a3q4_YOU

November 3, 2021

1 A3-Q4: Golf Driving Range

[24]:
```
import numpy as np
from copy import deepcopy
import matplotlib.pyplot as plt
g = 9.81
k = 0.5
```

[25]:
```
# Supplied functions
def Ground(d):
    '''
     h = Ground(d)

     Returns the height (in metres) of the ground at a horizontal distance
     d (metres) from the origin.
    '''
    return np.sin(d/3.) - 3.*np.sin(d/10.)

def GroundSlope(d):
    '''
     h = GroundSlope(d)

     Returns the slope of the ground at a horizontal distance
     d (metres) from the origin.
    '''
    return 1./3*np.cos(d/3) - 3./10*np.cos(d/10.)
```

1.1 (a) MyOde

[44]:
```
def MyOde(f, tspan, y0, h, event=(lambda t,y:1)):
    '''
     t,y = MyOde(f, tspan, y0, h, event=[])

     Numerically solves the initial value problem

        dy(t)/dt = f(t,y)
            y(0) = y0
```

```
    using the Modified Euler time-stepping method.

    Input
      f       a Python dynamics function with calling sequence
                 dydt = f(t, y)
      tspan   2-tuple giving the start and end times, [start, end]
      y0      initial state of the system (as a 1D vector)
      h       the time step to use (this is not adaptive time stepping)
      events  an event function with calling sequence
                 val = events(t, y)
              The computation stops as soon as a negative value is
              returned by the event function.

    Output
      t       1D vector holding time stamps
      y       an array that holds one state vector per row (corresponding
              to the time stamps)

      Notes:
          - t and y have the same number of rows.

          - The first element of t should be tspan[0], and the first
            row of y should be the initial state, y0.

          - The event function is NOT called until the second time step.

          - If the computation was stopped by the triggering of an event,
            then the last row of t and y should correspond to the
            time that linear interpolation indicates for the zero-crossing
            of the event-function.
'''

# Initialize output arrays, tlst and ylst
t = tspan[0]
y = deepcopy(y0)
i = t

tlst = []
ylst = []

tlst.append(t)
ylst.append(list(y))

# === YOUR CODE HERE ===
xpos = y0[0]
ypos = y0[1]
```

```
    xspeed = y0[2]
    yspeed = y0[3]

    while (i < tspan[1]):
        i += h
        xspeed_temp = xspeed + h * f(i, [xspeed,yspeed])[0]
        yspeed_temp = yspeed + h * f(i, [xspeed,yspeed])[1]
        xspeed = xspeed + (h/2)*(f(i, [xspeed,yspeed])[0]+f(i,␣
↪[xspeed_temp,yspeed_temp])[0])
        yspeed = yspeed + (h/2)*(f(i, [xspeed,yspeed])[1]+f(i,␣
↪[xspeed_temp,yspeed_temp])[1])
        xpos += xspeed
        ypos += yspeed
        if (event(i,[xpos,ypos,xspeed,yspeed]) < 0):
            break
        ylst.append(np.array([xpos,ypos,xspeed,yspeed]))
        tlst.append(i)

    # Return the time stamps, and corresponding solutions
    return tlst, np.array(ylst)
```

1.2 (b) Dynamics Function: `projectile`

[45]:
```
def projectile(t, z):
    dzdt = np.zeros_like(z)

    # === YOUR CODE HERE ===
    dzdt[0] = -k*z[0]
    dzdt[1] = -g-k*z[1]

    return dzdt
```

1.3 (c) Events Function: `projectile_events`

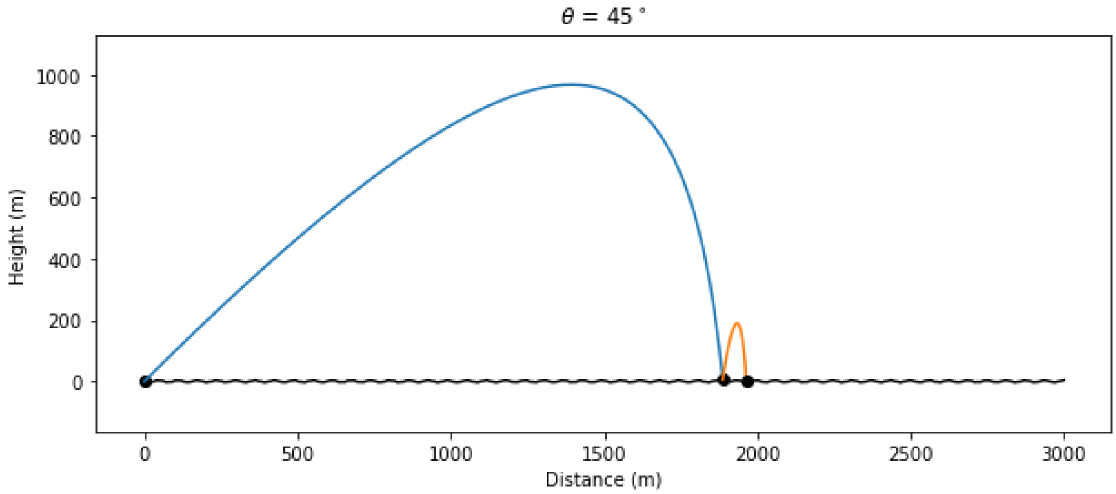
[46]:
```
def projectile_events(t, z):
    val = 1

    # === YOUR CODE HERE ===
    if (z[1] < Ground(z[0])):
        val = -1

    return val
```

1.4 (d) Two flights

[66]:
```python
# Here is the code for one sample flight.
theta = 45
S = 70
tspan = [0, 30]
h = 0.05
theta_rad = theta/180.*np.pi
yStart = np.array([0, 0, S*np.cos(theta_rad), S*np.sin(theta_rad)])
t,y = MyOde(projectile, tspan, yStart, h, projectile_events)
```

[74]:
```python
perp_angle = (np.arctan(GroundSlope(y[len(y)-1][0]))+np.pi/2) % (2*np.pi)
speed = np.sqrt(np.square(y[len(y)-1][2])+np.square(y[len(y)-1][3]))
angle = np.arctan2(y[len(y)-1][3],y[len(y)-1][2])
rev_angle = (angle+np.pi)%(2*np.pi)
new_angle = perp_angle + (perp_angle-rev_angle)
yStart2 = np.array([y[len(y)-1][0], y[len(y)-1][1], speed*np.cos(new_angle),␣
↪speed*np.sin(new_angle)])
t2,y2 = MyOde(projectile, tspan, yStart2, h, projectile_events)
```

[75]:
```python
# Plot the ground
x = np.linspace(-10, 3000, 300)
hills = Ground(x)
plt.figure(figsize=[10,4])
plt.plot(x,hills, 'k')
plt.axis('equal')
plt.plot([0],[0], 'ko')   # Plot initial ball position
plt.plot(y[:,0], y[:,1])  # Plot ball trajectory
plt.plot(y[-1,0], y[-1,1], 'ko')  # Plot final ball position
plt.plot(y2[:,0], y2[:,1])  # Plot ball trajectory
plt.plot(y2[-1,0], y2[-1,1], 'ko')  # Plot final ball position
plt.title(r'$\theta$ = '+str(theta)+'$^\circ$');
plt.xlabel('Distance (m)')
plt.ylabel('Height (m)');
```



[ ]:

```

```

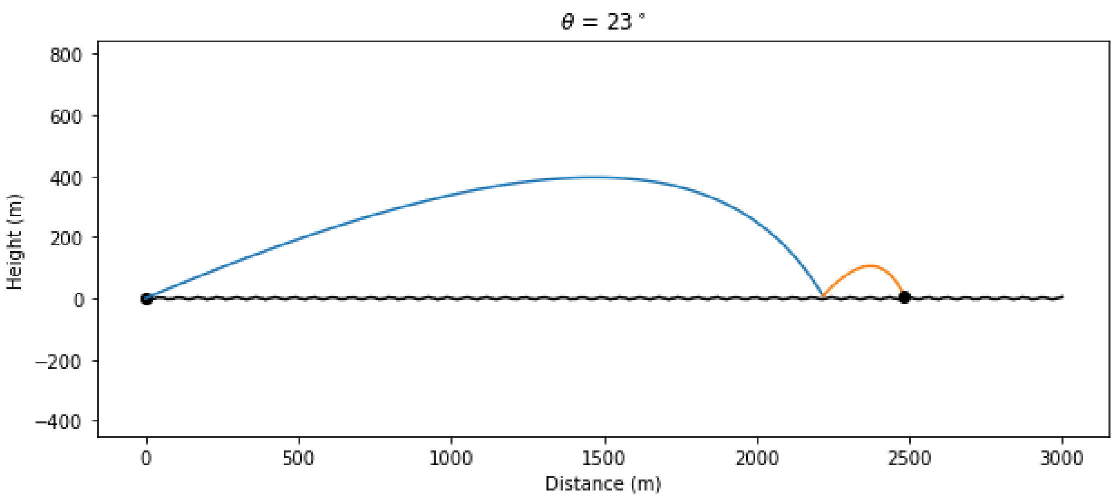
1.5 (e) Optimal θ

Double-click to answer here.

[79]:

```
theta = 23
S = 70
tspan = [0, 30]
h = 0.05
theta_rad = theta/180.*np.pi
yStart = np.array([0, 0, S*np.cos(theta_rad), S*np.sin(theta_rad)])
t,y = MyOde(projectile, tspan, yStart, h, projectile_events)

perp_angle = (GroundSlope(y[len(y)-1][0])+np.pi/2) % (2*np.pi)
speed = np.sqrt(np.square(y[len(y)-1][2])+np.square(y[len(y)-1][3]))
#print(y[len(y)-1][2])
#print(y[len(y)-1][3])
#print(speed)
#print(GroundSlope(y[len(y)-1][0])/np.pi*180)
#print(perp_angle/np.pi*180)
angle = np.arctan2(y[len(y)-1][3],y[len(y)-1][2])
rev_angle = (angle+np.pi)%(2*np.pi)
new_angle = perp_angle + (perp_angle-rev_angle)
yStart2 = np.array([y[len(y)-1][0], y[len(y)-1][1], speed*np.cos(new_angle),
 ↪speed*np.sin(new_angle)])
t2,y2 = MyOde(projectile, tspan, yStart2, h, projectile_events)

# Plot the ground
x = np.linspace(-10, 3000, 300)
```

```python
hills = Ground(x)
plt.figure(figsize=[10,4])
plt.plot(x,hills, 'k')
plt.axis('equal')
plt.plot([0],[0], 'ko')   # Plot initial ball position
plt.plot(y[:,0], y[:,1])  # Plot ball trajectory
plt.plot(y2[:,0], y2[:,1])  # Plot ball trajectory 2
plt.plot(y2[-1,0], y2[-1,1], 'ko')  # Plot final ball position
plt.title(r'$\theta$ = '+str(theta)+'$^\circ$');
plt.xlabel('Distance (m)')
plt.ylabel('Height (m)');
```



[ ]: