Status update

1 Status

- I have implemented some of the key machinery, so now I can play with it: weighted BDD transformations, building a simple MIP, creating BDDs (availability, covering, and their intersection), building CPP MIP and a network flow based on the intersection BDD.
- I ran several really simple examples, and stumbled upon a problem: I think my intersection BDD (unsurprisingly) blows up. It is just a little more dramatic than I expected – see the last section here.

2 A toy example (updated, again)

This is the same example updated according to our recent discussions.

2.1 Problem description

Let us consider a simple problem with two facilities and three customers, as depicted in Figure 1.

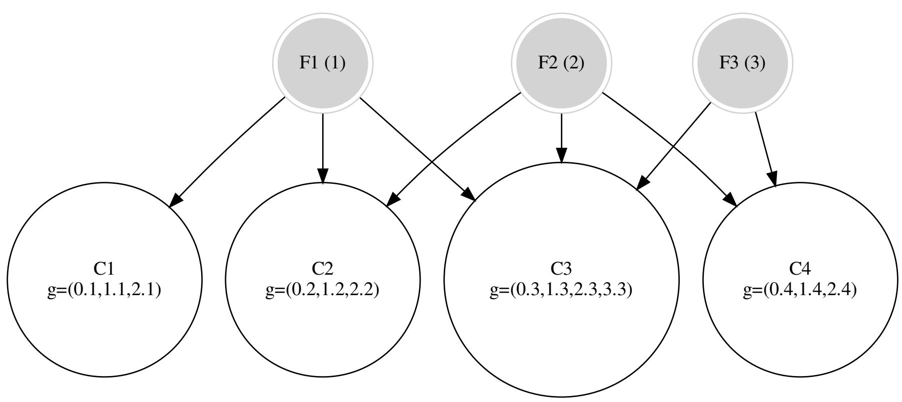


Figure 1: Problem description: facilty location with overlaps.
Facilities: numbers in parentheses indicate switch-on costs.
Consumers: overlap penalies are shown, $g = (0, 1, 2)$ would mean that for this consumer zero overlapping coverings imposed no additional cost, covering with one facility brought additional cost 1, with two facilities (i.e., actual overlap) brought cost 2. These numbers are chosen to be the same for all consumers (no restrictions to change this, just for simplicity).

I am representing this (or any other) problem in the following three ways.

2.2 Simple MIP

I generate a "naive" MIP right away:

$$\text{Minimize:} 1.0 + x_1 + 2.0x_2 + 3.0x_3 + v_1^1 + v_2^1 + 1.0v_2^2 + v_3^1 + 1.0v_3^2 + v_3^3 + 1.0v_4^1 + v_4^2$$

Subject To:

$$
\begin{aligned}
& -1.0x_1 + z_{1\to 1} = 0.0 \\
& -1.0x_1 + z_{1\to 2} = 0.0 \\
& -1.0x_1 + z_{1\to 3} = 0.0 \\
& -1.0x_2 + z_{2\to 2} = 0.0 \\
& -1.0x_2 + z_{2\to 3} = 0.0 \\
& -1.0x_2 + z_{2\to 4} = 0.0 \\
& -1.0x_3 + z_{3\to 3} = 0.0 \\
& -1.0x_3 + z_{3\to 4} = 0.0 \\
& -1.0z_{1\to 1} + b_1 = 0.0 \\
& -1.0z_{1\to 2} - 1.0z_{2\to 2} + b_2 = 0.0 \\
& -1.0z_{1\to 3} - 1.0z_{2\to 3} - 1.0z_{3\to 3} + b_3 = 0.0 \\
& -1.0z_{2\to 4} - 1.0z_{3\to 4} + b_4 = 0.0 \\
& b_1 - 1.0v_1^1 = 0.0 \\
& -1.0v_2^1 + v_2^2 \leq 0.0 \\
& b_2 - 1.0v_2^1 - 1.0v_2^2 = 0.0 \\
& -1.0v_3^1 + v_3^2 \leq 0.0 \\
& -1.0v_3^2 + v_3^3 \leq 0.0 \\
& b_3 - 1.0v_3^1 - 1.0v_3^2 - 1.0v_3^3 = 0.0 \\
& -1.0v_4^1 + v_4^2 \leq 0.0 \\
& b_4 - 1.0v_4^1 - 1.0v_4^2 = 0.0,
\end{aligned}
$$

where x are "locate" decisions, z are covering decisions (which are kind of dependent on each other, like we discussed), b are numbers of overlaps, and v are used to encode an arbitrary overlap penalty function.

Binary variables are: $[x_1, z_{1\to 1}, z_{1\to 2}, z_{1\to 3}, x_2, z_{2\to 2}, z_{2\to 3}, z_{2\to 4}, x_3, z_{3\to 3}, z_{3\to 4}, v_1^1, v_2^1, v_2^2, v_3^1, v_3^2, v_3^3, v_4^1, v_4^2]$ (and I kind of hope that Gurobi's `presolve` takes care of the redundant variables.)

2.3 CPP MIP

Now, I am generating two decision diagrams, as before:

Diagram	**Constraints incorporated**	**Costs incorporated**	**Figure**
Covering	Cover each consumer at least once	c_{ij} (covering) $g_j(n_j)$ (overlap)	2a
Availability	"Switch-on" and "covering" variables are consistent	f_i (switch-on)	2b

Which allows me to formulate the following MIP: The objective is:

$$
\begin{aligned}
\text{Minimize:} \quad & v^{A}_{0\to 1,h} + 2.0v^{A}_{6\to 8,h} + 3.0v^{A}_{14\to 16,h} + \\
& 1.1v^{C}_{0\to 1,h} + 0.1v^{C}_{0\to 1,l} + 1.2v^{C}_{2\to 4,h} + 0.2v^{C}_{2\to 4,l} + 2.2v^{C}_{3\to 4,h} + 1.2v^{C}_{3\to 4,l} + \\
& 1.3v^{C}_{7\to 10,h} + 0.3v^{C}_{7\to 10,l} + 3.3v^{C}_{9\to 10,h} + 2.3v^{C}_{9\to 10,l} + 2.3v^{C}_{8\to 10,h} + 1.3v^{C}_{8\to 10,l} + \\
& 2.4v^{C}_{12\to T,h} + 1.4v^{C}_{12\to T,l} + 1.4v^{C}_{11\to T,h} + 0.4v^{C}_{11\to T,l}.
\end{aligned}
$$

Here, for example, variable $v^{A}_{0\to 1,h}$ corresponds to the flow from node 0 to node 1 of diagram A (availability), along the "hi" ("yes") arc.

Legend.

- From each diagram, two types of constraints are generated:
  - *cont-at-*$\cdot$ are flow continuity constraints at a given node.
  - *bin-link-*$<k>$ are binary linking constraints (needed to link two BDDs – i.e., tangle network flow problems), one per layer, indexed with k.
- A denotes "Availability" diagram, C denotes "Covering" diagram.

All node numbers correspond to the diagrams and have nothing to do with customer and facility indices.

- Arc flow variables (continuous) v.
- Linking variables (binary): $\lambda_{z1-1}, \lambda_{z1-2}, \lambda_{z1-3}, \lambda_{z2-2}, \lambda_{z2-3}, \lambda_{z2-4}, \lambda_{z3-3}, \lambda_{z3-4}$.

Constraints from **covering BDD**:

Type	**Constraint**
cont-at-0	$-1.0v^A_{0\to 1,h} - 1.0v^A_{0\to 2,l} = -1.0$
bin-link-1	$\lambda_{z1-1} - 1.0v^A_{0\to 1,h} = 0.0$
cont-at-2	$v^A_{0\to 2,l} - 1.0v^A_{2\to 5,h} - 1.0v^A_{2\to 4,l} = 0.0$
cont-at-1	$v^A_{0\to 1,h} - 1.0v^A_{1\to 3,h} - 1.0v^A_{1\to 5,l} = 0.0$
bin-link-2	$\lambda_{z1-2} - 1.0v^A_{2\to 5,h} - 1.0v^A_{1\to 3,h} = 0.0$
cont-at-3	$v^A_{1\to 3,h} - 1.0v^A_{3\to 6,h} - 1.0v^A_{3\to 7,l} = 0.0$
cont-at-4	$v^A_{2\to 4,l} - 1.0v^A_{4\to 7,h} - 1.0v^A_{4\to 6,l} = 0.0$
cont-at-5	$v^A_{2\to 5,h} + v^A_{1\to 5,l} - 1.0v^A_{5\to 7,h} - 1.0v^A_{5\to 7,l} = 0.0$
bin-link-3	$\lambda_{z1-3} - 1.0v^A_{3\to 6,h} - 1.0v^A_{4\to 7,h} - 1.0v^A_{5\to 7,h} = 0.0$
cont-at-7	$v^A_{3\to 7,l} + v^A_{4\to 7,h} + v^A_{5\to 7,h} + v^A_{5\to 7,l} - 1.0v^A_{7\to 10,h} - 1.0v^A_{7\to 10,l} = 0.0$
cont-at-6	$v^A_{3\to 6,h} + v^A_{4\to 6,l} - 1.0v^A_{6\to 8,h} - 1.0v^A_{6\to 9,l} = 0.0$
bin-link-4	$\lambda_{z2-2} - 1.0v^A_{7\to 10,h} - 1.0v^A_{6\to 8,h} = 0.0$
cont-at-10	$v^A_{7\to 10,h} + v^A_{7\to 10,l} - 1.0v^A_{10\to 13,h} - 1.0v^A_{10\to 13,l} = 0.0$
cont-at-9	$v^A_{6\to 9,l} - 1.0v^A_{9\to 13,h} - 1.0v^A_{9\to 12,l} = 0.0$
cont-at-8	$v^A_{6\to 8,h} - 1.0v^A_{8\to 11,h} - 1.0v^A_{8\to 13,l} = 0.0$
bin-link-5	$\lambda_{z2-3} - 1.0v^A_{10\to 13,h} - 1.0v^A_{9\to 13,h} - 1.0v^A_{8\to 11,h} = 0.0$
cont-at-12	$v^A_{9\to 12,l} - 1.0v^A_{12\to 15,h} - 1.0v^A_{12\to 14,l} = 0.0$
cont-at-13	$v^A_{10\to 13,h} + v^A_{10\to 13,l} + v^A_{9\to 13,h} + v^A_{8\to 13,l} - 1.0v^A_{13\to 15,h} - 1.0v^A_{13\to 15,l} = 0.0$
cont-at-11	$v^A_{8\to 11,h} - 1.0v^A_{11\to 14,h} - 1.0v^A_{11\to 15,l} = 0.0$
bin-link-6	$\lambda_{z2-4} - 1.0v^A_{12\to 15,h} - 1.0v^A_{13\to 15,h} - 1.0v^A_{11\to 14,h} = 0.0$
cont-at-14	$v^A_{12\to 14,l} + v^A_{11\to 14,h} - 1.0v^A_{14\to 16,h} - 1.0v^A_{14\to 17,l} = 0.0$
cont-at-15	$v^A_{12\to 15,h} + v^A_{13\to 15,h} + v^A_{13\to 15,l} + v^A_{11\to 15,l} - 1.0v^A_{15\to 18,h} - 1.0v^A_{15\to 18,l} = 0.0$
bin-link-7	$\lambda_{z3-3} - 1.0v^A_{14\to 16,h} - 1.0v^A_{15\to 18,h} = 0.0$
cont-at-18	$v^A_{15\to 18,h} + v^A_{15\to 18,l} - 1.0v^A_{18\to F,h} - 1.0v^A_{18\to F,l} = 0.0$
cont-at-17	$v^A_{14\to 17,l} - 1.0v^A_{17\to F,h} - 1.0v^A_{17\to T,l} = 0.0$
cont-at-16	$v^A_{14\to 16,h} - 1.0v^A_{16\to T,h} - 1.0v^A_{16\to F,l} = 0.0$
bin-link-8	$\lambda_{z3-4} - 1.0v^A_{18\to F,h} - 1.0v^A_{17\to F,h} - 1.0v^A_{16\to T,h} = 0.0$
cont-at-T	$v^A_{17\to T,l} + v^A_{16\to T,h} = 1.0$
cont-at-F	$v^A_{18\to F,h} + v^A_{18\to F,l} + v^A_{17\to F,h} + v^A_{16\to F,l} = 0.0$

Constraints from **availability BBD**

Type	Constraint
cont-at-0	$-1.0v^C_{0\to 1,h} - 1.0v^C_{0\to 1,l} = -1.0$
bin-link-1	$\lambda_{z1-1} - 1.0v^C_{0\to 1,h} = 0.0$
cont-at-1	$v^C_{0\to 1,h} + v^C_{0\to 1,l} - 1.0v^C_{1\to 3,h} - 1.0v^C_{1\to 2,l} = 0.0$
bin-link-2	$\lambda_{z1-2} - 1.0v^C_{1\to 3,h} = 0.0$
cont-at-2	$v^C_{1\to 2,l} - 1.0v^C_{2\to 4,h} - 1.0v^C_{2\to 4,l} = 0.0$
cont-at-3	$v^C_{1\to 3,h} - 1.0v^C_{3\to 4,h} - 1.0v^C_{3\to 4,l} = 0.0$
bin-link-3	$\lambda_{z2-2} - 1.0v^C_{2\to 4,h} - 1.0v^C_{3\to 4,h} = 0.0$
cont-at-4	$v^C_{2\to 4,h} + v^C_{2\to 4,l} + v^C_{3\to 4,h} + v^C_{3\to 4,l} - 1.0v^C_{4\to 6,h} - 1.0v^C_{4\to 5,l} = 0.0$
bin-link-4	$\lambda_{z1-3} - 1.0v^C_{4\to 6,h} = 0.0$
cont-at-5	$v^C_{4\to 5,l} - 1.0v^C_{5\to 8,h} - 1.0v^C_{5\to 7,l} = 0.0$
cont-at-6	$v^C_{4\to 6,h} - 1.0v^C_{6\to 9,h} - 1.0v^C_{6\to 8,l} = 0.0$
bin-link-5	$\lambda_{z2-3} - 1.0v^C_{5\to 8,h} - 1.0v^C_{6\to 9,h} = 0.0$
cont-at-7	$v^C_{5\to 7,l} - 1.0v^C_{7\to 10,h} - 1.0v^C_{7\to 10,l} = 0.0$
cont-at-9	$v^C_{6\to 9,h} - 1.0v^C_{9\to 10,h} - 1.0v^C_{9\to 10,l} = 0.0$
cont-at-8	$v^C_{5\to 8,h} + v^C_{6\to 8,l} - 1.0v^C_{8\to 10,h} - 1.0v^C_{8\to 10,l} = 0.0$
bin-link-6	$\lambda_{z3-3} - 1.0v^C_{7\to 10,h} - 1.0v^C_{9\to 10,h} - 1.0v^C_{8\to 10,h} = 0.0$
cont-at-10	$v^C_{7\to 10,h} + v^C_{7\to 10,l} + v^C_{9\to 10,h} + v^C_{9\to 10,l} + v^C_{8\to 10,h} + v^C_{8\to 10,l} - 1.0v^C_{10\to 12,h} - 1.0v^C_{10\to 11,l} = 0.0$
bin-link-7	$\lambda_{z2-4} - 1.0v^C_{10\to 12,h} = 0.0$
cont-at-12	$v^C_{10\to 12,h} - 1.0v^C_{12\to T,h} - 1.0v^C_{12\to T,l} = 0.0$
cont-at-11	$v^C_{10\to 11,l} - 1.0v^C_{11\to T,h} - 1.0v^C_{11\to T,l} = 0.0$
bin-link-8	$\lambda_{z3-4} - 1.0v^C_{12\to T,h} - 1.0v^C_{11\to T,h} = 0.0$
cont-at-F	$0.0 = 0.0$
cont-at-T	$v^C_{12\to T,h} + v^C_{12\to T,l} + v^C_{11\to T,h} + v^C_{11\to T,l} = 1.0$

2.4 Intersection BDD

This is a little more tricky. First, I make 'availability' and 'covering' diagrams order-associated:

```
import varseq as vs
from BB_search import BBSearch

print(f"Size *before* alignment: {A.size()} + {C.size()} = {A.size() + C.size()} nodes.")
vs_A = vs.VarSeq(A.vars, [len(L) for L in A.layers[:-1]])
vs_C = vs.VarSeq(C.vars, [len(L) for L in C.layers[:-1]])
b = BBSearch(vs_A, vs_C)

status = b.search()
assert status == "optimal" or status == "timeout"

Ap = A.align_to(b.Ap_cand.layer_var, inplace=False)
Cp = C.align_to(b.Ap_cand.layer_var, inplace=False)

Ap.show(dir="reports/2021-02-23_Status_BM/", filename="A_aligned.dot")
Cp.show(dir="reports/2021-02-23_Status_BM/", filename="C_aligned.dot")
print(f"Size *after* alignment: {Ap.size()} + {Cp.size()} = {Ap.size() + Cp.size()} nodes.")
print(f"The order revised from \n A: {A.vars}, and\n C: {C.vars}...")
print(f"...to {Ap.vars}")
```

```
Size *before* alignment: 19 + 13 = 32 nodes.
```

```
Size *after* alignment: 22 + 14 = 36 nodes.
The order revised from
 A: ['z1-1', 'z1-2', 'z1-3', 'z2-2', 'z2-3', 'z2-4', 'z3-3', 'z3-4'], and
 C: ['z1-1', 'z1-2', 'z2-2', 'z1-3', 'z2-3', 'z3-3', 'z2-4', 'z3-4']...
...to ['z1-1', 'z1-2', 'z1-3', 'z2-2', 'z2-3', 'z3-3', 'z2-4', 'z3-4']
```

(This results in the diagrams depicted in Figures 3a and 3b, respectively.)
So, I can generate an intersection BDD (Figure 4) and the corresponding MIP:
The objective is:

$$\begin{aligned}\text{Minimize: } & 2.1v_{0\to1,h} + 0.1v_{0\to2,l} + 0.2v_{6\to9,h} + 0.2v_{6\to10,l} + 1.2v_{5\to9,h} + \\ & 1.2v_{5\to8,l} + 0.2v_{4\to9,h} + 0.2v_{4\to8,l} + 1.2v_{3\to7,h} + 1.2v_{3\to8,l} + 3.0v_{10\to15,h} + \\ & 3.0v_{7\to11,h} + v_{9\to14,h} + v_{8\to13,h} + 1.3v_{21\to26,h} + 0.3v_{21\to26,l} + 3.3v_{17\to24,h} + \\ & 2.3v_{17\to25,l} + 2.3v_{22\to24,h} + 1.3v_{22\to25,l} + 3.3v_{19\to26,h} + 2.3v_{19\to26,l} + \\ & 1.3v_{23\to27,h} + 0.3v_{23\to28,l} + 2.3v_{18\to26,h} + 1.3v_{18\to26,l} + 2.3v_{20\to27,h} + \\ & 1.3v_{20\to28,l} + 3.0v_{27\to33,l} + 3.0v_{24\to29,h} + 1.4v_{34\to F,h} + 0.4v_{34\to T,l} + \\ & 1.4v_{30\to F,h} + 0.4v_{30\to F,l} + 2.4v_{29\to T,h} + 1.4v_{29\to F,l} + 1.4v_{33\to T,h} + 0.4v_{33\to F,l} + \\ & 2.4v_{32\to F,h} + 1.4v_{32\to F,l} + 2.4v_{31\to F,h} + 1.4v_{31\to T,l},\end{aligned}$$

under the constraints, presented in Table 1. Obviously, here I have continuous variables only.

2.5 A quick cross-check

Of course, I'd like to cross-check somehow. E.g., I can just solve each of the three models and make sure the optimal objective coincide. Indeed:

```
Opt statuses are: 2, 2, 2
('optimal' is encoded by 2)
Optimal objectives are:
Simple MIP: 1.0
CPP MIP:    1.0
NF (linear):1.0
```

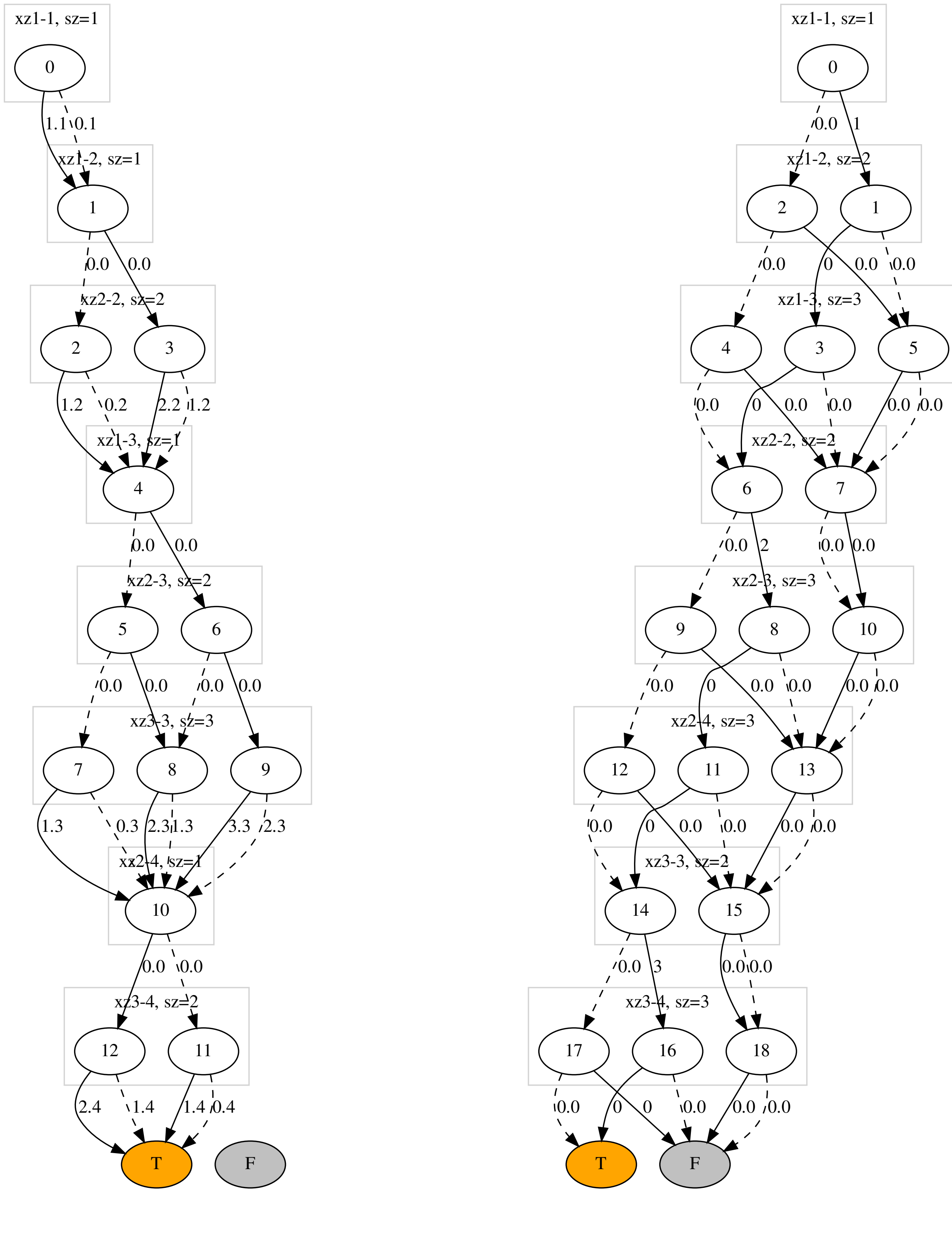
Here are, e.g., nonzero variables for the CPP MIP (`-1` encodes **True** terminal node).

```
A_vl0_2: 1.0
A_vl2_4: 1.0
A_vl4_6: 1.0
A_vl6_9: 1.0
A_vl9_12: 1.0
A_vl12_14: 1.0
A_vl14_17: 1.0
A_vl17_-1: 1.0
C_vl0_1: 1.0
C_vl1_2: 1.0
C_vl2_4: 1.0
C_vl4_5: 1.0
C_vl5_7: 1.0
```

```
C_vl7_10: 1.0
C_vl10_11: 1.0
C_vl11_-1: 1.0
```

3 The problem: intersection DD seems to blow up.

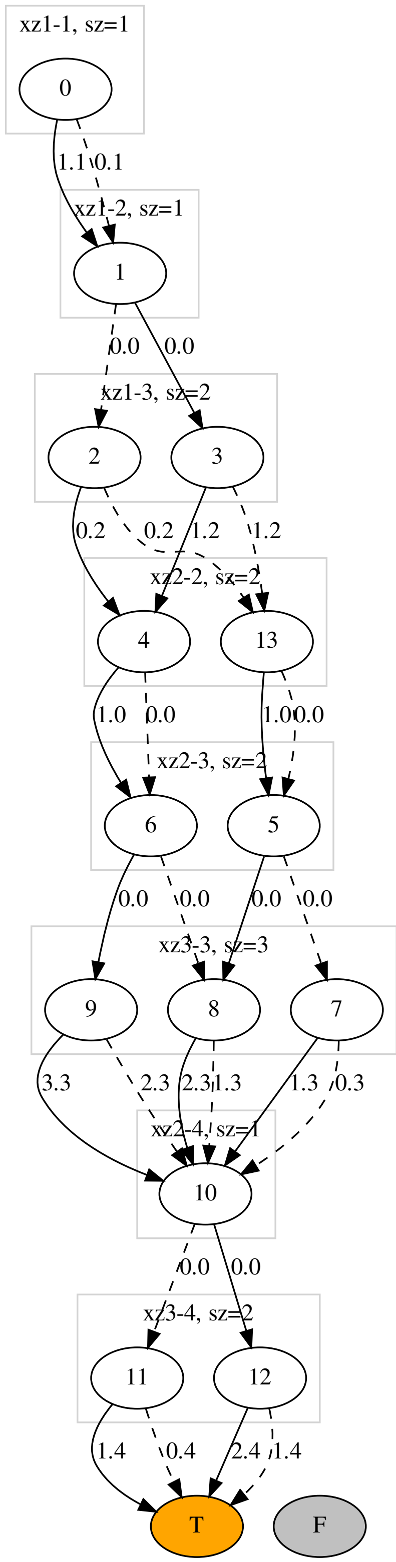
What I have done is a very simple experiment: I generated 15 random instances for different problem sizes – say, with number of facilities being $n = 3, 4, 5, 6$, and number of customers $m = 2n$ (in every case). What I have is: diagram sizes (A for availability and C for covering), along with the number of variables in the plain MIP grow reasonably (Figure 5a). However, intersection BDD just blows up (Figure 5b – note I had to draw it in **logarithmic** scale), and so does the runtime of what I am doing (Figure 6).



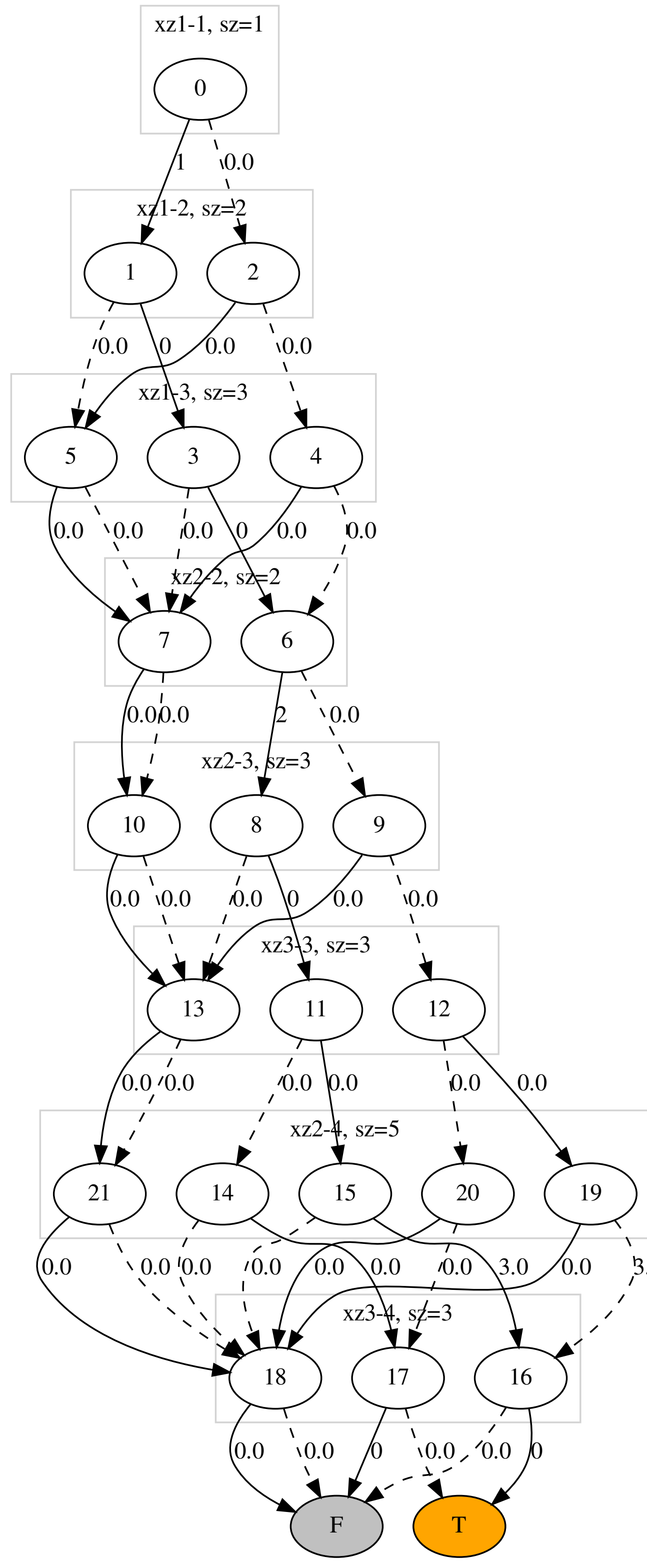
(a) Covering BDD

(b) Availability BDD

Figure 2: BDDs generated to encode the instance from Figure 1.



(a) Covering BDD



(b) Availability BDD

Figure 3: BDDs generated to encode the instance from Figure 1: after alignment.

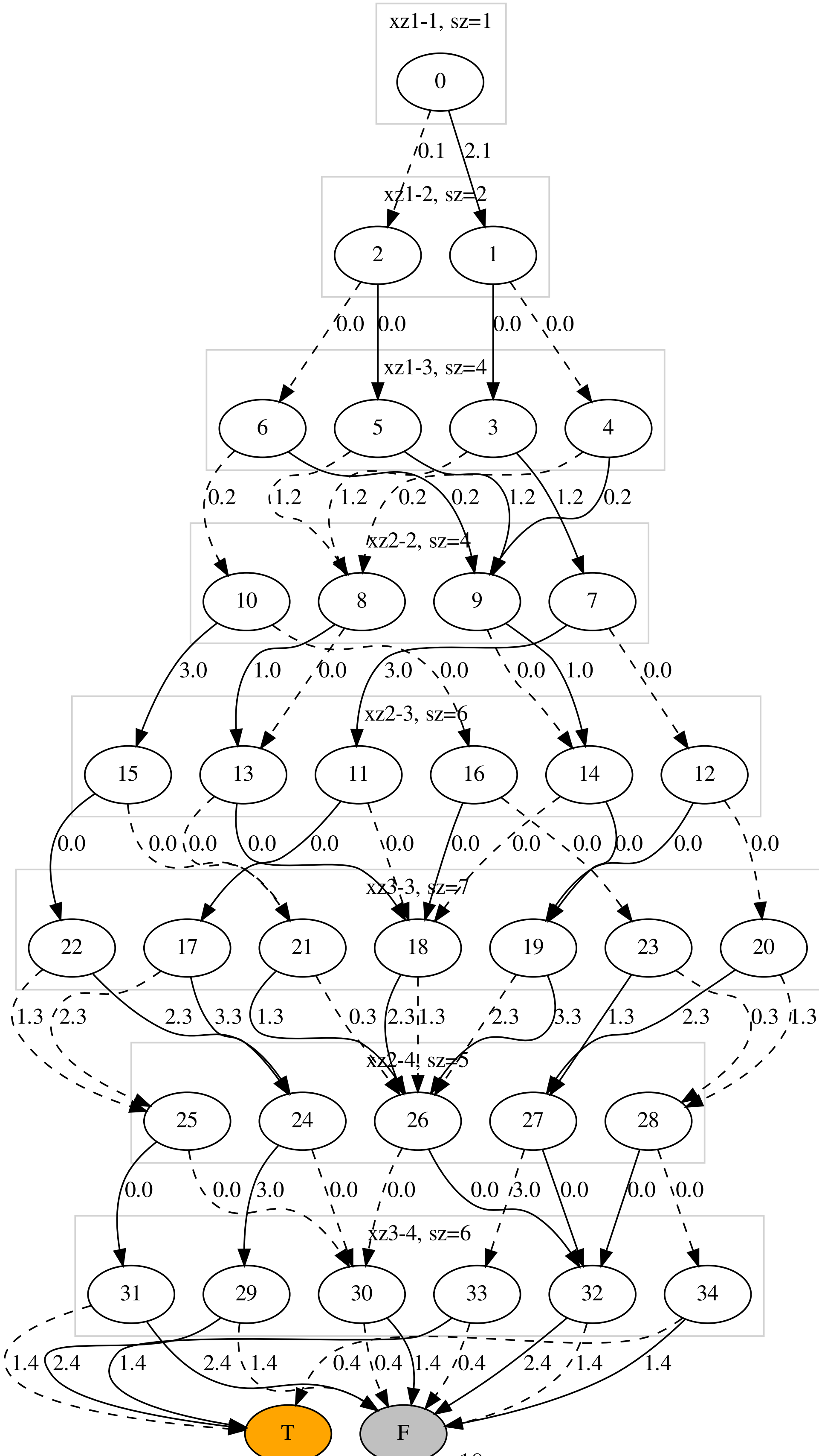
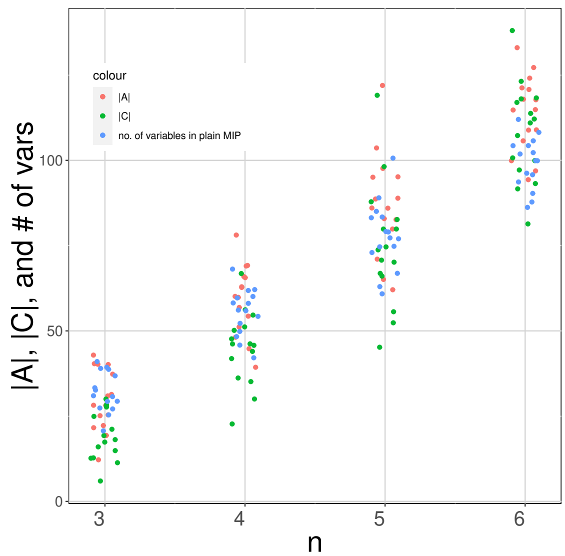
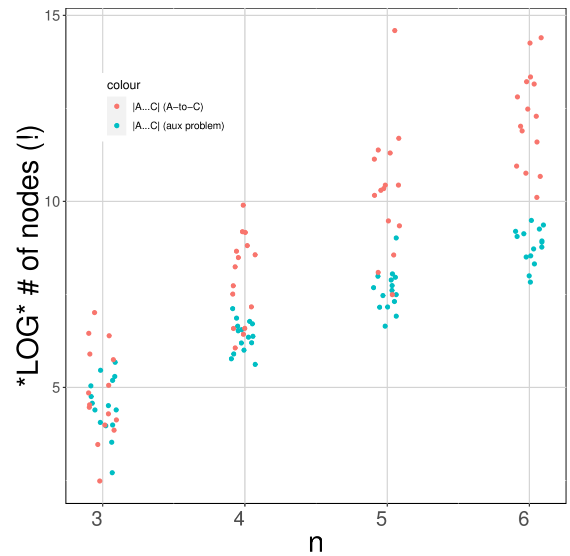

xz1-1, sz=1
0
xz1-2, sz=2
2
1
xz1-3, sz=4
6
5
3
4
xz2-2, sz=4
10
8
9
7
xz2-3, sz=6
15
13
11
16
14
12
xz3-3, sz=7
22
17
21
18
19
23
20
xz2-4, sz=5
25
24
26
27
28
xz3-4, sz=6
31
29
30
33
32
34
T
F

Table 1: Network flow constraints (for the intersection BDD).

Type	**Constraint**
cont-at-0	$-1.0v_{0\to1,h} - 1.0v_{0\to2,l} = -1.0$
cont-at-1	$v_{0\to1,h} - 1.0v_{1\to3,h} - 1.0v_{1\to4,l} = 0.0$
cont-at-2	$v_{0\to2,l} - 1.0v_{2\to5,h} - 1.0v_{2\to6,l} = 0.0$
cont-at-6	$v_{2\to6,l} - 1.0v_{6\to9,h} - 1.0v_{6\to10,l} = 0.0$
cont-at-5	$v_{2\to5,h} - 1.0v_{5\to9,h} - 1.0v_{5\to8,l} = 0.0$
cont-at-4	$v_{1\to4,l} - 1.0v_{4\to9,h} - 1.0v_{4\to8,l} = 0.0$
cont-at-3	$v_{1\to3,h} - 1.0v_{3\to7,h} - 1.0v_{3\to8,l} = 0.0$
cont-at-10	$v_{6\to10,l} - 1.0v_{10\to15,h} - 1.0v_{10\to16,l} = 0.0$
cont-at-7	$v_{3\to7,h} - 1.0v_{7\to11,h} - 1.0v_{7\to12,l} = 0.0$
cont-at-9	$v_{6\to9,h} + v_{5\to9,h} + v_{4\to9,h} - 1.0v_{9\to14,h} - 1.0v_{9\to14,l} = 0.0$
cont-at-8	$v_{5\to8,l} + v_{4\to8,l} + v_{3\to8,l} - 1.0v_{8\to13,h} - 1.0v_{8\to13,l} = 0.0$
cont-at-16	$v_{10\to16,l} - 1.0v_{16\to18,h} - 1.0v_{16\to23,l} = 0.0$
cont-at-11	$v_{7\to11,h} - 1.0v_{11\to17,h} - 1.0v_{11\to18,l} = 0.0$
cont-at-12	$v_{7\to12,l} - 1.0v_{12\to19,h} - 1.0v_{12\to20,l} = 0.0$
cont-at-14	$v_{9\to14,h} + v_{9\to14,l} - 1.0v_{14\to19,h} - 1.0v_{14\to18,l} = 0.0$
cont-at-15	$v_{10\to15,h} - 1.0v_{15\to22,h} - 1.0v_{15\to21,l} = 0.0$
cont-at-13	$v_{8\to13,h} + v_{8\to13,l} - 1.0v_{13\to18,h} - 1.0v_{13\to21,l} = 0.0$
cont-at-21	$v_{15\to21,l} + v_{13\to21,l} - 1.0v_{21\to26,h} - 1.0v_{21\to26,l} = 0.0$
cont-at-17	$v_{11\to17,h} - 1.0v_{17\to24,h} - 1.0v_{17\to25,l} = 0.0$
cont-at-22	$v_{15\to22,h} - 1.0v_{22\to24,h} - 1.0v_{22\to25,l} = 0.0$
cont-at-19	$v_{12\to19,h} + v_{14\to19,h} - 1.0v_{19\to26,h} - 1.0v_{19\to26,l} = 0.0$
cont-at-23	$v_{16\to23,l} - 1.0v_{23\to27,h} - 1.0v_{23\to28,l} = 0.0$
cont-at-18	$v_{16\to18,h} + v_{11\to18,l} + v_{14\to18,l} + v_{13\to18,h} - 1.0v_{18\to26,h} - 1.0v_{18\to26,l} = 0.0$
cont-at-20	$v_{12\to20,l} - 1.0v_{20\to27,h} - 1.0v_{20\to28,l} = 0.0$
cont-at-27	$v_{23\to27,h} + v_{20\to27,h} - 1.0v_{27\to32,h} - 1.0v_{27\to33,l} = 0.0$
cont-at-25	$v_{17\to25,l} + v_{22\to25,l} - 1.0v_{25\to31,h} - 1.0v_{25\to30,l} = 0.0$
cont-at-24	$v_{17\to24,h} + v_{22\to24,h} - 1.0v_{24\to29,h} - 1.0v_{24\to30,l} = 0.0$
cont-at-26	$v_{21\to26,h} + v_{21\to26,l} + v_{19\to26,h} + v_{19\to26,l} + v_{18\to26,h} + v_{18\to26,l}$ $-1.0v_{26\to32,h} - 1.0v_{26\to30,l} = 0.0$
cont-at-28	$v_{23\to28,l} + v_{20\to28,l} - 1.0v_{28\to32,h} - 1.0v_{28\to34,l} = 0.0$
cont-at-34	$v_{28\to34,l} - 1.0v_{34\to F,h} - 1.0v_{34\to T,l} = 0.0$
cont-at-30	$v_{25\to30,l} + v_{24\to30,l} + v_{26\to30,l} - 1.0v_{30\to F,h} - 1.0v_{30\to F,l} = 0.0$
cont-at-29	$v_{24\to29,h} - 1.0v_{29\to T,h} - 1.0v_{29\to F,l} = 0.0$
cont-at-33	$v_{27\to33,l} - 1.0v_{33\to T,h} - 1.0v_{33\to F,l} = 0.0$
cont-at-32	$v_{27\to32,h} + v_{26\to32,h} + v_{28\to32,h} - 1.0v_{32\to F,h} - 1.0v_{32\to F,l} = 0.0$
cont-at-31	$v_{25\to31,h} - 1.0v_{31\to F,h} - 1.0v_{31\to T,l} = 0.0$
cont-at-T	$v_{34\to T,l} + v_{29\to T,h} + v_{33\to T,h} + v_{31\to T,l} = 1.0$
cont-at-F	$v_{34\to F,h} + v_{30\to F,h} + v_{30\to F,l} + v_{29\to F,l} + v_{33\to F,l} + v_{32\to F,h} + v_{32\to F,l} + v_{31\to F,h} = 0.0$



(a) Diagram growth as number of facilities increases.



(b) Intersection diagram growth as number of facilities increases.

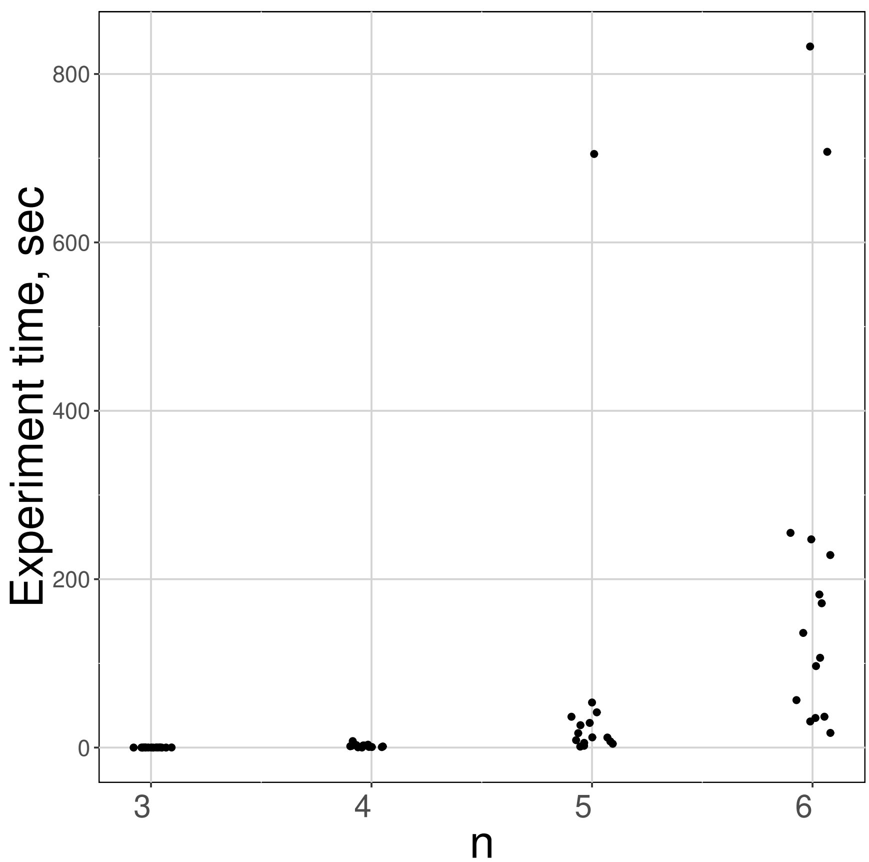


Figure 6: Experiment runtimes, seconds