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ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2024
(YEAR 4)

CANDIDATE NAME	

CLASS						INDEX NUMBER		

MATHEMATICS **4052/01**

Paper 1 **12 August 2024**

Candidates answer on the Question Paper. **2 hours 15 minutes (09:50 – 12:05)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

This document consist of **21** printed pages and **3** blank pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Solve the inequality $\dfrac{23-6x}{3} \le \dfrac{2+x}{6}$.

Answer …………………… [2]

2 **(a)** Write 0.0001087 in standard form.

Answer …………………… [1]

(b)

City	Population	Land area, km^2
Bangkok	1.07×10^7	7700.0
Ho Chi Minh City	2.12×10^7	30 595.0
Manila	1.35×10^7	24 317.7
Kuala Lumpur	8.46×10^6	8347.1

The table shows the population in some cities in ASEAN and their land areas.

Write down the city in the table that has the smallest population per km^2.

Answer …………………… [1]

[TURN OVER

3   Vikas takes a study loan of $12 000 that charges an interest rate of *r* % per year compound interest for 4 years.
The total amount that he would have paid for the loan at the end of 4 years is $12 688.

Find the value of *r*.

Answer r = ………………… [3]

4   **(a)**   Simplify $\left(\dfrac{81u^{20}}{v^{8}} \right)^{-\frac{3}{4}}$.

Answer …………………… [2]

(b)   Factorise $4x^{2} + 4 - y^{2} - x^{2}y^{2}$ completely.

Answer …………………… [3]

5 m and n are positive integers, where $m \neq n$.

Explain why $(m+n)^2 - (m-n)^2$ is not a prime number for all positive integer values of m and n.

…………………………………………………………………………………………

………………………………………………………………………………… [2]

6 The marked price of a smart watch at a shop is 25% above the cost price.
The shop gives a p % discount during a sale.

Calculate the value of p such that the shop makes neither a loss nor a profit from the sale of the smart watch.

Answer $p =$ ……………… [2]

[TURN OVER

7   The graph shows the average cost for a dozen of Grade A eggs in the year 2022.

Source: News Daily Mail UK
(https://www.dailymail.co.uk/news/article-11671431/Egg-prices-soar-60-2022-group-blaming-collusive-scheme-producers.html)

State one misleading feature of the graph and explain how this feature affects the reader's interpretation of the cost for a dozen of Grade A eggs.

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………… [2]

8   In triangle *XYZ*, *YZ* = 13 cm, *XY* = 20 cm and angle *YXZ* = 35°.

Find the two possible values of angle *XZY*.

Answer ……….. or ……….. [3]

9   The Mandai Wildlife Reserve conserves and promotes South-East Asian biodiversity.

(a)   The Mandai Wildlife Bridge is 140 m long.
It is represented by a length of 3.5 cm on a map.

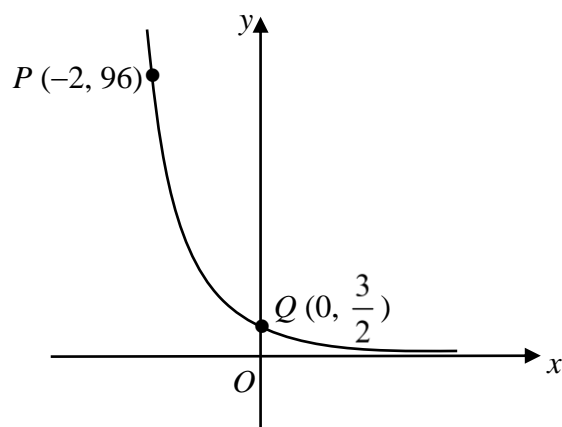Express the scale of the map in the form 1 : *r*.

Answer …………………… [1]

(b)   The Bird Paradise covers a land area of 17 hectares.

Find, in cm², the area representing the Bird Paradise on the same map.
(1 hectare = 10000 m²)

Answer ………………….cm² [2]

[TURN OVER

8

10 The sketch shows the graph of $y = ka^{-x}$.

The points $P\,(-2, 96)$ and $Q\,(0, \dfrac{3}{2})$ lie on the graph.

Find the value of k and of a.

Answer $k = \ldots\ldots\ldots\ldots\ldots$

Answer $a = \ldots\ldots\ldots\ldots\ldots$ [2]

---

11 $\dfrac{v}{w} = \dfrac{1}{v} + \dfrac{3v}{u}$

Rearrange the formula to make v the subject.

Answer $v = \ldots\ldots\ldots\ldots\ldots$ [3]

12 On a coordinate plane, P is the point $(0,12)$ and $\overrightarrow{PR} = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$.

    (a) Calculate $|\overrightarrow{PR}|$.

Answer …………………… [2]

    (b) Find the equation of the line *PR*.

Answer …………………… [2]

13 **(a)** Write down 252 as product of its prime factors in index notation.

Answer …………………… [1]

    (b) The highest common factor (HCF) of two numbers is 6.
The lowest common multiple (LCM) of the two numbers is 252.
One of the numbers is a perfect square.

Find the two numbers.
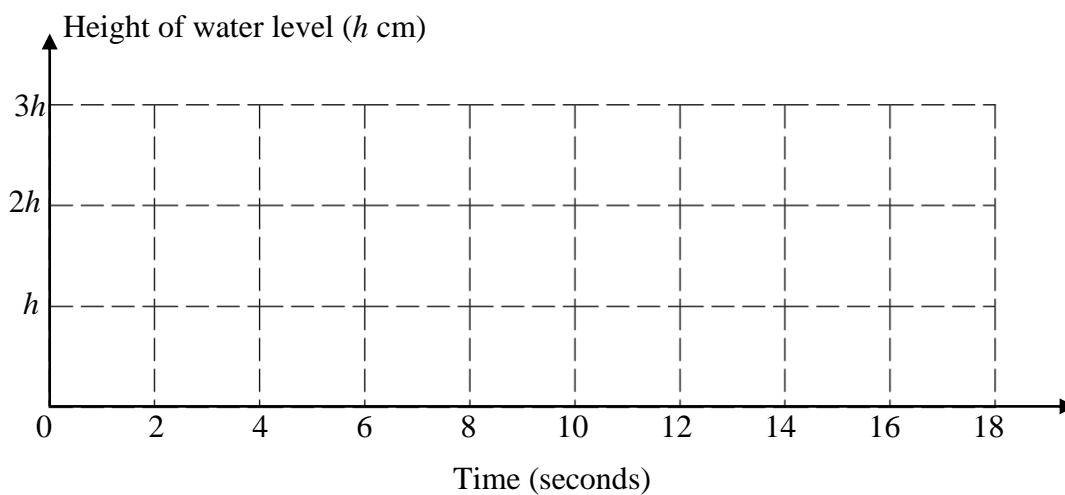
Answer ……… and ……… [2]

10

14

A container is a prism with a triangular cross-section.
The container has a height of $3h$ cm and uniform width.
Water is poured into the container at a constant rate.
It takes 18 seconds to fill the container.

(a) Find the time taken for the water level to reach a depth of $2h$.

Answer …………… seconds [2]

(b) On the axes in the answer space, sketch the graph of the height of the water level against time.

[3]

15 **(a)** The expression $x^2 + 3x - 7$ can be written in the form $\left(x + \dfrac{3}{2} \right)^2 + k$.

Find the value of k.

Answer $k =$ ……………… [2]

(b) Hence, explain why the equation $x^2 + 3x - 7 = p$ does not have solutions for some values of p.

………………………………………………………………………………………

……………………………………………………………………………………… [2]

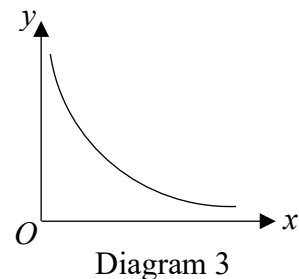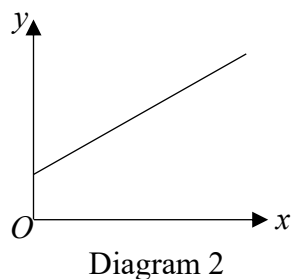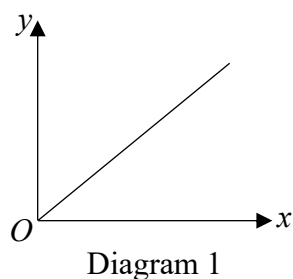
16 The sum of all reflex angles of a regular n sided polygon is $3060°$.

Find the number of sides of the regular polygon.

Answer ………………… [2]

---

17 **(a)** Which of the following diagrams represents a directly proportional relationship between two variables.

Diagram 1   Diagram 2   Diagram 3

Answer …………………… [1]

(b) The frequency, F hertz (Hz), of a sound wave is directly proportional to the square root of the tension, T newton (N), in a guitar string.

Calculate percentage F increases by when the tension of the guitar string is increased 9 times.

Answer …………………% [2]

18 Zara has only 2-dollar notes and 5-dollar notes in her wallet.
The number of 2-dollar notes to 5-dollar notes is in the ratio 4:5.
After Zara spends $9, the number of 2-dollar notes to 5-dollar notes in her wallet becomes 2:3.

Calculate number of 2-dollar notes in Zara's wallet initially.

Answer …………………….. [3]

19   11.8 is the mean of five positive integers.
The sum of their squares is 875.
Each of the positive integer is now multiplied by 2.

Find the new standard deviation.

Answer …………………… [3]

20 **(a)** ξ is the universal set and P, Q and R are three sets.

$\xi = \{j,k,l,m,n\}$

$P = \{j,k,l,n\}$

(i) It is given that $P \cap Q = \{l\}$ and $P' \cap Q \neq \varnothing$.

List the elements of Q.

Answer …………………… [1]

(ii) $R \subset P$. Xena claim's that one possible set is $R = \{j,k,l,n\}$.

Explain why Xena's claim is not true.

…………………………………………………………………………..

………………………………………………………………………….. [1]

(b) In a class of 23 students, 11 students speak French, and 7 students speak Spanish.

(i) Draw a Venn diagram to illustrate the case that there are some students who speak both languages.
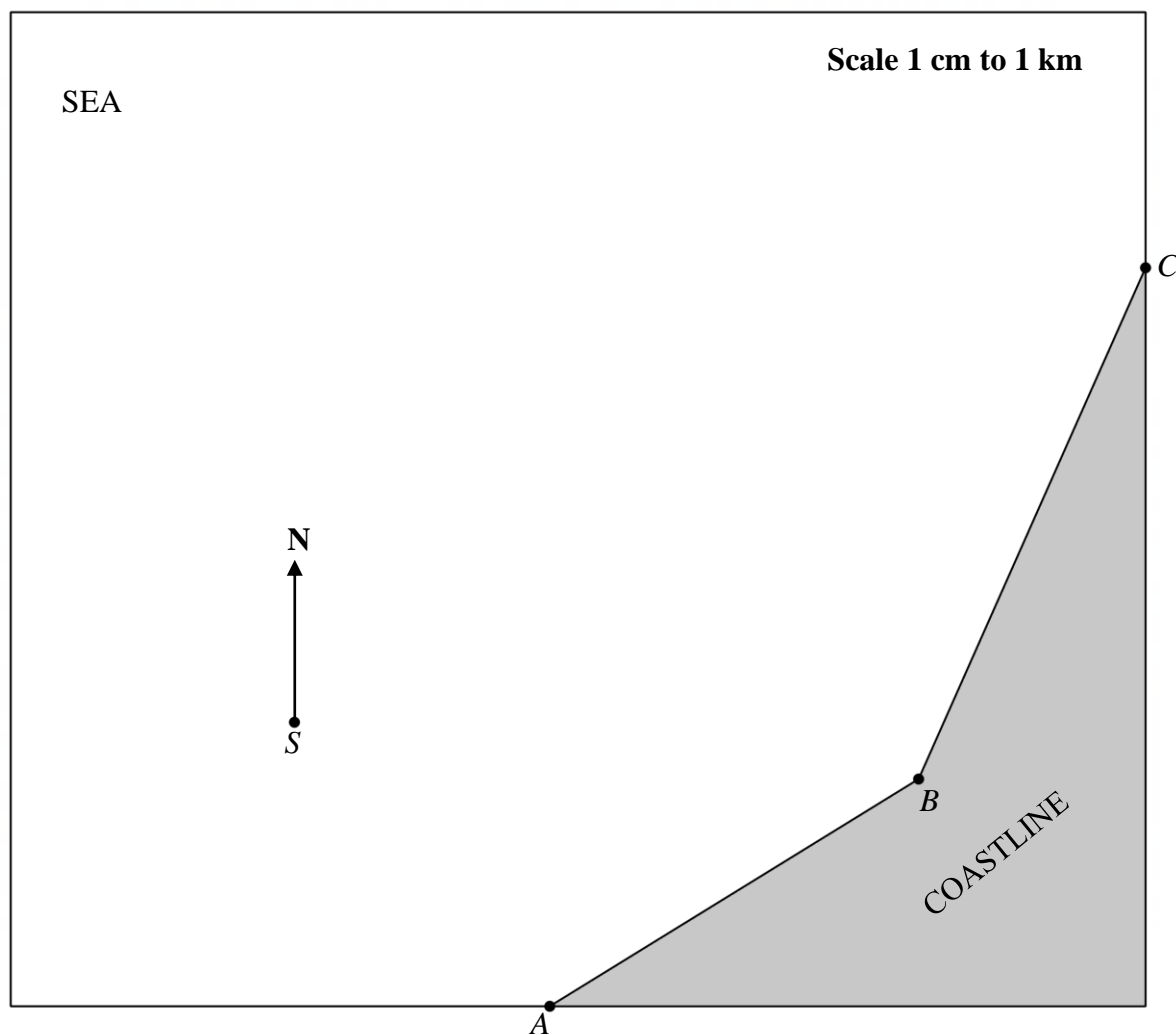
Answer

[1]

(ii) Calculate the maximum possible number of students who do not speak both languages.

Answer …………………… [1]

21

SEA

Scale 1 cm to 1 km

N

S

C

B

COASTLINE

A

In the scale drawing, *A*, *B* and *C* define a boundary between coastline and the sea.
B is a port and *S* is a ship.

(a)   *L* is a lighthouse that is located

  (i)   equidistant from *A* and *B*,

  (ii)   equidistant from *AB* and *BC*,

  Locate the lighthouse, *L*, on the scale drawing. [2]
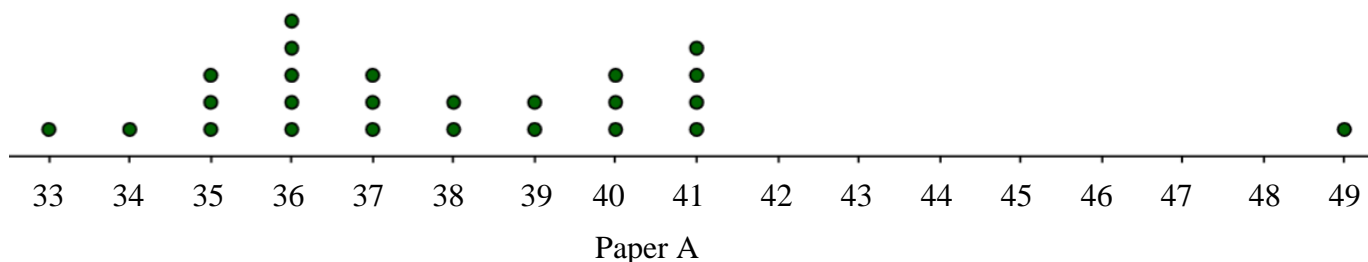
(b)   Ships must not sail within 2 km of the lighthouse nor the port.
  Ship, *S*, sailed on a bearing of $\theta°$.
  It passed between *B* and *L*.

  Estimate the range of $\theta°$.

  Answer ……. $< \theta° <$ ……. [2]

[TURN OVER

22 In a science examination, a class of 25 students took two papers, paper A and paper B.
Both papers were out of 50 marks.
The dot diagram shows the distribution of the marks for paper A.

Paper A

(a) Explain why mean is not an appropriate measure of central tendency to represent the distribution of marks for paper A.

………………………………………………………………………………

……………………………………………………………………………… [1]

(b) Calculate the interquartile range for paper A.

Answer …………………… [2]

(c) The table below summaries the distribution of the class for paper B.

Median	41
Interquartile Range	7

Make two comments comparing the distribution of marks for both papers.
Use figures to support your answer.

1 …………………………………………………………………………

………………………………………………………………………………..

2 …………………………………………………………………………

……………………………………………………………………………….. [2]

23 Han and Peng are training to take part in a triathlon.
Each training session involves training different segments, continuously in order.
For each session Han plans to swim 15 minutes, cycle 20 minutes and run 22 minutes.
For each session Peng plans to swim 20 minutes, cycle 25 minutes and run x minutes.

This information can be represented by the matrix $\mathbf{E} = \begin{pmatrix} 15 & 20 \\ 20 & 25 \\ 22 & x \end{pmatrix}$
$\begin{matrix} \text{Han} & \text{Peng} \\ & \\ & \end{matrix}$
$\begin{matrix} \text{swimming} \\ \text{cycling} \\ \text{running} \end{matrix}$

Han plans to train 3 sessions on the weekdays and 1 session on weekends.
Peng plans to train 1 session on weekdays and 2 sessions on weekends.

This information can be represented by the matrix $\mathbf{N} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$.

(a) Find, in terms of x, the matrix $\mathbf{T} = \mathbf{EN}$.

Answer $\mathbf{T} =$ [2]

(b) State what each element in the second column of matrix $\mathbf{T}$ represents.

………………………………………………………………...………………… [1]

(c) The total time ran by Han and Peng on the weekdays is equal to total time ran on the weekends.

Find the value of x.

Answer $x =$ ……………… [2]

[TURN OVER

(d) It is given that swimming burns 8 calories per minute, cycling burns
10 calories per minute and running burns 12 calories per minute.
This information can be represented by the matrix **C**.
Another Matrix **A** is such that all the elements are 1.
The elements of the matrix **B**, where **B** = **CTA**, represents the total number of
calories burnt by Han and Peng for a week.

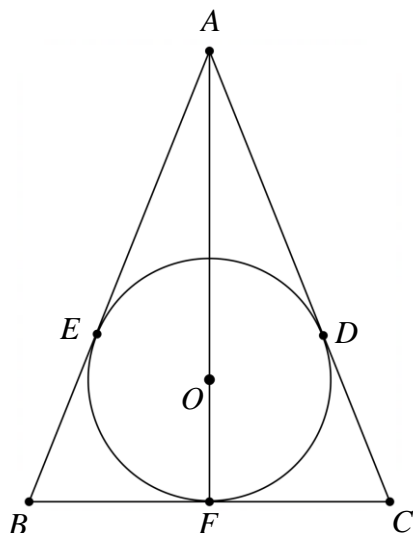Write down the matrix **C** and state the order of matrix **A**.

Answer **C** = [1]

Answer order of **A** = ……… [1]

---

24 A ball is thrown vertically upwards. Its height, h m, above ground at time t seconds
after being thrown is given by the formula $h = -1.44t^2 + 1.6t + 1.5$.

Find the time taken for the ball to return to the ground.

Answer …………… seconds [3]

25

The diagram shows a circle which passes through *D*, *E* and *F*.
AEB, *BFC* and *ADC* are tangents to the circle, with centre *O*.

(a) Show that triangle *ABF* is congruent to triangle *ACF*.
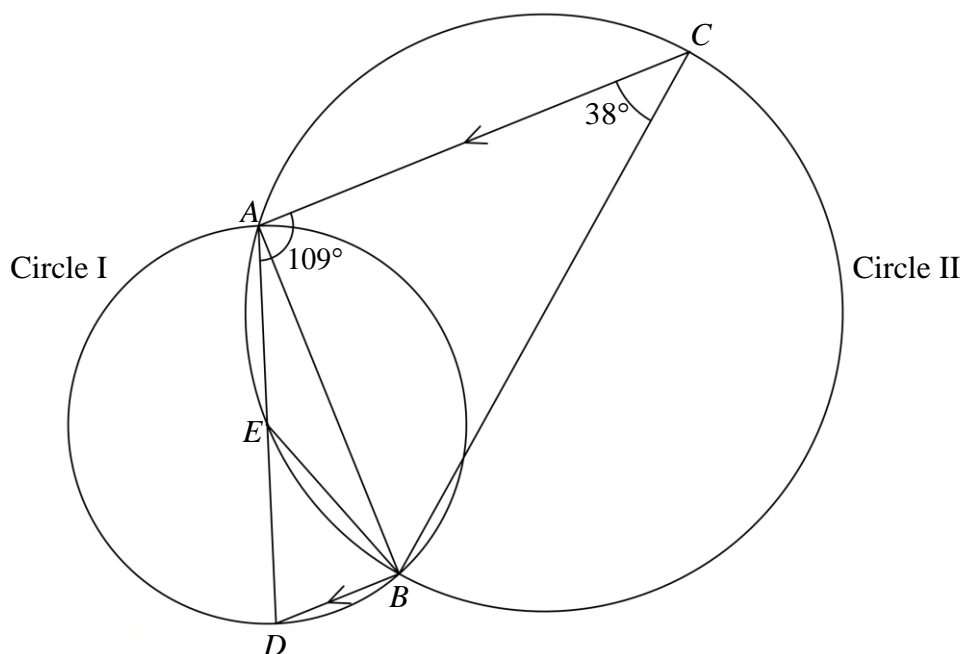Give a reason for each statement you make.

………………………………………………………………………………..

………………………………………………………………………………..

………………………………………………………………………………..

………………………………………………………………….. [3]

(b) *AB* = 25 cm and *BC* = 14 cm.

Find the radius of the circle.

Answer ………………….cm [3]

26

The diagram shows two circles that intersect at *A* and *B*.
A, *B* and *D* are three points on a circle I.
A, *E*, *B* and *C* are four points on a circle II.
AD is the diameter of the circle I.
BC is the diameter of the circle II.
CA is parallel to *BD*.
Angle $CAD = 109°$ and angle $ACB = 38°$.

(a) Find angle *DEB*.
Give a reason for each step of your working.

Answer Angle *DEB* = ……….. [2]

(b) Show that *E* is the centre of circle I.
Give a reason for each step of your working.

Answer

[3]

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Y2024 Y4 Prelim MA4052 Mark Scheme

1	$\dfrac{23-6x}{3} \le \dfrac{2+x}{6}$ $\dfrac{46-12x}{6} \le \dfrac{2+x}{6}$ $-13x \le 2-46$ $13x \ge 44$ $x \ge 3\dfrac{5}{13}$

2(a)	$0.0001087 = \underline{1.087 \times 10^{-4}}$
2(b)	**Manila**

3	$12000\left(1+\dfrac{r}{100}\right)^4 = 12688$ $\left(1+\dfrac{r}{100}\right)^4 = 1.057333$ $\left(1+\dfrac{r}{100}\right) = \sqrt[4]{1.057333}$ $r = (1.01403-1)(100)$ $r = 1.403$ $r = 1.40$

4(a)	$\left(\dfrac{81u^{20}}{v^8}\right)^{-\frac{3}{4}} = \left(\dfrac{v^8}{81u^{20}}\right)^{\frac{3}{4}}$ $= \dfrac{v^6}{\underline{27u^{15}}}$
4(b)	$4x^2 + 4 - y^2 - x^2y^2$ $= 4(x^2+1) - y^2(x^2+1)$ $= (4-y^2)(x^2+1)$ $= (2+y)(2-y)(x^2+1)$

5	$(m+n)^2 - (m-n)^2$
	$= m^2 + 2mn + n^2 - m^2 + 2mn - n^2$
	$= 4mn$
	4 is a factor of $(m+n)^2 - (m-n)^2$, **4 is not a prime number**.
	OR
	Any **prime number has exactly 2 factors,** 1 and itself.
	Given, $m \neq n$, $4mn$ **has more than 2 factors**, therefore **cannot be prime** number.

6	Let x be the cost price.
	$1.25x\left(1 - \dfrac{p}{100}\right) = x$
	$1 - \dfrac{p}{100} = \dfrac{1}{1.25}$
	$\dfrac{p}{100} = \dfrac{1}{5}$
	$\underline{p = 20}$

7	The vertical axis does not start from $0.
	Reader might <u>interpret</u> that price of eggs has <u>quadrupled/increased 4 times</u> between Jan/Feb to Dec, <u>when</u> in value the <u>price</u> of eggs has <u>risen by about $2 /doubled only.</u>

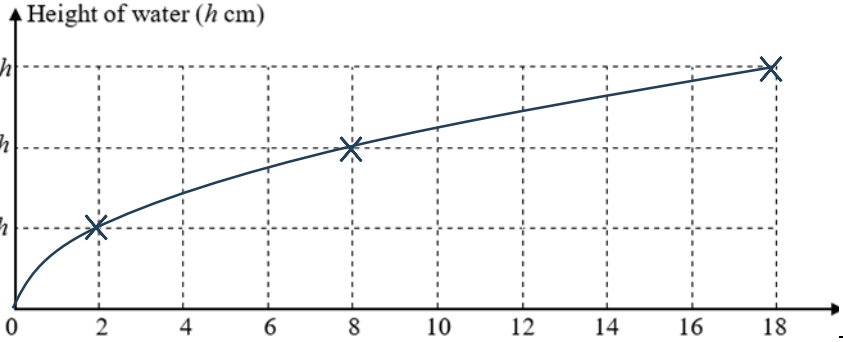| 8	By Sine Rule
	$\dfrac{\sin(\angle YXZ)}{13} = \dfrac{\sin(\angle XZY)}{20}$
	$\sin(\angle XZY) = \dfrac{20\sin(35°)}{13}$
	$\angle XZY = \sin^{-1}(0.882425)$
	$\angle XZY = 61.936°$

	$\angle XZY = 61.9°$
	Or
	$\angle XZY = 180° - 61.936°$ (By obtuse angle ratio)
	$\angle XZY = 118.1°$ (By obtuse angle ratio)

9(a)	Map scale $= \dfrac{140 \times 100}{3.5} = \dfrac{4000}{1}$
	Scale of map $= \underline{\textbf{1 : 4000}}$
9(b)	Scale of map $= 1 : 4000$
	Area scale of map $= 1 : (4000)^2$
	Area on map $= \dfrac{17 \times 10000 \times 100 \times 100}{(4000)^2}$
	$ = \underline{106.25 \ \textbf{cm}^2}$
	OR
	Area $= \dfrac{17 \times 10000}{(40)^2}$
	$ = 106.25 \ cm^2$

10	$y = ka^{-x}$
	At $Q\ (0, \dfrac{3}{2})$,
	$\dfrac{3}{2} = ka^{-0}$
	$\dfrac{3}{2} = k\,(1)$
	$k = \dfrac{3}{2}$
	At $P\ (-2, 96)$
	$96 = \dfrac{3}{2}a^{-(-2)}$
	$a^2 = 64$
	$a = \pm\sqrt{64}$
	As, $a \neq 8$ ($a > 1$ to be defined), $\underline{a = 8}$,

11	$\dfrac{v}{w} = \dfrac{1}{v} + \dfrac{3v}{u}$
	$\dfrac{v}{w} = \dfrac{u + 3v^2}{uv}$
	$uv^2 = uw + 3v^2 w$
	$uv^2 - 3v^2 w = uw$
	$v^2 (u - 3w) = uw$
	$v^2 = \dfrac{uw}{u - 3w}$
	$v = \pm \sqrt{\dfrac{uw}{u - 3w}}$
12(a)	$\left| \overrightarrow{PR} \right| = \sqrt{8^2 + (-10)^2}$
	$\left| \overrightarrow{PR} \right| = 2\sqrt{41}$ units
	OR
	$\left| \overrightarrow{PR} \right| = 12.8$ units
12(b)	Given $\overrightarrow{PR} = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$,
	Gradient $= \dfrac{-10}{8}$
	$= -\dfrac{5}{4}$
	Equation of PR, $y = mx + c$
	$y = -\dfrac{5}{4} x + 12$

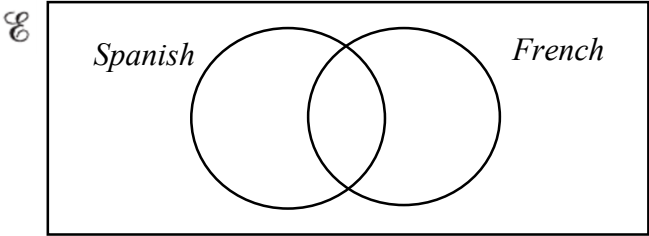| **13(a)**	$252 = 2^2 \times 3^2 \times 7$
13(b)	HCF $= 2 \times 3$
	LCM $= 2^2 \times 3^2 \times 7$
	First number $= 2^2 \times 3^2$ (perfect square)
	$= 36$
	Second number $= 2 \times 3 \times 7$
	$= 42$

14(a)	Given triangular prism, and constant width, time taken is proportional to cross-sectional area covered by water level.
	Time taken to fill to $2h = \dfrac{(2h)^2}{(3h)^2} \times 18$
	$= 8$ seconds

14(b)	

Height of water (h cm) vs Time (seconds)

15(a)	$x^2 + 3x - 7$
	$= \left(x + \dfrac{3}{2}\right)^2 - \left(\dfrac{3}{2}\right)^2 - 7$
	$= \left(x + \dfrac{3}{2}\right)^2 - \dfrac{37}{4}$
	$k = -\dfrac{37}{4}$

15(b)	Minimum value $y = x^2 + 3x - 7$ is $y = -\dfrac{37}{4}$
	Hence, the equation $x^2 + 3x - 7 = p$ has no solutions when $p < -\dfrac{37}{4}$ **as there is no intersection between** $y = x^2 + 3x - 7$ **and** $y = p$

16	Sum of all exterior angles $= 360°$
	Hence sum of all reflex angles
	$180°(n) + 360° = 3060°$
	$n = \dfrac{3060° - 360°}{180°}$
	$n = 15$
	OR
	Sum of all interior angles $= 180°(n-2)$
	Hence sum of all reflex angles
	$360°(n) - 180°(n-2) = 3060°$

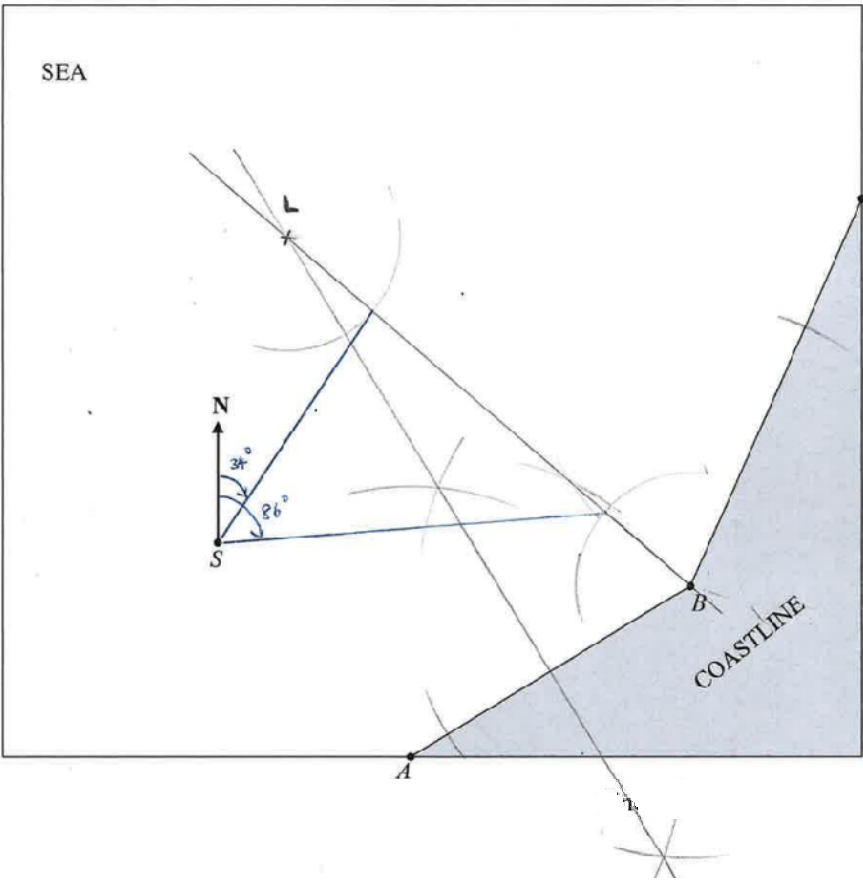|	
	$180°(n) + 360° = 3060°$ $$n = \frac{3060° - 360°}{180°}$$ $n = 15$

17(a)	Diagram 1
17(b)	$F = k\sqrt{T}$ $F_{new} = k\sqrt{T_{new}}$ $F_{new} = k\sqrt{9T}$ $F_{new} = 3k\sqrt{T}$ $F_{New} = 3F$ F increases by 200%

18	\$9 made up of 2, 2-dollars notes and 1, 5-dollars note. $$\frac{4x-2}{5x-1} = \frac{2}{3}$$ $12x - 6 = 10x - 2$ $x = 2$ $4x = 8$ Number of 2-dollar notes = 8

19	$\text{S.D.} = \sqrt{\dfrac{\sum x^2}{N} - \left(\bar{x}\right)^2}$ When all values are multiplied by 2, $\text{S.D.} = \sqrt{\dfrac{\sum (2x)^2}{N} - \left(2\bar{x}\right)^2}$ $= \sqrt{\dfrac{4\sum (x)^2}{N} - \left(2\bar{x}\right)^2}$ $= \sqrt{\dfrac{4(875)}{5} - \left(2 \times 11.8\right)^2}$ $= \sqrt{\dfrac{4(875)}{5} - \left(2 \times 11.8\right)^2}$ $= 11.9599$ $= 12.0 \ (3 \text{ sf})$

20(a)(i)	$Q = \{l, m\}$ or l, m
20(a)(ii)	R is **a proper subset** of P. There is **at least one element in P that is not an element of R.** If all elements of R also belong to P, then R is not a proper subset of P.
20(b)(i)	
20(b)(ii)	Max no. of students $= 23 - 11$ $= 12$

21		(a)
		(i) perpendicular bisector of A and B. **(ii)** angle bisector of A and B. **(b)** $034° < \theta° < 086°$

22(a)	Mean in this case is **distorted/skewed/affected by the outlier 49 marks (out of 50)**
22(b)	Lower quartile = 36 marks Upper quartile =40 marks Interquartile range = 40 – 36 $\qquad\qquad\qquad\quad$ = 4 marks
22(c)	Median for Paper B, 41 marks is higher than Paper A, 37 marks. Hence on average, students did better for Paper B than Paper A. Interquartile range for Paper A is 4 while interquartile range for Paper B is 7. Hence students' marks in Paper B has a greater spread than students' marks in Paper A.

23(a)	**T = EN** $$\mathbf{T} = \begin{pmatrix} 15 & 20 \\ 20 & 25 \\ 22 & x \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$ $$\mathbf{T} = \begin{pmatrix} 15\times3+20\times1 & 15\times1+20\times2 \\ 20\times3+25\times1 & 20\times1+25\times2 \\ 22\times3+x\times1 & 22\times1+x\times2 \end{pmatrix}$$ $$\mathbf{T} = \begin{pmatrix} 65 & 55 \\ 85 & 70 \\ 66+x & 22+2x \end{pmatrix}$$
23(b)	Elements in the second column of matrix T represent the **total number of minutes both Han and Peng spend swimming, cycling and running respectively on weekends.**
23(c)	$66 + x = 22 + 2x$ $x = 44$
23(d)	**B = CTA** $$\mathbf{B} = \begin{pmatrix} 8 & 10 & 12 \end{pmatrix} \begin{pmatrix} 65 & 55 \\ 85 & 70 \\ 66+x & 22+2x \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ $\mathbf{C} = \begin{pmatrix} 8 & 10 & 12 \end{pmatrix}$ Order of $\mathbf{A} = 2\times1$

24	t the time reaches the ground= time ball was above ground.
	$-1.44t^2 + 1.6t + 1.5 = 0$
	$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
	$t = \dfrac{-1.6 \pm \sqrt{(1.6)^2 - 4(-1.44)(1.5)}}{2(-1.44)}$
	$t = \dfrac{-1.6 \pm \sqrt{11.2}}{2(-1.44)}$
	$t = -0.60647$ or $t = 1.7175$
	as $t > 0$, $t = 1.72$ seconds

25(a)	AF is common length
	Angle BFA = Angle CFA = 90° (tangent $\perp$ radius)
	Angle EAO = Angle DAO (tangents from ext.pt)
	Hence triangle ABF is congruent to triangle ACF by ASA congruency test.
25(b)	By congruency and tangents from ext.pt,
	$BE = BF = FC = DC$.
	Considering triangle ABF,
	Angle $BFA = \sin^{-1}\left(\dfrac{7}{25}\right) = 16.260°$
	Considering triangle AEO,
	$\tan(16.260°) = \left(\dfrac{EO}{25-7}\right)$
	$EO = 18 \tan(16.260°)$
	$EO = 5.2499$ cm
	$EO = 5.25$ cm

26(a)	Angle $AEB = 180° - 38°$ (angles in opp.seg) $\qquad = 142°$ Angle $DEB = 180° - 142°$ (adj angles on st line) $\qquad = 38°$
26(b)	Angle $EAB = 109° - 90°$ (right angle in a semi-circle) $\qquad = 19°$ Angle $DEB = 38°$ $\dfrac{38°}{2} = 19°$ Angle $DEB = 2$ Angle EAB Hence, by angle at centre is twice the angle at circumference, E is the centre of circle I.
	OR Angle $AEB = 180° - 38°$ (angles in opp.seg) $\qquad = 142°$ Angle $EAB = 109° - 90°$ (right angle in a semi-circle) $\qquad = 19°$ Angle $ABE = 180° - 142° - 19°$ (angles sum of triangle) $\qquad = 19°$ Angle $EAB =$ Angle ABE *Hence, EA = EB,* by radius of circle, *E* is the centre of circle I OR Angle $CAB = 90°$ (right angle in a semi-circle) Angle $ABD =$ Angle $CAB = 90°$ (alt. angles of parallel lines) Angle $EAB = 109° - 90°$ (right angle in a semi-circle) $\qquad = 19°$ Consider triangle ADB Angle $ADB = 180 - 90° - 19°$ (angles sum of triangle) $\qquad = 71°$

Consider triangle *DEB*

Angle *ADB* = $180 - 71° - 38°$ (angles sum of triangle)

= 71°

Angle *EDB* = Angle *EBD*

Hence, ED = EB, by radius of circle, *E* is the centre of circle I

ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2024
(YEAR 4)

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS **4052/02**

Paper 2 **15 August 2024**

Candidates answer on the Question Paper. **2 hours 15 minutes (08:30 – 10:45)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total mark for this paper is 90.

This document consists of **20** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** In 2024, apples cost \$$x$ per kilogram and oranges cost \$$y$ per kilogram.
The total cost of 5kg of apples and 10kg of oranges is \$40.

(i) Show that $8 - 2y = x$.
Answer

[2]

(ii) In 2023, the cost, per kilogram, of apples were 9% lower than in 2024 while the cost of oranges, per kilogram were 9% higher than in 2024.

Find an expression, in terms of x and y, for the total cost of 3kg of apples and 2kg of oranges in 2023. Give your answer in its simplest form.
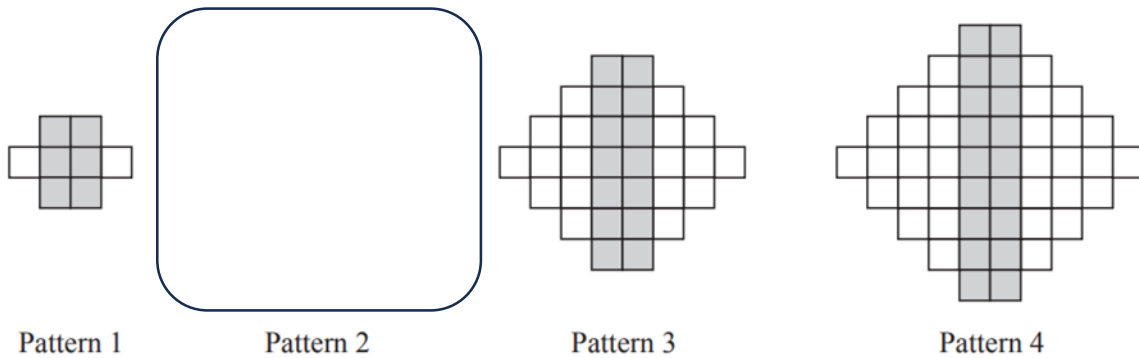
Answer ……………………...... [3]

(b) Write as a single fraction in its simplest form $\dfrac{x}{x^2-x-12}+\dfrac{3x}{4-x}$.

Answer …………………............ [4]

2 The diagram shows a sequence made by using grey and white squares.

| Pattern 1 | Pattern 2 | Pattern 3 | Pattern 4 |

(a) Draw Pattern 2 in the space above. [1]

(b) Complete the table for the next three pattern in the sequence.

Pattern Number	4	5	6
Number of grey squares	18		
Number of white squares	32		

[2]

(c) Find an expression, in terms of k, for the number of grey squares in Pattern k.
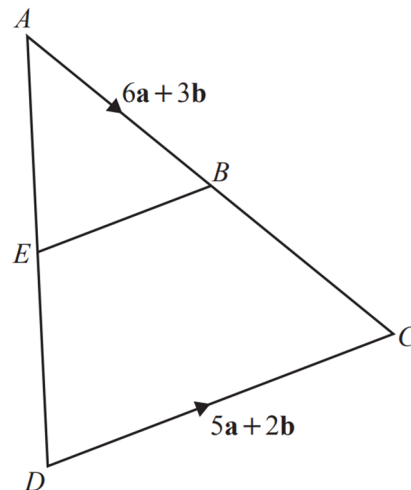
Answer …………………............... [2]

(d) Determine if any pattern can be formed with 100 grey squares.
Show your working.
Answer

[2]

3

In triangle *ACD*, *B* is the midpoint of *AC* and *E* is the midpoint of *AD*.
$\overrightarrow{AB} = 6\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{DC} = 5\mathbf{a} + 2\mathbf{b}$.

(a) Express $\overrightarrow{AD}$, as simply as possible, in terms of **a** and **b**.

Answer $\overrightarrow{AD} =$ ……………… [3]

(b) Show that triangles *AEB* and *ADC* are similar.

Answer

[2]

(c) *S* is a point on *AE* such $AS : SE = 2 : 3$.

It is given that $\overrightarrow{SC} = \dfrac{1}{k}\left(53\mathbf{a} + 26\mathbf{b}\right)$.

Find the value of *k*.

Answer   *k* =   ………………….   [2]

(d) Find the numerical value of $\dfrac{\text{area of triangle } AEB}{\text{area of } EBCD}$.

Answer   …………………………   [2]

(e) Use vectors to show that $\overrightarrow{EB}$ is parallel to $\overrightarrow{DC}$.

Answer

[2]

4 A tank is a closed cylinder of radius 5 m and height 8 m.
It is made of negligible thickness and stands with its base horizontal.

(a) The tank contains fuel to a depth of 3 m.

Find correct to the nearest cubic metre, the volume of the fuel in the tank.

Answer ………………… m^3 [1]

(b) 20 000 litres of fuel are added.

Find the increase in depth of the fuel in the tank. Give your answer in cm.
(1 litre = 1000 cm^3)

Answer ………………… cm [3]

(c) The outer curved surface and the top of the tank are to be painted.
The paint is sold in tins, each of which contains 5 litres.
One litre of paint can cover 7 m^2.
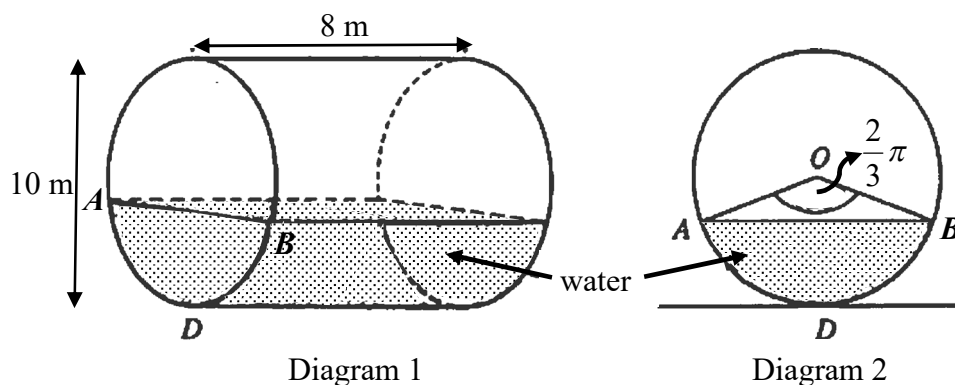
Calculate the number of tins required.

Answer ………………… tins [3]

(d) The tank is completely emptied of all fuel and then partially filled with water.
It is now placed with its curved surface on a horizontal floor.
Diagram 1 shows the dimensions of the tank.
Diagram 2 shows a circular end of the fuel tank. O is the centre of the circle
and D is vertically below O. AB represents the level of the water surface and
angle $AOB = \dfrac{2}{3}\pi$.

Diagram 1   Diagram 2

Calculate the area of the inside surface of the tank which is **not** in contact with
the water.

Answer   ………………………….   m^2 [5]
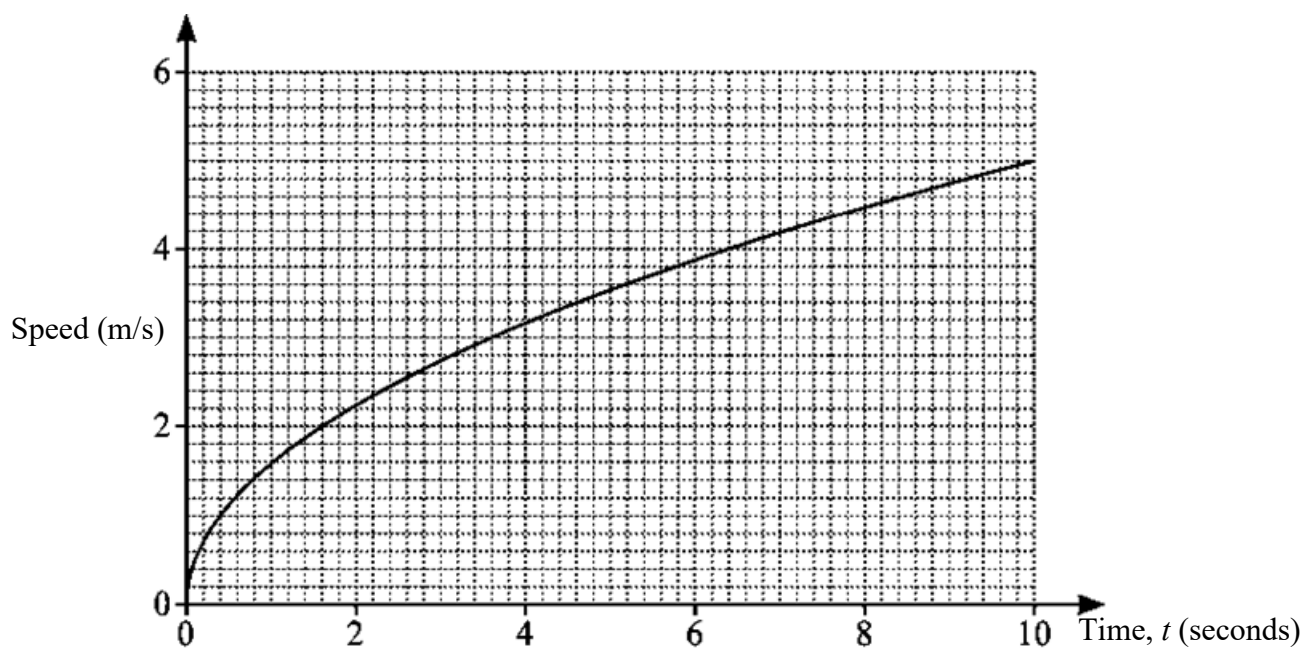
[TURN OVER

5 (a) (i) Convert 14 km/h to m/s.

Answer …………………………. m/s [1]

(ii) Haley rode a bike at a speed of 14 km/h for 1 hour 20 minutes from Kensington Town to Brood City.
After a 30 minutes rest at Brood City, she continued her 25 km journey to Bristol Plains at a speed of 10 km/h.

Calculate Haley's average speed, in km/h, for the entire journey.

Answer ……………………... km/h [4]

(b) The speed-time graph shows the first 10s for the motion of a car.

(i) **Using the graph above**, estimate the acceleration of the car at $t = 2$s.

Answer …………………………… m/s^2 [2]

(ii) State if you agree or disagree with the statement. Explain your choice.

Statement	Agree / Disagree	Explanation
The speed of the car is increasing at a decreasing rate.		

[2]

[TURN OVER

6 (a) A scientist researched on the effectiveness of a new drug to combat a disease.
The probabilities of a patient's condition upon receiving the drug are as follows:

Patient's condition	Probability
No effect	q
Improving but still ill	$2p$
Getting worst	$5p - \dfrac{4q}{5}$

There are three times as many patients whose condition was not affected by the drug as compared to those whose condition was getting worst.

Calculate the value of p and of q.

Answer p = …………………...

q = …………………... [4]

(b) Baloxavir marboxil (trade name Xofluza®) is the latest influenza (flu) antiviral drug.
The drug is immediately stopped once the patient is cured.
At most, each patient can only receive two doses of the drug.
The probabilities of a patient's condition after receiving each dose is as follows:

Patient's condition	Probability
Fully cured	$\dfrac{4}{7}$
Improving but still ill	$\dfrac{5}{14}$
Getting worst	$\dfrac{1}{14}$

(i) Draw a tree diagram showing the possible outcomes.

Answer

[3]

(ii) Calculate the probability that a patient remains ill despite 2 doses of Xofluza®.

Answer ……………………………… [3]

[TURN OVER

7 The manufacturing cost, y, of a watch when x watches are produced, is related by the equation $y = 20 + \dfrac{1200}{x}$.
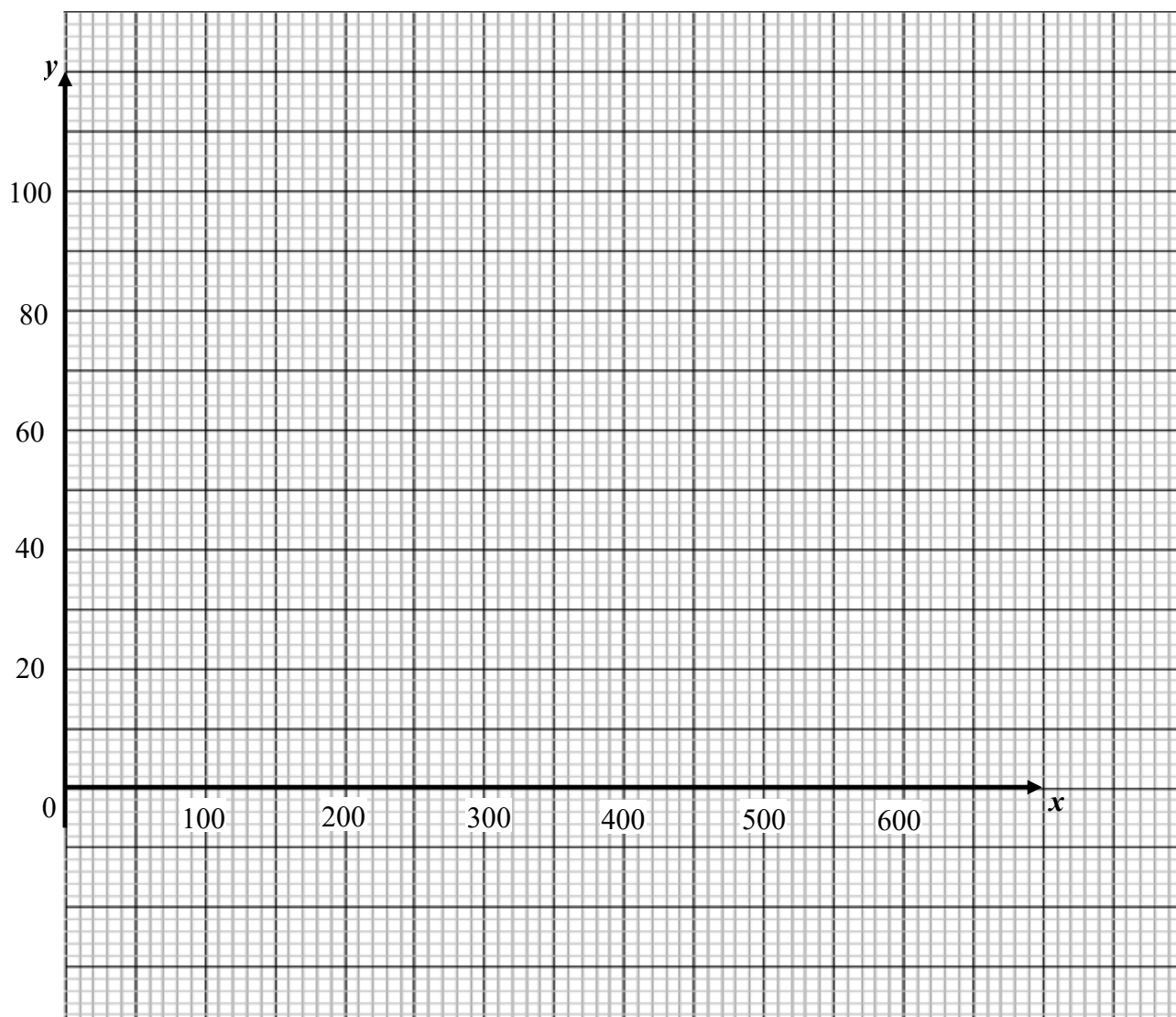
Some values of x and y are given in the table below.

x	15	30	60	150	300	400	600
y	100	60	40	28	24	q	22

(a) Find the value of q.

Answer q = ………………….. [1]

(b) On the grid below, draw the graph of $y = 20 + \dfrac{1200}{x}$ for $0 < x \leq 600$. [3]

Use your graph to find

(i) the manufacturing cost when 80 watches are produced,

Answer $ ………………………… [1]

(ii) the minimum number of watches to be produced for the manufacturing cost to be at most $25.
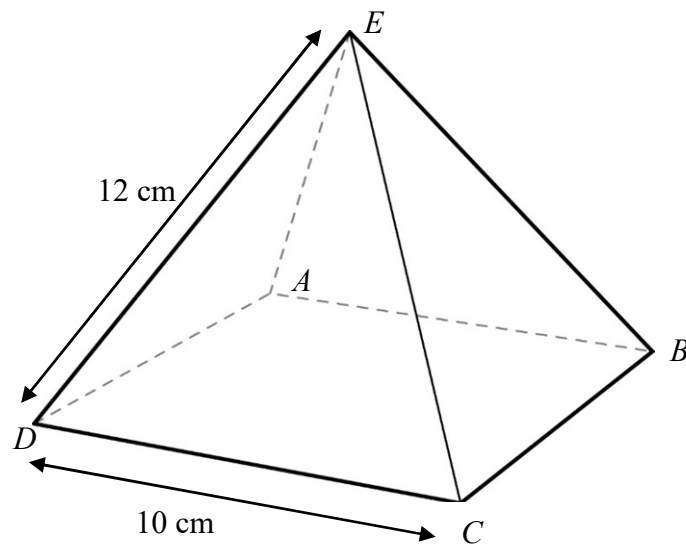
Answer …………………... watches [1]

(c) In order to sell x watches, the selling price of a watch needs to be $\left(90 - \dfrac{21x}{100}\right)$.

By inserting a suitable line, determine the range of number of watches that needs to be sold to ensure a profit.

Answer ……………………………… [3]

8 *ABCDE* is a right square-based pyramid. $DE = 12$ cm and $DC = 10$ cm.

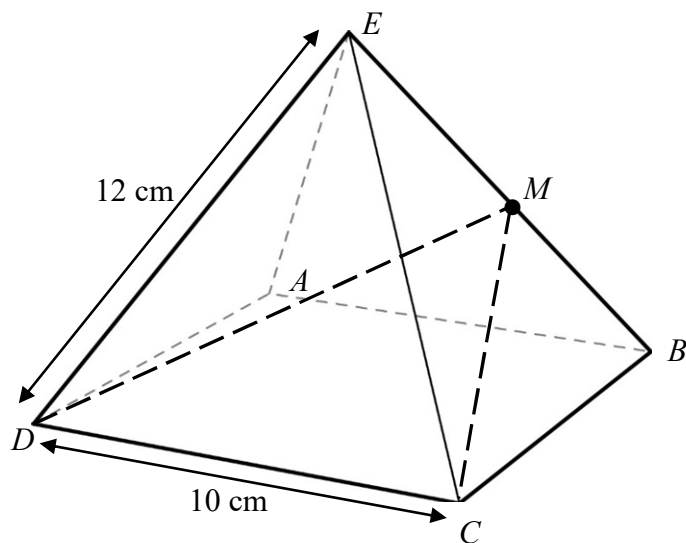**(a)** Show that the vertical height of the pyramid is 9.695 cm.

Answer

[4]

(b) Angle *DEC* = 49.2°.

Calculate angle *EDC*.

Answer …………………………….. [2]

(c)   *M* is the midpoint on *EB* such that angle *DMC* = 55.7° .
Angle *DEB* = 72.2° .

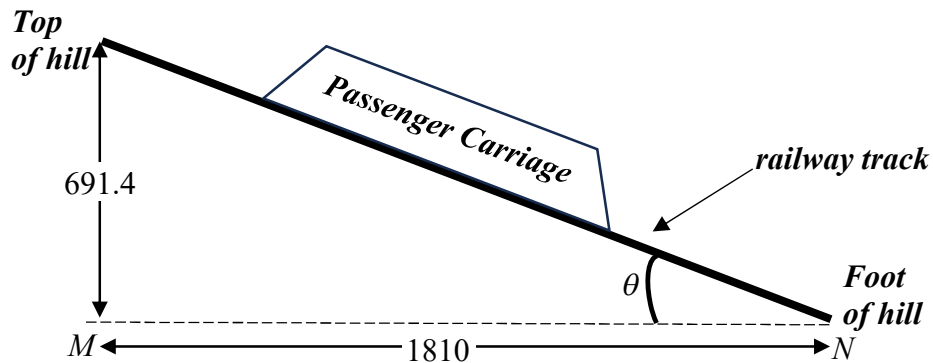Calculate the area of triangle *DMC* giving your answer correct to the nearest whole number.

Answer   …………………..   cm^2 [6]

[TURN OVER

9 The Penang Hill funicular railway which climbs the Penang Hill was built in 1923.

Supporting structures in the form of arches.

The diagram below shows a schematic drawing of the railway.

During the journey from the start (foot of the hill) to the end point (top of the hill), the passenger carriage makes a vertical gain of 691.4 m.
The horizontal distance between M and N is 1810 m.

(a) Calculate the angle of elevation, θ, of the Penang Hill funicular railway.

Answer θ = ……………………… [2]

(b) Calculate the length of the railway track.
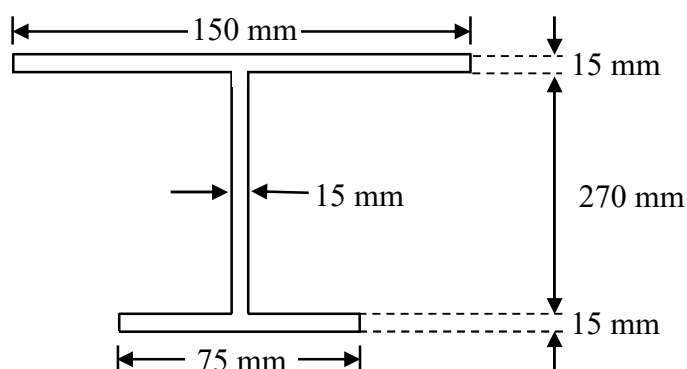
Answer ……………………... m [2]

(c) The railway is built on supporting structures in the form of arches which can withstand an overall maximum load of 140 tonnes.
The information table shows some technical parameters of the Penang Hill funicular railway.
(1 tonne = 1000kg)

	Railway track		Passenger Carriage	
Material used	1084 Hot rolled steel		5052 H-32 welded Aluminium	
Density of material	8.03 g/cm^3		2.68 g/cm^3	
Specifications	Shape	Asymmetric I-Beam	Mass of Carriage	14 500 kg
	Cross-sectional area (in mm^2)	*Refer to drawing*		

Drawing and measurements of the Asymmetric I-Beam

Some further details of the Penang Hill funicular railway are as follows:
- Operating hours are from 0900 till 1900.
- Ticket for the 2-way ride (up and down the hill) costs $45 per person.
- Rides up and down the hill are scheduled every hour, with each ride taking 30 minutes non-stop.
- Only 1 passenger carriage is used for the rides up and down the hill.
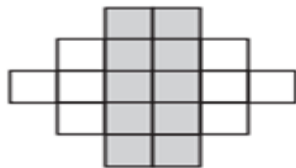
[Question 9 is continued on the next page.]

[TURN OVER

The government of Penang claims that the Penang Hill funicular railway receives a daily estimated total of at least $80 000 from the sale of tickets for the rides.

Assuming the average weight of a passenger is 75 kg, justify with calculations, if the claim made by the government of Penang is true.

[7]

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No	Solution	
1a i	$5x + 10y = 40$ $x + 2y = 8$ $8 - 2y = x (shown)$	
aii	Apple cost in 2023$=0.91x$ Orange cost in 2023 $= 1.09y$ Total cost in 2023 $= 3(0.91x) + 2(1.09y)$ $\qquad = 2.73x + 2.18y$	
iii	$\dfrac{x}{x^2 - x - 12} + \dfrac{3x}{4 - x}$ $= \dfrac{x}{(x-4)(x+3)} + \dfrac{3x}{4-x}$ $= \dfrac{x}{(x-4)(x+3)} - \dfrac{3x}{x-4}$ $= \dfrac{x - 3x(x+3)}{(x-4)(x+3)}$ $= \dfrac{-3x^2 - 8x}{(x-4)(x+3)}$	
2i	Pattern 2	
ii	<table><tr><td>Pattern Number</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Number of grey squares</td><td>18</td><td><u>22</u></td><td><u>26</u></td></tr><tr><td>Number of white squares</td><td>32</td><td><u>50</u></td><td><u>72</u></td></tr></table>	
iii	$4k + 2$	

1

iv	$4k + 2 = 100$ $4k = 98$ $k = 24.5(\text{ non-integer})$ Hence, it is not possible to have 100 grey squares.
3a	$\overrightarrow{AB} = 6\mathbf{a} + 3\mathbf{b}$ $\overrightarrow{AC} = 2\,(6\mathbf{a} + 3\mathbf{b})$ $\quad = 12\mathbf{a} + 6\mathbf{b}$ $\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$ $\overrightarrow{AD} = \overrightarrow{AC} - \overrightarrow{DC}$ $\quad = (12\mathbf{a} + 6\mathbf{b}) - (5\mathbf{a} + 2\mathbf{b})$ $\quad = 7\mathbf{a} + 4\mathbf{b}$
b	$\angle DAC = \angle EAB$ (common angle) $\dfrac{AE}{ED} = \dfrac{AB}{BC} = \dfrac{1}{2}$ Hence, by SAS similarity test, $\triangle$AEB is similar to $\triangle ADC$.

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c	$\overrightarrow{AS} + \overrightarrow{SC} = \overrightarrow{AC}$	
	$\overrightarrow{SC} = \overrightarrow{AC} - \overrightarrow{AS}$	
	$\phantom{\overrightarrow{SC}} = \overrightarrow{AC} - \dfrac{2}{5}\overrightarrow{AE}$	
	$\phantom{\overrightarrow{SC}} = \overrightarrow{AC} - \dfrac{2}{5}\left(\dfrac{1}{2}\overrightarrow{AD}\right)$	
	$\phantom{\overrightarrow{SC}} = \overrightarrow{AC} - \dfrac{1}{5}\overrightarrow{AD}$	
	$\phantom{\overrightarrow{SC}} = 2(6\mathbf{a}+3\mathbf{b}) - \dfrac{1}{5}(7\mathbf{a}+4\mathbf{b})$	
	$\phantom{\overrightarrow{SC}} = \dfrac{53}{5}\mathbf{a} + \dfrac{26}{5}\mathbf{b}$	
	$\phantom{\overrightarrow{SC}} = \dfrac{1}{5}(53\mathbf{a}+26\mathbf{b})$	
	$\Rightarrow k = \dfrac{1}{5}$	
d	$\dfrac{\text{area of } \Delta AEB}{\text{area of } \Delta ADC} = \left(\dfrac{1}{2}\right)^2$	
	$\phantom{\dfrac{\text{area of } \Delta AEB}{\text{area of } \Delta ADC}} = \dfrac{1}{4}$	
	$\dfrac{\text{area of } \Delta AEB}{\text{area of } EBCD} = \dfrac{1}{4-1}$	
	$\phantom{\dfrac{\text{area of } \Delta AEB}{\text{area of } EBCD}} = \dfrac{1}{3}$	
e	$\overrightarrow{AD} = 2\overrightarrow{AE}$	
	$\overrightarrow{AE} = \dfrac{1}{2}(7\mathbf{a}+4\mathbf{b})$	
	$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB}$	
	$6\mathbf{a}+3\mathbf{b} = \dfrac{1}{2}(7\mathbf{a}+4\mathbf{b}) + \overrightarrow{EB}$	
	$\overrightarrow{EB} = \dfrac{5}{2}\mathbf{a}+\mathbf{b}$	
	$\overrightarrow{DC} = 5\mathbf{a}+2\mathbf{b}$	
	$\phantom{\overrightarrow{DC}} = 2\left(\dfrac{5}{2}\mathbf{a}+\mathbf{b}\right)$	
	$\phantom{\overrightarrow{DC}} = 2\overrightarrow{EB}$	
	Since $\overrightarrow{DC} = k\overrightarrow{EB}$, where $k=2$ and do not have a common point, they are parallel.	

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4a	$V = \pi(5)^2(3)$ $= 75\pi$ or $235.619449 m^3$ $= 236$ m^3 (3sf)
b	$20000l = 20\ 000\ 000 cm^3 = 20 m^3$ $V_{new} = 20 + 235.619449$ $= 255.619449 m^3$ $\Rightarrow \pi(5)^2 h_{new} = 255.619449$ $h_{new} = 3.2546479 m$ $increase = 3.2546479 - 3$ $= 0.2546479 m$ $= 25.5 cm (3sf)$ OR $20000(1000) = \pi(500)^2(h)$ $h = \dfrac{20000000}{500^2 \pi}$ $= \underline{\mathbf{25.5\ cm}}$ (to 3 sf)
c	Area to be painted $= \pi(5)^2 + 2\pi(5)(8)$ $= 105\pi m^2$ $5l = 0.005 m^3$ $0.001 m^3$ rep $7m^2$ $0.005 m^3$ rep $35 m^2$ No. of tins needed $= \dfrac{105\pi}{35}$ $= 9.42477961$ $= 10$ tins (nearest integer)

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d		
	$$\text{reflex } \angle AOB = 2\pi - \frac{2}{3}\pi$$ $$= \frac{4\pi}{3}$$ $$\text{area of major sector AOB} = \frac{240°}{360°} \times \pi \times (5)^2$$ $$= \frac{50}{3}\pi m^2$$ $$\text{area of } \Delta AOB = \frac{1}{2}(5)^2 \sin\frac{2}{3}\pi$$ $$= \frac{25\sqrt{3}}{4}m^2$$ $$\text{major arc AB} = 2\pi(5) \times \frac{4\pi/3}{2\pi}$$ $$= \frac{20}{3}\pi m^2$$ $$\text{Dry curved area} = \frac{20}{3}\pi \times 8$$ $$= \frac{160}{3}\pi m^2$$ $$\text{Total area not in contact} = \frac{160}{3}\pi + 2\left(\frac{25\sqrt{3}}{4} + \frac{50}{3}\pi\right)$$ $$= \frac{160}{3}\pi + \frac{25\sqrt{3}}{2} + \frac{100}{3}\pi$$ $$= \frac{260}{3}\pi + \frac{25\sqrt{3}}{2}$$ $$= 293.9198m^2$$ $$= 294 \text{ m}^2 (3sf)$$	
5 **ai**	14000m in 60×60 s 14000m in 3600s $\frac{14000}{3600}$ m in 1s 3.8889m in 1s $\therefore 14km/h = 3.89m/s$ (accept $3\frac{8}{9}$)	

aii	
	Distance between Kensington and Brood $= 14 \times \dfrac{4}{3}$ $= 18\dfrac{2}{3} km$ Time taken from Brood to Bristol $= \dfrac{25}{10} hr$ Total time $= \dfrac{4}{3} + \dfrac{25}{10} + \dfrac{1}{2}$ $= \dfrac{13}{3} hrs$ Average speed $= \dfrac{25 + 18\dfrac{2}{3}}{13 \big/ 3}$ $= 10.07692 km/h$ $= 10.1$ km/h (3sf)	
bi	$0.5 - 0.6$	

ii	Statement	Agree / Disagree	Explanation	
	The velocity of the car is increasing at a decreasing rate.	**Agree**	The <u>gradient of the tangent</u> is decreasing.	

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6a

$$q + 2p + 5p - \frac{4q}{5} = 1$$

$$7p + \frac{q}{5} = 1 ---(1)$$

Let the total number of people be P

$$qP = 3(5p - \frac{4q}{5})(P)$$

$$q = 15p - \frac{12q}{5}$$

$$\frac{17}{5}q = 15p$$

$$p = \frac{17}{75}q ---(2)$$

sub (2) into (1):

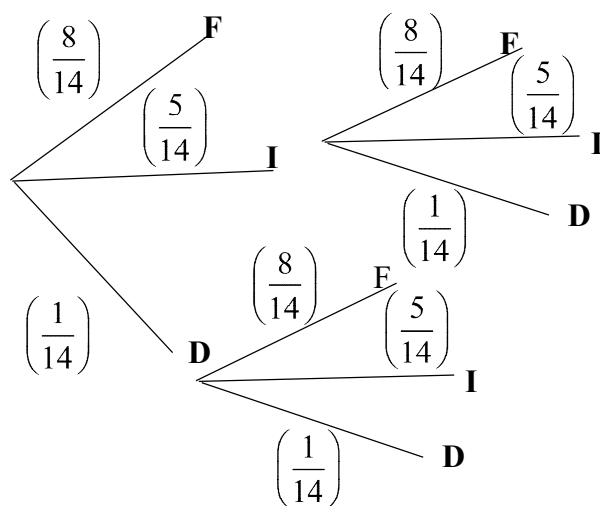$$7\left(\frac{17}{75}q\right) + \frac{q}{5} = 1$$

$$\frac{134}{75}q = 1$$

$$q = \frac{75}{134}$$

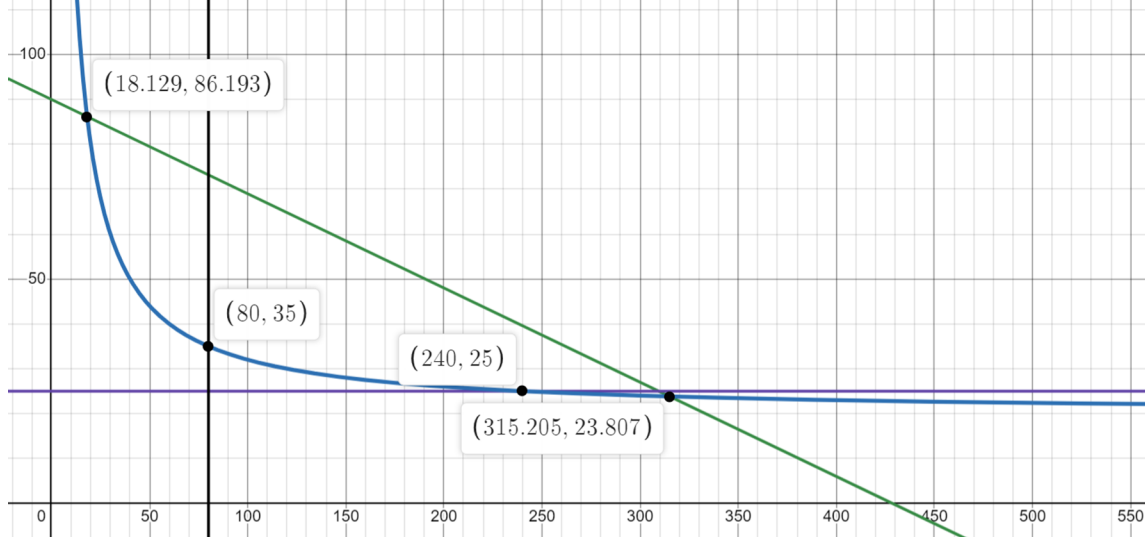$$\Rightarrow p = \frac{17}{75}\left(\frac{75}{134}\right)$$

$$= \frac{17}{134}$$

bi

B1 – for showing a tree diagram (award even if partially drawn) ;
B1 – all values in Tier 1, B1 – all values in Tier 2

7

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ii	P (ill after 2 doses) $$=\left(\frac{5}{14}\right)^2+\left(\frac{1}{14}\right)^2+\left(\frac{5}{14}\times\frac{1}{14}\right)+\left(\frac{1}{14}\times\frac{5}{14}\right)$$ $$=\frac{9}{49}$$
7a	23
b	**B1 – showing all plotting points** **B1 – labels of axis and equation of curve** **B1 – smooth curve passing through all points**
ci	$28 \le y \le 42$
ii	$232.5 \le x \le 247.5$
d	$15(+5) < x \le 320(\pm 5)$
8a	diagonal length of base $=\sqrt{10^2+10^2}$ $\qquad\qquad\qquad = \sqrt{200}cm$ hence, $$h^2+\left(\frac{\sqrt{200}}{2}\right)^2=12^2$$ $h^2+50=144$ $h^2=94$ $h=\sqrt{94}cm$ $\quad=9.69535971cm$ $\quad=9.695$ cm (shown)

8

| b | $\angle EDC = \dfrac{180^\circ - 49.2^\circ}{2}\,(\text{base}\angle\text{s of isos.}\Delta, ED = EC)$ | |
| | $\qquad = 65.4^\circ\,(1dp)$ | |

| c | By Cosine Rule, | |

$DM^2 = 12^2 + 6^2 - 2(12)(6)\cos 72.208409^\circ$

$DM = \sqrt{136}$

$= 11.6619038cm$

$\angle CBM = \dfrac{180^\circ - 49.24864^\circ}{2}\,(\text{base angles of isos.}\Delta,\ CM = BM)$

$= 65.37568^\circ$

$CM^2 = 6^2 + 10^2 - 2(6)(10)\cos 65.37568^\circ$

$= 85.999996863$

$CM = 9.27361833cm$

$Area = \dfrac{1}{2}(\sqrt{136})(9.2736183)\sin 55.7^\circ$

$= 44.67045755416cm^2$

$= 45\ cm^2\,(\text{nearest whole number})$

---Alternative Solution to 8c---

By Cosine rule,

$DM^2 = 12^2 + 6^2 - 2(12)(6)\cos 72.208409^\circ$

$DM = \sqrt{136}$

$= 11.6619038cm$

$\dfrac{\sqrt{136}}{\sin \angle DCM} = \dfrac{10}{\sin 55.7^\circ}$

$\sin \angle DCM = \dfrac{\sqrt{136}\sin 55.7^\circ}{10}$

$\angle DCM = 74.44807810^\circ$

$\angle MDC = 180^\circ - 55.7^\circ - 74.44807810^\circ\,(\text{sum of angles in }\Delta)$

$= 49.8519219^\circ$

$Area = \dfrac{1}{2}(10)(\sqrt{136})\sin 49.8519219^\circ$

$= 44.570667097cm^2$

$= 45\ cm^2\,(\text{nearest whole number})$

2024 MATH (MA) PAPER 2 PRELIM SOLUTIONS

Wednesday, 11 September 2024

9a	$\tan\theta = \dfrac{691.4}{1810}$ $\theta = \tan^{-1}\left(\dfrac{691.4}{1810}\right)$ $= 20.9063044°$ $= 20.9°$	
b	$l = \sqrt{(691.4)^2 + (1810)^2}$ $= 1937.5587763m$ $= 1940$ m (3sf)	

c	Maximum load for the supporting arches $= 140 \times 1000$ $\qquad\qquad = 140000 kg$ Cross-sectional area of track$=(150\times15)+(75\times15)+(270\times15)$ $\qquad\qquad = 7425 mm^2$ Mass of track $= \dfrac{7425}{100} \times 1937.585712 \times 100 \times 8.03$ $\qquad\qquad = 115524.1885 kg$ Maximum allowable weight of passengers $= 140000 - 115524.1885 - 14500$ $\qquad\qquad = 9975.8115 kg$ No.of passengers carriage can carry on each ride $= \dfrac{9975.8115}{75}$ $\qquad\qquad = 133.01082$ $\qquad\qquad = 133$ Maximum no. of single-rides in one day $= \dfrac{10 \times 60}{30}$ $\qquad\qquad = 20$ Maximum no. of two-way-rides in one day $= 10$ Maximum no. of passengers taking the rides each day $= 133 \times 10$ $\qquad\qquad = 1330$ Maximum money received from ticket sales each day $= 1330 \times 45$ $\qquad\qquad = \$59850\,(<\$80000)$ Hence, **the claim is false** as the maximum revenue from ticket sales is lesser than the stated $80 000	