Tarea semanal 5

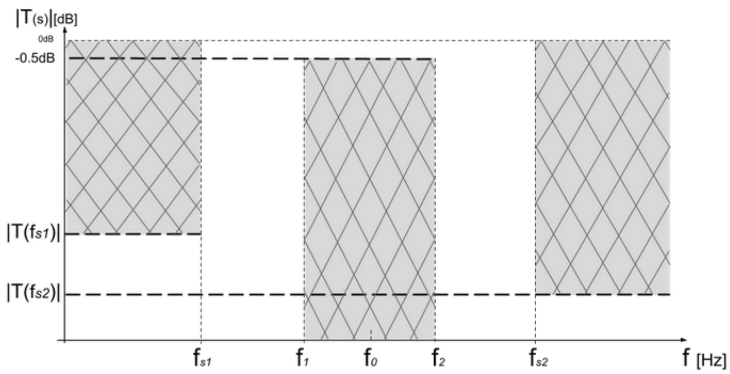
Consigna del ejercicio

Se pide diseñar un filtro pasabanda que cumpla con la siguiente plantilla:

- $\omega_0 = 2\pi\ 22$ kHz $\longrightarrow f_0 = 22\,kHz$
- $Q = 5$
- Aproximación Chebyshev con ripple de 0,5 dB

También se sabe que la transferencia del filtro debe ser:

- $|T(f_{S1})| = -16\,dB$ para $f_{S1} = 17\,kHz$
- $|T(f_{S2})| = -24\,dB$ para $f_{S2} = 36\,kHz$



Consignas de la actividad:

👉 Obtener la plantilla de diseño pasabanda normalizada

👉 Obtener la función transferencia normalizada del prototipo pasabajo que satisfaga el requerimiento del filtro pasabanda.

👉 Obtener la transferencia pasabanda normalizada

- $B = \frac{1}{Q} = \frac{1}{5} = f_2 - f_1$
- $f_0^2 = f_2 \cdot f_1$
- $f_0 = 1$ → Normalizado a 1

$$\Rightarrow \begin{cases} \frac{1}{5} = f_2 - f_1 \Rightarrow \frac{1}{5} = f_2 - \frac{f_0^2}{f_2} \quad (1) \\ f_0^2 = f_2 \cdot f_1 \Rightarrow f_1 = \frac{f_0^2}{f_2} \end{cases}$$

$$f_2^2 - \frac{1}{5} f_2 - 1 = 0$$

$$\Rightarrow \frac{1/5 \pm \sqrt{1/25 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{1/5 \pm 2{,}009}{2} \begin{cases} 1{,}105 \\ -0{,}\cancel{9}045 \end{cases}$$

$$f_1 = \frac{f_0^2}{f_2} \Rightarrow f_1 = \frac{1^2}{1{,}105} = 0{,}905$$

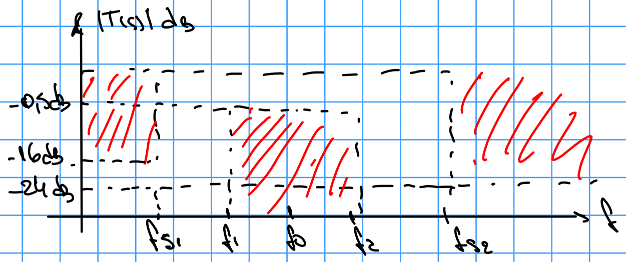
$$B = f_2 - f_1 = 1{,}105 - 0{,}905 = 0{,}2 = \frac{1}{5}$$

$f_0 = 1$
$f_1 = 0{,}905$
$f_2 = 1{,}105$
$f_{s1} = 0{,}772$
$f_{s2} = 1{,}636$ $\Rightarrow$

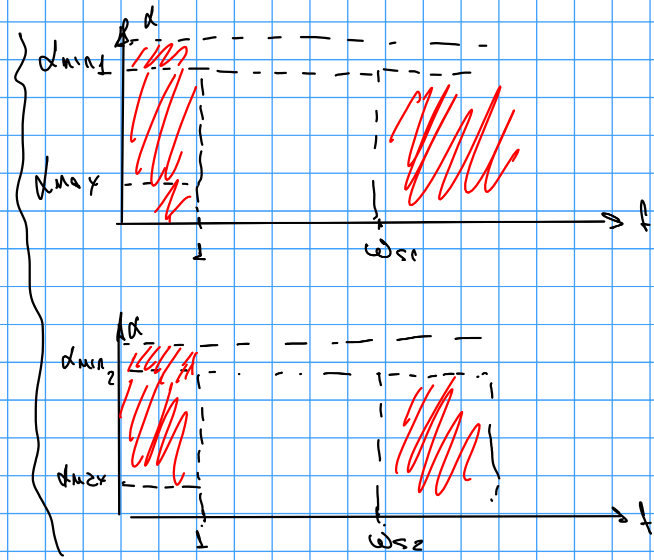
$\Omega_1 = 1/f_1 = 1{,}105$
$\Omega_{1m} = 1$
$\Omega_{s1} = Q \cdot \left(\frac{\omega_{s1}^2 - 1}{\omega_{s1}}\right) = -2{,}61$
$\Omega_{s1m} = -2{,}368$

$\Omega_2 = 1/f_2 = 0{,}905$
$\Omega_{2m} = 1$
$\Omega_{s2} = Q \cdot \left(\frac{\omega_{s2}^2 - 1}{\omega_{s2}}\right) = 5{,}123$
$\Omega_{s2m} = 5{,}663$

PLANTILLA PASABANDA NORMALIZADA.



PASA BAJOS NORMALIZADOS



PROTOTIPO PASBAJOS NORMALIZDO 1:

- $\alpha_{min\,1} = 16\,dB$
- $\alpha_{max} = 0{,}5\,dB$
- $\Omega_{s1m} = 1$
- $\Omega_{s2m} = 2{,}368$

$$|T_1(s)|^2 = \frac{1}{1+\varepsilon_1^2\,\omega_n^{2n}}$$

- $\varepsilon_1^2 = 10^{\frac{\alpha_{max}}{10}} - 1 \Rightarrow \varepsilon_1^2 = 10^{\frac{0{,}5}{10}} - 1$

$\varepsilon_1^2 = 0{,}122$

$\varepsilon_1 = 0{,}349$

- $10\log\left(1+\varepsilon_1^2 \cdot \omega_s^{2n}\right) = \alpha_{min\,1}$
- $10\log\left(1 + 0{,}122 \cdot 2{,}368^{2n}\right) = 16\,dB$

$n = 1 \longrightarrow \alpha_{min\,1} = 2{,}26\ dB$

$n = 2 \longrightarrow \alpha_{min\,1} = 8{,}39\ dB$

$n = 3 \longrightarrow \alpha_{min\,1} = 13{,}52\ dB$

$n = 4 \longrightarrow \alpha_{min\,1} = 20{,}84\ dB$

PROTOTIPO PASABAJOS NORMALIZADO 2:

- $\alpha_{min_2} = 24\,dB$
- $\alpha_{max} = 0{,}5\,dB$
- $\Omega_{2m} = 1$
- $\Omega_{s2m} = 5{,}661$

$$|T_2(s)|^2 = \frac{1}{1+\varepsilon_1^2\,\omega_n^2}$$

- $\varepsilon^2 = 10^{\frac{0{,}5dB}{10}} - 1 = 0{,}122$
- $\varepsilon_1 = 0{,}349$
- $10\log\left(1+0{,}122 \cdot 5{,}661^{2n}\right) = 24\,dB$

$n = 1 \longrightarrow \alpha_{min_2} : 6{,}91\ dB$

$n = 2 \longrightarrow \alpha_{min_2} : 21{,}01\ dB$

$n = 3 \longrightarrow \alpha_{min\,2} : 36{,}038\ dB$

POTOTIPO CHEBYSHEV DE ORDEN 3

$\varepsilon^2 = 0{,}122$

$n = 3$

$$|T(j\omega)|^2 = \frac{1}{1+C_n^2(\omega)} = T(s)\cdot T(-s)$$

$$C_3^2(\omega) = \varepsilon_1^2\left(4\omega^3 - 3\omega\right)^2 = \varepsilon_1^2\left(16\omega^6 - 2\cdot 4\omega^3\cdot 3\omega + 9\omega^2\right) = \varepsilon_1^2 16\omega^6 - \varepsilon_1^2 24\omega^4 + \varepsilon_1^2 9\omega^2$$

$$|T(j\omega)|^2 = \frac{1}{\varepsilon_1^2 16\omega^6 - \varepsilon_1^2 24\omega^4 + \varepsilon_1^2 9\omega^2 + 1} = T(s)\cdot T(-s)$$

$$|T(j\omega)|^2\Big|_{\omega=\frac{s}{j}} = \frac{1}{\varepsilon_1^2 16\omega^6 - \varepsilon_1^2 24\omega^4 + \varepsilon_1^2 9\omega^2 + 1} = \frac{1}{-s^6\varepsilon_1^2 16 - s^4\varepsilon_1^2 24 - s^2\varepsilon_1^2 9 + 1}$$

$$\frac{1/\varepsilon_1^2 16}{-s^6 - s^4\frac{3}{2} - s^2\frac{9}{16} + 1} = \frac{c}{s^3+s^2a+sb+c} \cdot \frac{c}{-s^3+s^2a-sb+c}$$

$$\frac{1/\varepsilon_1^2 16}{-s^6 - s^4\frac{3}{2} - s^2\frac{9}{16} + 1} = \frac{c^2}{-s^6 + 0s^5 + s^4(a^2-2b) + 0\cdot s^3 + s^2(2ac-b^2) + 0\cdot s + c^2}$$

$$c^2 = \frac{1}{\varepsilon^2 16} \Rightarrow C = \frac{1}{\varepsilon 4} \Rightarrow \varepsilon = 0{,}122 \Rightarrow \boxed{c = 2{,}05}$$

$$-\frac{3}{2} = a^2 - 2b \Rightarrow a^2 = \frac{3}{2} + 2b \Rightarrow b = \frac{a^2}{2} - \frac{3}{4}$$

$$-\frac{9}{16} = 2ac - b^2 \Rightarrow b^2 = 2ac + \frac{9}{16} \Rightarrow \left(\frac{a^2}{2} - \frac{3}{4}\right)^2 = \underbrace{4{,}01 \cdot a}_{\approx 4a} + \frac{9}{16}$$

$$\Rightarrow \frac{a^4}{4} - \cancel{2}\cdot\frac{a^2}{\cancel{2}}\cdot\frac{3}{4} + \cancel{\frac{9}{16}} = 4a + \cancel{\frac{9}{16}} \Rightarrow \frac{a^4}{4} - a^2 \cdot \frac{3}{4} - 4a = 0$$

$$a^3 - a \cdot 3 - 16 = 0 \Rightarrow a^3 - 3a = 16$$

$$a(a^2 - 3) = 16 \Rightarrow \boxed{a = 2{,}914}$$ por iteración

$$\Rightarrow \boxed{b = 3{,}495}$$

$$\Rightarrow T(s) = \frac{2{,}05}{s^3 + s^2 \cdot 2{,}914 + s \cdot 3{,}495 + 2{,}05} \Rightarrow$$

PASABAJOS PROTOTIPO NORMALIZADO
CHEBYSHEV ORDEN 3

$$T(s) = 2{,}05 \cdot \frac{1}{(s + 1{,}5)} \cdot \frac{1}{(s + 0{,}71068 + j0{,}9318)} \frac{1}{(s + 0{,}71068 - j0{,}9318)}$$

$$T(s) = 2{,}05 \frac{1}{(s + 1{,}5)} \frac{1}{(s^2 + s \cdot 0{,}71068 - \cancel{s\, j0{,}9318} + s \cdot 0{,}71068 + 0{,}71068^2 - \cancel{j\,0{,}9318 \cdot 0{,}71068} + \cancel{s\, j0{,}9318} + \cancel{j\,0{,}9318 \cdot 0{,}71068} + 0{,}9318^2)}$$

$$T(s) = 2{,}05 \frac{1}{(s + 1{,}5)} \frac{1}{(s^2 + 2 \cdot s \cdot 0{,}71068 + 0{,}71068^2 + 0{,}9318^2)} = \boxed{\frac{1{,}5}{(s + 1{,}5)} \frac{1{,}37}{s^2 + s \cdot 1{,}42 + 1{,}37}}$$

Pasabajos prototipo

DESNORMALIZACIÓN EN FRECUENCIA DEL PASABAJO.

$$T(s) = \frac{1{,}5}{\frac{s}{\Omega_2} + 1{,}5} \cdot \frac{1{,}37}{\left(\frac{s}{\Omega_2}\right)^2 + \frac{s}{\Omega_2} \cdot 1{,}42 + 1{,}37}$$

$$\Omega_2 = 1{,}105$$

$$T(s)|^2 = \frac{\Omega_2 \cdot 1{,}5}{s + \Omega_2 1{,}5} \frac{1{,}37}{\frac{s^2}{\Omega_2^2} + \frac{s}{\Omega_2} \cdot 1{,}42 + 1{,}37} = \frac{\Omega_2 \cdot 1{,}5}{s + 1{,}5\,\Omega_2} \cdot \frac{\Omega_2^3 \cdot 1{,}37}{s^2 \Omega_2 + s \cdot 1{,}42 \cdot \Omega_2^2 + 1{,}37 \cdot \Omega_2^3}$$

$$= \frac{\Omega_2 \cdot 1{,}5}{s + \Omega_2 1{,}5} \frac{\Omega_2^{\cancel{3}} \cdot 1{,}37}{\cancel{\Omega_2}(s^2 + s \cdot 1{,}42 \cdot \Omega_2 + 1{,}37 \cdot \Omega_2^2)} =$$

$$= \frac{1{,}105 \cdot 1{,}5}{s + 1{,}105 \cdot 1{,}5} \frac{1{,}105^2 \cdot 1{,}37}{s^2 + s \cdot 1{,}42 \cdot 1{,}105 + 1{,}37 \cdot 1{,}105^2} = \boxed{\frac{1{,}6575}{s + 1{,}6575} \cdot \frac{2{,}59}{s^2 + s \cdot 1{,}569 + 2{,}59}}$$

pasabajos desnormalizado

TRANSFORMACIÓN A PASABANDA:

$$T(s)\Big|_{s = Q \cdot \frac{s^2+1}{s}} = \frac{1{,}675}{Q\left(\frac{s^2+1}{s}\right) + 1{,}675} \frac{2{,}59}{\left(Q\left(\frac{s^2+1}{s}\right)\right)^2 + Q\left(\frac{s^2+1}{s}\right) \cdot 1{,}569 + 2{,}59}$$

$$= \frac{s\ 1{,}675/Q}{s^2 + s\,\frac{1{,}675}{Q} + 1} \frac{2{,}59}{Q^2\,\frac{(s^2+1)^2}{s^2} + Q \cdot \frac{s^2+1}{s} \cdot 1{,}569 + 2{,}59}$$

$$T(s)\Big|_{s=q\cdot\frac{s^2+1}{s}} = \frac{s\,1{,}675/q}{s^2+s\,1{,}675/q+1}\;\frac{s^3\,2{,}59\,1/q^2}{(s^2+1)^2 s+(s^2+1)\,s^2\,1{,}569/q+s^3\,2{,}59/q^2}$$

$$= \frac{s\,1{,}675/q}{s^2+s\,1{,}675/q+1}\;\frac{s^3\,2{,}59\,1/q^2}{(s^4+2s^2+1)s+s^4\,1{,}569/q+s^2\,1{,}569/q+s^3\,2{,}59/q^2}$$

$$= \frac{s\,1{,}675/q}{s^2+s\,1{,}675/q+1}\;\frac{s^3\,2{,}59\,1/q^2}{s^5+2s^3+s+s^4\,1{,}569/q+s^2\,1{,}569/q+s^3\,2{,}59/q^2}$$

$$= \frac{s\,1{,}675/q}{s^2+s\,1{,}675/q+1}\;\frac{s^{\cancel{3}\,2}\,2{,}59/q^2}{\cancel{s}\left(s^4+2s^2+1+s^3\,1{,}59/q+s\,1{,}569/q+s^2\,2{,}59/q^2\right)}$$

$$q=5 \Rightarrow = \frac{s\,0{,}335}{s^2+s\,0{,}335+1}\;\frac{s^2\,0{,}1036}{s^4+s^3\,0{,}318+s^2\,2{,}1036+s\,0{,}318+1}$$

$$\Rightarrow s^4+s^3\,0{,}318+s^2\,2{,}1036+s\,0{,}318+1=0 \Rightarrow \text{Raíces}: (-0{,}0905\pm j1{,}4659)\,(-0{,}0684\pm j0{,}8667)$$

(1) $\qquad$ (2)

(1) $\Rightarrow (s+0{,}0905+j1{,}4659)(s+0{,}0905-j1{,}4659) = s^2+s\,0{,}0905-\cancel{sj\,1{,}4659}+0{,}0905\,s+0{,}0905^2-\cancel{j1{,}4659\cdot0{,}0905}+\cancel{j1{,}4659\,s}+\cancel{j\,1{,}4659\cdot0{,}0905}+1{,}4659^2$

$$\Rightarrow \boxed{(s^2+s\,0{,}181+2{,}157)}$$

(2) $\Rightarrow (s+0{,}0684+j0{,}8667)(s+0{,}0684-j0{,}8667) = s^2+s\,0{,}0684-\cancel{j0{,}8667\,s}+s\,0{,}0684+0{,}0684^2-\cancel{j\,0{,}8667\cdot0{,}0684}+\cancel{j0{,}8667\,s}+\cancel{j\,0{,}8667\cdot0{,}0684}+0{,}8667^2$

$$\Rightarrow \boxed{(s^2+s\,0{,}1368+0{,}7558)}$$

$$T(s) = \frac{s\,0{,}335}{(s^2+s\,0{,}335+1)}\;\frac{s^2\,0{,}1036}{(s^2+s\,0{,}1368+0{,}7558)(s^2+s\,0{,}181+2{,}157)}$$

$\frac{\omega_{02}}{q_1}=0{,}181 \Rightarrow \omega_{01}=0{,}905$

$\frac{\omega_{01}}{q_1}=0{,}1368 \Rightarrow \omega_{02}=0{,}684$

$$\Rightarrow 0{,}1036=\frac{\omega_{01}}{q_1}\cdot\frac{\omega_{02}}{q_1}\cdot k_1\cdot k_2 \Rightarrow k_1\cdot k_2=\frac{0{,}1036\cdot q_1^2}{\omega_{01}\,\omega_{02}}=4{,}184$$

$$k_1=k_2 \Rightarrow k=\sqrt{4{,}184}=2{,}045$$

$$\to T(s)=\frac{s\,0{,}335}{(s^2+s\,0{,}335+1)}\;\frac{s\,0{,}1368\cdot2{,}045}{(s^2+s\,0{,}1368+0{,}7558)}\;\frac{s\,0{,}181\cdot2{,}045}{(s^2+s\,0{,}181+2{,}157)}$$

Prototipo pasabanda

$$\to T(s)\Big|_{s=\frac{s}{\omega_0}} \frac{s/\omega_0\,0{,}335}{\left(\frac{s^2}{\omega_0^2}+\frac{s}{\omega_0}0{,}335+1\right)}\;\frac{s/\omega_0\,0{,}1368\cdot2{,}045}{\left(\frac{s^2}{\omega_0^2}+\frac{s}{\omega_0}0{,}1368+0{,}7558\right)}\;\frac{s/\omega_0\,0{,}181\cdot2{,}045}{\left(\frac{s^2}{\omega_0^2}+\frac{s}{\omega_0}0{,}181+2{,}157\right)}$$

$$=\frac{\omega_0^3\,s/\omega_0\,0{,}335}{(s^2\omega_0+s\,\omega_0^2\,0{,}335+\omega_0^3)}\;\frac{\omega_0^3\,s/\omega_0\,0{,}1368\cdot2{,}045}{(s^2\omega_0+s\,\omega_0^2\,0{,}1368+0{,}7558\,\omega_0^3)}\;\frac{\omega_0^3\,s/\omega_0\,0{,}181\cdot2{,}045}{(s^2\omega_0+s\,\omega_0^2\,0{,}181+2{,}157\,\omega_0^3)}$$

$$=\frac{\omega_0^{\cancel{3}2}\,s/\cancel{\omega_0}\,0{,}335}{\cancel{\omega_0}(s^2+s\omega_0\,0{,}335+\omega_0^2)}\;\frac{\omega_0^3\,s/\cancel{\omega_0}\,0{,}1368\cdot2{,}045}{\cancel{\omega_0}(s^2+s\omega_0\,0{,}1368+0{,}7558\,\omega_0^2)}\;\frac{\omega_0^3\,s/\cancel{\omega_0}\,0{,}181\cdot2{,}045}{\cancel{\omega_0}(s^2+s\omega_0\,0{,}181+2{,}157\,\omega_0^2)}$$

$$=\frac{s\cdot\omega_0\cdot0{,}335}{(s^2+s\cdot\omega_0\cdot0{,}335+\omega_0^2)}\;\frac{s\cdot\omega_0\cdot0{,}1368\cdot2{,}045}{(s^2+s\cdot\omega_0\cdot0{,}1368+0{,}7558\cdot\omega_0^2)}\;\frac{s\cdot\omega_0\cdot0{,}181\cdot2{,}045}{(s^2+s\cdot\omega_0\cdot0{,}181+2{,}157\cdot\omega_0^2)}$$

$\omega_0=2\pi\cdot22\,\text{kHz}=138{,}23\times10^3$ krad/seg

$$\boxed{T(s)=\frac{s\cdot46{,}3\times10^3}{(s^2+s\cdot46{,}3\cdot10^3+19{,}1\times10^9)}\;\frac{s\cdot18{,}9\times10^3\cdot2{,}045}{(s^2+s\cdot18{,}9\times10^3+14{,}81\times10^9)}\;\frac{s\cdot25{,}02\times10^3\cdot2{,}045}{(s^2+s\cdot25{,}02\times10^3+41{,}2\times10^9)}}$$