I added an explanation for every step of the proof, in *italics* is the original text, below each paragraph there's a translation (along with the symbols it translates in parenthesis) and a small explanation to try and explain a little of what's going on.

We want to show that $\nexists (a, b) \in \mathbb{Z} : \frac{a}{b} = \sqrt{2}$ *and therefore that* $\sqrt{2} \notin \mathbb{Q}$*.*

Translation:
We want to show that there aren't ($\nexists$) any two integer numbers ($(a, b) \in \mathbb{Z}$) such that $\frac{a}{b} = \sqrt{2}$ and therefore that $\sqrt{2}$ is not a rational number ($\sqrt{2} \notin \mathbb{Q}$).
Sidenote:
A rational number is any number that can be expressed as the ratio of two integers, for example like $\frac{5}{2} = 2.5$ (integers are numbers like $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$).

Since:

$$\forall (a, b) \in \mathbb{Z}, (\frac{a}{b} = 2) \implies (a = 2b)$$

Translation:
For any two integer numbers ($\forall (a, b) \in \mathbb{Z}$), if $\frac{a}{b} = 2$ then it must be that $a = 2b$ ($(\frac{a}{b} = 2) \implies (a = 2b)$).
Sidenote:
That's just how equations work, you can imagine it like saying "if you can split 10 candies across 5 people so that everyone gets 2 candies, then if 5 people give you 2 candies you have 10 candies", change 10 with a and 5 with b and that's basically what it means.

Given the set of prime factors of b*,* $B = \{p_1^{k_1}, p_2^{k_2}, \ldots, p_n^{k_n}\}$*, we can define* $a = \prod x \ \forall x \in A$ *where* A *is:*

$$A = \begin{cases} \{2^{k+1}\} \cup (B \setminus \{2^k\}) & 2^k \in B \\ \{2^1\} \cup B & 2^k \notin B \end{cases}$$

Translation:
Given the set of prime factors of b, called B, we can define a to equal the product of all prime factors of A ($a = \prod x \ \forall x \in A$), which is defined as...
Explanation:
This is not as scary as it looks, I promise. Basically, any number greater than 1 can be either prime or composite. Examples of primes are 2, 3, 5, 7. Examples of composites are 4, 6, 8, 9, 10, 12. All composite numbers can be expressed as a product of prime numbers. You can think of it like ingredients to bake a cake. The flour and the eggs are food, just like the cake, but the cake is made up of flour

and eggs, while the flour is just made of flour and the eggs are just made of eggs. So the cake is the composite number and the flour and eggs are the prime numbers.
All this paragraph says is that, given the ingredients to the number b, we can define a to be those numbers plus an extra 2, as if we added extra sugar. What's really important is that **the difference in 2's contained in a and b must be 1** for $\frac{a}{b} = 2$ to be true.

Finally since $\forall m \in \mathbb{Z}, m^2 = \prod p^{2k}\ \forall p^k \in M$ where M is the set of prime factors of m, and ...

Translation:
Finally since for any integer number m ($\forall m \in \mathbb{Z}$), m^2 equals the product of all the prime factors of m with their exponents multiplied by 2 ($m^2 = \prod p^{2k}\ \forall p^k \in M$), and ...
Explanation:
This is unrelated to the previous paragraph, it's just another fact that needs to be laid out before we get to the final conclusion.
The only thing you need to understand is that the exponents (the k in 2^k) of the prime factors of any number are **always even** when you square said number.

... and $\forall (a,b) \in \mathbb{Z}, \exists c \in \mathbb{Z} : 2(a-b) = 2c \land \nexists c \in \mathbb{Z} : 2c = 1$:

Translation:
... and since for any integer numbers a and b ($\forall (a,b) \in \mathbb{Z}$) there exists another integer c ($\exists c \in \mathbb{Z}$) such that $2(a-b) = 2c$ and there isn't any integer c such that $2c = 1$:
Explanation:
We're almost there. It's 2:30, my sanity is slipping away. All this says is that when you substract two even numbers you always get an even number, so you can't ever get 1 by substracting even numbers.

$$\nexists (a,b) \in \mathbb{Z} : \frac{a^2}{b^2} = 2 \therefore \sqrt{2} \notin \mathbb{Q}$$

Translation:
There aren't any two integer numbers a and b ($\nexists (a,b) \in \mathbb{Z}$) such that $\frac{a^2}{b^2} = 2$ and therefore $\sqrt{2}$ is not rational ($\therefore \sqrt{2} \notin \mathbb{Q}$). **Explanation**:
The end is in sight! It might seem like a leap to go from that last step to here but actually we just proved everything we needed.
To recap: first we showed that for $\frac{a}{b}$ to equal 2, we want a to contain exactly one more 2 than b. Then we showed that when you square any number the number of 2's contained in the result will be even (remember 2^k?). Finally we showed that if you substract even numbers, you will always get another even number (which 1 isn't).
That's all, there aren't two integers which when squared equal 2.