INFORMED SEARCH STRATEGIES

- The intelligence of a system cannot be measured in terms of search capacity, but in the ability to use knowledge about the problem to reduce/mitigate the combinatorial explosion.

- If the system had some control on the order in which possible solutions are generated, then it would be useful to use this order so that solutions have a high chance to appear before.

- Intelligence, for a system with limited processing capacity is the wise choice of what to do next ..... ".

INFORMED STRATEGIES

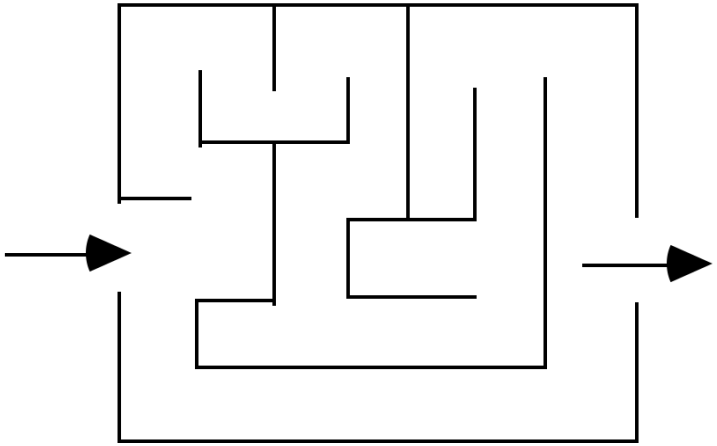
- **Newell-Simon**:
  - Uninformed search methods in a search space of depth d and branching factor b have space complexity proportional b^d to find a goal in one of the leaves.
  - This is unacceptable for certain complex problems. So we can resort to expanding the nodes using heuristic domain knowledge (evaluation functions) to decide which node to expand first.

INFORMED STRATEGIES

- Evaluation functions **give a computational estimate of the effort** to reach the final state.
- The time spent to evaluate a node by means of a heuristic function should correspond to a reduction in the size of the space explored.
  - Trade-off between the time it takes to solve the problem (baseline) and time spent in reasoning on top of it (meta-level).
  - Uninformed search strategies have no meta-level activities.

- PROBLEMS:
  - How to find the correct evaluation functions, ie how to select the most "promising nodes"?
  - It is difficult to numerically characterize the empirical knowledge about the problem.
  - Not always the obvious choice is the best.

EXAMPLE



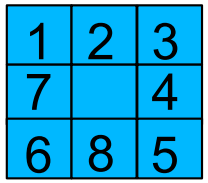
- Heuristic Choice: always move to reduce the distance from the exit.
- In the case of this maze we would choose a longer way.

EXAMPLE: GAME OF 8

- The distance from the goal can be estimated based on the number of boxes out of place: *state* *goal*

  - Left: distance 2
  - Right: distance 4
  - High: distance 3

- It might be that the heuristics does not estimate the exact distance from the solution



Distance = 3 but 4 moves are needed to reach a solution

BEST FIRST SEARCH

Best first search uses ***evaluation functions*** that compute a number that represents the desirability of the node expansion.

Best-first means that the chosen node is the one considered as the most desirable.

QueuingFn = Insert successors in descending order of desirability.

Special cases:

- ***Greedy search or hill climbing***
- ***A* search***

BEST-FIRST SEARCH

- BEST-FIRST: Try to move to the maximum (resp. minimum) of a function that "estimates" the desirability (resp. cost) to reach the goal.

- Greedy:
  1. Let L be a list of the initial nodes of the problem, ranked according to their distance from the goal (ascending order);

  2. If L is empty fail; otherwise let n be the first node of L.

  3. If n is the goal return solution (the path to reach the goal)

  4. Otherwise remove n from L and add to L all the children of n. Then order the entire list L based on an estimate of the relative distance from the goal. Return to step 2

BEST-FIRST SEARCH

function BEST-FIRST-SEARCH(*problem*, EVAL-FN) **returns** a solution sequence

  inputs: *problem*, a problem

    Eval-Fn, an evaluation function

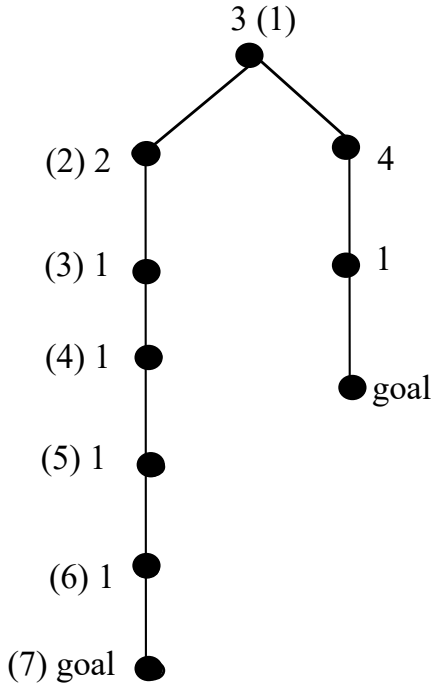
  Queueing-Fn ← a function that orders nodes by EVAL-FN

  return GENERAL-SEARCH(*problem*, *Queueing-Fn*)

It uses the general search algorithm and **evaluation function *EvalFn***

BEST FIRST SEARCH

- The best-first search is not optimal in the sense that it is not guaranteed to find the best solution, or **the best path** toward a solution (remember the maze?).
- This is because the best-first technique tries to find as soon as possible a node with 0 distance from the goal and not the node with the lowest depth.



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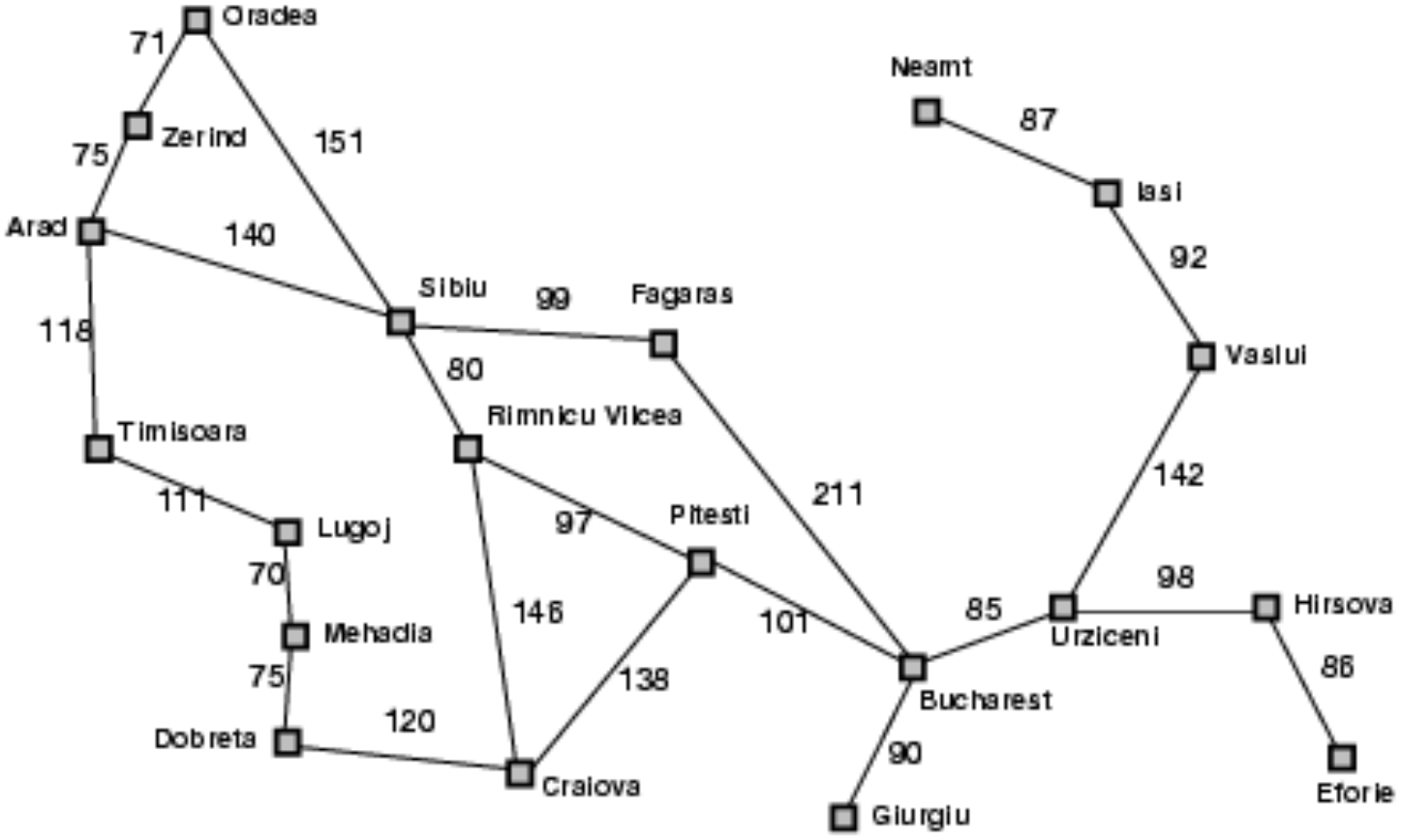

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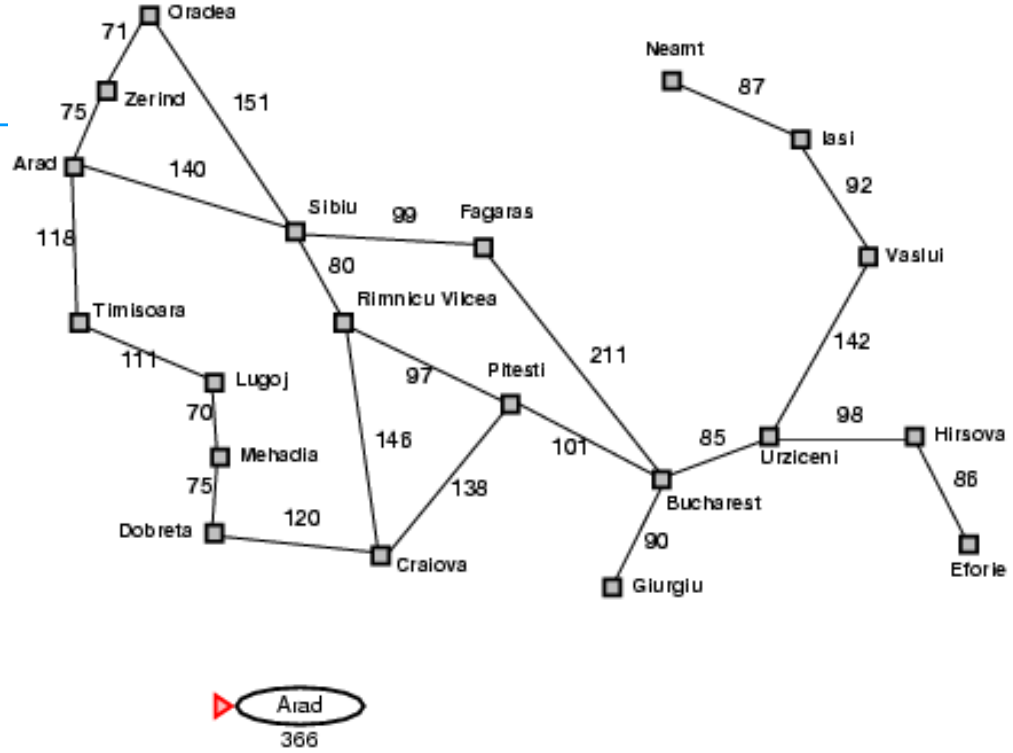
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

BEST FIRST SEARCH

- Evaluation Function *f (n)* = *h (n)* (heuristic)
- h(n) = Estimate of the cost from *n* to the *goal*
- eg, $h_{SLD}(N)$ = Straight-line distance between *n* and Bucharest
- The greedy best-first search expands the node that seems closer to the goal

EXAMPLE



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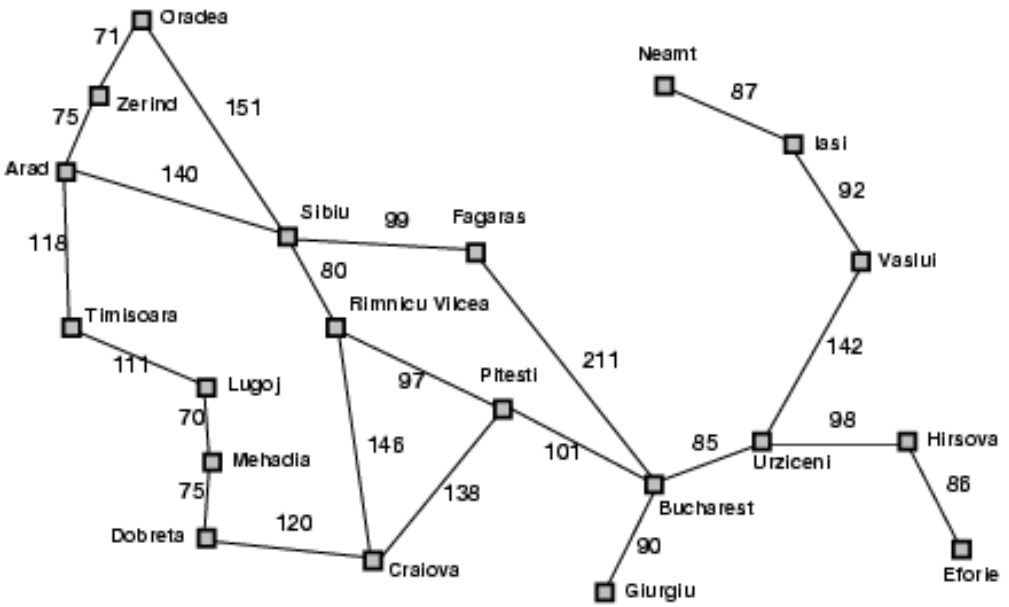
EXAMPLE



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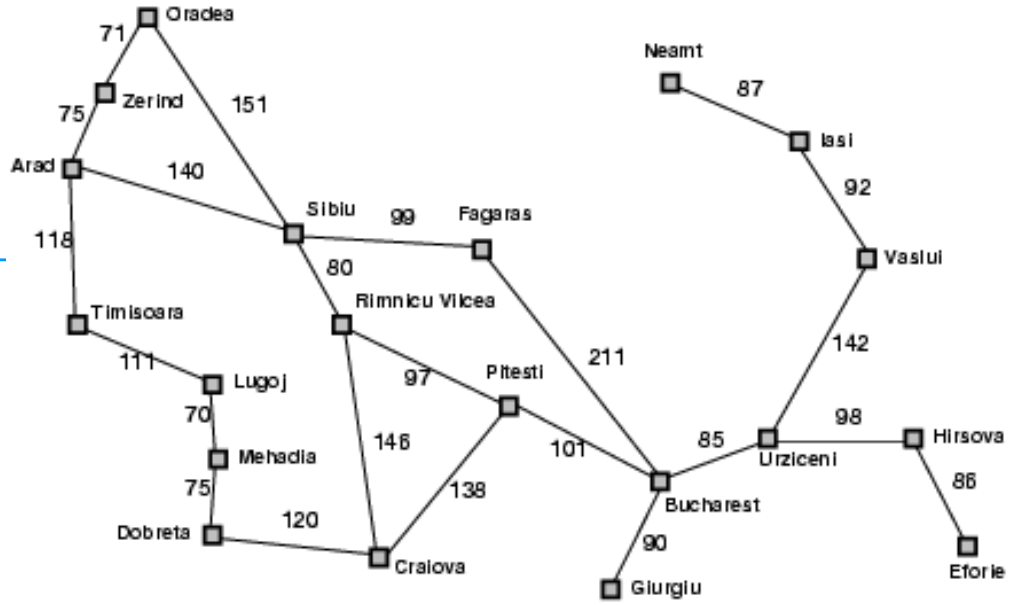
EXAMPLE



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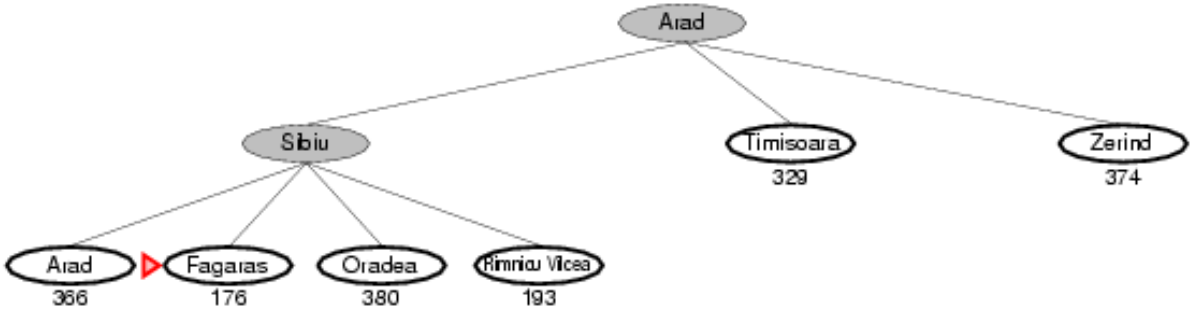
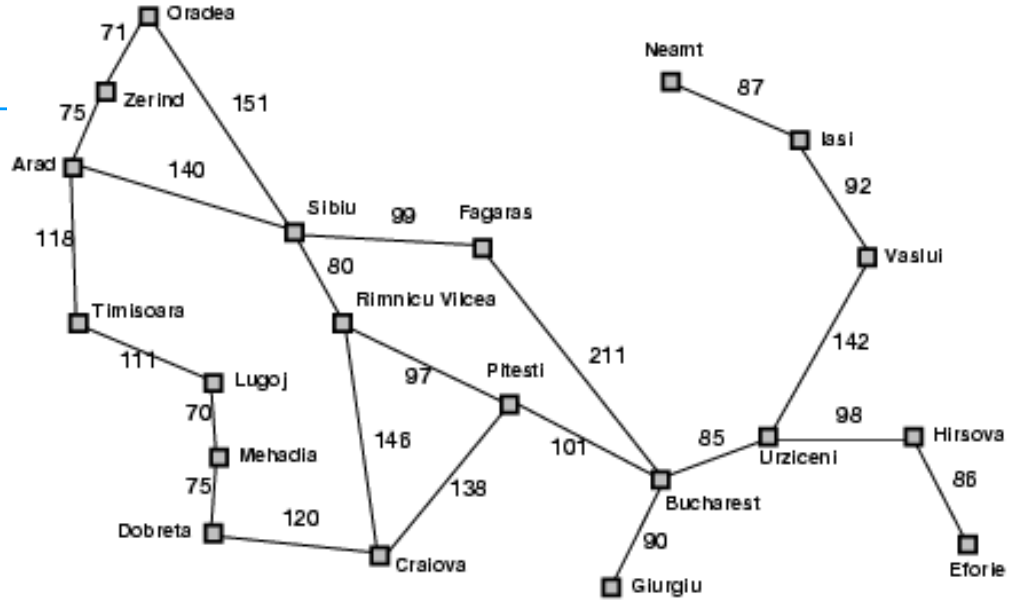


EXAMPLE



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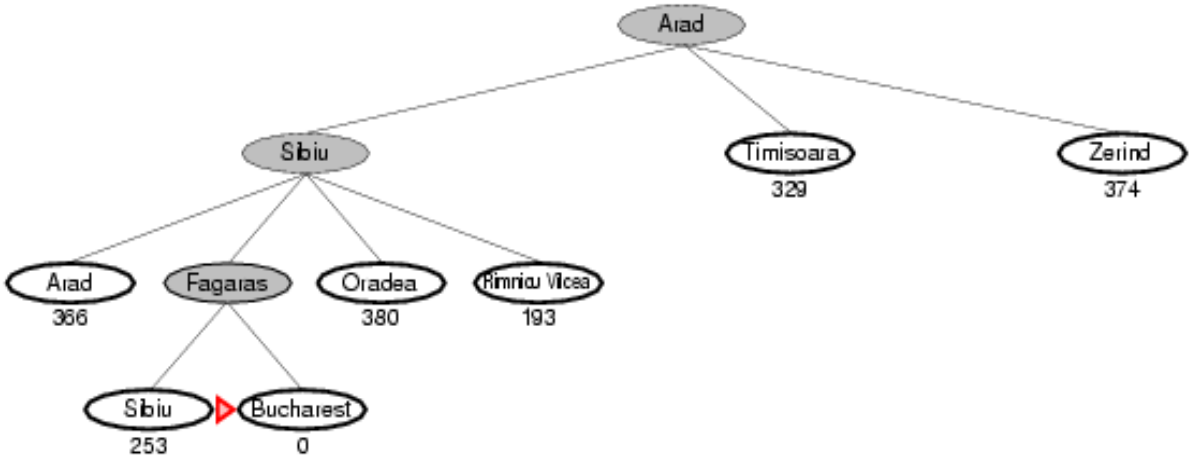


BEST-FIRST SEARCH

- Best-first search is not optimal and may be incomplete (the same pittfall as the depth-first search).
- In the worst case, if the depth is d and the branching factor b the maximum number of expanded nodes will be b^d. (Time complexity).
- In addition, it keeps in memory all nodes, the spatial complexity coincides with the temporal one.
- With a good heuristic function, the spatial and temporal complexity can be reduced substantially.

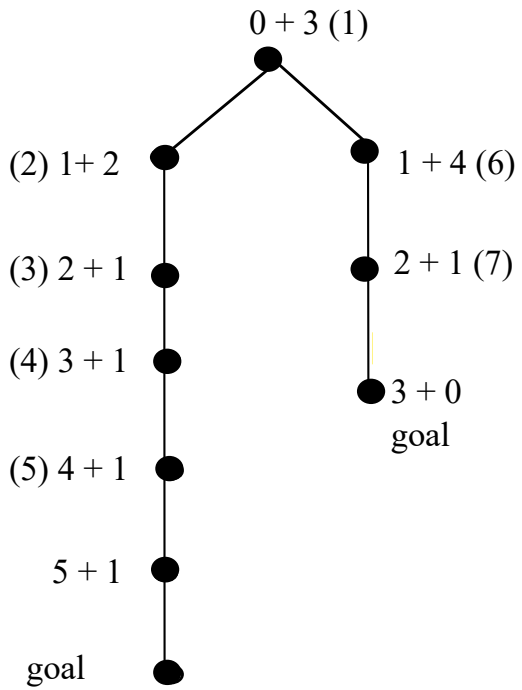
A* ALGORITHM

- Instead of only considering the distance to the goal, also consider the "cost" in reaching the node n from the root.
- We expand nodes for increasing values of f(n)

  $f(n) = g(n) + h'(n)$
- Where g (n) is the depth of the node, and h'(n) the estimated distance from the goal.
- We choose the node to expand as the one for which this sum is smaller.

A* ALGORITHM

- Basically we try to combine the benefits of depth-first search (efficiency) with the ones of uniform cost search (optimality and completeness).

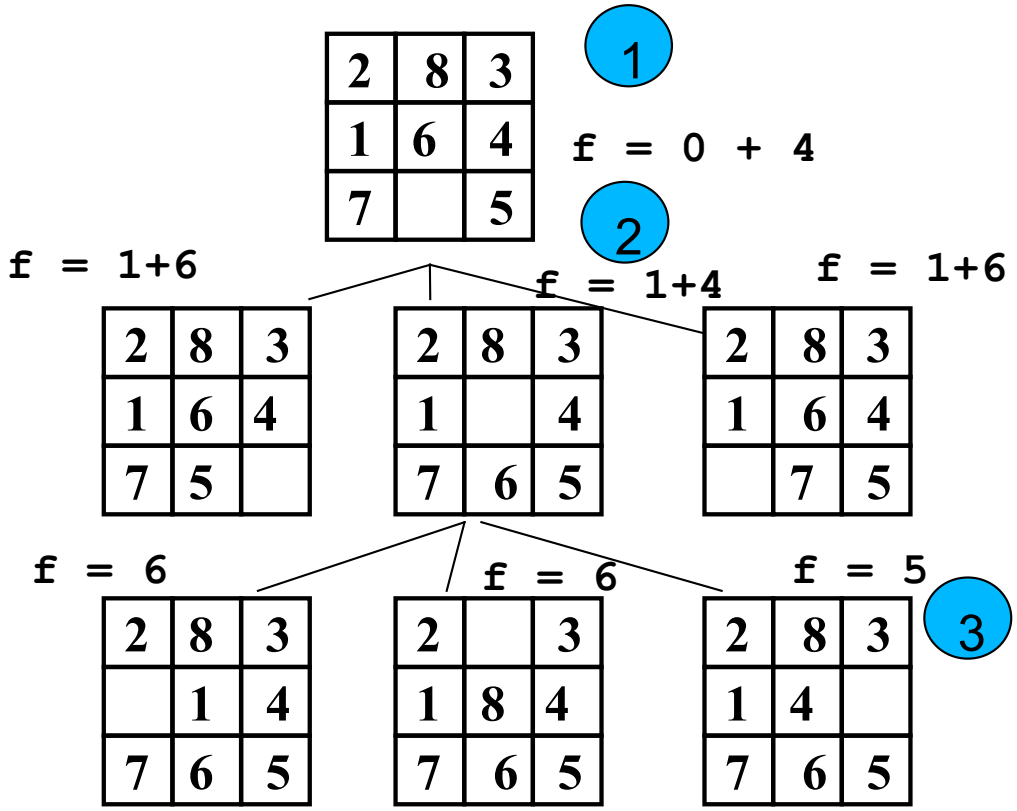


A* ALGORITHM

1. Let L be a list initial nodes of the problem.
2. If L is empty fail; otherwise et n be the node for which g (n) + h'(n) is minimal.
3. If n is the goal return solution (the path to reach it)
4. Otherwise remove n from L and add to L all the children of n labeled with g(n) + h'(n). Return to step 2

- Note:
  - the algorithm does not guarantee to find the optimal path. In the example, if the node with label 5 was the goal this would have been reached before the goal on the right (optimal).

EXAMPLE



f (n) = g (n) + h '(n)
where:

- g (n) = depth of the node n;
- h '(n) = number of tiles in the wrong place

etc….

Goal

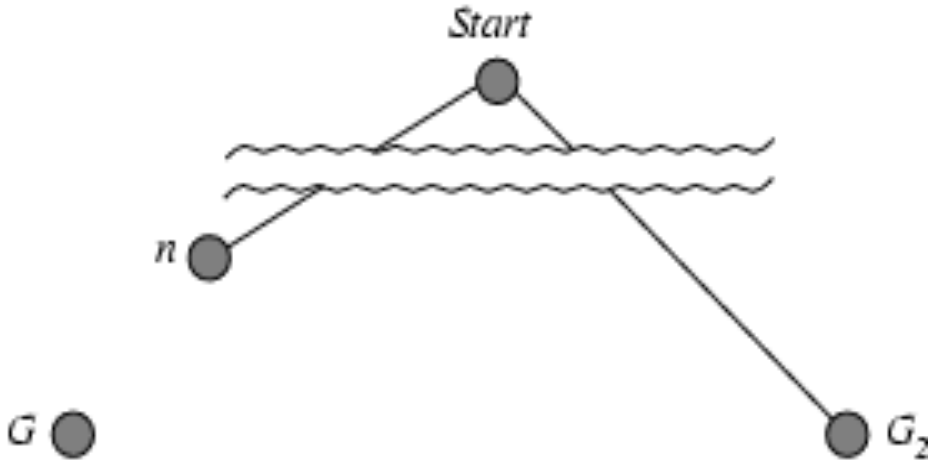
1	2	3
8	4	
7	6	5

A* ALGORITHM

- A * does not guarantee to find the optimal solution (it depends on the heuristic function).
- Suppose you indicate with h(n) the true distance between the current node and the goal.
- The heuristic function h'(n) is **optimistic** that if we always have h '(n) <= h (n).
- This heuristic function is said to be **feasible**.
- Theorem:
  - If h '(n) <= h (n) for each node, then the A * algorithm always finds the optimal path to the goal
- Obviously the perfect heuristic h '= h is always feasible.
- If h '= 0 we always obtain a feasible heuristic function $\Rightarrow$ breadth-first search

A* ALGORITHM OPTIMALITY PROOF

- Suppose you have created a sub-optimal goal G_2 *and* to have it in the queue. Let *n* be an unexpanded node in the queue such that *n* is on the shortest path to the optimal goal *G*.



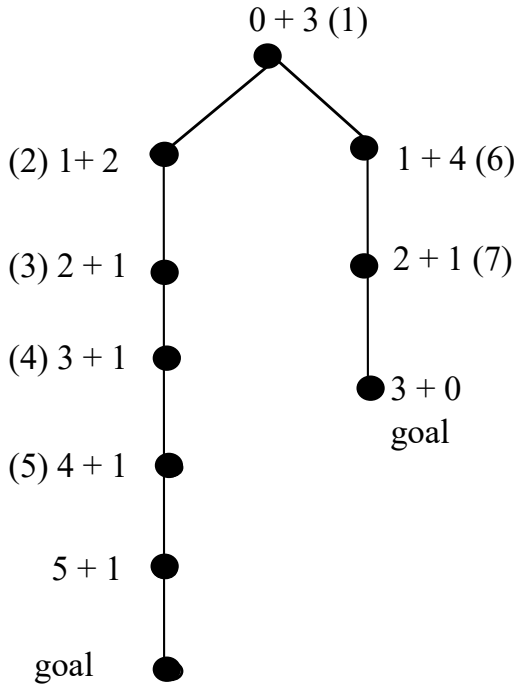
A* ALGORITHM OPTIMALITY PROOF

- $f(G_2) = g(G_2)$ as $h(G_2) = 0$
- $g(G_2) > g(G)$ as G_2 is sub-optimal
- $f(G) = g(G)$ as $h(G) = 0$
- $f(G_2) > F(G)$ from above

- $h'(n) \leq h(n)$ as h' is feasible permissible
- $g(n) + h'(n) \leq g(n) + h(n) = f(G)$ (n is on the path to G)
- $f(n) \leq f(G)$

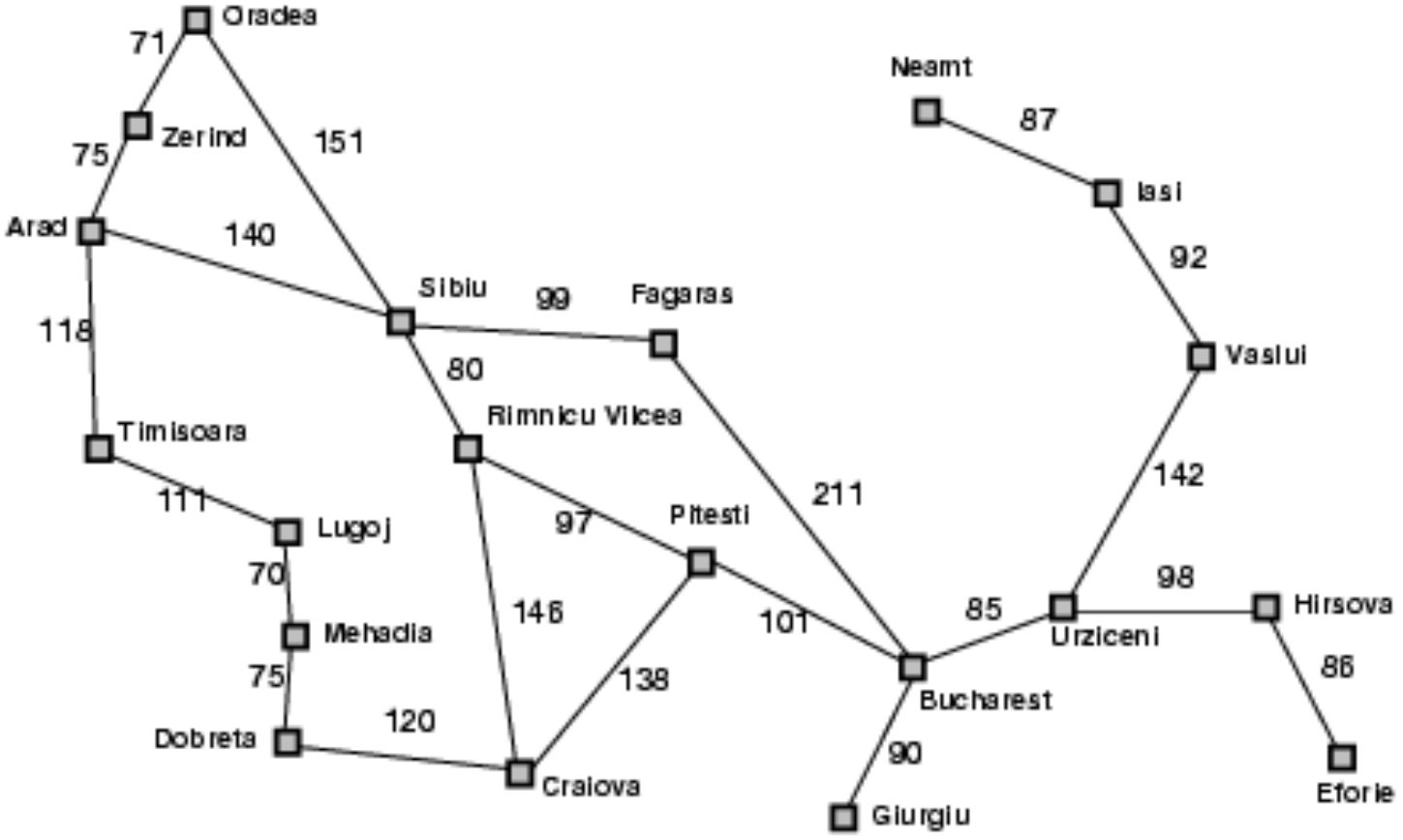
then $f(G_2) > f(n)$, and A* will never select G_2 for expansion.

EXAMPLE: SEARCH WITH FEASIBLE HEURISTIC FUNCTION



- The previous choice for the game of 8 (counting the tiles in the wrong position as h') is always feasible because every move can reduce the distance to at most one unit

EXAMPLE



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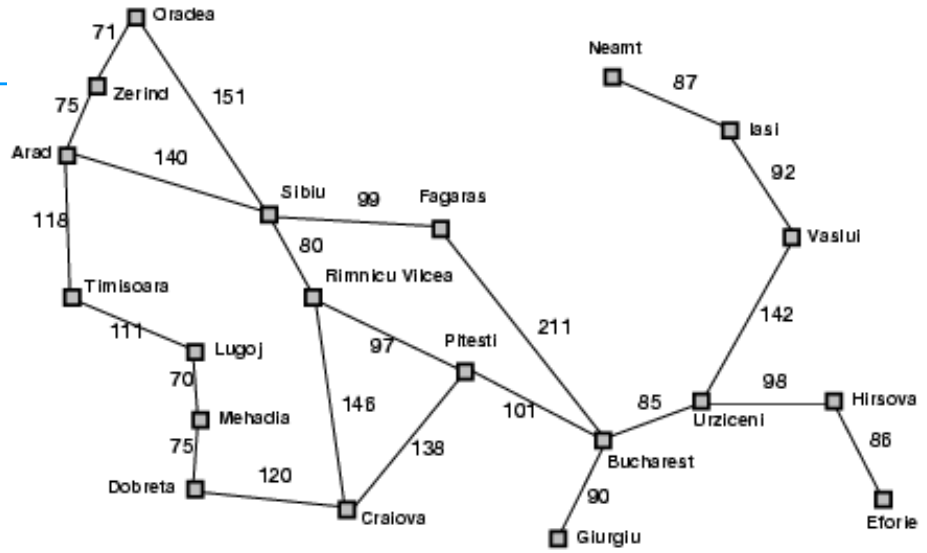
A* EXAMPLE



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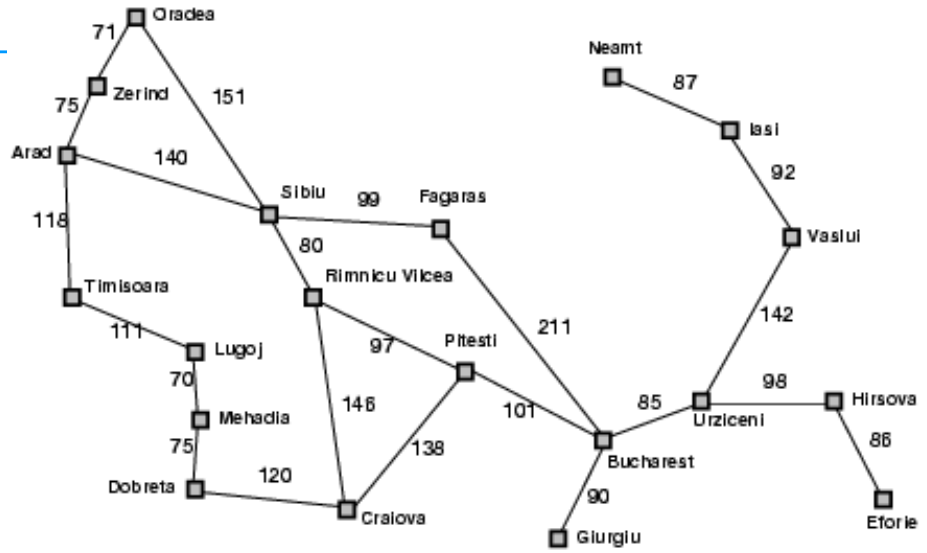

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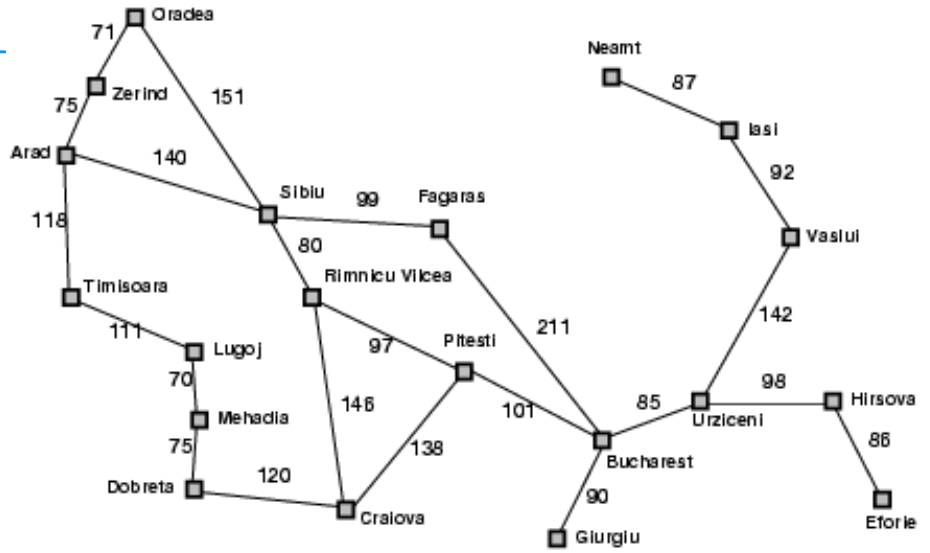

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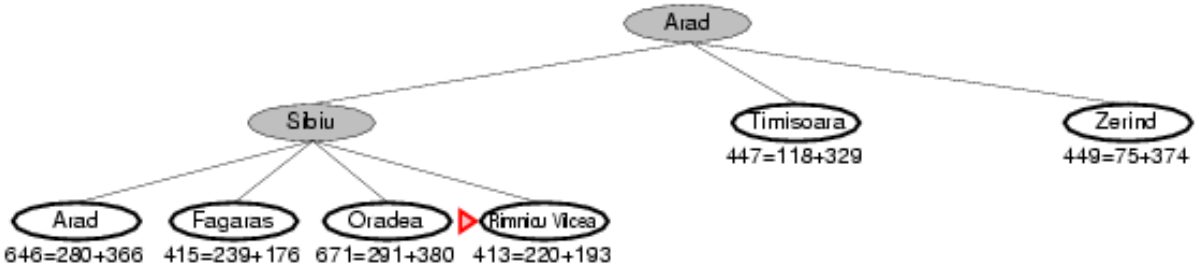

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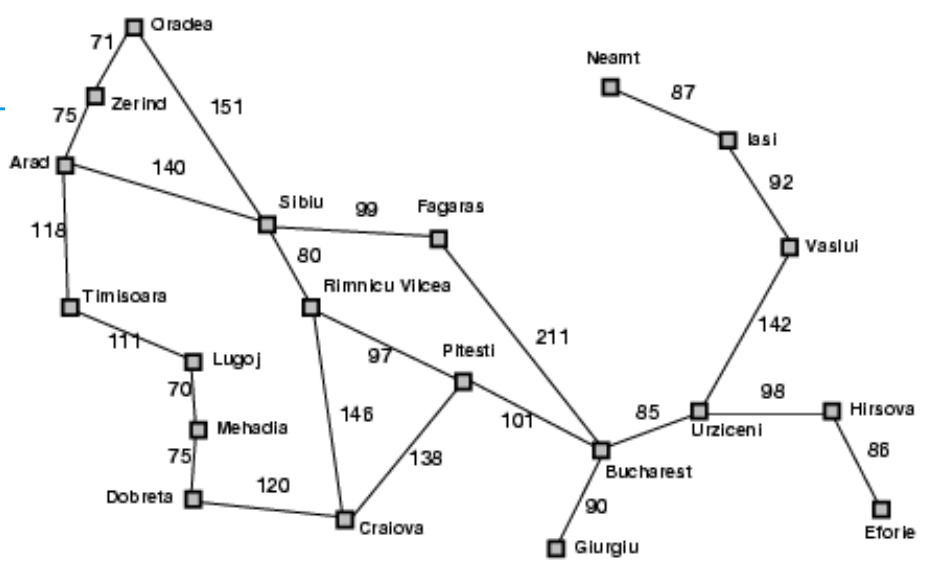

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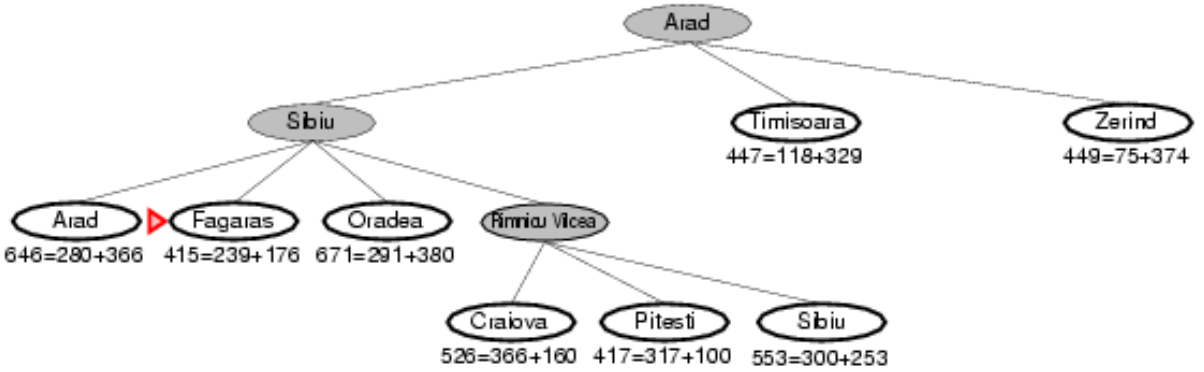

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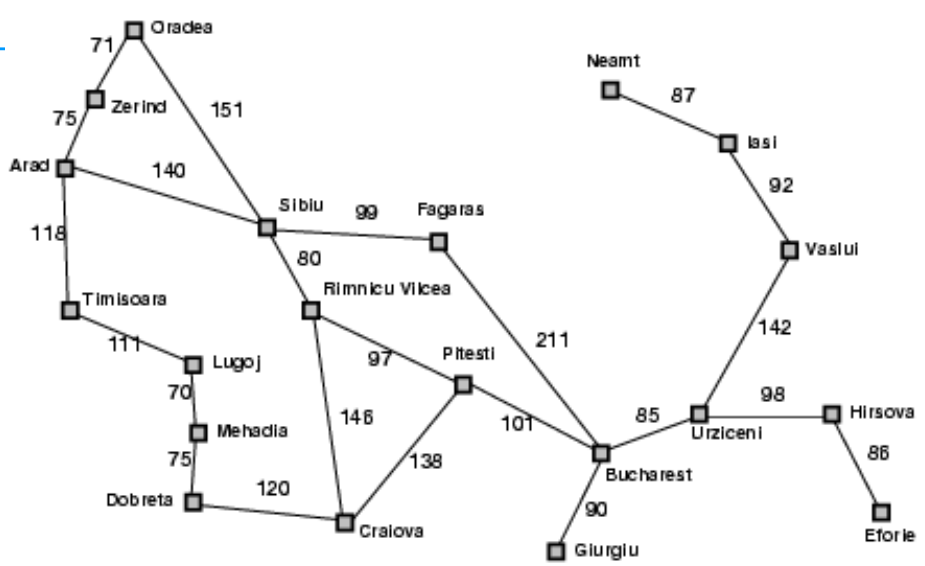

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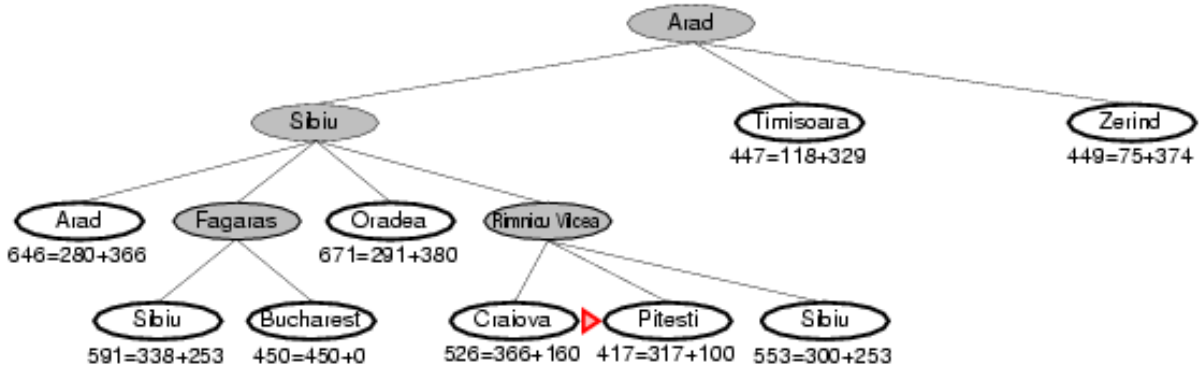

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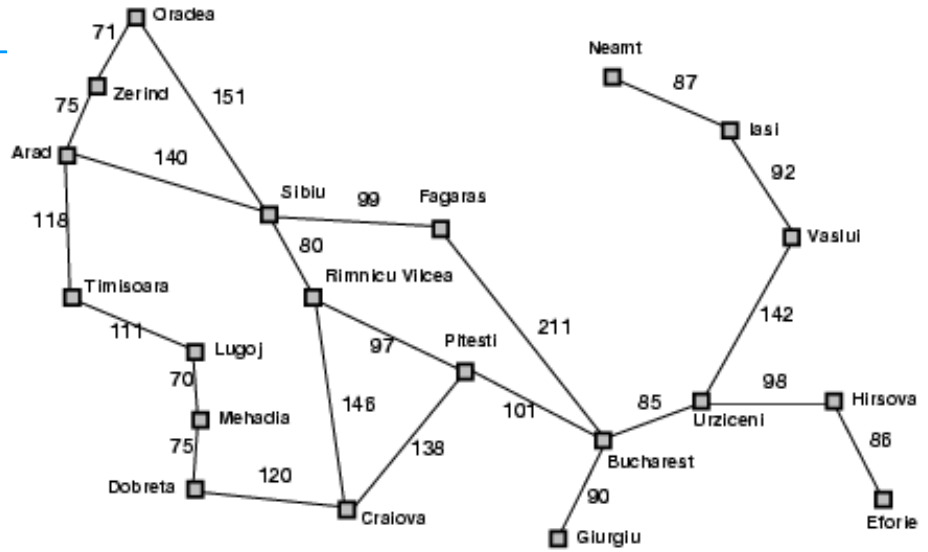

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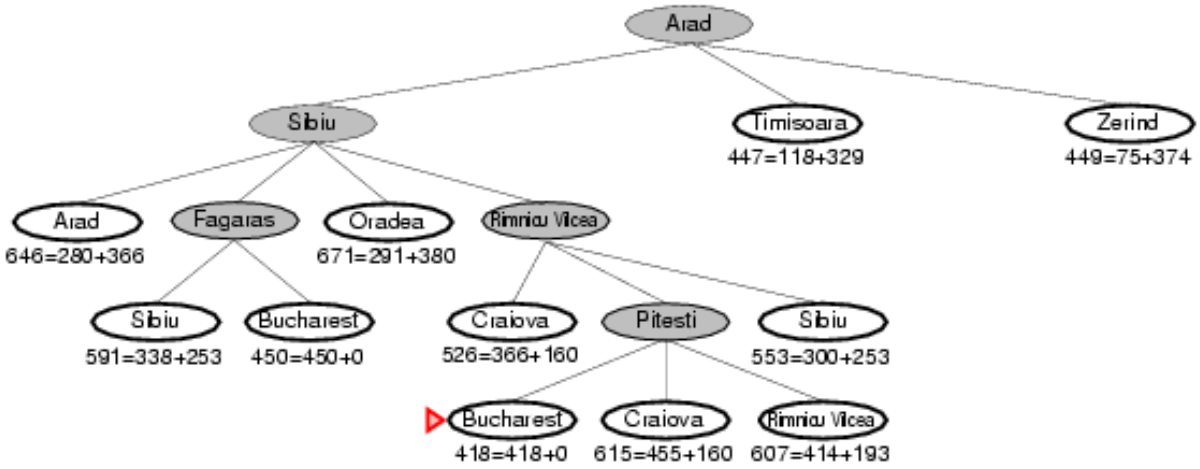
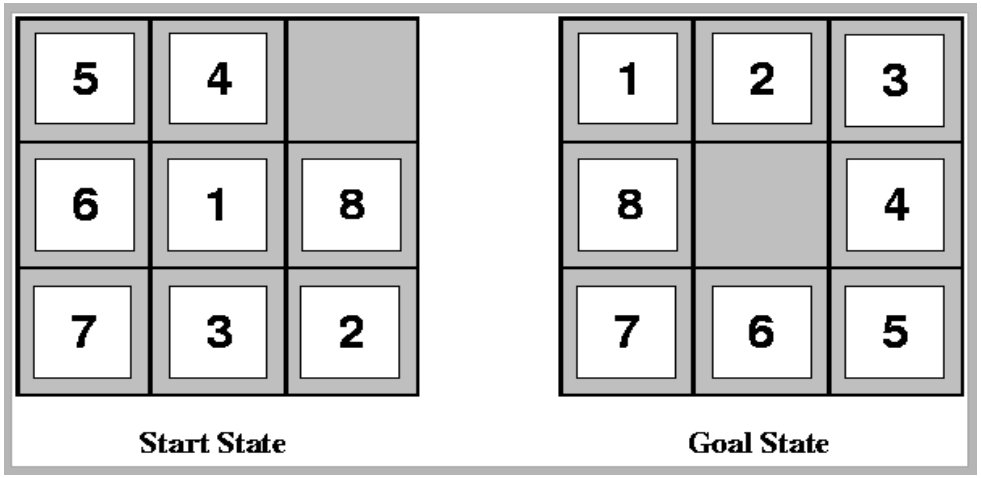


ELIGIBLE FUNCTIONS HEURISTICS



We can define different heuristics. For example:

h_1= Number of tiles that are out of place (h_1= 7)

h_2= The sum of the distances from the initial and final position of each tile. The distance is a sum of the horizontal and vertical distances (**Manhattan distance**). The tiles 1 to 8 in the initial state have a distance h_2=2+3+3+2+4+2+0+2=18

Both heuristics are eligible

ELIGIBLE FUNCTIONS HEURISTICS

- As h_1'<= h_2' then h_2' is better
- It is better to use a heuristic function with higher values, provided it is optimistic.
- How to invent heuristics?
- Often the cost of an exact solution of a relaxed problem is a good heuristic for the original problem.
- If the problem definition is described in a formal language you can build relaxed problems automatically.
- Sometimes you can use the maximum among different heuristics.
- h '(n) = max (h_1 (n), h_2(n) ... h_m(n)).

EXAMPLE: GAME OF 8

- Description: a tile can move from square A to square B if A is adjacent to B and B is empty.

- Relaxed problems remove some conditions
  - A tile can move from square A to square B if A is adjacent to B. (manhattan distance)
  - A tile can move from square A to square B if B is empty.
  - A tile can move from square A to square B in one hop (tiles out of place).

FROM TREES TO GRAPHS

- We have assumed so far that the search space is a tree and not a graph. It is therefore not possible to achieve the same node from different paths.

tree has only one arc to enter in each node, and it can have multiple children if a node has 2+ enter arc, we are using a graph

- This assumption is of course simplistic: think of the game of 8, the missionaries and cannibals etc.

- How to extend the previous algorithms for dealing with graphs?

GRAPH-SEARCH

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

If we in the graph we founf 2 pattern/different cost for achieving the same node, we select the one with the lower weight

SEARCH IN GRAPH WITH A*

- Two lists: **open and closed nodes**
  - **Closed nodes**: expanded nodes are removed from the list to avoid further examination;
  - **Open nodes**: Nodes still to be examined.

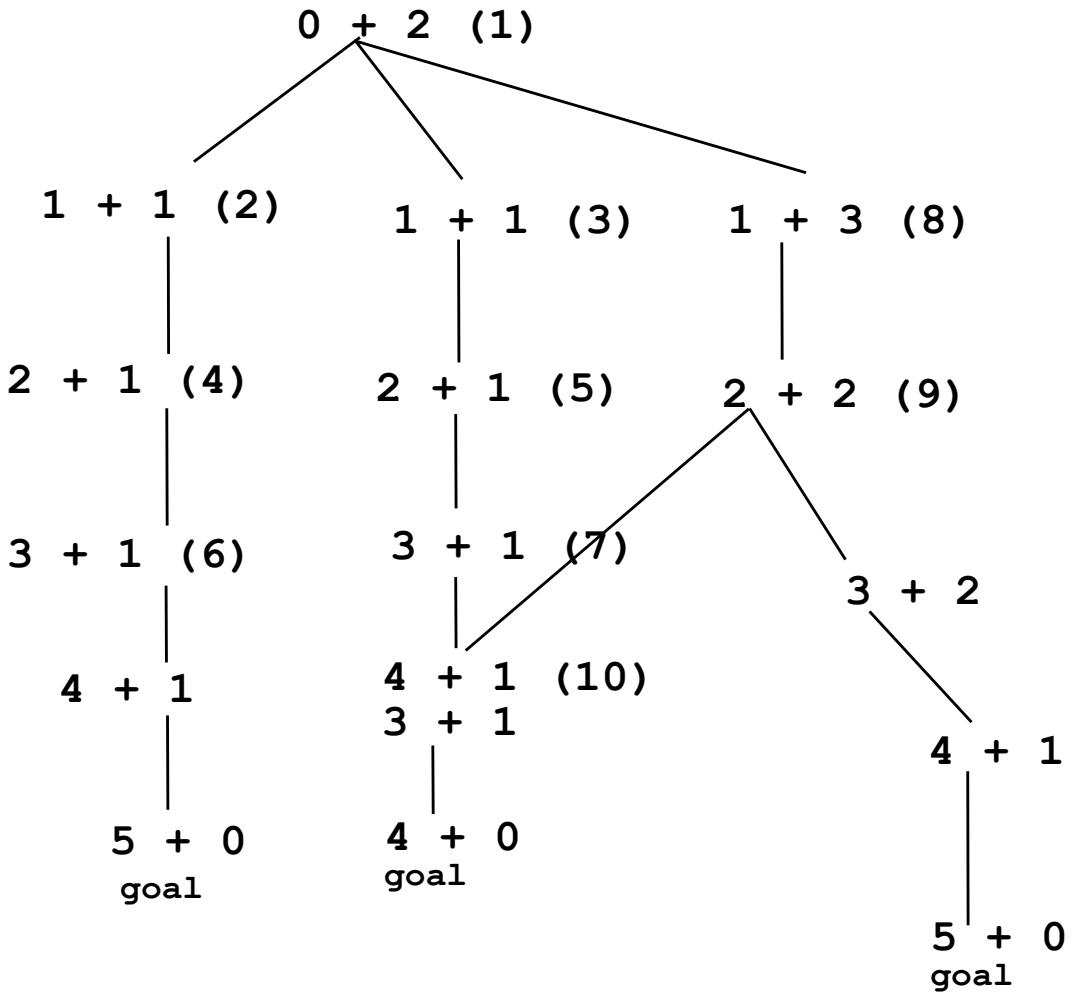
- A* searching in a graph instead of a tree.

- Changes:
  - The graph can become a tree with repeated nodes
  - Add the list of closed nodes and assume that g(n) evaluates the minimum distance of node n from the starting node.

A* ALGORITHM FOR GRAPH

- Let L_a be the open list of initial nodes of the problem.
- Let n be the node for which g (n) + h '(n) is minimal. If the list is empty, fail;
- If n is the goal then stop and return the path to reach it
- Otherwise remove n from L_a, enter it in the list of closed nodes L_c and add to L_a all the children of n, labelling them with the cost from the starting node to n.
- If a child node is already in L_a, do not add it, but update is label in case its cost (root to node) is smaller than the one it had.
- If a child node is already in L_c, do not add it to L_a, but if its cost is better, update its cost and the one of its children and descendant.
- Returns to step 2

EXAMPLE


0 + 2 (1)
1 + 1 (2)
1 + 1 (3)
1 + 3 (8)
2 + 1 (4)
2 + 1 (5)
2 + 2 (9)
3 + 1 (6)
3 + 1 (7)
3 + 2
4 + 1
4 + 1 (10)
3 + 1
4 + 1
5 + 0
goal
4 + 0
goal
5 + 0
goal

CONSISTENT HEURISTIC (MONOTONE)

- A further condition of h: consistency (or monotonicity)
- Definition: a heuristic is' consistent if for each node *n*, any successor *n'* of *n* generated by each action *a*,

  h (n) = 0 if the corresponding status is the goal

  $h(n) \leq c(n, a, n') + h(n')$ otherwise

- With monotonicity, we are guaranteed to find the shortest path from the root to the goal

CONSISTENT HEURISTIC (MONOTONE)

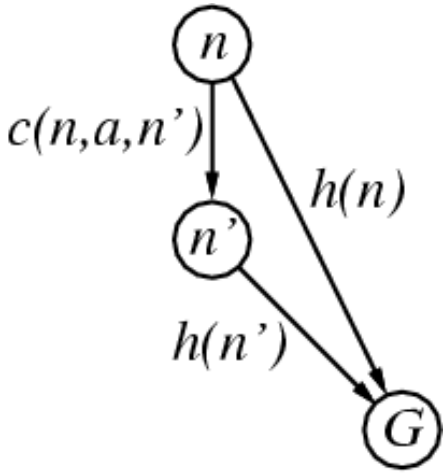
- if *h* is consistent we have that:

  f (n') = G (n') + h (n')

  = G (n) + c (n, a, n') + h (n')

  ≥ g (n) + h (n)

  = f(n)



- *f(n)* never decreases along a path.

- Theorem : if *h(n)* is consistent then A* using `GRAPH-SEARCH` is optimal

For the exam
Theoratical question --> open question in english --> ex: When heuristic is consider admissible and consistent
Excercise in research