MECH 539: Computational Aerodynamics
Department of Mechanical Engineering, McGill University

Project #3: Murman-Cole Scheme for the Transonic Small Disturbance Equation
Due 17th March, 2023

Solve the transonic small disturbance (TSD) theory over a circular arc airfoil at various Mach numbers using the Murman-Cole method. The TSD equation is simplified from the two-dimensional full potential equations and can be written as

$$\left[\left(1 - M_\infty^2\right) - (\gamma + 1)M_\infty^2 \frac{\phi_x}{U_\infty}\right] \phi_{xx} + \phi_{yy} = 0$$

where $\gamma = 1.4$ is the specific heat ratio for air, the gas constant, $\mathrm{R} = 287.058$ J kg^{-1} K^{-1}, the freestream static temperature and pressures are $T_\infty = 293$ K and $p_\infty = 100$ kN/m^2, and lastly, $(x, y) \in [0, 50.0]^2$ spans the two-dimensional domain, and the following boundary conditions hold:

$$\phi(x, y) = 0, \;\; \forall \;\; (x, 50), (j, 0), \text{ and } (j, 50) \tag{1}$$

$$\frac{\partial \phi}{\partial n} = 0 \;\; \forall \;\; (x, 0) \notin 20 \leq x \leq 21 \tag{2}$$

$$\frac{\partial \phi}{\partial n} = U_\infty \frac{dy}{dx} \;\; \forall \;\; (x, 0) \in 20 \leq x \leq 21. \tag{3}$$

The airfoil is defined by the following equation for a circular-arc:

$$y(x) = (t/c)(-2x^2 + 82x - 840), \;\; \forall (x, 0) \in 20 \leq x \leq 21$$

where, $t/c = 0.08$ is the thickness ratio, and x_c is the position of the mid chord. Employ a constant grid spacing in the x direction over the airfoil but an exponential or polynomial stretching of the grid along the x direction before and after the airfoil surface. Along the y-direction, employ an exponential or polynomial stretching. You may initialize the flow with, $\phi = 0$. Either specify the freestream Mach number or velocity as in input parameter. Use the isentropic equations to evaluate Mach, velocity, or pressure in the domain.

Provide the following in a written report:

1. Solve the TSD using the Murman-Cole method on a stretched grid and use at a minimum 60 points in the x-direction, with at least 40 grid points, and at least 20 points in the y-direction. The first grid spacing adjacent to the airfoil in the y-direction should be $dy = \frac{t/c}{2}$. Use the Gauss-Seidel method to solve the equation along each column (y-direction).

2. Provide a plot of the pressure coefficient along the airfoil surface with the negative pointing upwards. Vary the freestream Mach number between $[0.80, 0.90]$ with 0.02 increments. For each case, provide a convergence plot of the L_∞-norm, surface pressure coefficient as a function of x, and pressure contour for $x \in [20, 21]$ and $y \in [0, 1]$. Discuss your findings. A four order reduction in the residual is sufficient for the Gauss-Seidel method. Ensure that the residual reaches at least 1×10^{-4}.

3. Vary the grid size, by doubling it in each direction as well as the number of points on the airfoil surface. Produce a coarse, medium, and fine grid. Plot the surface coefficient of pressure as a function of x on the airfoil surface for the three grids on the same plot at Mach 0.88. Discuss your findings. Does the shock location change.

4. Now solve the equations using line implicit Gauss-Seidel at a Mach number of 0.86 and compare the convergence as a function of iterations and CPU time to that achieved by the standard Gauss-Seidel approach. Both approaches must reach machine accuracy.

5. For a select grid size, plot the coefficient of pressure along the airfoil surface for Mach number between $[0.80, 0.90]$ with 0.02 increments on the same plot. Discuss your observations. What is the effect of increasing the Mach number.