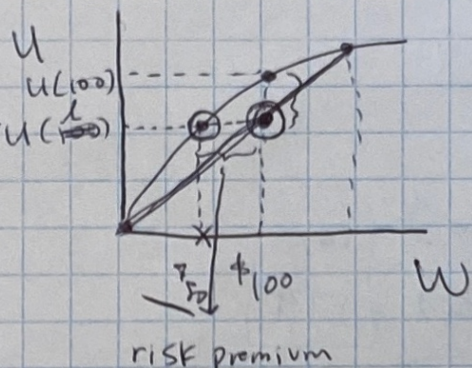
Lecture 2: Moral Hazard

$$\mathcal{L} = \int u(H_t, z_t)\, e^{-\rho t}\, dt + \lambda\big[\underbrace{p_h h_t + p_z z_t \le Y_t + y(H_t)}\big] + \Psi\big[G(h_{it}) - \delta H\big]$$

Solve for z_t
→ gets rid of λ and z

$$\frac{Y_t + y(H_t) - p_h h_t}{p_z} = z_t$$

RISK AVERSION: $u'' < 0$



$EU(l) = p_1(200) + p_2(0)$

$$\boxed{EU(l) \le u(E[l])}$$

P-A problem

$u_p = Q(e) - w$
$u_A = \sqrt{w} - g(e)$
$= \sqrt{w} - e$

$$Q = \begin{cases} 400 & p \\ 100 & 1-p \end{cases} \qquad \begin{array}{l} p = 2/3 \text{ if } e = 1 \\ p = 1/3 \text{ if } e = 0 \end{array}$$

Agent has a reservation wage $u_A \ge 10$

Problem: $\max_w \; Q(e) - w$ s.t. (1) PARTICIPATION $u_A \ge 10$
(2) INCENTIVE COMP.
$u_A(e^*) \ge u_A(e)$

SOLUTION SET:
1. Set of contracts that satisfy (1)
2. Cheapest way to do that
3. Pick e to maximize principal's profits

CASE 1: e is observable.

① Consider both values of e

- Suppose $e = 0$

$EV_A \geq 10$

$$\underset{\substack{\uparrow \\ q=100}}{\tfrac{1}{3}}(\sqrt{w} - 0) + \underset{\substack{\uparrow \\ Q=400}}{\tfrac{2}{3}}(\sqrt{w} - 0) \geq 10$$

$\sqrt{w} \geq 10$

$w \geq 100$

- suppose $e = 1$

$\Rightarrow \sqrt{w} - 1 \geq 10$

$w^* \geq (11)^2$

$w^* = 121$

② cheapest for principal $w^* = 100$

③ which is better?

$EV_P(e=0)$: $\frac{2}{3}(100-100) + \frac{1}{3}(400-100) = \boxed{100}$

⊛ $EV_P(e=1)$: $\frac{2}{3}(400-121) + \frac{1}{3}(100-121) = \boxed{179}$

optimal contract is $\boxed{\begin{array}{ll} w^* = 121 \text{ when } e^* = 1 \\ w^* = -\infty \quad e^* = 0 \end{array}}$

CASE 2: e is UNOBSERVED

If $q = 400$, w_H

$q = 100$, w_L

① Suppose $e = 0$

$EV_A \geq 10$

$$\underbrace{\tfrac{2}{3}(\sqrt{w_L} - 0) + \tfrac{1}{3}(\sqrt{w_H} - 0)}\geq 10$$

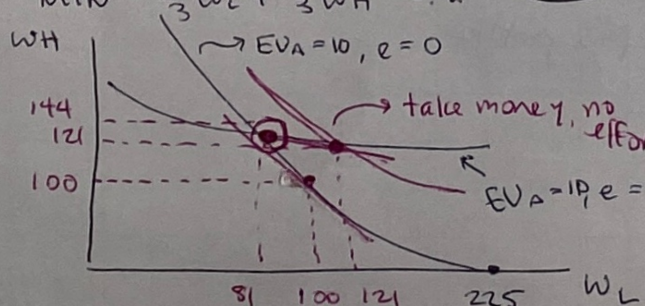
MIN $\frac{2}{3}w_L + \frac{1}{3}w_H$ s.t.

Solve for $w_H(w_L)$

$\Rightarrow 4(w_L - 30\sqrt{w_L} + 225) = 10$

If $e=1$: $\frac{2}{3}(\sqrt{w_H} - 1) + \frac{1}{3}(\sqrt{w_L} - 1) = 10$

$\Rightarrow$ solve for $w_H(w_L)$



$\boxed{\begin{array}{l} w_L^* = 81 \\ w_H^* = 144 \end{array}}$

$U_p(e=0) = 100$

$U_p(e=1) = \underline{177}$

ZECKHAUSER

$$EU = pU(w-s) + (1-p)U(w)$$

$$\downarrow$$

$$u'' < 0$$

$$EU = pU(\underbrace{w-s-h+f(h)}) + (1-p)U(w)$$

$$\Downarrow$$

$$\max_h \; w-s-h+f(h)$$

$$\boxed{f'(h^*) = 1}$$

$$EU = pU(w-\pi-s+\tau+f(h^*)-h^*) + (1-p)U(w-\pi)$$

FULL INSURANCE

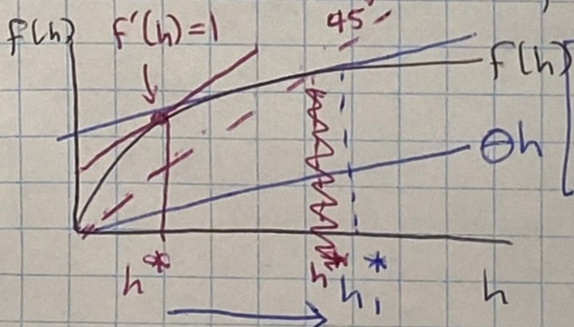
$$w-\pi-s+\tau+f(h^*)-h^* = w-\pi$$

$$\hookrightarrow \boxed{\tau^* = s+h^*-f(h^*)}$$

$$\boxed{\pi^* = p \cdot \tau^*}$$

s is not verifiable, but <u>h is</u>.

Coinsurance $(1-\theta)h = \tau$, $\theta < 1$



$$\boxed{\frac{\partial h^*}{\partial \theta} < 0}$$

$$\max_{\pi,\,\theta} \quad (1-p)U(W-\pi) + pU(W-\pi-S+f(\underline{h(\theta)}) \cancel{-h(\theta)} - \theta h(\theta))$$

h is endogenous   coinsurance

$$\text{s.t.} \quad \pi = p(1-\theta)h(\theta)$$

So ex-post: $\underline{f'(h(\theta)) = \theta}$

↳ means more care demanded

FIRST ORDER CONDITIONS $\mathcal{L} = (1-p)U(W-\pi) + pU(W-\pi-S+f(h(\theta)) - \theta h(\theta)) + \lambda(\pi - p(1-\theta)h(\theta))$

$$\mathcal{L}_\pi = -(1-p)U'(W-\pi) - pU'(W-\pi-S+f(h(\theta)) - \theta h(\theta)) + \lambda = 0$$

$$\mathcal{L}_\theta = pU'(W-\pi-S+f(h(\theta)) - \theta h(\theta))\Big[\underbrace{f'(h(\theta))\cdot h'(\theta) - h(\theta) - \theta h'(\theta)}_{Z(\theta)}\Big] - \lambda p\big[(1-\theta)h'(\theta) - h(\theta)\big] = 0$$

$$\Downarrow$$

$$\underbrace{U'(\text{sick}, h^*)\,Z(\theta)}_{P_h} = \lambda\Big[\underbrace{(1-\theta)h'(\theta) - h(\theta)}_{P_\pi(\theta)}\Big]$$

Change in premiums

MORAL HAZARD = RISK PROTECTION