Power Minimal Optimal Control for Quadrotor UAVs in SE(3)

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***Abstract*— A problem that continues to persist for quadrotor UAVs is their low endurance relative to other prominent UAV platforms. There exist three main approaches to solving this problem: improving hardware efficiency, designing new hybrid systems, and developing software solutions that focus on optimal planning and control. Of the three, software solutions have received the least attention. Existing methods are either heavily constrained, challenging to implement on physical systems, or require significant off-line computation. In this paper we present a method for finding power minimal, smooth trajectories for a quadrotor in SE(3) using exiting models for power and energy consumption. We show that the problem can be formulated as a constrained, continuous time optimal control problem with a polynomial cost function that can be solved numerically. Through simulation we show that our approach, when compared to existing methods, reduces total energy consumption by INSERT RESULTS HERE% on average.**

Fig. 1. Example of a quadrotor flying with a constant forward velocity where the pitch angle is small enough that the dynamics about each axis are decoupled when linearized.

I. INTRODUCTION

The last decade has seen quadrotor helicopters explode in popularity. From an emerging unmanned aerial vehicle (UAV) concept to a prominent research and commercial platform [1], [2] quadrotors have become the nearly ubiquitous aerial robot. Their relative low cost and the simplicity of their dynamics when near hover [3] has made them popular in numerous applications [4], [5], [6].

No matter the application, if the quadrotor is autonomous, all its motion is occurring near hover. This condition introduces the quadrotor platform's greatest weakness, power efficiency. In [1], Kumar et al. state that the power required for a quadrotor to maintain hover is approximately 200 W per kg. Additionally, due to current LiPo battery technology a quadrotor's battery can be 25-30% of its total mass. These two purely hardware constraints restrict the size of an area that can be explored, the number of images that can be captured by an onboard camera, the mass of potential payloads, etc.. The problem of quadrotor power efficiency therefore creates a practical limit on the platform's utility.

Currently, most approaches to solving this problem can be classified in to three categories, listed in descending order of prevalence:

1) hardware focused optimization
2) algorithm (software) focused optimization
3) development of bio-inspired, hybrid systems

Traditionally, the bulk of work done to increase quadrotor efficiency is in the first category. Efforts here consist mostly of reducing the weight of materials such as the airframe, sensors, and power electronics. The second category has grown in popularity in recent years due to the increased capabilities of onboard computers. The most common methods here involve incorporating vehicle power consumption in to cost functions of existing optimal planning and control algorithms. Finally, the third category often produces novel systems that increase efficiency by tranisitioning to another dynamic mode such as perching, walking, or rolling. In [7], Karydis et al. provide a thorough review of some of the most promising work in all three categories. Category one and three are not addressed further in this paper as the scope falls entirely in category two.

In this paper we present a solution for finding power optimal quadrotor trajectories between two configurations in SE(3) that minimize the total amount of power required, P_{tot}, to complete. Our solution differs from current methods by allowing for a continuous action space and removing fixed altitude constraints present in current methods. The remainder of the paper is organized as follows: Section II broadly reviews similar approaches already in the literature; Section III describes the dynamic model of the quadrotor including power consumption; Section IV details the formulation of our optimal control problem; Section V presents the results from our experiments in simulation; and Section VI contains discussion of future work and conclusions from this work.

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II. RELATED WORK

Several promising approaches for increasing small UAV endurance through software have been made in the last four years. Di Franco and Buttazzo present one of the earliest successful methods in [8] where they use energy as an additional optimization criteria for coverage problems. They fit energy models to empirical data collected from four basic maneuvers: take-off, straight and level flight, level turn, and landing. Their algorithm then finds a set of nodes at a fixed altitude that maximizes coverage with a given resolution. The edges of the graph are one of the four basic maneuvers they have an energy model for. An optimization scheme then finds the constant velocity that minimizes the energy consumed by each edge.

The major drawback of this approach is that the UAV must come to a stop at each node consuming a large amount of energy while accelerating and decelerating. Our approach alleviates this issue by allowing for smooth trajectories.

One of the most promising approaches comes from Morbidi et. al [9] where they leverage a brushless DC motor model to find minimum energy paths with respect to the angular acceleration of the rotors in continuous time. Their approach has relatively lax constraints, the initial and final angular acceleration of the rotors must be equal, and has a polynomial objective function.

$$E = \int_{t_i}^{t_f} \sum_{j=1}^{4} (c_0 + c_1\omega_j(t) + c_2\omega_j(t)^2 + c_3\omega_j(t)^3 + c_4\omega_j(t)^4 + c_5\dot{\omega}_j(t)^2)\ dt$$

With a clever change of variables the optimization problem becomes simple enough that it can be solved numerically. An issue with this method is that it has only been validated in simulation and there is currently no method for determining the polynomial coefficients of the objective function for a real system. While our method is also only currently validated in simulation it only relies on constants that have established empirical methods for determining them.

Taking a step up in complexity are approaches more similar to ours that aim to greater increase endurance gains by including aerodynamic effect considerations. Ware and Roy [10] incorporate urban wind data to find more efficient trajectories between two points in the urban canopy layer. Using the wind's prevailing speed and direction above the surrounding buildings as the input to a CFD solver they generate a grid with 1 m resolution. Similar to [8] they create a graph for a fixed altitude such that each interior node has eight edges. Using the wind vector, v_w, from the CFD solution they can then choose an upper and lower bounded ground velocity, v_g, for the quadrotor such that it minimizes:

$$E_i = \frac{T(v_i + v_\infty \sin\alpha)(v_g - v_w)\|d\|}{v_g}$$

where $\|d\|$ is the Euclidean distance between the two nodes. They address the acceleration problem encountered by [8] by constraining the change in velocity, Δv_g, between edges. Their simulation results show that wind aware planning uses less power than wind naive planning and highlights the importance of having an estimate of the local wind field. In our work we also assume that the local wind field is known, however we focus on a general, time varying wind field (more similar to [11]) rather than a domain specific solution.

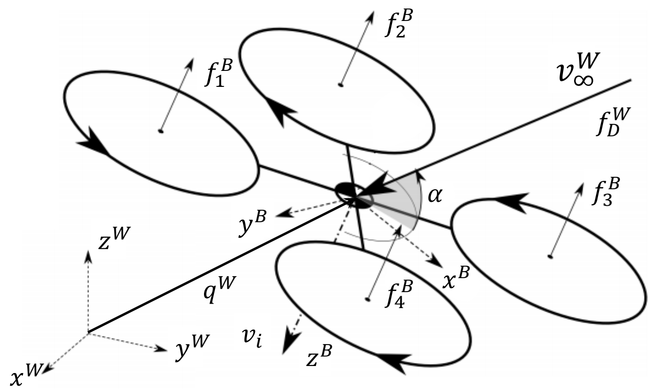

Fig. 2. Definition of our coordinate system. Frame W is defined as a NEU inertial frame. Frame B is attached to the quadrotor's center of mass. Keeping with convention, z^B points down so that positive pitch angles, θ, correspond to pitching up. We define α as the angle between v_∞^W and the plane of the rotors and v_i so that its direction is always equivalent to z^B. Diagram adapted from [12]

Most recently, Tagliabue et. al [12] presented a model-free control approach that uses an extremum seeking controller t0 converge to the velocity that minimizes energy consumption on a given trajectory. The extremum seeking controller takes the voltage and current draw measured at the battery terminals as inputs and ouputs a velocity for the flight controller to track. This approach requires little knowledge about your system and environment which they demonstrate by attaching packages to the quadrotor completely altering its dynamics and aerodynamics. Theirs is one of the only methods to be verified empirically on a physical system, however the experiments did not include wind disturbance. A potential downside to this approach is that it minimizes energy consumed on a predetermined trajectory rather than determining a minimal energy trajectory. For practical applications, combining this method with a method similar to ours would likely produce better results.

III. Quadrotor and Power Model

In this section we derive our version of the standard quadrotor dynamic model given in [13] and [14]. We also derive a model for power consumption based on existing models for general rotorcraft given in [15].

A. Quadrotor dynamics

As seen in Fig. 2. we define two frames of reference, W the world or inertial frame and B the body fixed frame attached the quadrotor at its center of mass. We also define R, a rotation matrix that describes the orientation of B in W; $q^w = \left(x^W,\, y^W,\, z^W\right)$, a vector that gives the Cartesian position of B in W; and $\omega_{W,B}^B$, a vector that describes the angular velocity of B with respect to W in the body frame. Since we constrain the quadrotor to always be within ${}^{\pi}/_{6}$ of hover, R can be parameterized by the Euler angles $\Theta^w = \left(\phi^W,\, \theta^W,\, \psi^W\right)$ corresponding to roll, pitch, and yaw respectively.

Assuming all four rotors are identical and neglecting blade flapping then each rotor will produce a thrust $f_j^B \propto \sigma_j$ where

σ_j is the spin rate of the rotor. Additionally, each rotor will produce a torque, τ_j^B, around each axis of B. Torques about x^B and y^B are moments proportional to σ_j whereas torques about z^B are pure torques proportional to the difference in spin rate between a rotor and its counter-rotor (i.e. $\sigma_2 - \sigma_4$ and $\sigma_1 - \sigma_3$).

We can then describe the translational and rotational dynamics of the quadrotor as a set of Newton-Euler equations:

$$m\ddot{q}^W = R\sum f_j^B + f_D^W - mg^W \tag{1}$$

$$I^B \dot{\omega}_{W,B}^B = -\omega_{W,B}^B \times I^B \omega_{W,B}^B + \sum \tau_j^B \tag{2}$$

Where I^B is the rotational inertia matrix, which is diagonal when the quadrotor is axisymmetric, the gravity vector is $g^W = (0,\, 0,\, 9.8066)$ ms^{-2}, and m is the mass of the quadrotor in kg.

B. Drag model

Following [12] and [16] we adopt an isotropic drag model for simplicity.

$$f_D^W = \left(\mu_1 v_\infty + \mu_2 v_\infty^2\right) \frac{v_\infty}{\|v_\infty\|} \tag{3}$$

Where μ_1 and μ_2 are experimentally determined drag coefficients selected to make (3) approximate a more complex drag model such as the one presented in [17], [18], or [19]. As shown in Fig. 2. we assume that f_D^W is always co-linear with v_∞ and acts at the quadrotor's center of mass producing no moments.

C. Power consumption model

We base our model for power consumption off Leishman's [15] model for aerodynamic power required for level flight and maneuvers. By assuming that the total aerodynamic power required is the sum of the power required to maintain level flight and the power to climb in altitude if needed.

$$P_{req} = \kappa T \left(v_i + v_\infty^B|_z\right) + \dot{q}_z^W m\|g\| \tag{4}$$

Where κ is an experimentally determined correction factor, T is the sum of all the individual rotor thrusts f_j, $v_\infty^B|_z$ is the component of v_∞ orthogonal to the rotor plane, $\dot{q}_z^W$ is the vertical component of the quadrotor's velocity in the world frame, and v_i is the velocity induced by air flowing through the rotors and is defined implicitly by [15] as:

$$\|v_i\| = \frac{v_h^2}{\sqrt{(v_\infty \cos\alpha)^2 + (v_\infty \sin\alpha)^2}} \tag{5}$$

where v_h is the induced velocity at hover. This expression for $\|v_i\|$ can be manipulated in to a quartic function and solved numerically [2].

To convert P_{req} to electrical power consumed, P, we join [10], [20], and [12] in assuming that all losses due to electrical components can be aggregated in to one efficiency term η. Thus:

$$P = \frac{1}{\eta} P_{req} \tag{6}$$

IV. Optimal Control Problem

In this section we show how given two points in SE(3) q_i, q_f; physical properties I_B, m; aerodynamic constants μ_1, μ_2, κ; and a known time varying wind field $v_w(t,\, x,\, y,\, z)$ we find a power optimal trajectory by solving

$$\begin{aligned} \min_{v_\infty} \quad & \sum_{t=i}^{f} \kappa T \left(v_i + v_\infty^B|_z\right) + \dot{q}_z^W m\|g\| \\ \text{s.t.} \quad & v_\infty = \dot{q}_B^W - v_w \end{aligned} \tag{7}$$

From (4) we can see that choosing v_∞ as our decision variable is the most practical solution

V. Results and Discussion

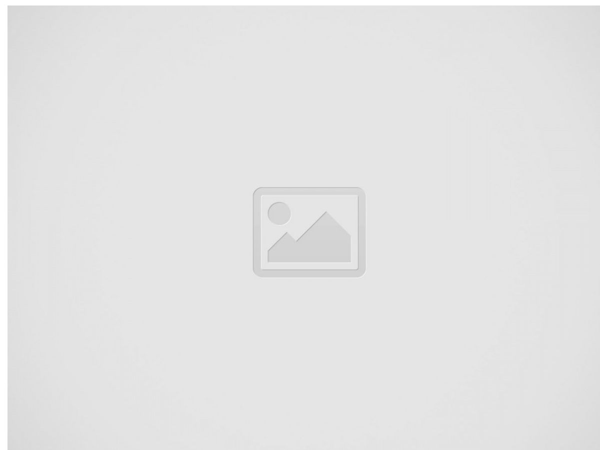

Fig. 3. Simulation results.

A. Simulation setup

VI. Conclusions and Future Work

INSERT CONCLUSIONS HERE.

ACKNOWLEDGMENT

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