Chapter 1: Limit Points and Sequences

- **Point**: element of the real numbers
- **Point Set**: collection of points
- **Linearly Ordered**: if a, b, c $\in$ M, then
  - if a $<$ b and b $<$ c, then a $<$ c and
  - only one of the following is true
    * a $<$ b
    * b $<$ a or
    * a $=$ b
- R is linearly ordered
- If p is a point, there is a point less than p and greater than p
- If p and q are two points, there is a point between then (e.g. (p+q)/2)
  - 'Two points' implies that they are not the same point
- If a $<$ b and c is a point, then a+c $<$ b+c
- If a $<$ b and c $>$ 0, then $a \cdot c < b \cdot c$. If c $<$ 0 then $a \cdot c > b \cdot c$
- If x is a point, then x is an integer, or $\exists$ an integer n s.t. $n < x < n + 1$
- **Open Interval**: If O is an open interval, then O is the set containing all points between two points a and b. Denoted (a,b).
- **Closed Interval**: If C is an open interval, then C is the set containing all points between two points a and b, and a and b themselves. Denoted [a,b].
- **Limit Point**: If M is a point set and p is a point, then p is a limit point of M if *every* open interval containing p contains a point in M different from p.

Problem 1

Show that if M is the open interval (a,b), and p is in M, then p is a limit point of M.

Proof. To show that p is a limit point of M, we need to show that if we have an open interval containing p, we also have a different point p in the interval that is also in M. If we construct a new open interval (c,d) that contains p, we need to show that the new interval also contains another point from M. We can choose this point to be (p+x)/2, where x = min(b,d). We know that this point is also in M, because it is greater than p (which is in M), but less that the highest point in M. Therefore we have found a point in the same interval that is not p. □

Problem 2

Show that if M is the closed interval [a,b] and p is not in M, then p is not a limit point of M.

Proof. Since p is not in M, it is not between or equal to a and b, and therefore any interval that contains p is not within M. □

Problem 3

Show that if M is a point set having a limit point, then M contains 2 points. Must M countain 3 points? 4 points?

Proof. □