Hierarchical Topic Models

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UC San Diego, Dept. of Computer Science and Engineering

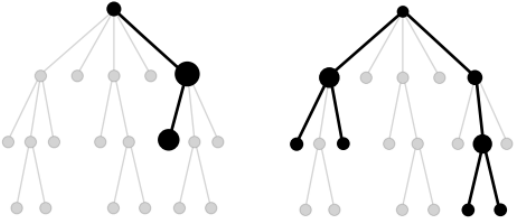

Image Source: Paisley et al [16]

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- How can we navigate these collections efficiently?
- Typical task: find sets of documents that share the same topic or subject matter
- Natural language can be both redundant and ambiguous
- Superficial attributes of documents aren't enough
- We need a notion of *latent semantics*, or underlying meaning

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- Similar to the Dewey Decimal System or Library of Congress Classification

Example: Documents as Mixtures of Topics

$$\begin{aligned} \text{Topics:} \quad \theta_1 &= \text{“sports”} \\ \theta_2 &= \text{“medicine”} \end{aligned}$$

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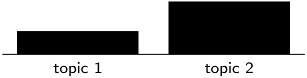

“Flat” Topic Models

Probabilistic Latent Semantic Analysis

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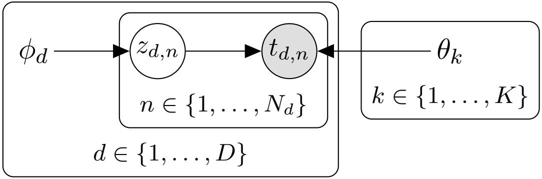

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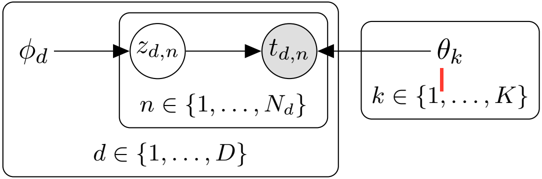

- Infer values of θ_k, ϕ_d using maximum likelihood

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- Allows Bayesian posterior inference

Latent Dirichlet Allocation: The Model

$\theta_k \sim \text{Dirichlet}(\alpha)$	for each topic k
$\phi_d \sim \text{Dirichlet}(\beta)$	for each document d
$z_{d,n} \sim \text{Categorical}(\phi_d)$	for the n^{th} word in document d
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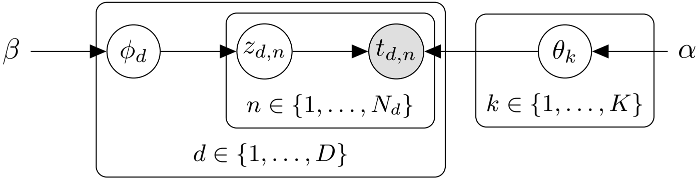

Bayesian Inference Algorithms

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- Need approximate inference methods

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- Repeatedly update $\vec{z}$ by iterating through latent variables, updating z_k by sampling from its *complete conditional*:

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- For the remaining latent variables, we compute a modified form of the complete conditionals:

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- Running Gibbs sampling based on these distributions yields an estimate for

$$p(\vec{z}_{\mathsf{subset}} \mid \vec{x})$$

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- Minimizing reversed KL corresponds to maximizing *evidence lower bound* (ELBO):

$$\mathsf{ELBO} = E_q[\log p(\vec{z}, \vec{x})] - E_q[\log q(\vec{z})]$$

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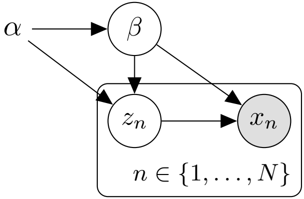
- For exponential-family models, the update rule for z_k is

$$\nu_k = E_q[\eta_k(\vec{z}_{-k}, \vec{x})]$$

where η_k denotes the natural parameters of the complete conditional of z_k

Stochastic Variational Inference: Context

- Generic model with local (per-observation) and global variables:



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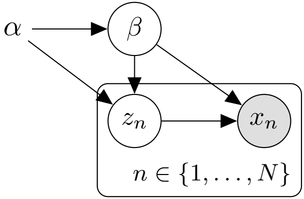

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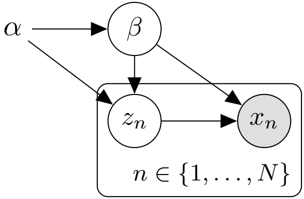

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- For local variables, the update rule is the same as in CAVI:

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Learning Topic Hierarchies

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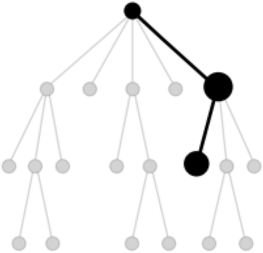

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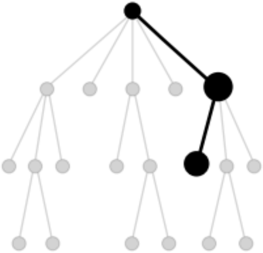
- Idea: Each document samples a path from an infinite tree
- Within each document, we can only select nodes (ie, topics) from the sampled path



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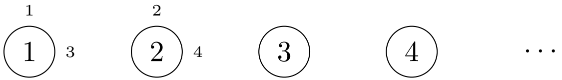

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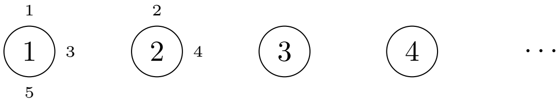

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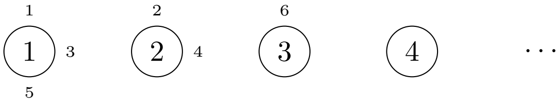

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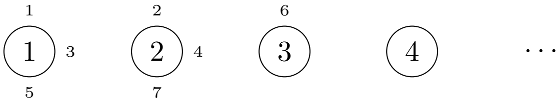

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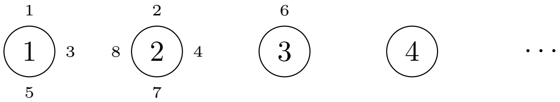

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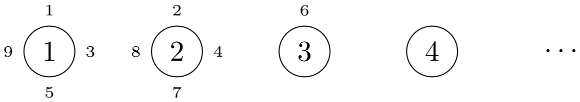

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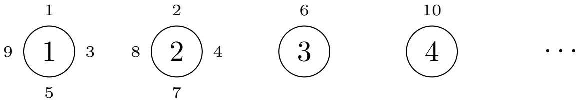

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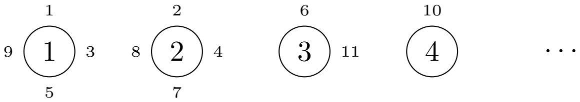

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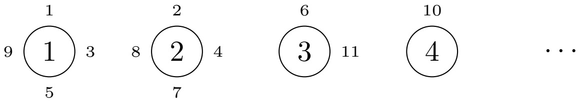

- Parameter α: As $\alpha \to \infty$, number of occupied tables increases

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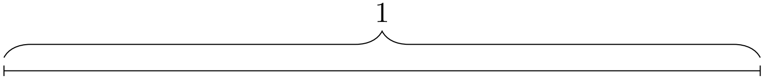

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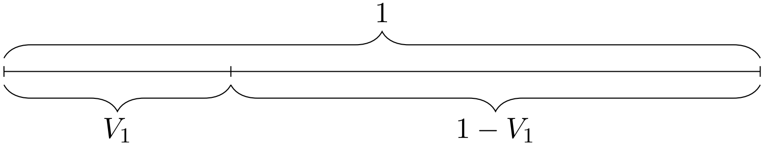

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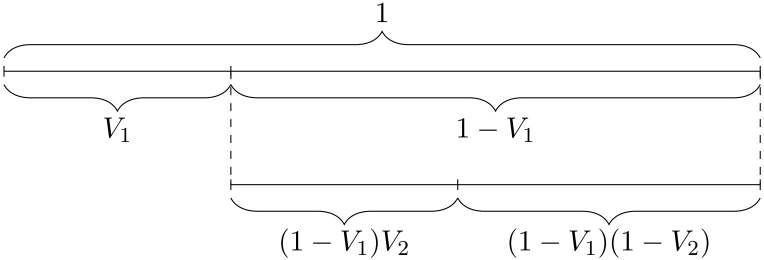

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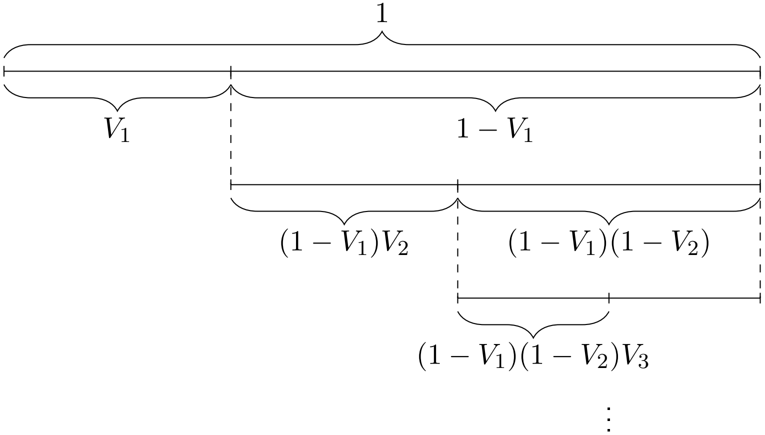

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- This process repeats indefinitely at new restaurants

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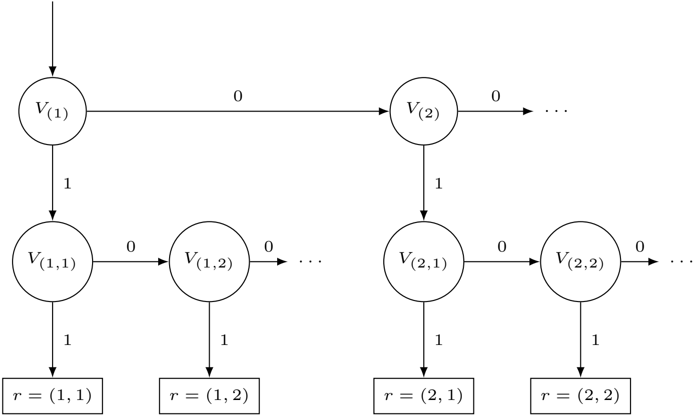
Nested CRP: A Finite-Depth Example



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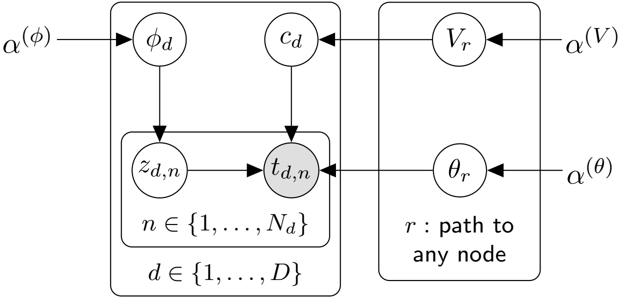

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- Griffiths et al [11]: Finite depth, uses order-dependent "restaurant analogy" formulation to avoid tracking infinitely many paths; each sampling step may grow or shrink the tree
- Blei et al [3]: Uses lazy evaluation; if final layer is ever sampled, then start tracking one extra layer

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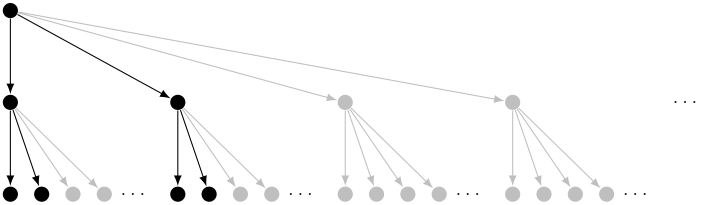

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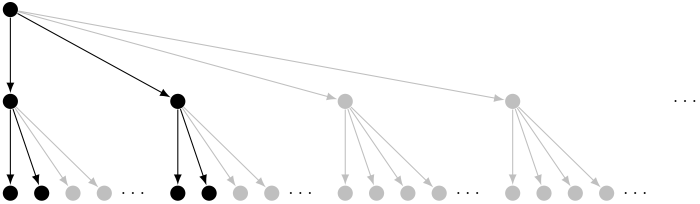

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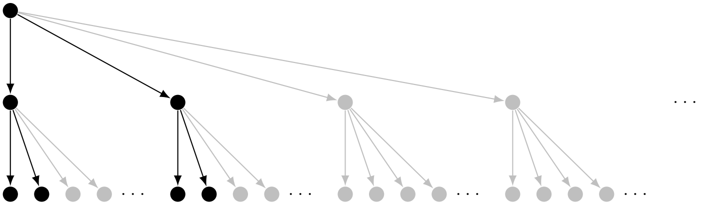

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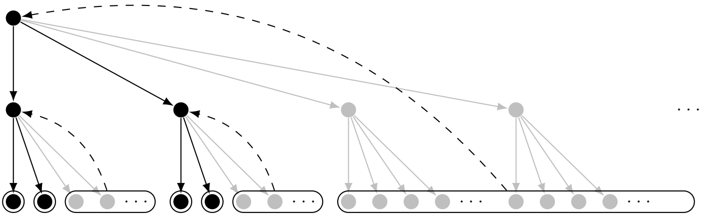

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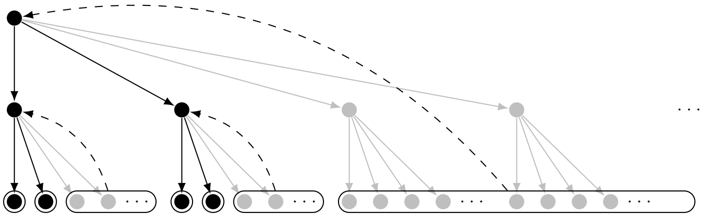
- If one equivalence class becomes sufficiently likely, add a representative path from it



Nested Hierarchical Dirichlet Process

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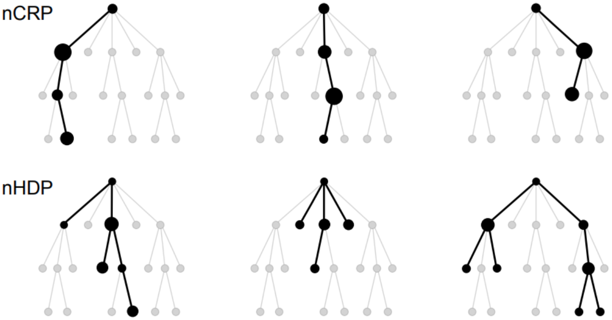
- Handles "hybrid" topics better than NCRP



Source: Paisley et al [16]

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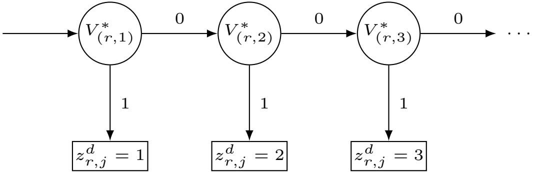
- Together, $z^d_{r,j}$, $V^d_{r,j}$, and U^d_r define a document-specific distribution over nodes

NHDP Topic Model: Selecting Indices

- For each document d, for each node r, and for each $j \geq 1$, select $z^d_{r,j}$:



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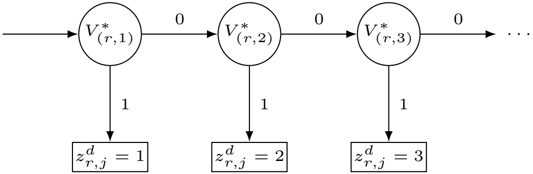
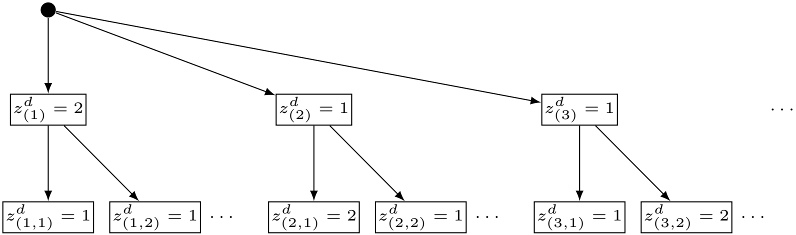
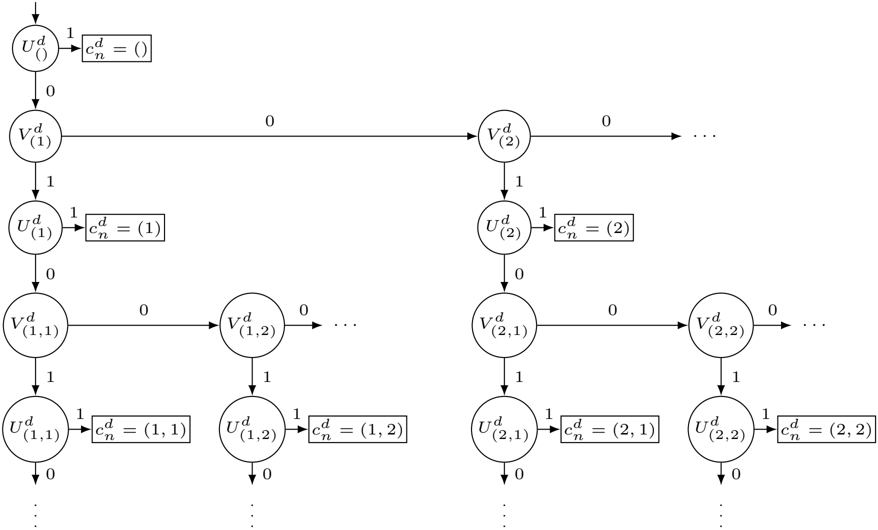


- Result defines how branches are permuted and copied (per document):



NHDP Topic Model: Path Propagation

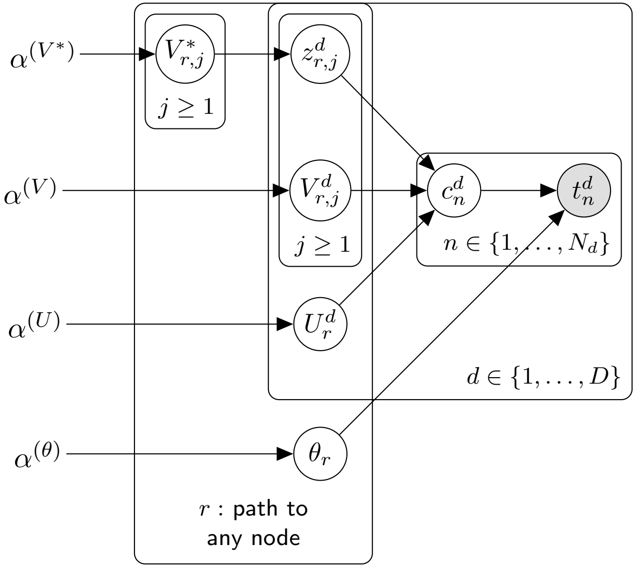
Visualizing $V_{r,j}^d$ and U_r^d, ignoring branch permutations
(i.e., assuming $z_{r,j}^d = j$)



NHDP Topic Model: Conditional Distributions

$$
\begin{aligned}
\theta_r &\sim \text{Dirichlet}(\alpha^{(\theta)}) \\
V^*_{r,j} &\sim \text{Beta}(1, \alpha^{(V^*)}) \\
V^d_{r,j} &\sim \text{Beta}(1, \alpha^{(V)}) \\
U^d_r &\sim \text{Beta}(\alpha^{(U)}_1, \alpha^{(U)}_2) \\
z^d_{r,j} &\sim \sum_{k \geq 1} \left(V^*_{r,k} \prod_{i=1}^{k-1} (1 - V^*_{r,i}) \right) \delta_k \\
c^d_n &\sim \sum_{r:\text{path}} A(r, V^d, z^d)\, B(r, U^d)\, \delta_r \\
A(r, V^d, z^d) &= \prod_{m=0}^{\text{len}(r)-1} \sum_{k \geq 1} \mathbb{1}\left[z^d_{r[1:m],k} = r[m+1] \right] \left(V^d_{r[1:m],k} \prod_{i=1}^{k-1} \left(1 - V^d_{r[1:m],i}\right) \right) \\
B(r, U^d) &= U^d_r \prod_{m=0}^{\text{len}(r)-1} \left(1 - U^d_{r[1:m]}\right) \\
t^d_n &\sim \text{Categorical}(\theta_{c^d_n})
\end{aligned}
$$

NHDP Topic Model: Plate Diagram



NHDP: Stochastic Variational Inference

- Use a finite-depth, finite-width tree

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- Remainder of algorithm is a standard application of stochastic variational inference

Directions for Future Research

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- Frameworks for Bayesian non-parametric inference

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