Aproximación de Bessell:

Schaum Cap. 10

$$X \longrightarrow \boxed{\quad} \longrightarrow Y$$

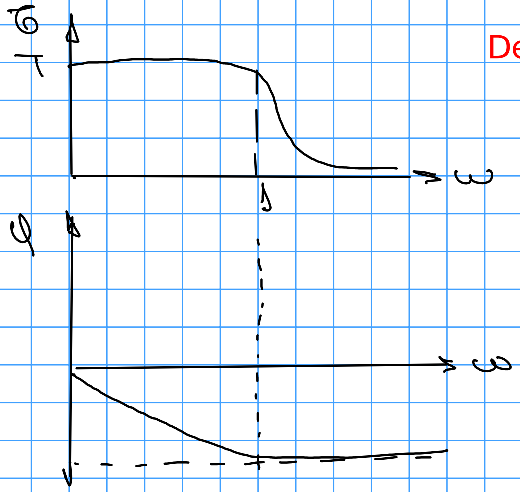
$$\frac{Y}{X} = M(\omega) \cdot e^{j\varphi(\omega)}$$

$$= 1 \cdot e^{j\varphi}$$

$M(\omega) = 1 \Rightarrow$ Se busca modificar la fase del filtro.

Esquema de Delay:

Demora que varía con la frecuencia en forma negativa.



Asintótica

$$H(j\omega) = e^{-j\,\frac{\omega}{1/T}} \to \Omega_\omega$$

$\frac{1}{T} = \Omega_\omega$: Norma en frecuencia.

Storch 1954

$$H(j\omega)\Big|_{\omega = \frac{s}{j}} = e^{-j\frac{s}{j}\cdot T} = e^{-sT}$$

$$H(s) = e^{-sT} = \frac{P(\omega)}{Q(\omega)}$$

→ Se busca encontrar la forma polinomial de H(s)

→ e^{-sT} : Exponencial compleja.

$$\operatorname{senh} x = \frac{e^{x} - e^{-x}}{2} \longrightarrow \operatorname{senh}(s) = s + \frac{s^3}{3!} + \frac{s^5}{5!} + \frac{s^7}{7!} + \dots$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \longrightarrow \cosh(s) = 1 + \frac{s^2}{2!} + \frac{s^4}{4!} + \frac{s^6}{6!} + \dots$$

$$e^{x} = \operatorname{senh} x + \cosh x$$

$$H(s) = \frac{1}{\operatorname{senh}(s) + \cosh(s)}$$

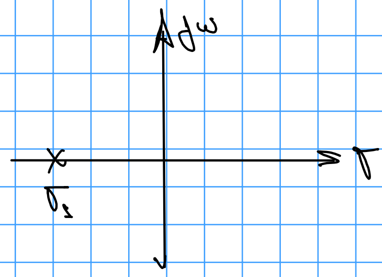
$$\operatorname{cotgh}(s) = \frac{\cosh(s)}{\operatorname{senh}(s)} = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\cfrac{5}{s} + \cfrac{1}{\ddots}}}$$

$\underset{\sim}{n=1}$

$$\coth(s) = \frac{1}{s} = \frac{\cosh s}{\operatorname{senh} s}$$

$$H(s) = \frac{1}{s+1}$$

$$\therefore\ H_{B1}(s) = \frac{1}{s+1}$$



$\underset{\sim}{n=2}$

$$\coth = \frac{1}{s} + \frac{s}{3} = \frac{3+s^2}{3s}$$

$$H(s) = \frac{K}{s^2+3s+3}$$ → Ganancia unitaria continua $K=3$

$$H(s) = \frac{3}{(s^2+3s+3)} \rightarrow B_2(s)$$

$$H_{Bn}(s) = \frac{B_{n0}}{B_n(s)}$$ → Genérico

$\underset{\sim}{n=3}$

$$\coth(s) = \frac{1}{s} + \frac{1}{\frac{3}{s}+\frac{s}{5}} = \frac{1}{s} + \frac{5s}{15+s^2} = \frac{15+s^2+5s^2}{s^3+15s} = \frac{6s^2+15}{s^3+15s}$$

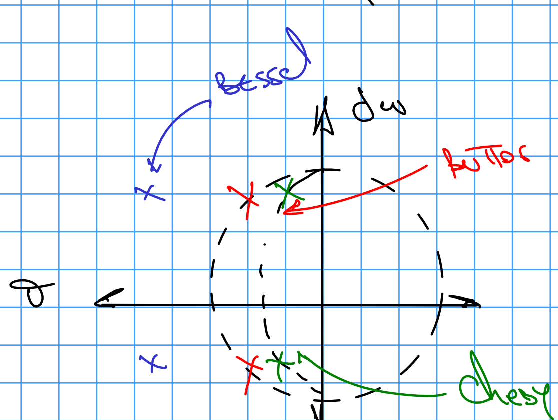
$$H_3(s) = \frac{15}{s^3+6s^2+15s+15}$$

Para n=2

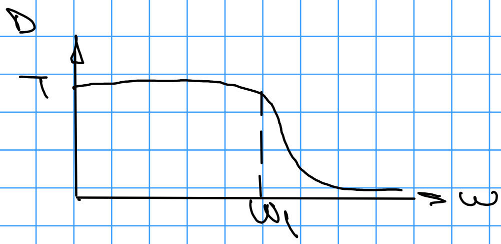
$$\bullet\ H(s) = \frac{\omega_0^2}{s^2 + s\cdot\frac{\omega_0}{Q} + \omega_0^2}$$

$$\begin{cases} \omega_0^2 = 3 \\ \frac{\omega_0}{Q} = 3 \Rightarrow Q = \frac{\sqrt{3}}{3} < \frac{\sqrt{2}}{2} \end{cases}$$

Q de Bessel de orden 2 es menor al Q de Butter de mismo orden.



MÉTODOS DE DISEÑO DE BESSEL.



. Ripple en banda de paso.
. % de errror en la demora.
. Atenuación en Ws.

$KT\big|_{\omega=\omega_1}$

$D_{\omega_1} = 0{,}95 \cdot T$

ERROR DE DELAY