

```

1  4.
2  a)
3  p(0) = 0, p(1) = 1, p(4) = 2, p(9) = 3
4  c1 = 0
5  c1 + c2 + c3 + c4 = 1
6  c1 + 4c2 + 16c3 + 64c4 = 2
7  c1 + 9c2 + 81c3 + 729c4 = 3
8  | 1 0 0 0 ||c1| |0|
9  | 1 1 1 1 ||c2| |1|
10 | 1 4 16 64 ||c3| = |2|
11 | 1 9 81 729 ||c4| |3|
12 c1 = 0
13 c2 = 37/30
14 c3 = -1/4
15 c4 = 1/60
16 p(x) = (37/30)x - (1/4)x^2 + (1/60)x^3
17
18 b)
19 L0(x) = (x-1)(x-4)(x-9)/((-1)(-4)(-9)) = -(x-1)(x-4)(x-9)/36
20 L1(x) = (x)(x-4)(x-9)/((1)(1-4)(1-9)) = x(x-4)(x-9)/24
21 L2(x) = (x)(x-1)(x-9)/((4)(4-1)(4-9)) = -x(x-1)(x-9)/60
22 L3(x) = (x)(x-1)(x-4)/((9)(9-1)(9-4)) = x(x-1)(x-4)/360
23
24 p(x) = y0*L0(x) + y1*L1(x) + y2*L2(x) + y3*L3(x)
25 p(x) = x(x-4)(x-9)/24 - x(x-1)(x-9)/30 + x(x-1)(x-4)/120
26
27 c)
28 p(x) = a0 + a1(x) + a2(x)(x-1) + a3(x)(x-1)(x-4) + a4(x)(x-1)(x-4)(x-9)
29 a0 = 0
30 1 = a0 + a1
31 a1 = 1
32 2 = a0 + 4a1 + 12a2
33 12a2 = -2
34 a2 = -1/6
35 3 = 9a1 + 72a2 + 360a3
36 3 = 9 - 12 + 360a3
37 a3 = 1/60
38
39 p(x) = x - (1/6)(x)(x-1) + (1/60)(x)(x-1)(x-4)
40
41 d)
42 x=0.5
43 interpolated:
44 p(0.5) = (37/30)0.5 - (1/4)0.5^2 + (1/60)0.5^3 = 0.55625
45 actual:
46 p(0.5) ~= 0.70711
47 absolute error = 0.15086
48 relative error = 0.21335
49
50 x=2
51 interpolated:
52 p(2) = (37/30)2 - (1/4)2^2 + (1/60)2^3 = 8/5
53 actual:
54 p(2) = sqrt(2) ~= 1.41421
55 absolute error = 8/5-sqrt(2) ~= 0.18579
56 relative error ~= 0.13137
57
58 x=7
59 interpolated:
60 p(7) = (37/30)7 - (1/4)7^2 + (1/60)7^3 = 21/10 = 2.1
61 actual:
62 p(2) = sqrt(7) ~= 2.64575
63 absolute error ~= 0.54575
64 relative error ~= 0.20627
65
66 x | interpolated | actual | absolute error | relative error
67 0.5 | 0.55625 | 0.70711 | 0.15086 | 0.21335
68 2 | 8/5 | 1.41421 | 0.18579 | 0.13137
69 7 | 2.1 | 2.64575 | 0.54575 | 0.20627

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