1. $L(\lambda_1, x, y) = 3 - 8x + 6y + \lambda_1 \cdot (x^2 + y^2 = 36)$

$L'_x = -8 + \lambda_1 \cdot 2x = 0 \qquad x = \frac{8}{\lambda_1 \cdot 2} = \frac{4}{\lambda_1}$

$L'_y = 6 + \lambda_1 \cdot 2y = 0 \qquad y = \frac{-6}{\lambda_1 \cdot 2} = -\frac{3}{\lambda_1}$

$L'_{\lambda_1} = x^2 + y^2 - 36 = 0 \qquad \frac{16}{\lambda^2} + \frac{9}{\lambda^2} - 36 = 0 \qquad \frac{25}{\lambda^2} = 36 \qquad \lambda^2 = \frac{25}{36} \qquad \lambda = \frac{5}{6}, -\frac{5}{6}$

$\left(\frac{24}{5}, \frac{18}{5}, \frac{5}{6}\right), \left(-\frac{24}{5}, \frac{18}{5}, -\frac{5}{6}\right)$

$L''_{xx} = 2\lambda_1 \qquad L''_{yy} = 2\lambda_1 \qquad L''_{\lambda\lambda} = 0$

$L''_{xy} = 0 \qquad L''_{x\lambda} = 2x \qquad L''_{y\lambda} = 2y$

$$\begin{pmatrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{pmatrix} = -8\lambda_1 (x^2 + y^2)$$

2. $L(\lambda_1, x, y) = 2x^2 + 12xy + 32y^2 + 15 + \lambda_1 (x^2 + 16y^2 - 64)$

$L'_x = 4x + 12y + 2\lambda_1 x = 0 \qquad x(4 + 2\lambda_1) = -12y \qquad x = \frac{-12y}{4 + 2\lambda_1}$

$L'_y = 12x + 64y + 32\lambda_1 y = 0 \qquad y(64 + 32\lambda_1) = -12x \qquad y = \frac{-12x}{64 + 32\lambda}$

$L'_\lambda = x^2 + 16y^2 = 64$

$$\frac{-12}{4 + 2\lambda_1} \cdot \frac{-12x}{64 + 32\lambda_1} + 64 \cdot \left(\frac{-12x}{64 + 32\lambda_1}\right)^2 =$$

$$x = \frac{-12}{4 + 2\lambda} \cdot \frac{-12x}{64 + 32\lambda} = \frac{144x}{16(4 + 2\lambda)^2}$$

$16(4 + 2\lambda)^2 = 144$

$4 + 2\lambda = \frac{12}{4} = 3$

$\lambda = -\frac{1}{2} \qquad x = 0 \qquad y = 0 \qquad \left(0, 0, -\frac{1}{2}\right)$

$L''_{xx} = 4 + 2\lambda \qquad L''_{yy} = 64 + 32\lambda \qquad L''_{\lambda\lambda} = 0$

$L''_{xy} = 12 \qquad L''_{x\lambda} = 2\lambda \qquad L''_{y\lambda} = 32$

$$\begin{pmatrix} 0 & 2\lambda & 32 \\ 2\lambda & 4 + 2\lambda & 12 \\ 32 & 12 & 64 + 32\lambda \end{pmatrix} = 0 - 2\lambda\left(2\lambda \cdot (64 + 32\lambda) - 12 \cdot 32\right) + 32\left(2\lambda \cdot 12 - 32 \cdot (4 + 2\lambda)\right)$$

$= 1 \cdot (2 \cdot 80 - 384) + 32 \cdot (12 - 32 \cdot 5) =$

$-224 - 7536 = -1760 < 0$ – т. минимум

3. $\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial u}{\partial y} = 2y \qquad \frac{\partial u}{\partial z} = 2z$

$\frac{\partial u}{\partial x}\Big|_{(8,-12,9)} = 16 \qquad \frac{\partial u}{\partial y}\Big|_{(8,-12,9)} = -24 \qquad \frac{\partial u}{\partial z}\Big|_{(8,-12,9)} = 18$

$|\bar{c}| = \sqrt{81+64+144} = 17$

$\bar{c}_0 = \left(-\frac{9}{17}, \frac{8}{17}, -\frac{12}{17}\right)$

$\frac{\partial u}{\partial l}\Big|_{n_0} = -\frac{9}{17} \cdot 16 + \frac{8}{17} \cdot (-24) - \frac{12}{17} \cdot 18 = (-144 - 192 - 218)/17 = -\frac{552}{17}$

4. $\frac{\partial u}{\partial x} = 2x \cdot e^{x^2+y^2+z^2} \qquad \frac{\partial u}{\partial y} = 2y + e^{x^2+y^2+z^2} \qquad \frac{\partial u}{\partial z} = 2z \cdot e^{x^2+y^2+z^2}$

$\frac{\partial u}{\partial x}\Big|_{L} = -32 \cdot e^{441} \qquad \frac{\partial u}{\partial y}\Big|_{L} = 8 \cdot e^{441} \qquad \frac{\partial u}{\partial z}\Big|_{L} = -26 \cdot e^{441}$

$|\bar{d}| = \sqrt{16+169+256} = 21$

$\bar{d}_0 = \left(\frac{4}{21}, -\frac{13}{21}, -\frac{16}{21}\right)$

$\frac{\partial u}{\partial d}\Big|_{L} = \frac{4}{21} \cdot (-32 e^{441}) - \frac{13}{21} \cdot 8 \cdot e^{441} - \frac{16}{21} \cdot (-26 \cdot e^{441}) =$

$\frac{e^{441}}{21} \cdot (-128 - 104 - 416) = \frac{648 \cdot e^{441}}{21}$