Lecture 1 : Jan 11, 2023

(x_1, x_2) goods

- MARKET: (y, p_1, p_2)
- PREFERENCES:

$$u(x_1, x_2) \quad - u'(x_1) \geq 0$$

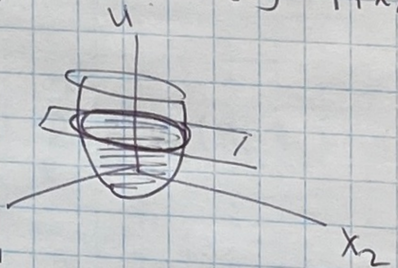
$$- u''(x_1) \leq 0$$

$$\max_{(x_1, x_2)} u(x_1, x_2) \quad \cancel{x_1^* = x_2^* = 0}$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq y$$

EXAMPLE: $u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$

$$\mathcal{L} = x_1^{1/3} x_2^{2/3} + \lambda (y - p_1 x_1 - p_2 x_2)$$



$$\mathcal{L}_{x_1} = \frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{3} x_1^{1/3-1} x_2^{2/3} - \lambda p_1 = 0 \quad (1)$$

$$\mathcal{L}_{x_2} = \frac{\partial \mathcal{L}}{\partial x_2} = \frac{2}{3} x_1^{1/3} x_2^{2/3-1} - \lambda p_2 = 0 \quad (2)$$

$$\mathcal{L}_{\lambda} = \frac{\partial \mathcal{L}}{\partial \lambda} = y - p_1 x_1 - p_2 x_2 = 0 \quad (3)$$

1. Solve (1) and (2) for λ
2. Set those equal to each other

$$\frac{1}{3p_1} \left(\frac{x_2}{x_1}\right)^{2/3} = \lambda = \frac{2}{3p_2} \left(\frac{x_1}{x_2}\right)^{1/3}$$

$$\cancel{3} p_2 x_2 = 2 \cdot \cancel{3} p_1 \cdot x_1$$

$$\boxed{x_2 = 2\left(\frac{p_1}{p_2}\right) x_1}$$

$$y - p_1 x_1 - p_2 \left[2\left(\frac{p_1}{p_2}\right) x_1\right] = 0$$

$$\boxed{x_1^* = \frac{y}{3p_1}}$$

$$x_2^* = \frac{2}{3}\left(\frac{y}{p_2}\right)$$

$U = u(H_t, Z_t)$ — flow

$H_t \rightarrow$ Stocks

$$H_t = f(h_t)$$

ASSUMPTIONS: $u_H > 0$ $u_{HH} < 0$

$u_Z > 0$ $u_{ZZ} < 0$

$u_{HZ} > 0$

Choice variables: (H_t, Z_t)

$$\underset{(H_t, Z_t)_{t=0}^{T}}{\text{MAX}} (u_t)_{t=0}^{T} \text{ s.t. } \theta \leq \bar{\theta}$$

$$\text{s.t. } p_h h_t + p_z z_t \leq w T^w + y(H_t)$$

1. Discrete
2. Continuous

$$\underset{\{H_t, Z_t\}_{t=0}^{T}}{\text{MAX}} \sum_{t=0}^{T} \beta^t u(H_t, Z_t) \text{ s.t. } \underline{p_h h_t + p_z z_t \leq w T^w + y(H_t)}$$

for all t

$$\underset{\{H_t, Z_t\}}{\text{MAX}} \int_0^T e^{-\rho t} u(H_t, Z_t)\, dt \quad \text{~~where~~ s.t. "}$$

$$\beta = \frac{1}{1+\rho} \qquad \dot{H} = \frac{\partial H}{\partial t} = G(h_t) - \delta H$$

$$\mathcal{L} = \int_0^T e^{-\rho t} u(H_t, Z_t)\, dt + \psi\left(G(h_t) - \delta H\right)$$

$$\mathcal{L} = \int_0^T e^{-\rho t} u\left(H_t, \frac{1}{p_z}\left(w T^w + y(H_t) - p_h h_t\right)\right) + \psi\left(G(h_t) - \delta H\right)$$

$$\frac{\partial \mathcal{L}}{\partial h}: e^{-\rho t} u\left(H_t, \frac{1}{p_z}\left(w T^w + y(H_t) - p_h h_t\right)\right) \cdot u_z \cdot \left(-\frac{p_h}{p_z}\right) + \psi \frac{\partial G}{\partial h} = 0$$

$$\boxed{\frac{p_h}{p_z} e^{-\rho t} \frac{\partial U}{\partial z} = \psi \frac{\partial G}{\partial h}}$$