February 16, 2021: CPP from BDDs (simple example), updated.

1 Objective and general notes.

$$\begin{aligned}\text{Minimize:}\quad & 0.1v^A_{0\to1,h} + 0.2v^A_{9\to11,h} + 0.2v^A_{10\to13,h} + 0.3v^A_{20\to22,h} + 0.3v^A_{21\to24,h} + \\ & v^C_{0\to1,h} + 2.0v^C_{3\to4,h} + v^C_{3\to4,l} + v^C_{2\to4,h} + \\ & 2.0v^C_{8\to10,h} + v^C_{8\to10,l} + 3.0v^C_{9\to10,h} + 2.0v^C_{9\to10,l} + v^C_{7\to10,h} + \\ & v^C_{11\to T,h} + 2.0v^C_{12\to T,h} + v^C_{12\to T,l}\end{aligned}$$

Here, for example, variable $v^A_{0\to1,h}$ corresponds to the flow from node ⓪ to node ① of diagram A (availability), along the "hi" ("yes") arc.

Legend.

- From each diagram, two types of constraints are generated:
  - *cont-at-⊙* are flow continuity constraints at a given node.
  - *bin-link-<k>* are binary linking constraints (needed to link two BDDs – i.e., tangle network flow problems), one per layer, indexed with k.
- A denotes "Availability" diagram, C denotes "Covering" diagram.

All node numbers correspond to the diagrams and have nothing to do with customer and facility indices.

- Arc flow variables (continuous) v.
- Linking variables (binary): $[\lambda_{x1}, \lambda_{z1-1}, \lambda_{z1-2}, \lambda_{z1-3}, \lambda_{x2}, \lambda_{z2-2}, \lambda_{z2-3}, \lambda_{z2-4}, \lambda_{x3}, \lambda_{z3-3}, \lambda_{z3-4}]$.

  Note: All this is generated automatically. That is,
  - **Input:** "customer proximity" lists S_j (customers vs. facility proximity), (no covering costs c_{ij} anymore), switch-on costs f_i, overlap penalty $g_j(n_j)$.
  - **Output:** (weighted) BDDs: covering and availability. CPP MiP (what is below: already a snapshot of a Gurobi model object: output of `gurobipy.Model.display()` converted to LaTeX with a semi-automatic procedure.

2 Constraints from Availability diagram.

Type	Constraint
A: cont-at-0	$-1.0v^A_{0\to1,h} - 1.0v^A_{0\to2,l} = -1.0$
A: bin-link-1	$\lambda_{x1} - 1.0v^A_{0\to1,h} = 0.0$
A: cont-at-1	$v^A_{0\to1,h} - 1.0v^A_{1\to3,h} - 1.0v^A_{1\to5,l} = 0.0$
A: cont-at-2	$v^A_{0\to2,l} - 1.0v^A_{2\to5,h} - 1.0v^A_{2\to4,l} = 0.0$
A: bin-link-2	$\lambda_{z1-1} - 1.0v^A_{1\to3,h} - 1.0v^A_{2\to5,h} = 0.0$
A: cont-at-3	$v^A_{1\to3,h} - 1.0v^A_{3\to6,h} - 1.0v^A_{3\to8,l} = 0.0$
A: cont-at-4	$v^A_{2\to4,l} - 1.0v^A_{4\to8,h} - 1.0v^A_{4\to7,l} = 0.0$
A: cont-at-5	$v^A_{1\to5,l} + v^A_{2\to5,h} - 1.0v^A_{5\to8,h} - 1.0v^A_{5\to8,l} = 0.0$
A: bin-link-3	$\lambda_{z1-2} - 1.0v^A_{3\to6,h} - 1.0v^A_{4\to8,h} - 1.0v^A_{5\to8,h} = 0.0$
A: cont-at-8	$v^A_{3\to8,l} + v^A_{4\to8,h} + v^A_{5\to8,h} + v^A_{5\to8,l} - 1.0v^A_{8\to10,h} - 1.0v^A_{8\to10,l} = 0.0$
A: cont-at-6	$v^A_{3\to6,h} - 1.0v^A_{6\to9,h} - 1.0v^A_{6\to10,l} = 0.0$
A: cont-at-7	$v^A_{4\to7,l} - 1.0v^A_{7\to10,h} - 1.0v^A_{7\to9,l} = 0.0$
A: bin-link-4	$\lambda_{z1-3} - 1.0v^A_{8\to10,h} - 1.0v^A_{6\to9,h} - 1.0v^A_{7\to10,h} = 0.0$
A: cont-at-9	$v^A_{6\to9,h} + v^A_{7\to9,l} - 1.0v^A_{9\to11,h} - 1.0v^A_{9\to12,l} = 0.0$
A: cont-at-10	$v^A_{8\to10,h} + v^A_{8\to10,l} + v^A_{6\to10,l} + v^A_{7\to10,h} - 1.0v^A_{10\to13,h} - 1.0v^A_{10\to13,l} = 0.0$
A: bin-link-5	$\lambda_{x2} - 1.0v^A_{9\to11,h} - 1.0v^A_{10\to13,h} = 0.0$
A: cont-at-11	$v^A_{9\to11,h} - 1.0v^A_{11\to14,h} - 1.0v^A_{11\to16,l} = 0.0$
A: cont-at-12	$v^A_{9\to12,l} - 1.0v^A_{12\to16,h} - 1.0v^A_{12\to15,l} = 0.0$
A: cont-at-13	$v^A_{10\to13,h} + v^A_{10\to13,l} - 1.0v^A_{13\to16,h} - 1.0v^A_{13\to16,l} = 0.0$
A: bin-link-6	$\lambda_{z2-2} - 1.0v^A_{11\to14,h} - 1.0v^A_{12\to16,h} - 1.0v^A_{13\to16,h} = 0.0$
A: cont-at-14	$v^A_{11\to14,h} - 1.0v^A_{14\to17,h} - 1.0v^A_{14\to19,l} = 0.0$
A: cont-at-16	$v^A_{11\to16,l} + v^A_{12\to16,h} + v^A_{13\to16,h} + v^A_{13\to16,l} - 1.0v^A_{16\to19,h} - 1.0v^A_{16\to19,l} = 0.0$
A: cont-at-15	$v^A_{12\to15,l} - 1.0v^A_{15\to19,h} - 1.0v^A_{15\to18,l} = 0.0$
A: bin-link-7	$\lambda_{z2-3} - 1.0v^A_{14\to17,h} - 1.0v^A_{16\to19,h} - 1.0v^A_{15\to19,h} = 0.0$
A: cont-at-17	$v^A_{14\to17,h} - 1.0v^A_{17\to20,h} - 1.0v^A_{17\to21,l} = 0.0$
A: cont-at-18	$v^A_{15\to18,l} - 1.0v^A_{18\to21,h} - 1.0v^A_{18\to20,l} = 0.0$
A: cont-at-19	$v^A_{14\to19,l} + v^A_{16\to19,h} + v^A_{16\to19,l} + v^A_{15\to19,h} - 1.0v^A_{19\to21,h} - 1.0v^A_{19\to21,l} = 0.0$
A: bin-link-8	$\lambda_{z2-4} - 1.0v^A_{17\to20,h} - 1.0v^A_{18\to21,h} - 1.0v^A_{19\to21,h} = 0.0$
A: cont-at-20	$v^A_{17\to20,h} + v^A_{18\to20,l} - 1.0v^A_{20\to22,h} - 1.0v^A_{20\to23,l} = 0.0$
A: cont-at-21	$v^A_{17\to21,l} + v^A_{18\to21,h} + v^A_{19\to21,h} + v^A_{19\to21,l} - 1.0v^A_{21\to24,h} - 1.0v^A_{21\to24,l} = 0.0$
A: bin-link-9	$\lambda_{x3} - 1.0v^A_{20\to22,h} - 1.0v^A_{21\to24,h} = 0.0$
A: cont-at-24	$v^A_{21\to24,h} + v^A_{21\to24,l} - 1.0v^A_{24\to27,h} - 1.0v^A_{24\to27,l} = 0.0$
A: cont-at-23	$v^A_{20\to23,l} - 1.0v^A_{23\to27,h} - 1.0v^A_{23\to26,l} = 0.0$
A: cont-at-22	$v^A_{20\to22,h} - 1.0v^A_{22\to25,h} - 1.0v^A_{22\to27,l} = 0.0$
A: bin-link-10	$\lambda_{z3-3} - 1.0v^A_{24\to27,h} - 1.0v^A_{23\to27,h} - 1.0v^A_{22\to25,h} = 0.0$
A: cont-at-25	$v^A_{22\to25,h} - 1.0v^A_{25\to T,h} - 1.0v^A_{25\to F,l} = 0.0$
A: cont-at-26	$v^A_{23\to26,l} - 1.0v^A_{26\to F,h} - 1.0v^A_{26\to T,l} = 0.0$
A: cont-at-27	$v^A_{24\to27,h} + v^A_{24\to27,l} + v^A_{23\to27,h} + v^A_{22\to27,l} - 1.0v^A_{27\to F,h} - 1.0v^A_{27\to F,l} = 0.0$
A: bin-link-11	$\lambda_{z3-4} - 1.0v^A_{25\to T,h} - 1.0v^A_{26\to F,h} - 1.0v^A_{27\to F,h} = 0.0$
A: cont-at-T	$v^A_{25\to T,h} + v^A_{26\to T,l} = 1.0$
A: cont-at-F	$v^A_{25\to F,l} + v^A_{26\to F,h} + v^A_{27\to F,h} + v^A_{27\to F,l} = 0.0$

3 Constraints from Covering diagram.

Type	Constraint
C: cont-at-0	$-1.0v^C_{0\to1,h} - 1.0v^C_{0\to1,l} = -1.0$
C: bin-link-1	$\lambda_{z1-1} - 1.0v^C_{0\to1,h} = 0.0$
C: cont-at-1	$v^C_{0\to1,h} + v^C_{0\to1,l} - 1.0v^C_{1\to3,h} - 1.0v^C_{1\to2,l} = 0.0$
C: bin-link-2	$\lambda_{z1-2} - 1.0v^C_{1\to3,h} = 0.0$
C: cont-at-3	$v^C_{1\to3,h} - 1.0v^C_{3\to4,h} - 1.0v^C_{3\to4,l} = 0.0$
C: cont-at-2	$v^C_{1\to2,l} - 1.0v^C_{2\to4,h} - 1.0v^C_{2\to4,l} = 0.0$
C: bin-link-3	$\lambda_{z2-2} - 1.0v^C_{3\to4,h} - 1.0v^C_{2\to4,h} = 0.0$
C: cont-at-4	$v^C_{3\to4,h} + v^C_{3\to4,l} + v^C_{2\to4,h} + v^C_{2\to4,l} - 1.0v^C_{4\to6,h} - 1.0v^C_{4\to5,l} = 0.0$
C: bin-link-4	$\lambda_{z1-3} - 1.0v^C_{4\to6,h} = 0.0$
C: cont-at-6	$v^C_{4\to6,h} - 1.0v^C_{6\to9,h} - 1.0v^C_{6\to8,l} = 0.0$
C: cont-at-5	$v^C_{4\to5,l} - 1.0v^C_{5\to8,h} - 1.0v^C_{5\to7,l} = 0.0$
C: bin-link-5	$\lambda_{z2-3} - 1.0v^C_{6\to9,h} - 1.0v^C_{5\to8,h} = 0.0$
C: cont-at-8	$v^C_{6\to8,l} + v^C_{5\to8,h} - 1.0v^C_{8\to10,h} - 1.0v^C_{8\to10,l} = 0.0$
C: cont-at-9	$v^C_{6\to9,h} - 1.0v^C_{9\to10,h} - 1.0v^C_{9\to10,l} = 0.0$
C: cont-at-7	$v^C_{5\to7,l} - 1.0v^C_{7\to10,h} - 1.0v^C_{7\to10,l} = 0.0$
C: bin-link-6	$\lambda_{z3-3} - 1.0v^C_{8\to10,h} - 1.0v^C_{9\to10,h} - 1.0v^C_{7\to10,h} = 0.0$
C: cont-at-10	$v^C_{8\to10,h} + v^C_{8\to10,l} + v^C_{9\to10,h} + v^C_{9\to10,l} + v^C_{7\to10,h} + v^C_{7\to10,l} - 1.0v^C_{10\to12,h} - 1.0v^C_{10\to11,l} = 0.0$
C: bin-link-7	$\lambda_{z2-4} - 1.0v^C_{10\to12,h} = 0.0$
C: cont-at-11	$v^C_{10\to11,l} - 1.0v^C_{11\to T,h} - 1.0v^C_{11\to T,l} = 0.0$
C: cont-at-12	$v^C_{10\to12,h} - 1.0v^C_{12\to T,h} - 1.0v^C_{12\to T,l} = 0.0$
C: bin-link-8	$\lambda_{z3-4} - 1.0v^C_{11\to T,h} - 1.0v^C_{12\to T,h} = 0.0$
C: cont-at-T	$v^C_{11\to T,h} + v^C_{11\to T,l} + v^C_{12\to T,h} + v^C_{12\to T,l} = 1.0$
C: cont-at-F	$0.0 = 0.0$