Increasing Quadrotor Endurance Through Momentum Analysis

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***Abstract*— Determining an aircraft's velocity that maximizes its range or endurance is a long solved problem in aerodynamics and flight mechanics. However, due to the dynamic limitations of quadrotors a decade ago these solutions have been widely ignored in robotic applications. We present experimental validation that incorporating solutions derived from momentum analysis can increase a quadrotor's endurance by as much as 20%. We focus on quadrotors with a camera payload and demonstrate the advantages of orbiting a point-of-interest rather than hovering above it.**

I. INTRODUCTION

The last decade has seen quadrotor helicopters explode in popularity. From an emerging unmanned aerial vehicle (UAV) concept to a prominent research and commercial platform [1], [2] quadrotors have become the nearly ubiquitous aerial robot. Their relative low cost and the simplicity of their dynamics when near hover [3] has made them popular in numerous applications [4], [5], [6].

The condition of operating near hover also introduces the quadrotor platform's greatest weakness, power efficiency. The power required to keep a quadrotor at hover is approximately 200 W per kg [1]. Since quadrotors are rarely flown dynamically far away from hover when used in application, the problem of power efficiency creates a practical limit on their utility. This limit restricts the size of an area that can be explored, the number of images that can be captured by a camera, the mass of potential payloads, etc.. In robotics, maximizing endurance is mostly thought of as a design problem handled by manufacturers. In this paper we present evidence that choices about trajectory and velocity also have a meaningful effect on flight time.

Determining an aerial vehicle's endurance is a common problem in flight mechanics. Solutions for fixed wing and rotor aircraft maximum endurance in steady, forward flight are well known and widely used [7], [8]. Maximum endurance is achieved by traveling at the relative velocity, V_∞, where the power required, P_{req}, to overcome the drag force, D, is at a minimum. We call this velocity V_{me}.

II. RELATED WORK

A. Rotorcraft Flight Mechanics

The bulk of our work is supported by the momentum analysis methods derived by [8]. These methods provide closed form, non-dimensional expressions for a rotorcraft's power required to maintain straight level flight (1) (excluding the unnecessary term for tail rotor power) and power required to maintain a constant, level turn (2) where ϕ is the bank angle.

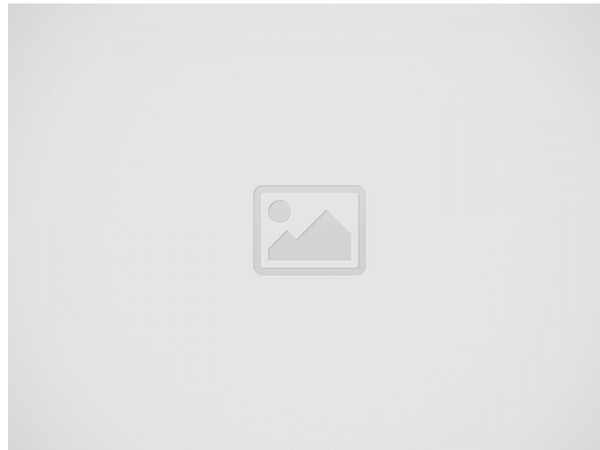

Fig. 1. Coordinate system and flow diagram illustrating induced velocity, force balance at equilibrium, etc..

$$P_{req} = \frac{kC_W^2}{2\sqrt{\lambda^2+\mu^2}} + \frac{\sigma C_{d_0}}{8}(1+K\mu^2) + \frac{1}{2}(\frac{f}{A})\mu^3 + \lambda_c C_W \tag{1}$$

$$C_P = \frac{k(\frac{C_W}{cos\phi})^2}{2\sqrt{\lambda^2+\mu^2}} + \frac{\sigma C_{d_0}}{8}(1+K\mu^2) + \frac{1}{2}(\frac{f}{A})\mu^3 + \lambda_c C_W \tag{2}$$

We simplify equations 1 and 2 to dimensioned versions specific to our quadrotor using empirical values for K and k. **NEED TO VERIFY THAT THESE HOLD AT SCALE**. These equations are covered in more detail in Section III.

B. Drone Trajectory Optimization and Control

Biggest difference between the majority of quadrotor work is that wind is treated as a disturbance that needs to be rejected[9]. Our work lays the foundation for using wind disturbance as an "energy source" similar to autonomous water vehicles.

C. Dynamic Soaring

In similar spirit to our work is the climbing strategies proposed by [10]. They propose exploiting certain wind conditions to improve the efficiency of rotor vehicles most inefficient maneuver. Combined with our work the majority of common quadrotor mission trajectories can be created piece-wise efficiently.

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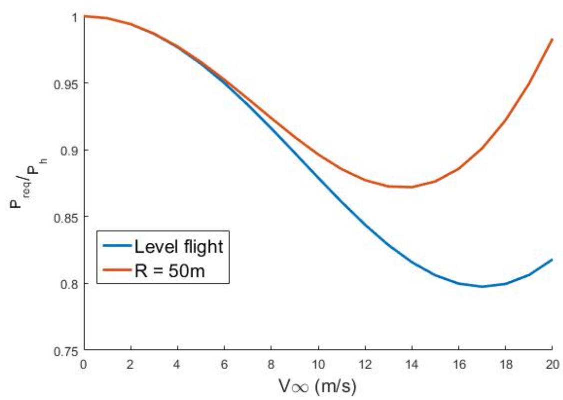

Fig. 2. We compare the theoretical power ratio for our vehicle configuration using momentum theory analysis. If our vehicle was flown straight and level at 17m/s we could expect to fly approximately 21% longer. Even with losses associated with overcoming centripetal acceleration we can expect to fly 13% longer by flying at 13m/s while turning with a radius of 50m.

D. Electric Vehicles

III. Flight Mechanic Preliminaries

In this section we review the flight mechanic principles necessary for understanding our work. We show derivations for calculating the following:

1) P_{req} - the power required to maintain level flight
2) V_{mp} - the linear velocity where P_{req} is minimized.
3) $P_{req_{turn}}$ - the power required to maintain a level turn

Additionally, we show that for level flight V_{mp} is equivalent to the velocity that maximizes endurance.

A. Power Required P_{req}

Using the standard quadrotor model given in [11] and [12] with the coordinate frames illustrated in I a simple force balance shows that the pitch angle, θ, can be written as a function of V. Assuming a windless environment we can say $V = V_\infty$, the relative velocity of the flow, and write (3).

$$\theta(V_\infty) = \frac{\rho v_\infty^2 f}{2mg} \tag{3}$$

B. Equations Likely Needed

$$P_{req} = \frac{kT^2}{2\rho A V_\infty} + (\frac{1}{2}\rho V_\infty^2 S_{ref} C_{D_f}) V_\infty \tag{4}$$

IV. Experiments

To determine the effectiveness of a trajectory, $\mathcal{T}_c$ we empirically compare the time it takes to deplete 1 Ah of charge while flying $\mathcal{T}_c$ versus the time it takes to deplete the same charge at hover. The experiment consists of two main phases: hover flights and trajectory flights where these two phases are flown by the same quadrotor alternately. The parameters defining $\mathcal{T}_c$ (i.e. v_Q, R_t, type of yaw tracking) vary between iterations while hover parameters remain unchanged. Each iteration begins by replacing the used battery with a fully charged one and determining if local wind speeds are below moderate which we define below. The experiments were designed this way to control for two primary factors, wind and battery variations.

A. Controlling for wind variations

From equations [eqno?] and decades of aerodynamic research we know that aircraft performance is susceptible to wind disturbances. Extensive work has been done in flight control to reduce wind induced error in the absence of a human pilot. [13] and [9] focus on the quadrotor platform while [14] looks at the more traditional fixed-wing aircraft. In [13] and [14] wind disturbance is categorized by

$$W = \frac{|V_w|}{|V_Q|} * 100\% \tag{5}$$

As stated in section III, in this paper we are not concerned with controlling against moderate or severe wind disturbance. Therefore, before each flight test we measure the local wind speed to ensure $W < 20\%$ or below moderate conditions.

B. Mitigating battery issues

C. Quadrotor platform

For this experiment we flew the DJI Matrice 100 built to the specifications provided by [15]. With the exception that we use the Optor visual inertial sensor instead of the discontinued Intel ZR300. All modifications made to their ROS package is available at **LINK TO GIT**.

D. Measuring consumption

The DJI SDK reports battery voltage, SoC, and capacity at 10Hz. This was a major reason for selecting this platform as there is no need for additional boards or modules.

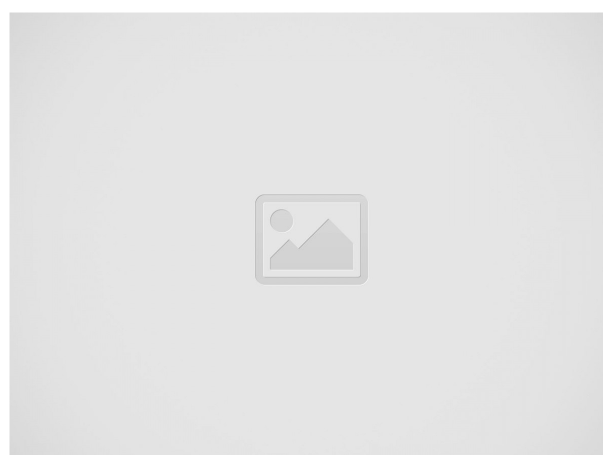

Fig. 3. Graphic detailing setup.

V. Conclusions

INSERT CONCLUSIONS HERE.

Appendix

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

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