We want to show that $\nexists(a,b) \in \mathbb{Z} : \frac{a}{b} = \sqrt{2}$ and therefore that $\sqrt{2} \notin \mathbb{Q}$. Since:

$$\forall(a,b) \in \mathbb{Z}, (\frac{a}{b} = 2) \implies (a = 2b)$$

Given the set of prime factors of b, $B = \{p_1^{k_1}, p_2^{k_2}, \ldots, p_n^{k_n}\}$, we can define $a = \prod x \ \forall x \in A$ where A is:

$$A = \begin{cases} \{2^{k+1}\} \cup (B \setminus \{2^k\}) & 2^k \in B \\ \{2^1\} \cup B & 2^k \notin B \end{cases}$$

Finally since $\forall m \in \mathbb{Z}, m^2 = \prod p^{2k} \ \forall p^k \in M$ where M is the set of prime factors of m, and $\forall(a,b) \in \mathbb{Z}, \exists c \in \mathbb{Z} : 2(a-b) = 2c \land \nexists c \in \mathbb{Z} : 2c = 1$:

$$\nexists(a,b) \in \mathbb{Z} : \frac{a^2}{b^2} = 2 \therefore \sqrt{2} \notin \mathbb{Q}$$