How to Interpret Nonlinear Regression Coefficients

Summary

Name	**Model**	**Interpretation**
Level-level (basic regression)	$y = \beta_0 + \beta_1 x + \epsilon$	As x increases one unit, y changes by b_1 units
Level-log (logarithmic)	$y = \beta_0 + \beta_1 \ln(x) + \epsilon$	As x increases 1%, y changes by (b_1/100) units
Log-level (exponential)	$\ln(y) = \beta_0 + \beta_1 x + \epsilon$	As x increases 1 unit, y changes by (b_1*100)%
Log-log (elasticities)	$\ln(y) = \beta_0 + \beta_1 \ln(x) + \epsilon$	As x increases 1%, y changes by b_1%
Quadratic model	$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$	As x increases 1 unit, y changes $b_1 + 2b_2 x$

Extended Examples

Suppose we have three explanatory variables:

x_1	a continuous variable
$\ln(x_2)$	the natural log of a continuous variable
x_3	a dummy variable that equals 1 or 0

Suppose we have three regression models. Each model uses the same explanatory variables listed above, but each uses a different dependent variable:

y_1	a continuous variable
$\ln(y_2)$	the natural log of a continuous variable
y_3	a dummy variable that equals 1 or 0

Below each model is a description of how to interpret each regression coefficient.

Model 1: $\boldsymbol{y_1 = \beta_0 + \beta_1 x_1 + \beta_2 ln(x_2) + \beta_3 x_3 + \epsilon}$

β_1 = a one-unit change in x_1 generates a β_1 unit change in y_1
β_2 = a 100% change in x_2 generates a β_2 unit change in y_1
β_1 = a switch in x_3 from 0 to 1 generates a β_3 unit change in y_1

Model 2: $\boldsymbol{ln(y_2) = \beta_0 + \beta_1 x_1 + \beta_2 ln(x_2) + \beta_3 x_3 + \epsilon}$

β_1 = a one-unit change in x_1 generates a 100*β_1 percent change in y_2
β_2 = a 100% change in x_2 generates a 100*β_2 percent change in y_2
β_1 = a switch in x_3 from 0 to 1 generates a 100*β_3 percent change in y_2

Model 3: $\boldsymbol{y_3 = \beta_0 + \beta_1 x_1 + \beta_2 ln(x_2) + \beta_3 x_3 + \epsilon}$

β_1 = a one-unit change in x_1 generates a 100*β_1 percentage point change in the probability y_3 occurs
β_2 = a 100% change in x_2 generates a 100*β_2 percentage point change in the probability y_3 occurs
β_1 = a switch in x_3 from 0 to 1 generates a 100*β_3 percentage point change in the probability y_3 occurs