369-6



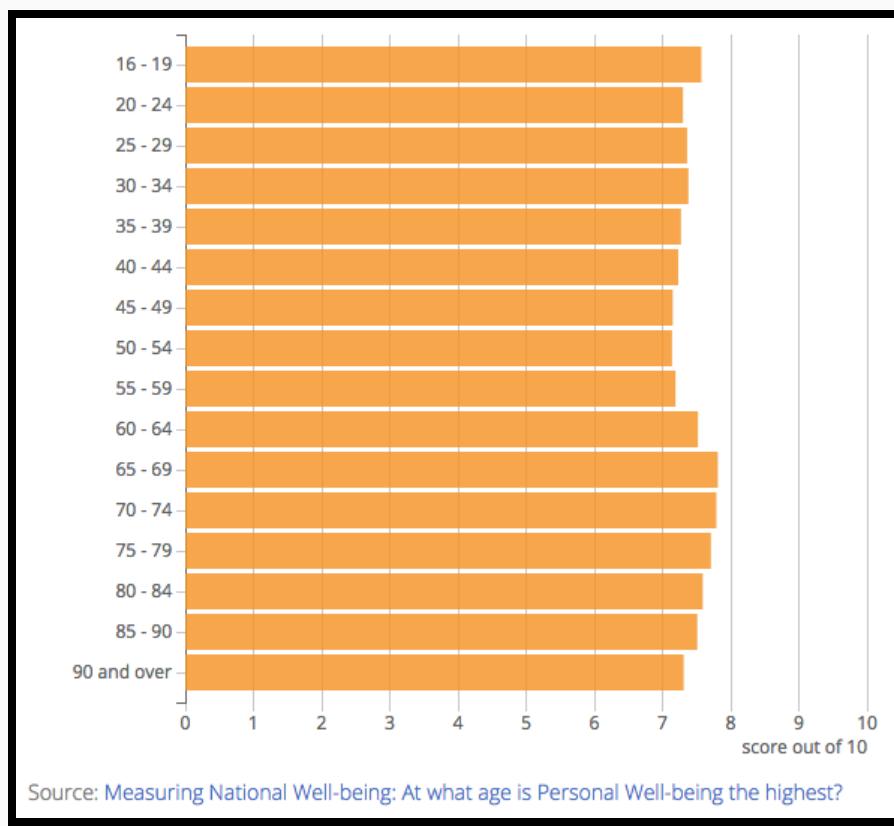
# What does this graph tell us?

- From 1 to 10, how happy were you yesterday?



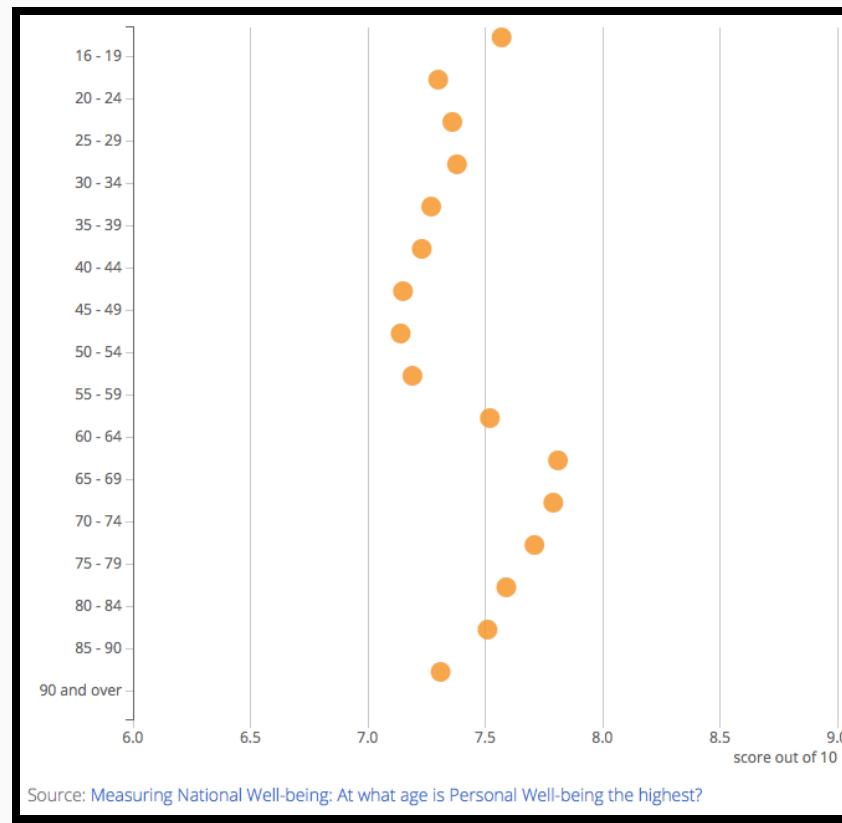
# What does this graph tell us?

- From 1 to 10, how happy were you yesterday?



# What does this graph tell us?

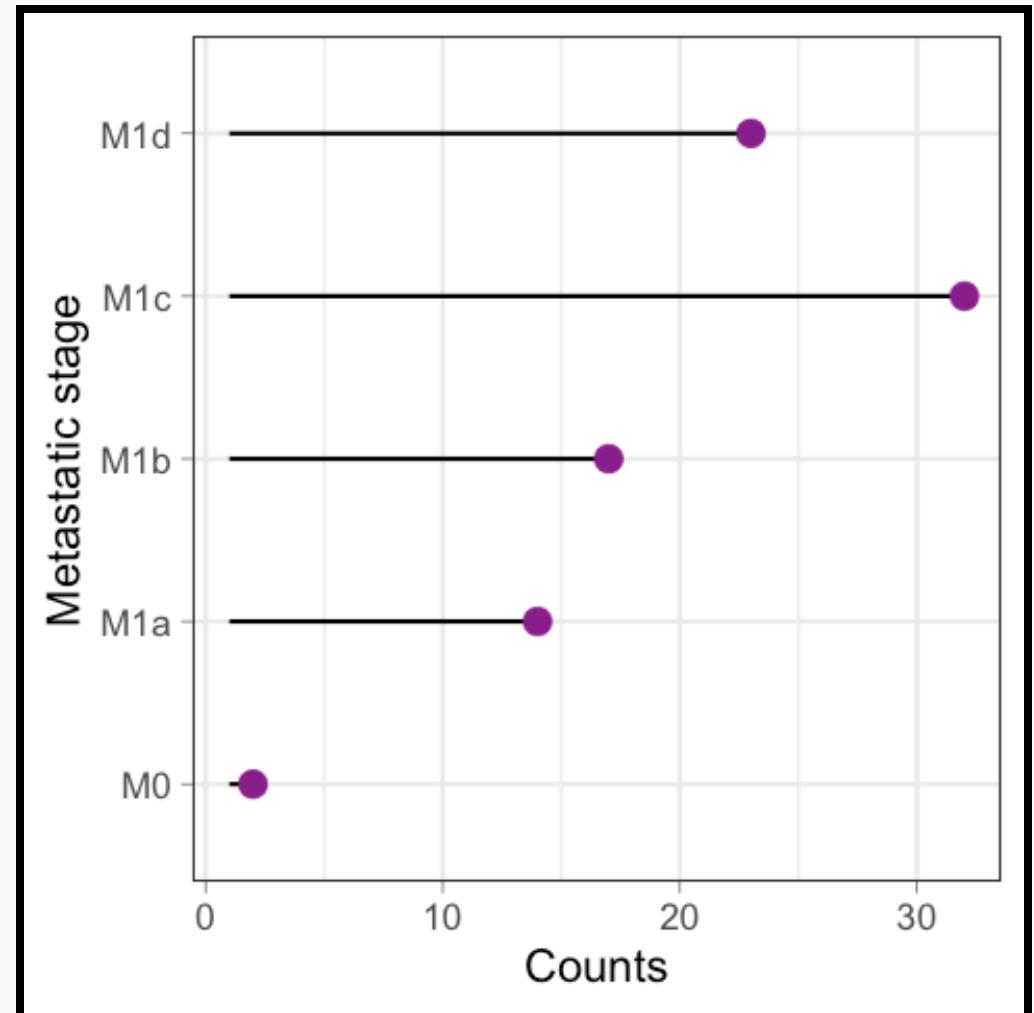
- From 1 to 10, how happy were you yesterday?



# Lollipop chart

- categorical data
  - absolute frequency
  - relative frequency

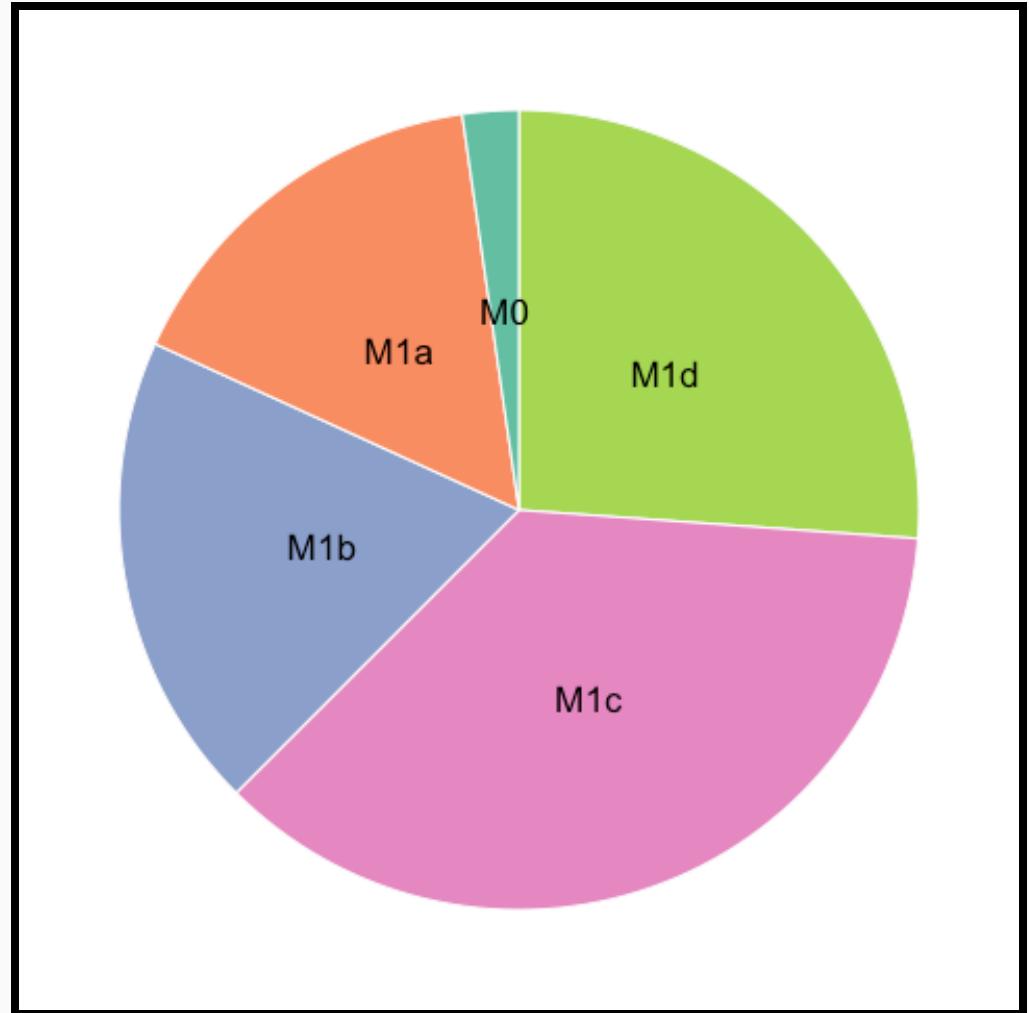
Visconti A., et al., *Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma*, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Pie chart

- categorical data
  - relative frequency

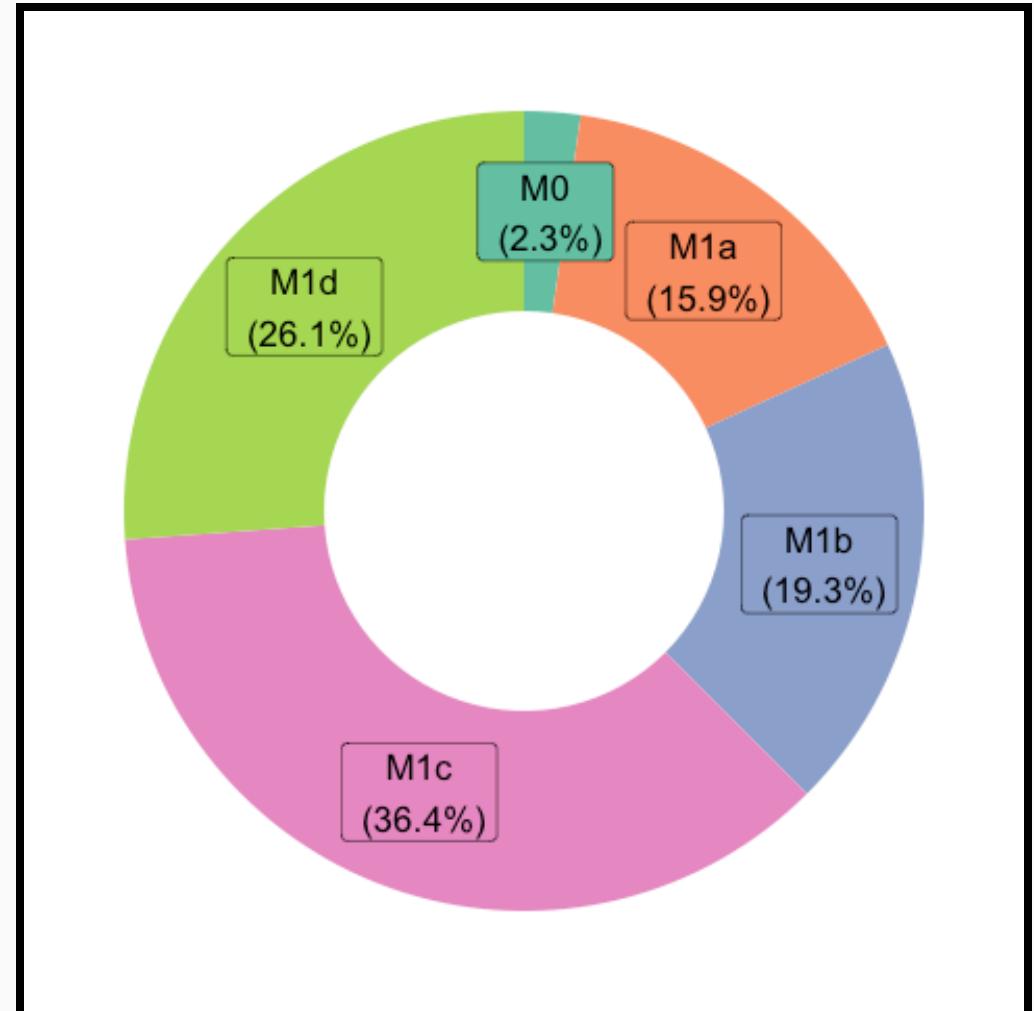
Visconti A., et al., *Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma*, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Donut chart

- categorical data
  - relative frequency

Visconti A., et al., *Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma*, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Waffle chart

- categorical data
  - relative frequency

Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Infographics

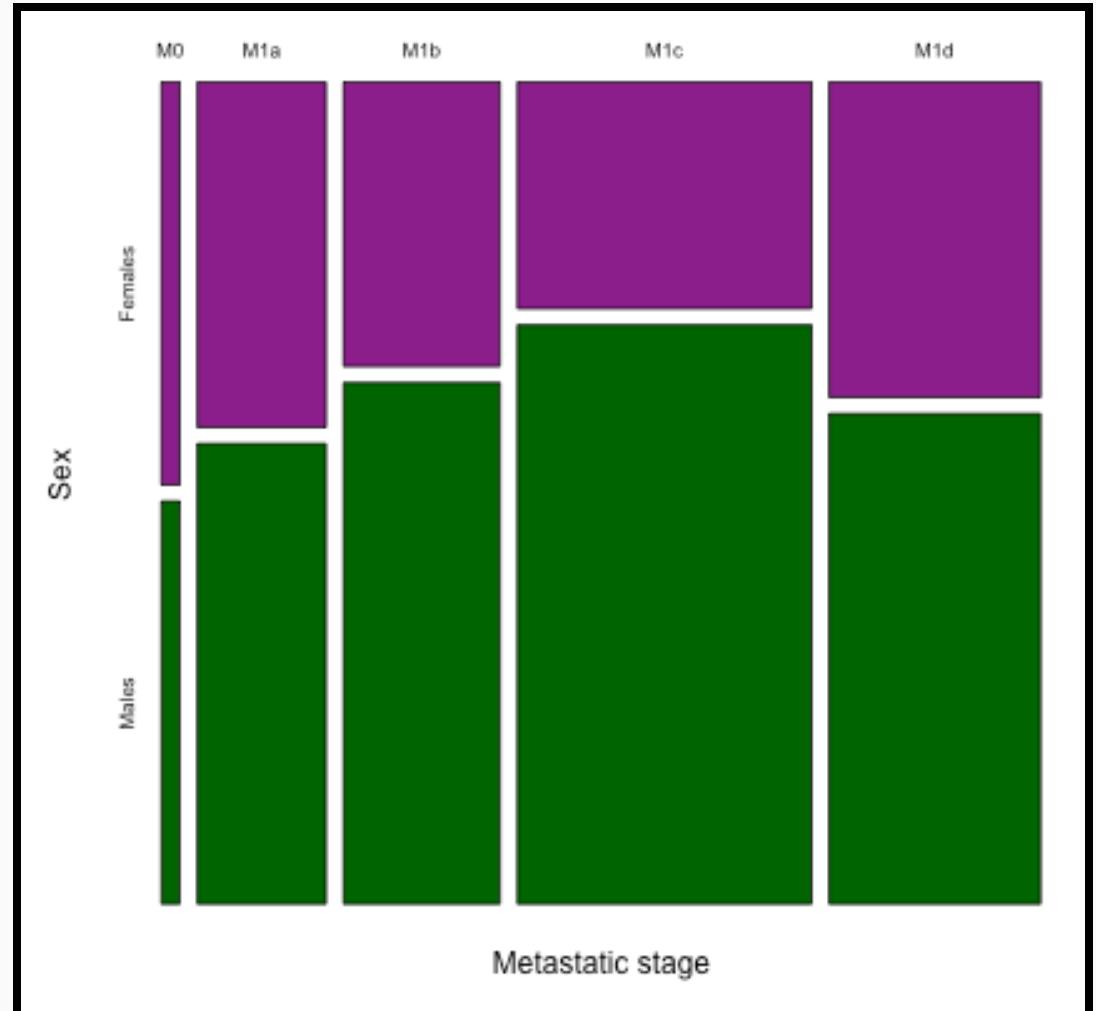


Spiegelhalter, D., *The Art of Statistics: Learning From Data*, Pelican, 2019

# Mosaic plot

- categorical data
  - relative frequency

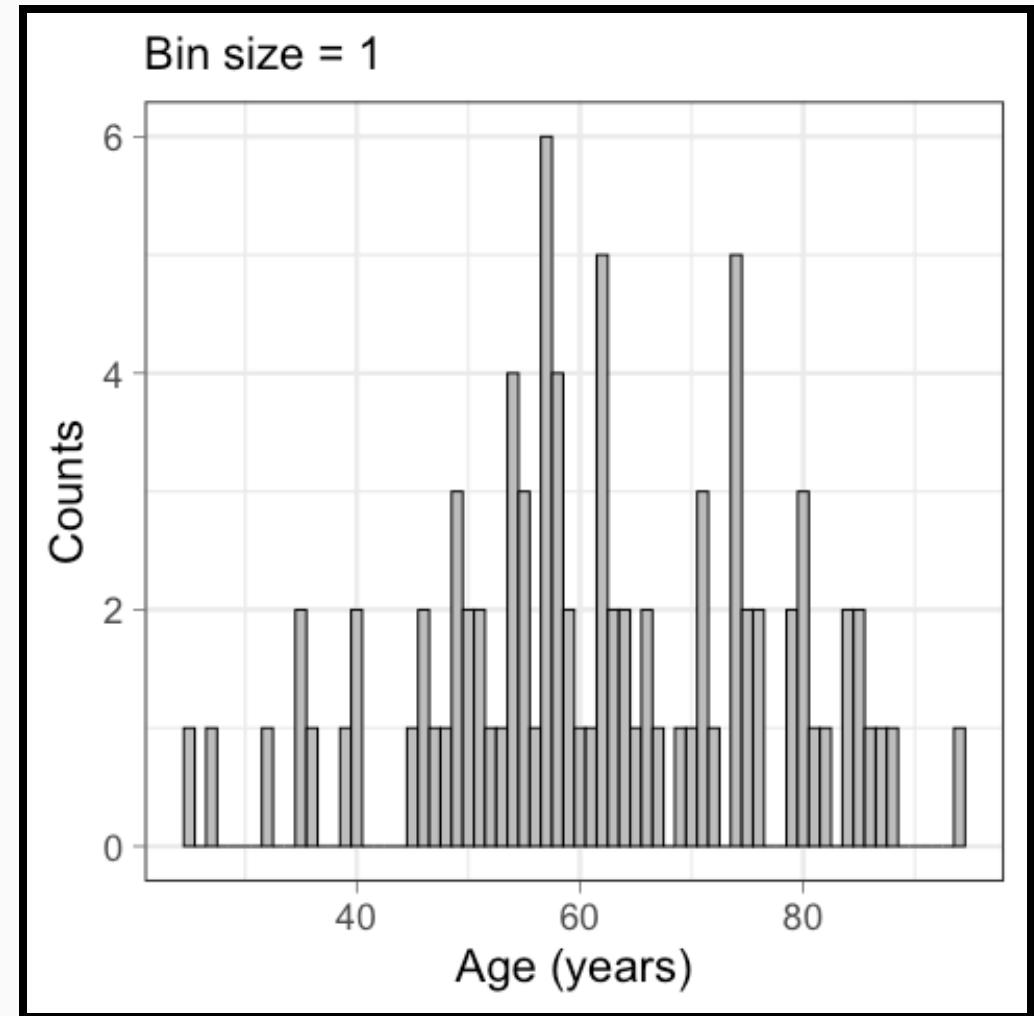
Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Histogram

- numerical data

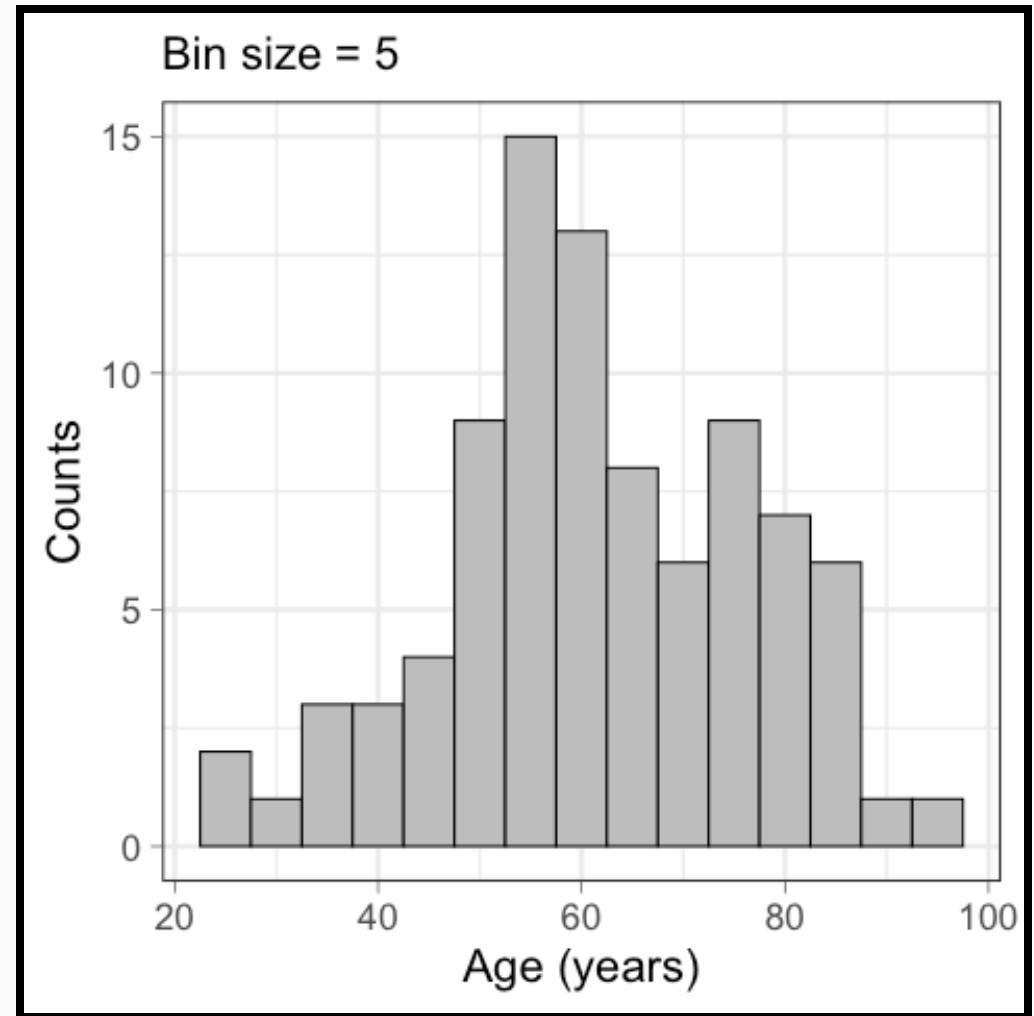
Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Histogram

- numerical data

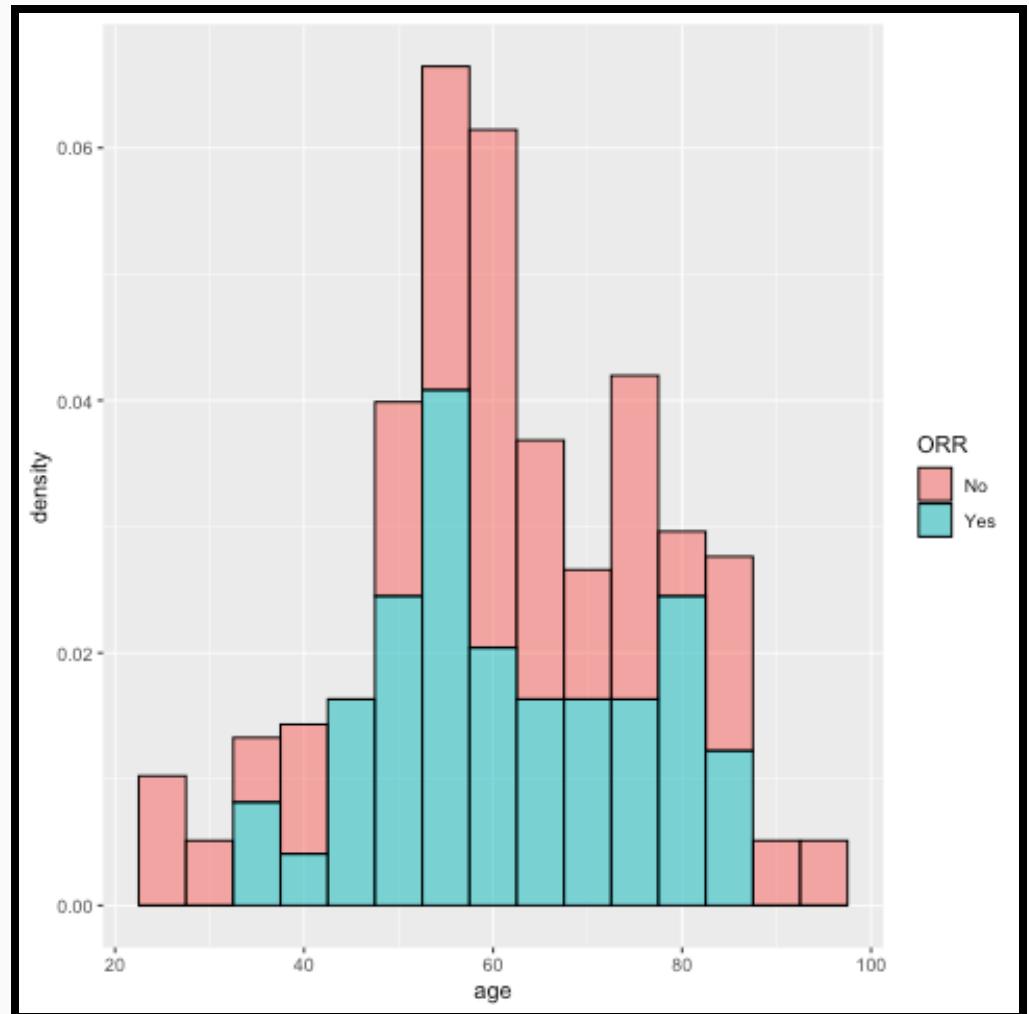
Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Histogram

- numerical data

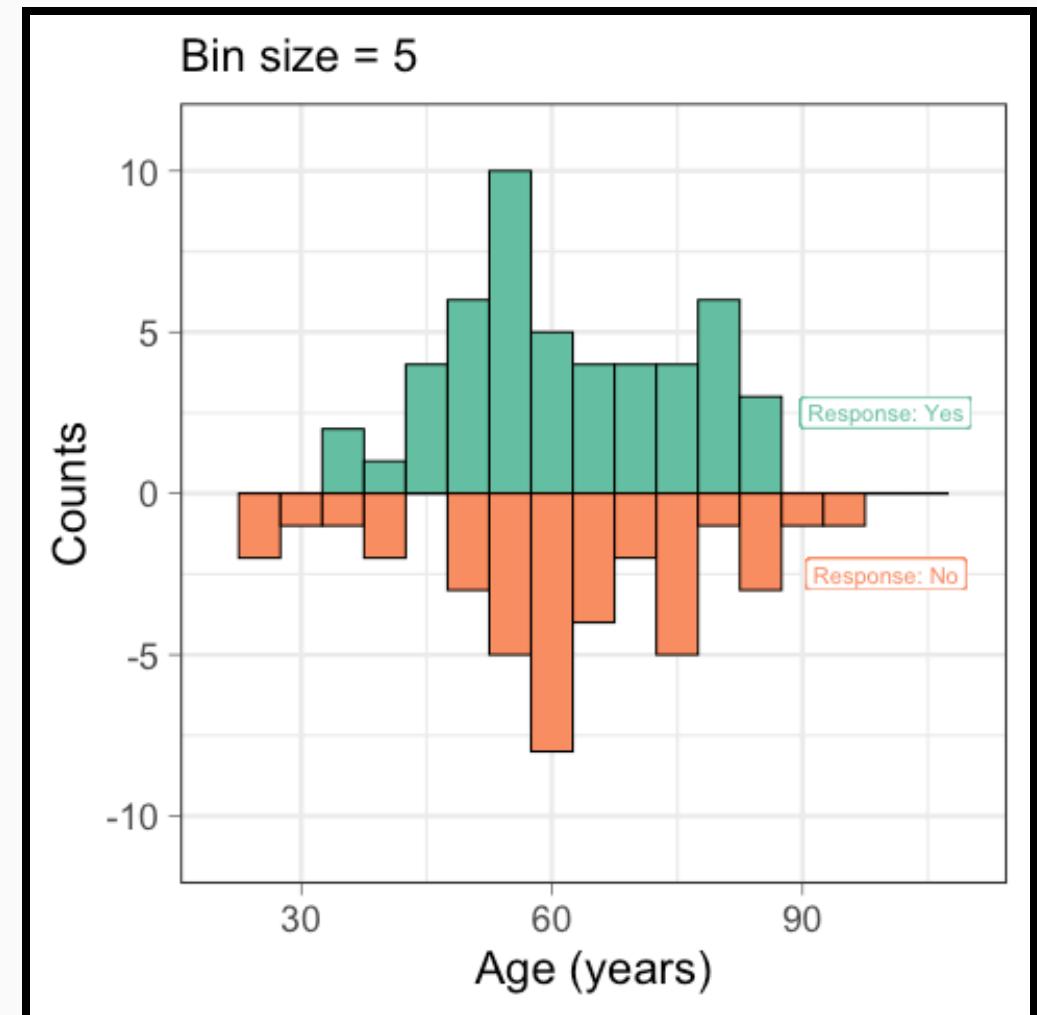
Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



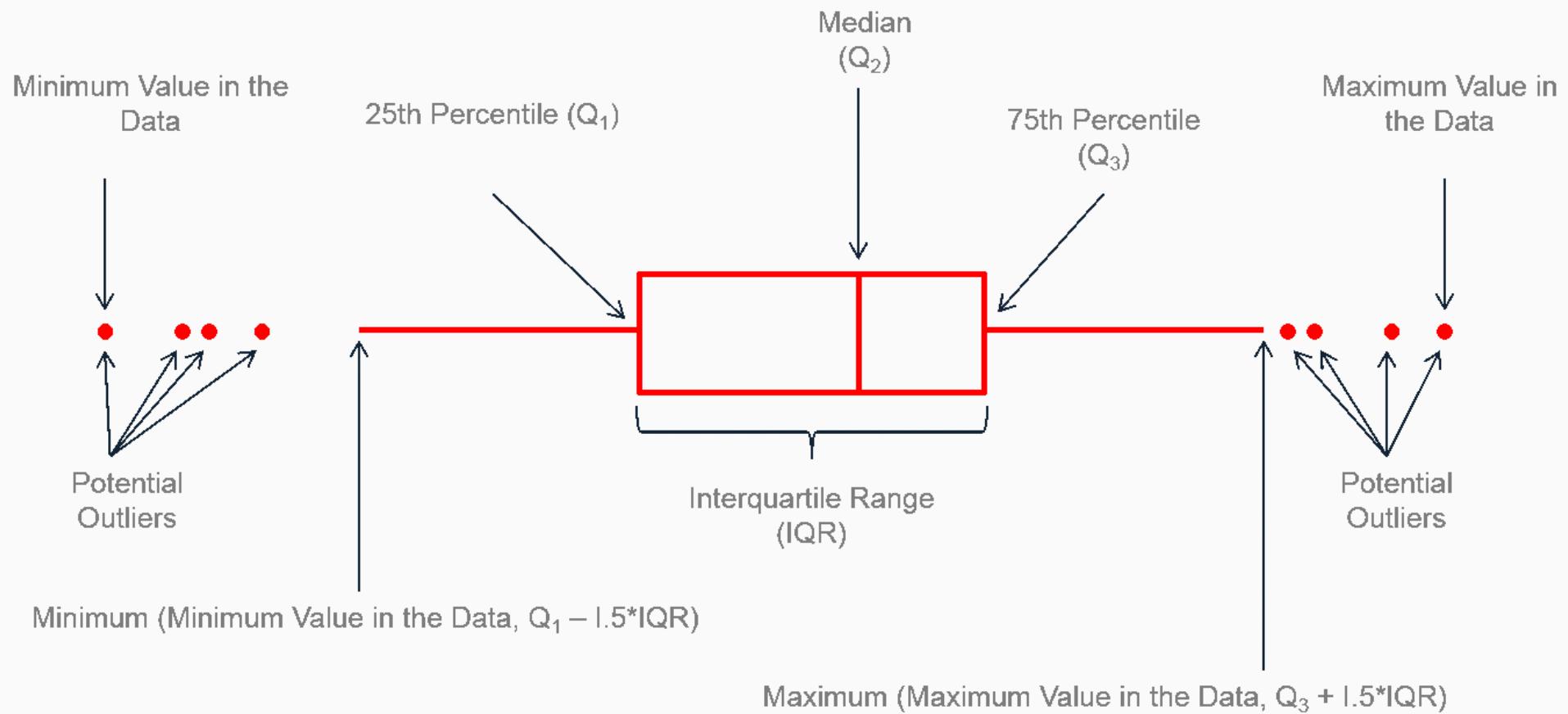
# Miami plot/Mirror histogram

- numerical data

Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



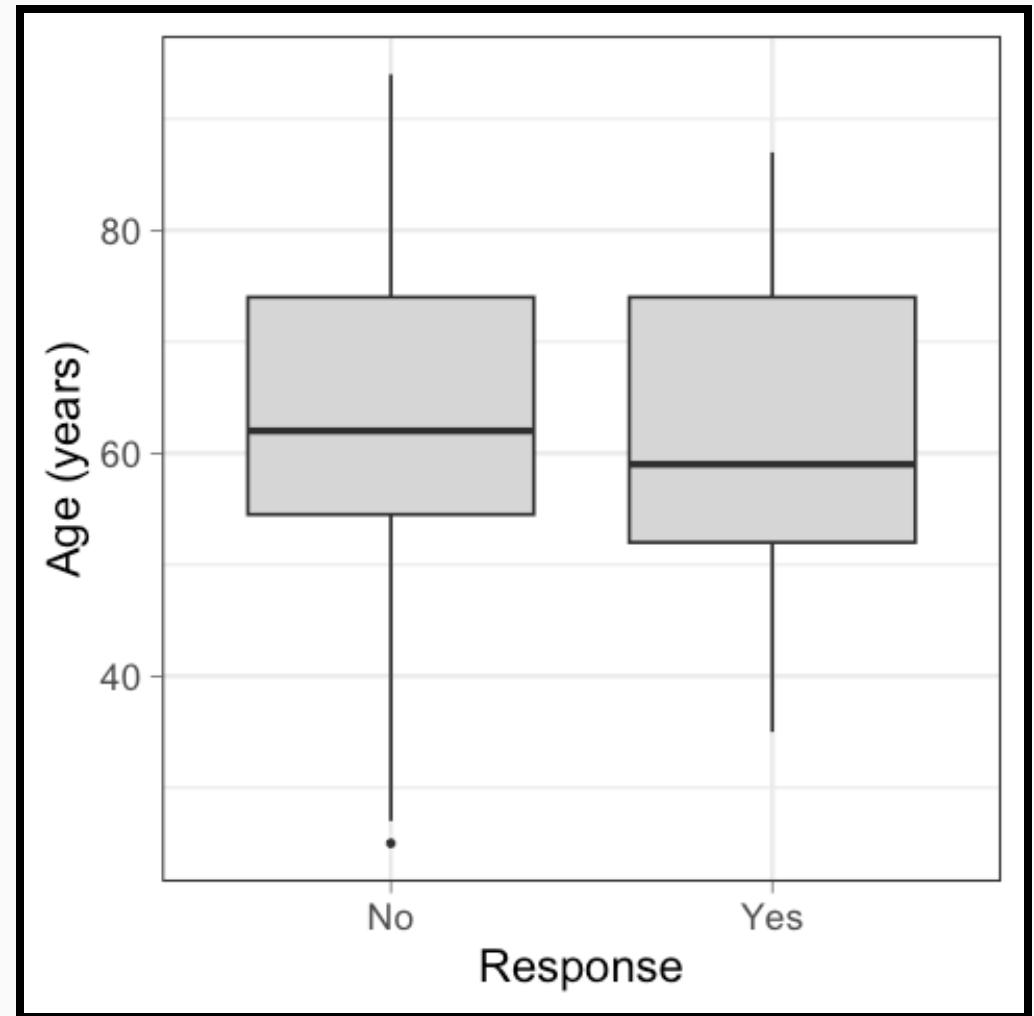
# Boxplot



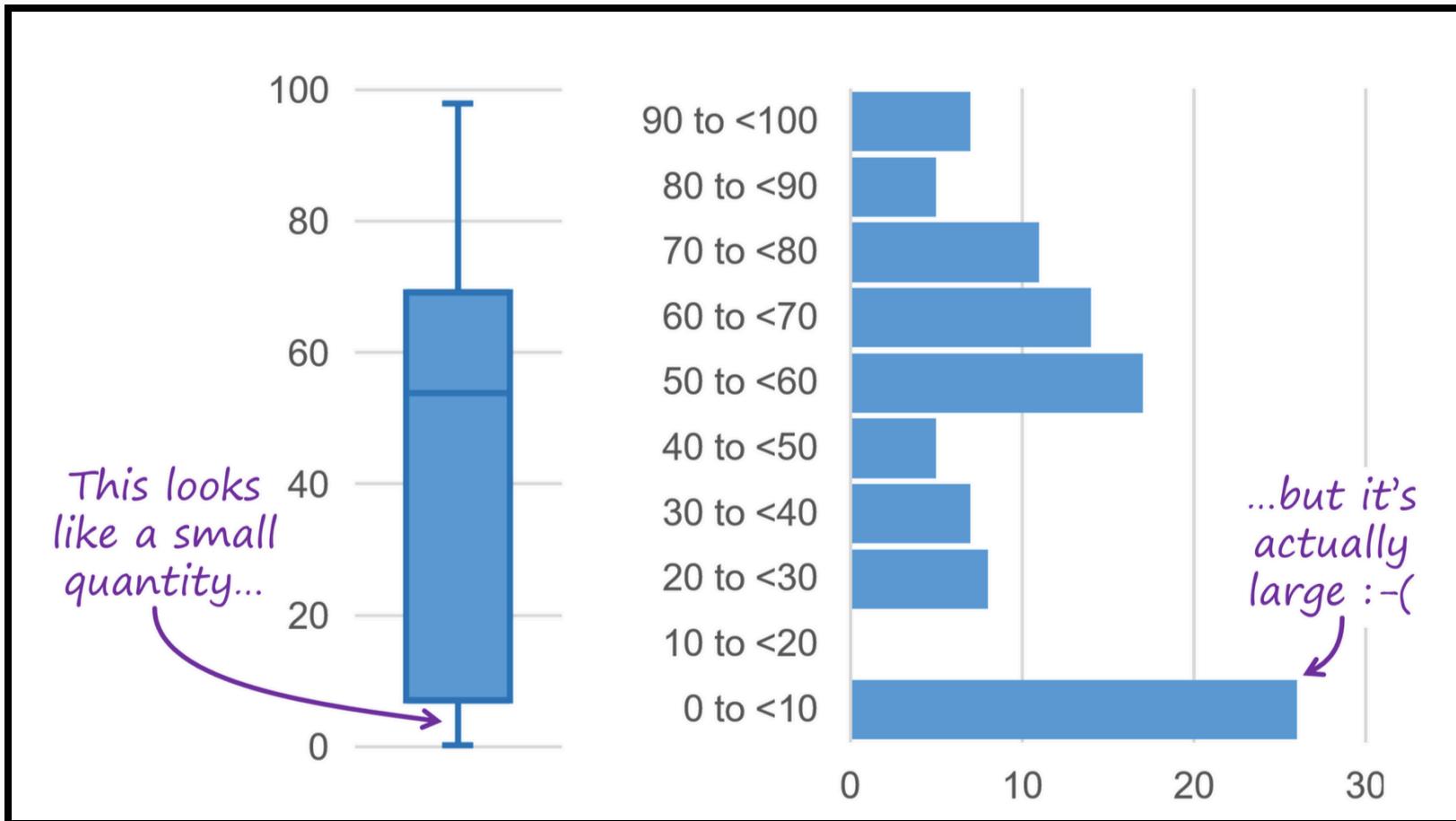
# Boxplot

- numerical data

Visconti A., et al., Total serum *N*-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



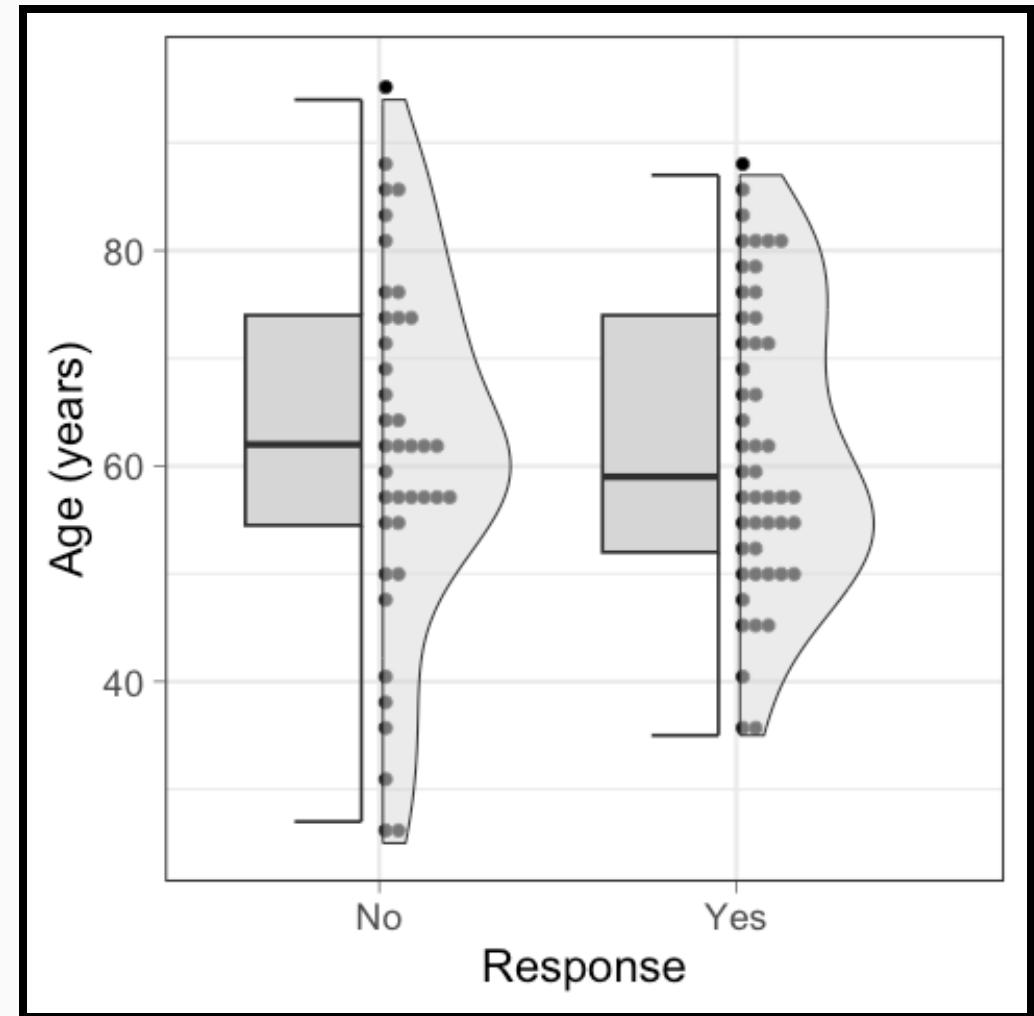
# Boxplot



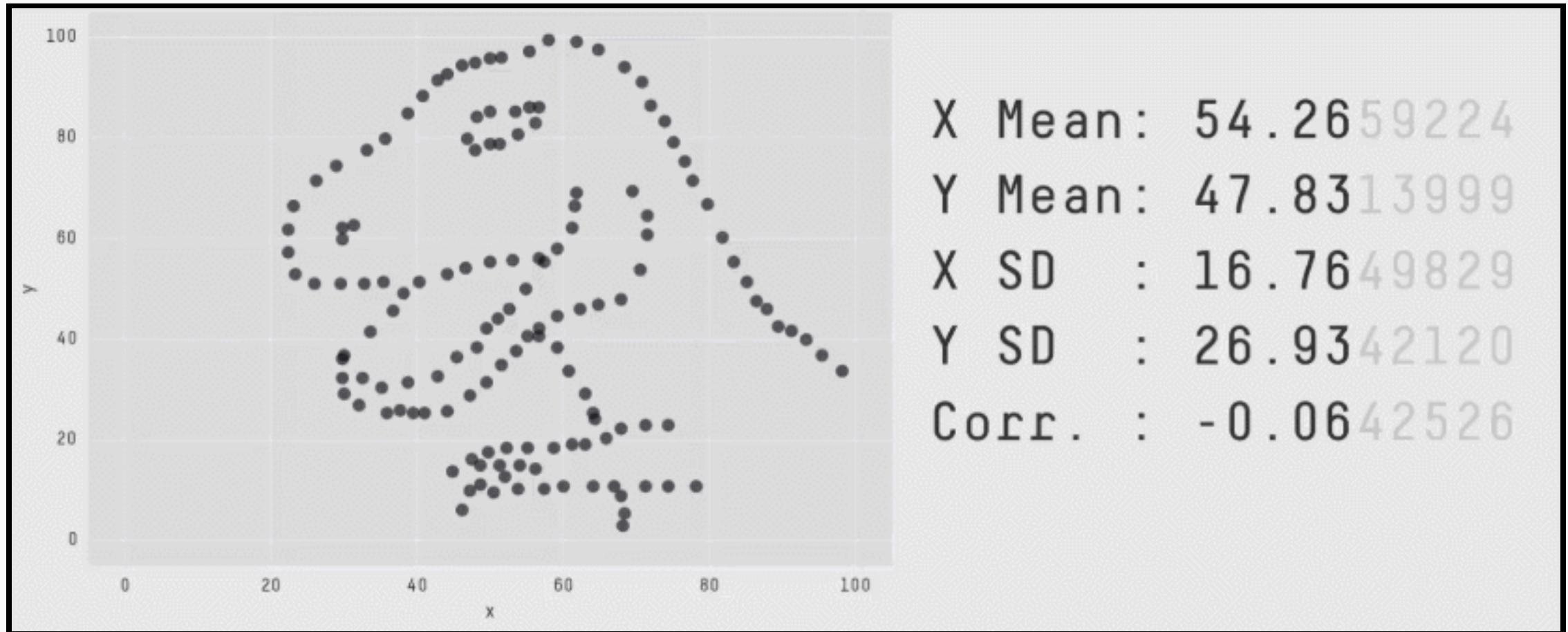
# Boxplot

- numerical data

Visconti A., et al., Total serum N-glycans associate with response to immune checkpoint inhibition therapy and survival in patients with advanced melanoma, BMC Cancer, 2023 doi:10.1186/s12885-023-10511-3



# Data visualisation: DataSaurus Dozen



# Summary

- Data come in different types
- Categorical variables can be summarized with absolute and relative frequencies
- Numerical variables can be summarized with measures of central tendency and dispersion, remembering that some of these measures are influenced by asymmetrical distribution and/or outliers
- Variables can be summarised with multiple graphical representations (but some are better than others), and visualising your data is always a good idea
- Samples are summarised by statistics, populations by parameters

# See you tomorrow

