1 Introduction

1.1 What is analysis?

- **Analysis**: the rigorous study of objects such as real numbers, complex numbers, etc.

1.2 Why do analysis?

- To understand how these objects work so that you can apply them intelligently

2 The Natural Numbers

2.1 The Peano Axioms

- **Axiom 2.1** 0 is a natural number
- **Axiom 2.2** If n is a natural number, then $n++$ is also a natural number
- **Axiom 2.3** 0 is not the successor of any natural number
  - This prevents "wrap-around"
- **Axiom 2.4** Different natural numbers must have different successors: if n, m are natural numbers and $n \neq m$, then $n++ \neq m++$. Equivalently, if $n++ = m++$ then $n = m$
  - This prevents incrementation hitting a "ceiling"
- **Axiom 2.5 (Principle of mathematical induction)** Let $P(n)$ be any property pertaining to a natural number n. Suppose that $P(0)$ is true, and suppose that whenever $P(n)$ is true, $P(n++)$ is also true. Then $P(n)$ is true $\forall$ natural n.
  - This prevents "rogue" elements (like rational numbers) from being in the naturals
- This definition is *axiomatic* and not *constructive*, they lay out what you can do with the naturals and properties they have, rather than what they are

2.2 Addition

- **Def 2.2.1 (Addition)** Let m be a natural number. To add zero to m, we define $0 + m := m$. Now by induction, suppose we know how to add n to m. Adding $n++$ to m can the be defined as

$$(n++) + m := (n + m)++$$

- **Lemma 2.2.2** For any natural number n, $n + 0 = n$
  - This cannot be proven from $0 + m = m$, as we haven't proven commutativity