Align-BDD application: on joint UFLP.

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1 Summary and questions TBD

- weak point: CPP MIP
- weak point: feeding more info to the DD-based methods.
- weak point: not that much variety in graph topologies.
- question: not equivalent to another UFLP? I guess not (consider j10 and j16).

2 Joint-UFLP formulation

We next illustrate the performance of our algorithms in the context of a specific application. We will consider the following variant of the uncapacitated facility location problem, UFLP (Owen and Mark S. Daskin, 1998; ReVelle, Eiselt, and M. S. Daskin, 2008), which we call the "joint uncapacitated facility location problem," j-UFLP. We have designed this problem specifically to highlight a possible application of our heuristic. Therefore, it comprises two interconnected, but comparable subproblems, which can be represented as BDDs.

To introduce j-UFLP, let us first consider a simple UFLP formulation. This problem considers a set of N points. At each point i, we can locate a facility at a cost given by c_i, covering all points in a set given by S_i, where $i \in S_i$. (Set S_i might refer to customers that are sufficiently close to location i according to some specified metric, like distance or travel time.) We also define a cost function $f_i : \mathbb{N}_0 \to \mathbb{R}$, so that $f_i(a)$ would indicate a cost of covering point i exactly a_i times. Note that we do not imply any properties of f_i, such as convexity or concavity. This UFLP variant minimizes the total cost and can be formulated as follows:

$$\min \sum_{i=1}^{N} \Big(c_i x_i + f_i(a_i) \Big) \tag{1a}$$

$$\text{s.t. } a_i = \sum_{j \in S_i} x_i \qquad \text{for all } i = 1, \ldots, N, \tag{1b}$$

$$x_i \in \{0, 1\} \qquad \text{for all } i = 1, \ldots, N. \tag{1c}$$

In our numerical experiments, we consider j-UFLP, which comprises two such problems, interconnected by having certain common variables. For a set of common variables $J \subseteq \{1, \ldots, N\}$, the j-UFLP is formulated as follows:

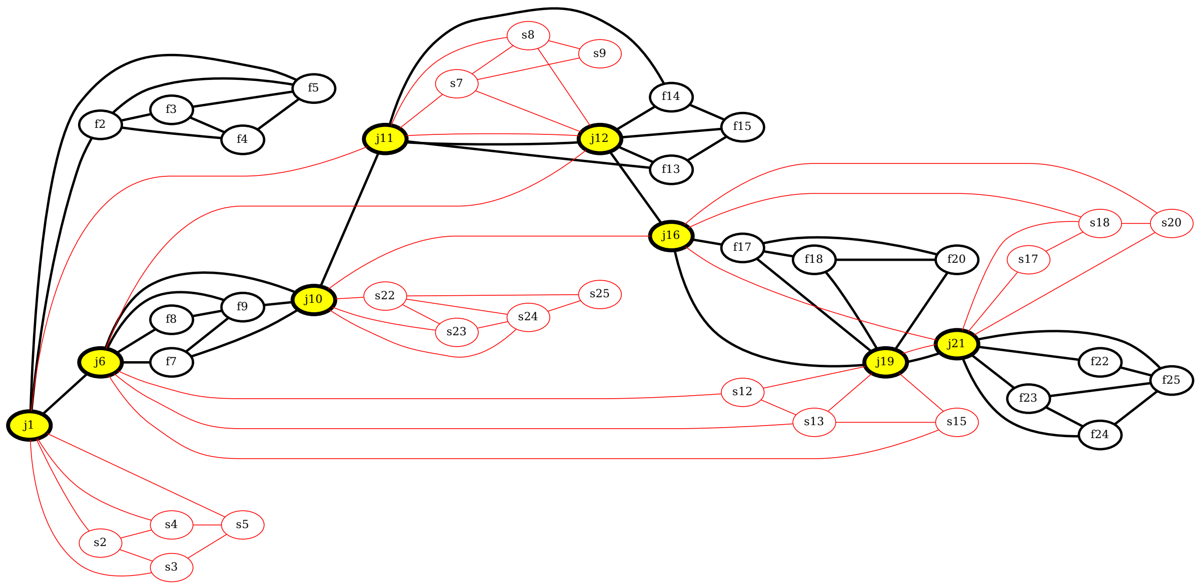

Figure 1: Sample j-UFLP instance graph.

$$\min \sum_{i=1,t=1,2}^{N} \left(c_i^t x_i^t + f_i^t(a_i^t) \right) \tag{j-UFLP}$$

$$\text{s.t. } a_i^t = \sum_{j \in S_i^t} x_i^t \qquad \text{for all } i = 1, \ldots, N, t = 1, 2, \tag{2a}$$

$$x_i^t \in \{0, 1\} \qquad \text{for all } i = 1, \ldots, N, t = 1, 2, \tag{2b}$$

$$x_j^1 = x_j^2 \qquad \text{for all } j \in J. \tag{2c}$$

Further, to make this problem hard for an MIP solver, we induce the following special structure for the underlying graph. We assume that each sub-problem graph consists of n subgraphs, M nodes each, and that there is at most one arc between every two subgraphs. Moreover, we assume J to be the set of endpoints of such arcs between the subgraphs. An example of such instance is presented in Figure 1. A subgraph corresponding to the first subproblem is depicted with thick black lines, while the one corresponding to the second subproblem has thin red lines. Nodes that are common for the two subgraphs (corresponding to conditions (3d)) are marked with background color (nodes `j1`, `j6`, `j10`–`j12`, `j16`, `j19`, and `j21`). Note that overlaps are calculated independently: e.g., locating a facility at `j10` does contribute to a_{16}^2, but not a_{16}^1.

A naive MIP reformulation for (2) implies introducing new binary variables $y_{i,a}^t$ indicating whether point i in subproblem t was covered at least a times. Therefore, the equivalent problem is:

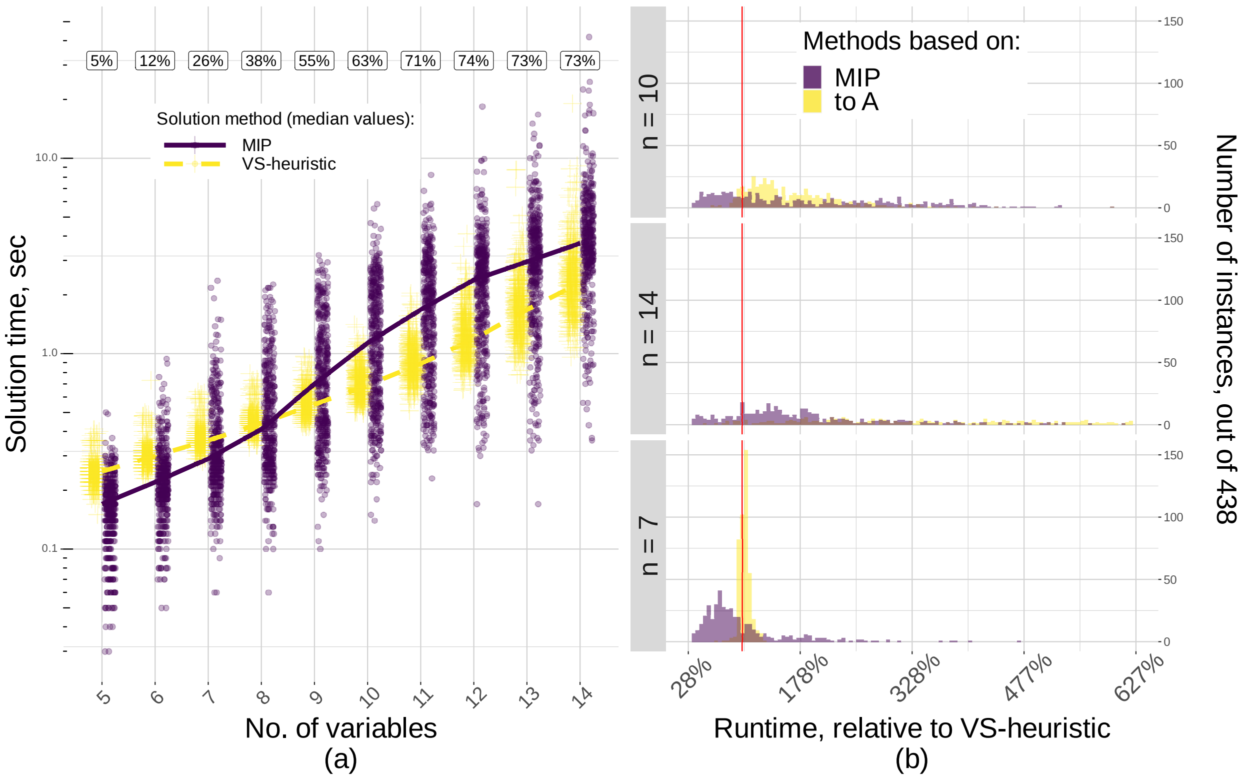

Figure 2: Numerical performance of various heuristics for j-UFLP.

$$\min \sum_{i=1,t=1,2}^{N} \left(c_i^t x_i^t + \sum_{a=0}^{|S_i^t|} q_{i,a}^t y_{i,a}^t \right) + C \qquad \text{(j-UFLP-MIP)}$$

$$\text{s.t.} \sum_{a=1}^{|S_i|} y_{i,a}^t = \sum_{j \in S_i^t} x_i^t \qquad \text{for all } i = 1, \ldots, N, t = 1, 2, \qquad (3a)$$

$$y_{i,a}^t \geq y_{i,a+1}^t \qquad \text{for all } i = 1, \ldots, N, t = 1, 2, a = 0, \ldots, |S_i|, \qquad (3b)$$

$$x_i^t \in \{0, 1\} \qquad \text{for all } i = 1, \ldots, N, t = 1, 2, \qquad (3c)$$

$$x_j^1 = x_j^2 \qquad \text{for all } j \in J, \qquad (3d)$$

where $q_{i,a} = f_i^t(a) - f_i^t(a-1)$ and $C = \sum_{i,t} f_i^t(0)$ are constants.

3 Numerical performance

4 References

References

Owen, Susan Hesse and Mark S. Daskin (Dec. 1998). "Strategic Facility Location: A Review". In: *European Journal of Operational Research* 111.3, pp. 423–447. ISSN: 03772217. DOI: 10.1016/S0377-2217(98)00186-6 (cit. on p. 1).

ReVelle, C. S., H. A. Eiselt, and M. S. Daskin (Feb. 2008). “A Bibliography for Some Fundamental Problem Categories in Discrete Location Science”. In: *European Journal of Operational Research* 184.3, pp. 817–848. ISSN: 0377-2217. DOI: `10.1016/j.ejor.2006.12.044` (cit. on p. 1).