

③ E-шаг: Найдем $P(Z_{nk} = 1 | x_n, y_n)$

Ф-ла байеса:

$$P(Z_{nk} = 1 | x_n, y_n) = \frac{P(y_n | Z_{nk} = 1, x_n) \tilde{\pi}_k}{\sum_{j=1}^K P(y_n | Z_{nj} = 1, x_n) \pi_j} \quad \textcircled{=}$$

$$\textcircled{=} \tilde{\pi}_k \frac{\mathcal{N}(y_n | x_n^T \omega, \beta_k) \mathcal{N}(\omega | \mu_k, \alpha I)}{\sum_{j=1}^K \pi_j \mathcal{N}(y_n | x_n^T \omega, \beta_j) \mathcal{N}(\omega | \mu_j, \alpha I)}$$

Вычисляем $\forall n, k$.

M-шаг: Обновляем $\pi_k, \beta_k, \mu_k, \alpha$ исходя из макс. правдоподобия

$$\mathcal{L} = \sum_{n=1}^N \sum_{k=1}^K \left(\ln \pi_k + \ln \mathcal{N}(y_n | x_n^T \omega, \beta_k) + \ln \mathcal{N}(\omega | \mu_k, \alpha I) \right) P(Z_{nk} | x_n, y_n)$$

\Rightarrow Обновляем пар-ы:

$$\pi_k = \frac{1}{N} \sum_{n=1}^N P(Z_{nk} = 1 | x_n, y_n)$$

$$\beta_k = \frac{\sum_{n=1}^N P(Z_{nk} = 1 | x_n, y_n) \|y_n - x_n^T \omega\|^2}{\sum_{n=1}^N P(Z_{nk} = 1 | x_n, y_n)}$$

$$\mu_k = \frac{\sum_{n=1}^N P(Z_{nk} = 1 | x_n, y_n) \omega}{\sum_{n=1}^N P(Z_{nk} = 1 | x_n, y_n)}$$

$$\alpha = \frac{\sum_{n=1}^N P(Z_{nk} = 1 | x_n, y_n) \|\omega - \mu_k\|^2}{D \cdot N}, \text{ где } \omega \in \mathbb{R}^D$$

Повторяем до сходимости.