Regression Analysis of Apartment Building Evaluation in Toronto

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Introduction

Analysis Goal

The goal of this analysis is to use multiple linear regression models in order to predict the evaluation score of a building. This prediction will be based on three independent variables: the number of units in a building, the age of the building, and the property type of the building. This analysis is conducted using the Apartment Building Evaluation data set provided by the Toronto open data portal (*1. "Open Data"*). The analysis is performed using buildings that have been built in the last 40 years (i.e., after 1980), and evaluated in 2020. The reason is that a great proportion of buildings have been built during the 1950-1970's baby boom and are concentrated in specific wards of Toronto.

This analysis is relevant to Torontonians who are owner of an apartment building property or seeking to buy or rent a property in Toronto. Additionally, the analysis is relevant to people seeking social housing in Toronto. This is because the data provides information about private properties, Toronto Community Housing, but also social housing.

The apartment building evaluation program is called "RentSafeTO". It was implemented in 2017 by the city of Toronto in order to protect Torontonians by making sure they are provided with information to live in the best conditions possible. During evaluations, Bylaw Enforcement Officers inspect common areas, mechanical and security systems, parking and exterior grounds (*2. "City of Toronto"*). Buildings are then provided with a percentage score. A higher score implies a building respects safety regulations and is well maintained.

Now having introduced the importance of the "RentSafeTO" program which aims to improve the well being of Torontonians, we can formulate a research question for our analysis: ***Can the evaluation score of a building be predicted such that there exists a linear relationship where the evaluation score is dependent on the number of units, property type, and age of the building?***

Terminology

In order to understand the following analysis, let's define the terms that will be used extensively.

- **Private Property**: These are buildings operated by private owners and/or investment firms. They are run in a for-profit manner.
- **TCHC**: The acronym stands for Toronto Community Housing. It is a social housing provider in Canada wholly owned by the City of Toronto that operates in a non-profit manner. TCHC has 2,100 buildings and 50 million square feet of residential space, which represent a $9 billion public asset (*3. "TCHC, Who We Are ."*).
- **Social Housing**: This represents all other Toronto social housing providers that are not TCHC.

Hypothesis

Before exploring the data and computing the regression model, let's formulate three hypotheses for potential results.

1. Intuitively, buildings with a younger age should be easier to maintain than older ones. Therefore, we expect younger buildings to have a higher evaluation score.

2. Supposing that buildings with higher evaluation scores are more in demand, we would expect their value to increase. Therefore, it is expected that properties of type private have a higher evaluation score. My reasoning behind this is that private property owners have incentives to see their assets go up in value which happens when demand for an asset is high.

3. Finally, I hypothesize that the direction of the relationship between evaluation score and the number of units in a building is positive. Intuitively, buildings with more units would be harder to maintain which would negatively affect the evaluation score. On the other hand, the maintenance cost should be less challenging to handle for buildings with more units. This is because the cost of maintaining common areas can be divided among more people such that maintenance fees per unit are lower. This would positively affect the relationship. I hypothesize that the latter economic reason has a stronger effect; thus, the relationship would be positive.

The purpose of the following section is to showcase the data to get a better understanding of what we are working with.

Data

Collection Process

The data that will be used for this analysis are published on the Toronto Open Data Portal by the Toronto Municipal Licensing & Standards (*1. "Open Data"*). The data is refreshed daily. For this analysis, the data was downloaded locally on October 20, 2021. Therefore, no entry added after this date will be captured by our analysis. The data provides an entry for each year a building was evaluated. These entries contain the overall score evaluation (as a percentage), geographic information (Ward, Latitude, Longitude, Address), the property type, number of units and storeys, but also detailed score evaluation (out of five) about specific areas evaluated (Entrance Lobby, Stairwell, Elevator, Security, Parking Area…). The formula to calculate the overall score is as follows: sum of all assigned detailed scores during the evaluation $\div$ (number of unique items reviewed $\times$ 5).

Cleaning Process

In the original data, each entry corresponds to a building evaluation made in a certain year. A building could appear more than once in the data if it was evaluated in different years. For this analysis, we only use buildings that were evaluated in 2020. Thus, we remove all observations which do not have an evaluation year of 2020. In the cleaned data frame, a building will not appear twice, as it can only be evaluated once a year.

As mentioned in the introduction, we are only interested in buildings built in 1980 or after. All other observations for buildings built before 1980 are removed. Additionally, observations where the number of units is below 248 are removed. This is because, among each property type, Social Housing has the lowest max at 248. The rational for doing this is that we want to make sure our linear regression model is comparable given a property type.

The original data contains a variable Year Built. This variable is removed, but used to create a new variable named Age where Year Built is subtracted from 2021. For example, a given observation in the cleaned data frame will not display the variable Year Built being 1980, but instead will display the variable Age being 31.

Finally, we are only interested in the overall score, the property type, the age, and the number of units in a given building. Thus, we can remove all other variables and are left with four variables in the cleaned data frame.

Definition of the variables that will be used extensively

This subsection serves as reference for the definition of the variables that are used throughout the analysis.

- **Age**: This variable describes how old a building is. If a building was built in 2005, its age will be 16.
- **Score**: This variable describes the evaluation percentage score for a given building that was evaluated in 2020. A low score implies that a building is poorly maintained.
- **Type**: This variable describes the property type of a building. There are three possible values for this variable: Private, TCHC, and Social Housing. These were defined previously in the terminology section.
- **Number of Units**: This variable provides the number of units in a given building.

Numerical Summary Statistics

Firstly, let's consider that there are 103 observations in our cleaned data frame. Additionally, we have one categorical variable named Type. This variable contains 23 observations for Private properties, 26 for TCHC properties, and 54 for other Social Housing properties. Let's compute a numerical summary for each continuous variable we have.

Table 1: Numerical Summaries of Continuous Variables

Statistic	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Age	26.427	11.365	2	18	29	36	41
Score	81.301	8.870	61	75	81	87.5	100
Number of Units	92.544	69.017	10	28.5	83	152.5	248

The first variable summary statistic provided by Table 1 is Age. It has a mean of 26.427 which differs from the median of 29. This can be interpreted in the following way: most of the building in our data were built in the 1990's. Age has a minimum of 2 which means that the last property built was in 2019.

The summary statistic also provides a better idea of the evaluation score system. It has a mean of 81.301 and a standard deviation of 8.87. In this context, the standard deviation is quite low so the buildings would be compared by very few percentage points when using the evaluation score.

The minimum number of units is 10. This makes sense because the "RentSafeTO" program only evaluates buildings with 10 units or more.

The Number of Units has a mean of 92.544 and a remarkably high standard deviation of 69. For this reason, let's compute the means of each continuous variable conditional on the property type to see if they differ greatly.

Table 2: Mean Summary by Property Type

Type / Mean	Age	Score	Number of Units
Private	18.70	86.30	84.04
Social Housing	27.91	80.17	78.30
TCHC	30.19	79.23	129.65

First, notice that Private properties are on average younger than both TCHC properties and other Social Housing organizations.

Secondly, the mean evaluation score of private properties stands at 86.304. It is way above the mean evaluation score of both TCHC properties, and other Social Housing. This is in line with hypothesis 2.

Finally, the mean for Private and Social Housing properties are quite close standing at 84.043 and 78.296. These two values are quite far from the mean number of units for TCHC properties which stands at 129.654. This explains the high standard deviation founded in Table 1 for the number of units not conditional on the property type.

Visualizing the Data

Let's use three scatter plots for each of the property types mapping Evaluation Score dependent on the number of units in a building. A continuous scale representing Age is added.

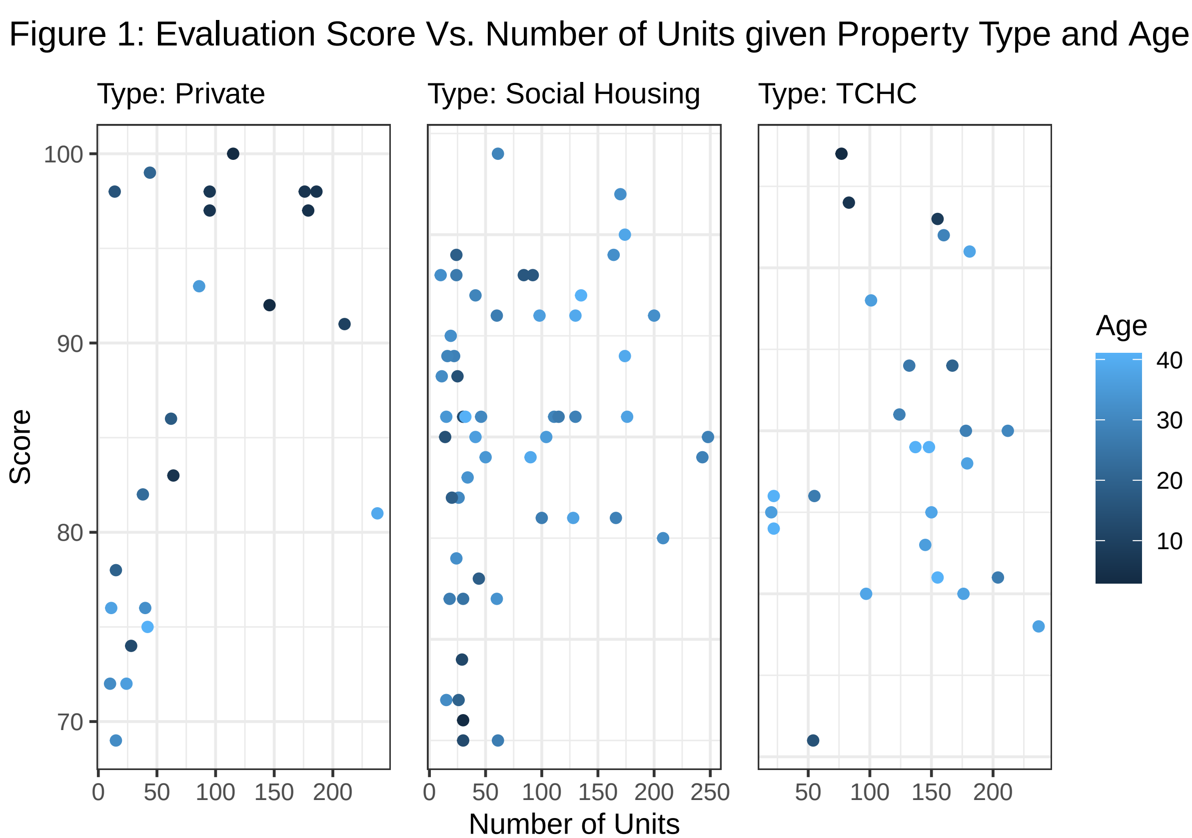


Figure 1 contains a lot of information but is nevertheless easy to understand. Each of the three scatter plots represent observations for each of the three property types: Private, Social Housing, and TCHC. The dependent variable on the y-axis is the evaluation score of a building. The independent variable on the x-axis is the number of units in a building. The continuous scale represents the age of buildings. A darker blue means a building is younger while a lighter blue means a building is older.

A multiple linear regression model is appropriate to predict the evaluation score as we took care of the issue regarding outliers in the data section. A multiple linear regression of the dependent variable Score on the following independent variables number of units, Age, and Property Type will be performed. The number of units should be considered as our main independent variable of interest while the two other independent variables age and type should be considered as control variables. This will be defined more precisely in the next section called 'Methods' where the multiple linear regression model is introduced.

Methods

The aim of this section is to introduce multiple linear regression, assumptions, and selection techniques to compare models.

Multiple Linear Regression Models

Multiple linear regression generalizes the simple linear regression model by allowing for many terms in a mean function rather than just one intercept and one slope (*4. Weisberg, Sanford*).

The simple linear regression model is defined as follows: $y = \beta_0 + \beta_1 x + \epsilon$. The goal of the simple linear regression model was to estimate the β_0 and β_1 parameters to obtain a function that predicts the independent variable y given any x. The Greek letter ϵ is called the error term. It will not be rigorously defined here, but the intuition behind ϵ is that it captures the inaccuracy in the model. It explains the difference between the theoretical value of the model and the actual observed results (*5. Holmes, Alexander, et al.*).

The multiple linear regression model is simply an expansion of the simple linear regression model where we can add as many $x_1, x_2, ..., x_k$ as we wish. It follows that for each x_i added predictor, we need to estimate an additional β_i. The estimate of the β parameters is found using ordinary least-squares. Here is a reference for the reader interested in the mathematics of OLS (*4. Weisberg, Sanford*). For our analysis, the estimation of the parameters will be handled by the R statistical programming software. The multiple linear regression model is as follows:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \epsilon$$

The main idea behind adding $x_2, ..., x_k$ is to explain the part of y that has not already been explained by x_1. Now, to relate this theory to our original problem, we will introduce four linear regression models. First, a simple linear regression of Score on Number of Units. Then for the second model, we add a second independent variable Age to create a multiple linear regression model. The third model is an extension of the second where we add the independent variable Type.

- **Model 1**: $Score_i = \beta_0 + \beta_1 NumUnits_i + \epsilon_i$

The interpretation of this model is simple: if the number of units increases by one, then the score would increase by β_1.

- **Model 2**: $Score_i = \beta_0 + \beta_1 NumUnits_i + \beta_2 Age_i + \epsilon_i$

The variable Type we will add to the third model is categorical. This means we need to add two binary variables for the three categories.

- **Model 3**: $Score_i = \beta_0 + \beta_1 NumUnits_i + \beta_2 Age_i + \beta_3(Type = SH) + \beta_4(Type = TCHC) + \epsilon_i$

In the case of an observation where the Type is Social Housing, we set $SH = 1$ and $TCHC = 0$. If Type were to be Private, we set $SH = 0$ and $TCHC = 0$.

- **Model 4**: $Score_i = \beta_0 + \beta_1 NumUnits_i + \beta_2 Age_i + \beta_3(Type = SH) + \beta_4(Type = TCHC) + \beta_5(Type = SH) \cdot Age_i + \beta_6(Type = TCHC) \cdot Age_i + \epsilon_i$

In this model, we have introduced an interaction effect by multiplying the type variable by the age variable. If dependence between these variables exists, model 4 should provide more accurate responses than model 3.

Assumptions

Readers not interested in the specifics of linear regression may skip this section. Before introducing selection techniques, let's introduce a few assumptions about the linear regression models:

1. *The model is linear in parameters.* This assumption is met for all four models.
2. *Random Sampling, i.e.,* $(x_{1i}, ..., x_{ki}, y_i)$*,* $i = 1, ..., n$ *are i.i.d.* This assumption is met as we have used all the existing observations in our setting.
3. *No Perfect Collinearity.* This assumption is met for all four models as our variables do not constitute any linear combination of each other, and do not sum to a constant.
4. $E(\epsilon | x_1, ..., x_k) = 0$. We will check if this assumption is met in the 'Results' section.

All these assumptions were retrieved from the Wooldridge's textbook referenced in the Bibliography section (*6. Wooldridge, Jeffrey M.*).

Model Selection Techniques

There are three techniques that will be used to determine which of the four models is the most appropriate.

- **F-Test**: This is an hypothesis test which is formulated as follows; $H_0 : \beta_1, ..., \beta_k = 0$. This says that none of the predictors are linearly related to the response. If we can reject the null hypothesis, then at least one predictor is linearly related to the response. We will perform this test on each model, and reject the null hypothesis if we find a p-value below 0.05 (*7. "STA302/1001"*). In other words, if a model has a p-value greater than 0.05, it will not be considered.
- **AIC**: The equation of this test is as follows; $AIC = 2k - 2ln(\hat{L})$. The number of predictors is k, so more predictors increases the value of AIC. $\hat{L}$ is the maximum value of the likelihood function. Therefore, if our model fits well, the second term of AIC would be higher. This means AIC would be lower. Thus, when comparing models, we are looking for the one with the lowest AIC (*8. "Akaike Information Criterion."*).
- $\mathbf{R^2_{adj}}$: This test is called the adjusted coefficient of determination. A higher value is better when comparing it between models. It is used to measure the proportion of the variation in the dependent variable that is predictable from the independent variables (*9. "Coefficient of Determination."*). As opposed to R^2, R^2_{adj} takes into account the size of the model. This means adding predictors brings down R^2_{adj}, but if the model fits better with the added predictors, R^2_{adj} goes up.

The computation of all these test statistics will be handled by R. Let's finally compute the results of all the methodologies introduced.

Results

Numerical Results

The next table outputs the results of the estimations of parameters (along with their standard errors in parenthesis) for each model, and the test statistics to compare them.

Table 3: Estimation of Parameters for Linear Regression Models

	Model 1	Model 2	Model 3	Model 4
(Intercept)	78.760	86.126	87.464	93.887
	(1.440)	(2.212)	(2.349)	(2.966)
Number of Units	0.027	0.032	0.037	0.021
	(0.012)	(0.012)	(0.012)	(0.012)
Age		−0.295	−0.230	−0.502
		(0.071)	(0.074)	(0.115)
Type: Social Housing			−3.808	−21.443
			(2.087)	(4.475)
Type: TCHC			−6.134	−5.040
			(2.471)	(4.864)
Age × Type: Social Housing				0.719
				(0.171)
Age × Type: TCHC				0.091
				(0.173)
N	103	103	103	103
F	4.830	11.533	7.582	9.489
p	0.030 254 81	3.113×10^{-5}	2.303×10^{-5}	4×10^{-8}
AIC	742.1	727.6	725.2	709.0
AdjustedR-Squared	0.036	0.171	0.205	0.333

Interpretation of estimated parameters

The number of units parameter varies a little between the 4 models. It is estimated at 0.027 in Model 1. As we add variables, it goes up to 0.032 and 0.037 in Model 2 and 3. Interestingly, it goes down to 0.021 in Model 4 after the interaction variable is added. The number of units parameter in Model 4 can be interpreted in the following way: Holding all other variables constant, increasing the number of units by 1 will increase the apartment evaluation score by 0.021.

Model 2, 3, and 4 introduce a negative relationship between the Age of a building and the evaluation score. This means that as a building gets older, the evaluation score goes down.

Now, there is a nuance in Model 4 concerning the estimated parameter of the interaction variable $Age \times Type : SocialHousing$. Its estimate is 0.719 while the estimation of the parameter Age is -0.502. Intuitively, this means that properties of type Social Housing have an evaluation score that increases as they get older. The mathematical reason for this is that we can factor the age variable in Model 4. Given Type is Social housing we have $\beta_2 Age_i + \beta_6 Age_i \times (1) = (\beta_2 + \beta_6) Age_i$. Plugging in the estimates we get $(-0.502 + 0.719) Age_i = 0.217 Age_i$. Thus, we can say that holding everything else constant and given a property of type Social Housing, a 1 year increase in age will increase the evaluation score by 0.217.

Comparing Models Using the test statistics

The letter N is simply the number of observations we used to run our models. The F-statistic for each model is shown in Table 3. Its only relevance is to calculate the p-value. Every model has a p-value which is less than 0.05. Therefore, we can reject the hypothesis that no relationship exists between the response and at least one of the variables. So far, all 4 models are valid so let's now check the AIC statistic.

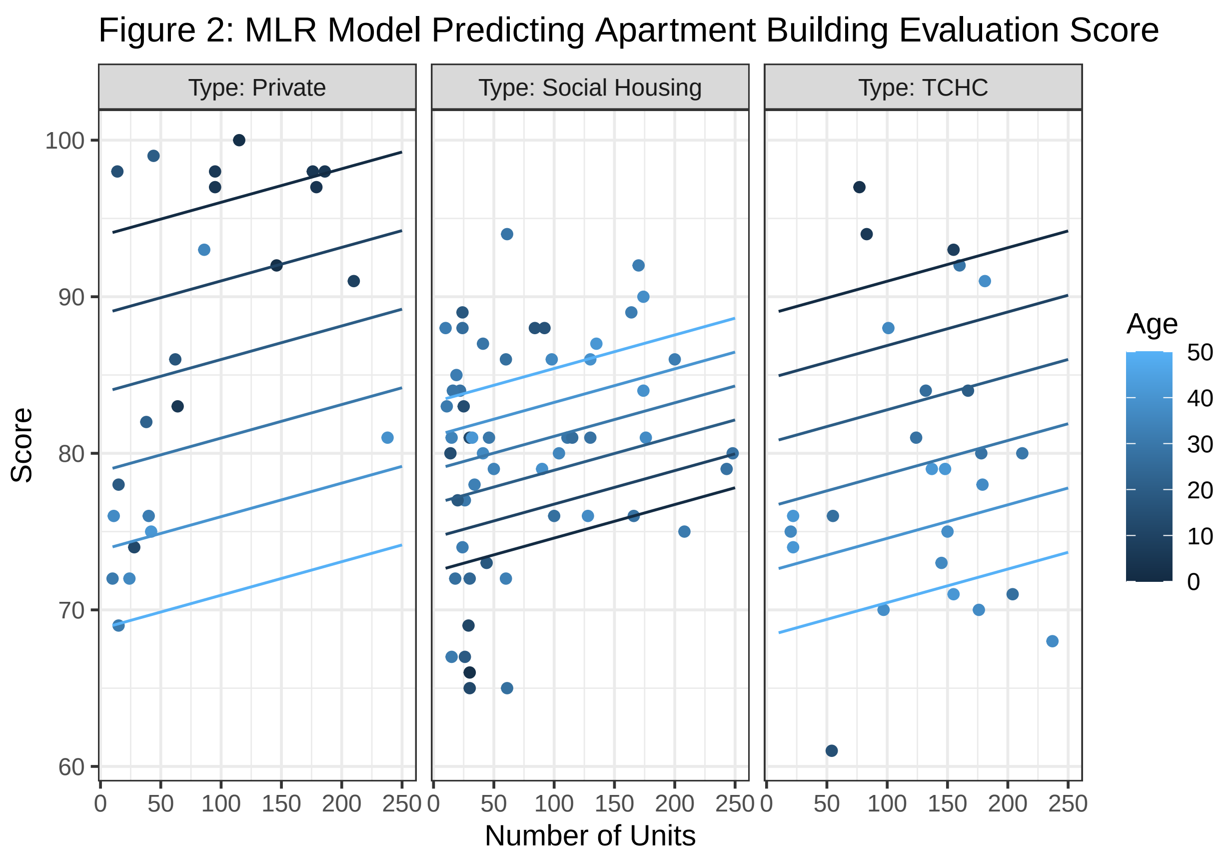
Recall that when comparing models, the one with the lower AIC is more appropriate (given the models have been run on the same data set). Every time we add a variable from Model 1 through Model 4, the AIC statistic decreases. This means adding variables does not decrease the efficiency of the models, and the models fit better and better. Consequently, Model 4 is the most appropriate according to the AIC statistic.

It was explained in the 'Methods' section why a higher R^2_{adj} is better. Model 4 has the highest R^2_{adj}, so we can say it is the most appropriate according to that test.

Finally, I conclude that Model 4 is the most appropriate since it is the number one performer in the three previous tests.

Graphical Results

Let's graphically interpret the results of Model 4 by fitting regression lines over the scatter plots from the Data section.



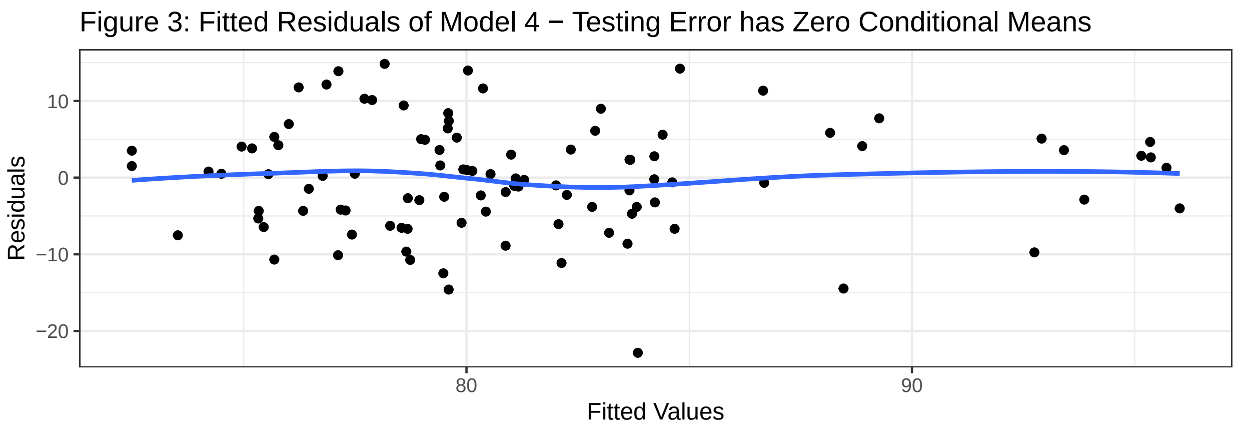
Given any property type, the relationship between number of units and evaluation score is positive.

Looking at the Age continuous scale and understanding that each regression line on one scatter plot represents a different age, we can state that recently built Private properties are expected to have a higher evaluation score than Social Housing and TCHC.

The nuance we previously stated is confirmed graphically. Light blue (older) lines are higher than dark blue (younger) lines given a Social Housing property type.

Testing Hypothesis that the Error has Zero Conditional Means

Finally, I want to check graphically if hypothesis 4 holds, i.e. $E(\epsilon|x_1,...,x_k) = 0$.



Assumption 4, $E(\epsilon|x_1,...,x_k) = 0$, holds. Even though the regression on residuals is not exactly a horizontal line equal to zero, there is no concerning pattern and the divergence really is negligible.

Conclusions

In this analysis, we have used a multiple linear regression model to answer the following research question: ***Can the evaluation score of a building be predicted such that there exists a linear relationship where the evaluation score is dependent on the number of units, property type, and age of the building?*** The answer is yes. The multiple linear regression model was valid as its assumptions were met, and relevant relationships were found between evaluation score given the number of units, property type, and the age of a building.

The first hypothesis stating that we expect younger buildings to have a higher evaluation score is partially met. Results have shown that this is true only for properties of type private and TCHC. Not for properties of type Social housing.

The second hypothesis stating that it was expected properties of type private have a higher evaluation score is true especially for recently built buildings.

The last hypothesis stating that the direction of the relationship between evaluation score and the number of units in a building is positive is true for any observation. It does not depend on the property type or age.

If there is one takeaway for this analysis, it is the last hypothesis being true. The big picture is the following: It is economically easier for apartments with many units to maintain common areas as the cost is divided among each unit. Apartments with few units implies that the fees paid per unit are high.

Weaknesses

The first obstacle faced in this analysis was that different apartment types are built in bulk around the same time periods. This is what happened during the baby boom from 1950 to 1970. Including these buildings in our analysis would have made our model flawed.

Next Steps and Discussion

Some future work of relevance would be to reproduce this analysis on buildings evaluated in 2021 once the year is done. This analysis about buildings evaluated in 2020 would be compared to the future one. More specifically, differences in the models and relationships would be compared.

Another step to be taken is to analyze if Model 4 can be improved by adding dependent variables to the model.

Finally, to summarize this report, we have shown using a multiple linear regression model that there exists a positive relationship between a building's evaluation score and the number of units in this building.

All analysis for this report was programmed using `R version 4.1.1`.

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