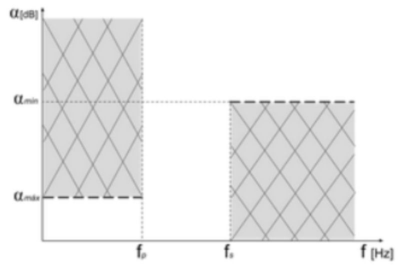
Tarea semanal 3: Torres 141884-1.

A partir de la siguiente plantilla, sabiendo que:



αmáx [dB]	αmín [dB]	fp [Hz]	fs [Hz]
1	12	1500	3000

- Obtener la transferencia para máxima planicidad en la banda de paso utilizando los conceptos de partes de función. **Recordar que**: $|T(j\omega)|2 = T(j\omega)\cdot T(-j\omega) = T(s)\cdot T(-s)|s = j\omega$
- Obtener el diagrama de polos y ceros, y un bosquejo de la respuesta en frecuencia.
- Implementar el circuito normalizado con estructuras pasivas separadas mediante buffers.
- Obtenga el circuito que cumpla con la plantilla requerida si dispone de capacitores de 100nf.
- Proponga una red que se comporte igual a la hallada en 4) pero con resistores, capacitores y opamps.

. $\omega_p \rightarrow 1$

. $\omega_s \rightarrow 2$

. $\varepsilon^2 = 10^{\alpha_{max}/10} - 1 = 0{,}258925411$

. $10 \log(1 + \varepsilon^2 . \omega_s^{2n}) = \alpha_{min}$

$n=1 \rightarrow 3{,}08$

$n=2 \rightarrow 7{,}11$

$n=3 \rightarrow \boxed{12{,}44}$

$j = \sqrt{-1}$, $j^2 = -1$, $j^3 = -j$, $j^4 = 1$, $j^5 = j$, $j^6 = -1$

$$|T(j\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}} = T(s) * T(-s)$$

$$|T(s)|^2_{s=j\omega} = \frac{1}{1+\varepsilon\left(\frac{s}{j}\right)^6} \Rightarrow T(s) = \frac{1/\varepsilon^2}{1/\varepsilon^2 - s^6} = \frac{c}{(s^3+a.s^2+bs+c)} \cdot \frac{c}{(-s^3+as^2-bs+c)}$$

$$(s^3+as^2+bs+c)(-s^3+as^2-bs+c)$$

$-s^6 + as^5 - bs^4 + cs^3$
$-as^5 + a^2s^4 - abs^3 + acs^2$
$-bs^4 + abs^3 - b^2s^2 + cbs$
$-cs^3 + acs^2 - bcs + c^2$

$\Rightarrow -s^6 + s^5(a-a) + s^4(a^2-2b) + s^3(ab-ab+c-c) + s^2(2ac-b^2) + s(cb-bc) + c^2$

$-s^6$ — $-s^6$
$s^5(0)$ — $s^5(a-a)$
$s^4(0)$ — $s^4(a^2-2b) \rightarrow a^2-2b=0 \Rightarrow \frac{a^2}{2} = b$
$s^3(0)$ — $s^3(ab-ab+c-c)$
$s^2(0)$ — $s^2(2ac-b^2) \rightarrow 2ac-b^2=0 \Rightarrow 2ac = b^2$
$s(0)$ — $s(cb-bc) \rightarrow c^2 = 1/\varepsilon^2 \Rightarrow c = 1/\varepsilon \cong c=2$
c^2 — $1/\varepsilon^2$

$$\begin{cases} 2.a.c = b^2 \rightarrow 2.a.2 = \left(\frac{a^2}{2}\right)^2 \Rightarrow 4.a = \frac{a^4}{4} \Rightarrow 16 = a^3 \Rightarrow \sqrt[3]{16} = a = \sqrt[3]{2.2^3} = \boxed{2.\sqrt[3]{2} = a} \\ \frac{a^2}{2} = b \longrightarrow \frac{(2\sqrt[3]{2})^2}{2} = b \Rightarrow \frac{\cancel{2}^2 . 2^{2/3}}{\cancel{2}} = b \Rightarrow b \Rightarrow \boxed{2.2^{2/3} = b} \\ c^2 = 1/\varepsilon^2 \Rightarrow c^2 = 1/(0{,}258824411) \Rightarrow c = 1{,}965226\ldots \cong 2 \end{cases}$$

$$T(s) = \frac{c}{s^3 + a.s^2 + b.s + c} = \frac{2}{s^3 + 2\sqrt[3]{2}.s^2 + 2.2^{2/3}.s + 2} \cong \frac{2}{s^3 + 2{,}52.s^2 + 3{,}17.s + 2}$$

python $\rightarrow$ $$T(s) = \frac{1}{(s + 1{,}26)} \cdot \frac{2}{(s + 0{,}629 \pm j\,1{,}0931)}$$

$$\left.T(s)\right|_{s=-1,2} \cong 0{,}096$$

	1	2,52	3,17	2
		-1,2	-1,584	-1,9032
-1,2	1	1,32	1,586	0,0968

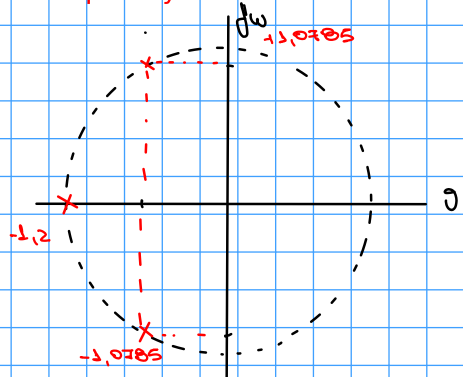
A mano $\rightarrow$ $$T(s) = \frac{1}{(s+1{,}2)} \cdot \frac{2}{s^2 + 1{,}32.s + 1{,}586}$$

$$\frac{-1{,}32 \pm \sqrt{1{,}32^2 - 4.1.1{,}586}}{2.1}$$

$$T(s) = \frac{1}{(s+1{,}2)} \quad \frac{2}{(s + 0{,}66 \pm j\,1{,}0785)}$$

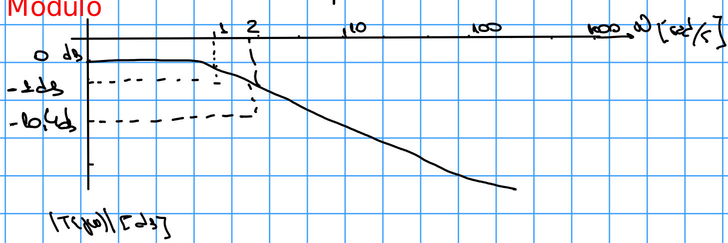
$$\frac{-1{,}32 \pm \sqrt{-4{,}654}}{2} = \frac{-1{,}32 \pm j2{,}157}{2} \longrightarrow -0{,}66 \pm j\,1{,}0785$$

Diagrama de polos y ceros



$$|T(s)| = \left| \frac{1}{s+1{,}2} \cdot \frac{2}{s^2 + 1{,}32.s + 1{,}586} \right|$$

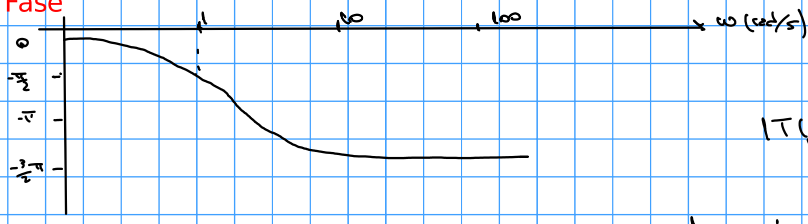
Módulo



$$\left.|T(s)|\right|_{s=j\omega} = \left|\frac{1}{j\omega + 1{,}2}\right| \left|\frac{2}{(j\omega)^2 + 1{,}32\,j\omega + 1{,}586}\right|$$

$$\left.|T(s)|\right|_{s=j\omega} = \frac{1}{|j\omega + 1{,}2|} \quad \frac{2}{|-\omega^2 + 1{,}32\,j\omega + 1{,}586|}$$

Fase



$$\left.|T(s)|\right|_{s=j\omega} = \frac{1}{\sqrt{1{,}2^2 + \omega^2}} \quad \frac{2}{\sqrt{(1{,}586 - \omega^2)^2 + (1{,}32.\omega)^2}}$$

$$|T(j\omega)|_{dB} = 20 \log\left(\frac{1}{\sqrt{1{,}2^2+\omega^2}} \cdot \frac{2}{\sqrt{(1{,}586-\omega^2)^2 + 1{,}74\,\omega^2}}\right)$$

$$|T(j\omega)|_{dB} = 20 \log\left(\frac{1}{\sqrt{1{,}2^2+\omega^2}}\right) + 20 \log\left(\frac{2}{\sqrt{(1{,}586-\omega^2)^2 + 1{,}74\,\omega^2}}\right)$$

$$|T(j\omega)|_{dB} = \cancel{20 \log 1} - 10 \log(1{,}44 + \omega^2) + 20 \log 2 - 10 \log\left[(1{,}586 - \omega^2)^2 + 1{,}74\,\omega^2\right]$$

$$|T(j\omega)|_{dB} = 20 \log 2 - 10 \log(1{,}44 + \omega^2) - 10 \log\left[(1{,}586 - \omega^2)^2 + 1{,}74.\omega^2\right]$$

$$|T(j\omega)|_{\omega=0} = 20 \log 2 - 10 \log 1{,}44 - 10 \log 2{,}515 = 0{,}43\,dB$$

$$|T(j\omega)|^{dB}_{\omega=\omega_p} = 20\log 2 - 10\log(1{,}44+1) - 10\cdot\log\left[(1{,}586-1^2)^2+1{,}74\cdot 1^2\right] =$$

$$= 20\log 2 - 10\log 2{,}44 - 10\log 2{,}083 = \boxed{-1{,}04\,dB}$$

$$|T(j\omega)|^{dB}_{\omega=\omega_s} = 20\log 2 - 10\log(1{,}44+2) - 10\log\left[(1{,}586-2^2)^2+1{,}74\cdot 2^2\right] =$$

$$= 20\cdot\log 2 - 10\log 3{,}44 - 10\log 12{,}787 = \boxed{-10{,}412\,dB}$$

Desnormalización en frecuencia.

$$T(S) = \frac{k_1\cdot W_{o1}}{S+W_{o1}}\cdot\frac{k_2\cdot W_{o2}^2}{S^2+S\frac{W_{o2}}{Q}+W_{o2}^2} = \frac{1}{S+1{,}2}\,\frac{2}{S^2+1{,}32S+1{,}586}$$

$W_{o1} = 1{,}2$ $\qquad$ $k_1\cdot W_{o1} = 1 \Rightarrow k_1 = \frac{1}{1{,}2} = 0{,}833$

$W_{o2}^2 = 1{,}586 \Rightarrow W_{o2} = 1{,}26$ $\qquad$ $k_2\cdot W_{o2}^2 = 2 \Rightarrow k_2 = \frac{2}{1{,}586} = 1{,}26$

$\frac{W_{o2}}{Q} = 1{,}32 \Rightarrow Q = \frac{W_{o2}}{1{,}32} = \frac{1{,}26}{1{,}32} = 0{,}954$ $\qquad$ $k = k_1\cdot k_2 = 1{,}049$

$$T(s) = k\,\frac{1{,}2}{s+1{,}2}\,\frac{1{,}586}{s^2+1{,}32s+1{,}586}\;;\quad T(s)\Big|_{s=\frac{s}{\omega p}} = k\cdot\frac{1{,}2}{\frac{s}{\omega p}+1{,}2}\,\frac{1{,}586}{\left(\frac{s}{\omega p}\right)^2+1{,}32\left(\frac{s}{\omega p}\right)+1{,}586}$$

$$T(s) = k\cdot\frac{1{,}2\,\omega p}{s+1{,}2\,\omega p}\,\frac{1{,}586}{\frac{s^2}{\omega p^2}+\frac{s}{\omega p}\cdot 1{,}32+1{,}586} = k\cdot\frac{1{,}2\,\omega p}{s+1{,}2\,\omega p}\,\frac{1{,}586}{\frac{s^2\omega p+1{,}32\,\omega p^2 s+1{,}586\,\omega p^3}{\omega p^3}}$$

$$T(s) = k\,\frac{1{,}2\,\omega p}{s+1{,}2\,\omega p}\,\frac{1{,}586\cdot\omega p^{\cancel{3}}}{\cancel{\omega p}\left(s^2+1{,}32\,\omega p\cdot s+1{,}586\,\omega p^2\right)} = k\,\frac{1{,}2\,\omega p}{s+1{,}2\,\omega p}\,\frac{1{,}586\cdot\omega p^2}{s^2+1{,}32\,\omega p\cdot s+1{,}586\,\omega p^2}$$

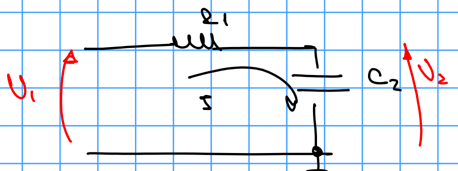
$$\omega p = 2\pi\cdot 1500\,Hz = 9.425\times 10^3\ rad/seg$$

$$T(s) = k\cdot\frac{11310}{s+11310}\,\frac{140.885\times 10^6}{s^2+12441\cdot s+140.885\times 10^6}$$

Desarrollo circuital:

$$T(s) = \frac{1}{s+1{,}2}\cdot\frac{2}{s^2+1{,}32s+1{,}586} = \frac{k_1\cdot W_{o1}}{S+W_{o1}}\cdot\frac{k_2\cdot W_{o2}^2}{S^2+S\frac{W_{o2}}{Q}+W_{o2}^2}$$

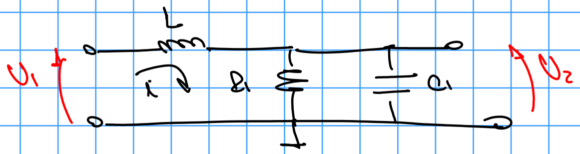
Filtro de pimer orden.



$$\frac{V_2}{V_1} = \frac{1/sC}{R+1/sC} = \frac{1/sC}{\frac{sCR+1}{sC}} = \frac{1/RC}{s+1/RC}$$

$$\frac{1/RC}{s+1/RC} = \frac{k_1\cdot W_{o1}}{s+W_{o1}} \Rightarrow \begin{cases} W_{o1} = 1/RC \Rightarrow C = 1/R\cdot W_{o1} \\ k_1 = 1/RC\cdot W_{o1} \Rightarrow k_1 = 1/R^2\cdot W_{o1}^2 \end{cases}$$

Filtro de segundo orden.



$Y_{RC} = \frac{1}{R_1} + sC_1$

$Z_{RC} = \frac{1}{Y_{RC}} = \frac{1}{1/R_1 + sC_1}$

. $V_1 = i\,(Z_L + Z_{RC}) \Rightarrow i = \frac{V_1}{Z_L + Z_{RC}}$

. $V_2 = i \, . \, Z_{RC}$

. $Z_{RC} = R \parallel 1/sC = \frac{R \, . \, 1/sC}{R + 1/sC}$

. $Z_{RC} = \frac{R \, . \, 1/sC}{\frac{sRC+1}{sC}}$

$Z_{RC} = \frac{R}{sRC+1} = \frac{1}{sC + \frac{1}{R}}$

$V_2 = V_1 \, . \, \frac{Z_{RC}}{Z_{RC} + Z_L} \Rightarrow \frac{V_2}{V_1} = \frac{Z_{RC}}{Z_{RC} + Z_L} = \frac{1}{1/R_1 + sC_1} \, . \, \frac{1}{\frac{1}{1/R_1 + sC_1} + sL}$

$\frac{V_2}{V_1} = \frac{1}{\frac{1}{R_1} + sC_1} \, . \, \frac{1/R + sC}{1 + (1/R_1 + sC_1) \, . \, sL} = \frac{1}{1 + \frac{sL}{R_1} + s^2 LC_1}$

$\frac{V_2}{V_1} = \frac{1}{s^2 + s \, . \, \frac{1}{RC_1} + \frac{1}{LC_1}} = \frac{K_2 \, . \, \omega_{o2}^2}{s^2 + s \, . \, \frac{\omega_{o2}}{Q} + \omega_{o2}^2}$

. $K_2 \, . \, \omega_{o2}^2 = 1$. $\frac{\omega_{o2}}{Q} = \frac{1}{RC_1} \Rightarrow Q = \omega_{o2} \, . \, RC_1$

. $\omega_{o2}^2 = \frac{1}{LC_1} \Rightarrow \omega_{o2} = \frac{1}{\sqrt{LC_1}}$

. $K_2 = \frac{1}{\omega_{o2}^2}$; $L = \frac{1}{\omega_{o2}^2 \, . \, C_1}$; $C_1 = \frac{Q}{R_1 \, . \, \omega_{o2}}$