HAD6750 Lecture 1: Demand for Health Care

Alex Hoagland, PhD

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1 Simple Demand Model (Two Goods)

Setup

Two goods: (x_1, x_2)
Prices: (p_1, p_2)
Preferences: $u(x_1, x_2)$
Income: y

Utility maximization

$$\max_{x_1,x_2} u(x_1, x_2) \quad \text{s.t.} \quad p_1x_1 + p_2x_2 \leq y.$$

Example: Cobb–Douglas

$$u(x_1, x_2) = x_1^{1/3}x_2^{2/3}.$$

- What does this mean?
- Define **indifference curves** here
- What would the **unconstrained optimum be?**

We find the **constrained equilibrium** using a Lagrangian:

$$\mathcal{L} = x_1^{1/3}x_2^{2/3} + \lambda\left(y - p_1x_1 - p_2x_2\right).$$

FOCs:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1} &= \frac{1}{3}x_1^{-2/3}x_2^{2/3} - \lambda p_1 = 0, \\
\frac{\partial \mathcal{L}}{\partial x_2} &= \frac{2}{3}x_1^{1/3}x_2^{-1/3} - \lambda p_2 = 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= y - p_1x_1 - p_2x_2 = 0.
\end{aligned}$$

Solving yields:

$$x_1^* = \frac{y}{3p_1}, \qquad x_2^* = \frac{2y}{3p_2}.$$

Model use cases

Three things a model is useful for:

1) **Equilibrium** (what do x_1^*, x_2^* on their own tell us?)
2) **Comparative statics** (how does equilibrium change if I vary exogenous parameters?)
3) **Counterfactuals** (how would outcomes change under a policy or state change?)

Comparative statics for the Cobb–Douglas demands

From $x_1^* = \frac{y}{3p_1}$ and $x_2^* = \frac{2y}{3p_2}$:

$$\frac{\partial x_1^*}{\partial p_1} = -\frac{y}{3p_1^2} < 0, \qquad \frac{\partial x_2^*}{\partial p_1} = 0,$$

$$\frac{\partial x_1^*}{\partial y} = \frac{1}{3p_1}, \qquad \frac{\partial x_2^*}{\partial y} = \frac{2}{3p_2}.$$

- What do these tell us?
- What is important here about signs, magnitudes?
- How could these be shaped into **testable implications** for an empirical exercise?
- Counterfactual example: compare an "old" state $(p_1, p_2, y) = (1, 2, 100)$ to a "new" state $(p_1, p_2, y) = (10, 2, 100)$ and compute how demand changes.

2 Grossman Notes (Health Capital Framework)

Preferences and health production

Instantaneous utility:

$$U_{it} = u(H_{it}, Z_{it})$$

with $u_H > 0$, $u_Z > 0$, and concavity conditions $u_{HH} < 0$, $u_{ZZ} < 0$.

- If we assume $u_{HZ} > 0$, what are we assuming?

Health production (sketch):

$$H_{it} = f(h_{it}) \quad \text{with} \quad f'(h_{it}) > 0, \;\; f''(h_{it}) < 0.$$

- Graph this, what are we saying?

The notes also distinguish health as a *stock* and a *flow* concept. H_{it} depends on h_{it} and $H_{i,t-1}$, while Z_{it} does not depend on $Z_{i,t-1}$.

Unconstrained problem ("cone point")

A stylized example appears as:

$$\max_{H,Z} \; u(H, Z) \quad \text{e.g.} \quad u = H^\alpha Z^{1-\alpha}.$$

- What is the obvious unconstrained optimum here?

Adding constraints

1) **Time constraint:** written as $\Theta = \bar{\Theta}$. Lagrangian form:

$$\mathcal{L} = u(H, Z) + \lambda(\bar{\Theta} - \Theta).$$

2) **Convert to budget constraints:** notes list income and a monetary constraint, including:

$$Y_t = wT^w,$$

and a budget inequality of the form

$$p_H h_{it} + p_Z z_{it} \leq wT^w + y(H_{it}),$$

where h_{it} denotes health investment/inputs and $y(H_{it})$ is written as a function of health.

3) **Constrained Problem:** The constrained problem is given by the Lagrangian now:

$$\mathcal{L} = u(H, Z) + \lambda(wT^w + y(H_{it}) - p_H h_{it} - p_Z z_{it})$$

- How would you solve this?
- What will be the main intuitions of the FOCs? (MB=MC, what does that mean in context here?)

Dynamic versions

Discrete time. Discounted lifetime utility written as:

$$U(H, Z) = \sum_{t=0}^{T\to\infty} \beta^t u(H_t, Z_t).$$

The notes include a per-period constraint with savings S_t (same constraint each period but with an S_t included).

Continuous time. Lifetime utility:

$$U(H, Z) = \int_0^T e^{-\rho t} u(H_t, Z_t)\, dt \qquad (\text{with } \beta = \tfrac{1}{1+\rho}).$$

Constraints (as recorded):

$$\begin{aligned} p_Z z_t + p_h H_t + S_t \geq wT^w + y(H_t) \qquad &\forall t \text{ (now infinitely many constraints)}\ , \\ \dot{H}_t = G(h_t) - \delta H_t \text{ (this is our } &\textbf{law of motion}\text{)}\ , \\ h_t \geq 0.& \end{aligned}$$

A continuous-time Lagrangian is written with multipliers $\lambda_1, \lambda_2, \lambda_3$:

$$\mathcal{L} = \int_0^T e^{-\rho t} u(H_t, Z_t)\, dt + \lambda_1[\cdots] + \lambda_2\big(G(h_t) - \delta H_t\big) + \lambda_3(h_t).$$

How can we solve this? Plug in the budget constraints in each period! (See slides.)

3 Empirical Application: Darden & Kaestner (2022) — Smoking and Selection

Model highlights the **problem** → generates **hypotheses** → dictates **research design**.

Motivation / selection issues

- Are estimates of the social burden of smoking wrong because they rely on cross-sectional data?
- Two fundamental selection problems:
  (1) Smoking → health → probability of death.
  (2) Health → quitting smoking.
- Implication: cross-sectional estimates may capture costs for relatively "healthy" smokers, producing a *big downward bias*.

Model Primitives

1) Health stock law of motion (age a). The notes record:

$$H_a = \underbrace{\big(1-\delta(c_{a-1}, m_{a-1}, a-1)\big)}_{\text{depreciation}} \cdot H_{a-1} + \underbrace{\gamma(H_{a-1})}_{\text{value of } I} \; \underbrace{I(H_{a-1})}_{\text{Inpatient care}} \; \mathbf{1}(H_{a-1} < \bar{H}) + \varepsilon_a.$$

State discretization. A discrete health state S_a is defined based on H_a:

$$S_a = \begin{cases} 2 & \text{"healthy" if } H \geq \bar{H}, \\ 1 & \text{"inpatient" if } 0 \leq H < \bar{H}, \\ 0 & \text{"dead" if } H < 0. \end{cases}$$

2) Choice variables. Conditional on (S_a, H_a), the choice variables are (c_a, m_a), chosen to maximize $u^S(x_a, c_a)$ (with x_a annotated as outside consumption).

- what is u^S capturing? **State-dependent utility**
- Can let x_a be defined as relative purchasing power given c_a, m_a, income (w_a) and prices:

3) Budget/constraint.

$$x_a = w_a - p_c c_a - p_m m_a - p_I I(H_a).$$

Problem Statement/(Constrained) Objective Function

This is all the information we need to write the model's **problem statement**, here written as a Bellman equation (deferring the mechanics of this to future lectures):

$$V^S(H_a, a) = \max_{(c_a, m_a)} \left[u^S(\boldsymbol{x_a}, c_a) + \beta \sum_{j=1}^{2} p_j(S_{a+1} = j)\, \mathbb{E} V^j(H_{a+1}, a+1) \right] \; s.t. \; ? \; .$$

- Why are there no constraints here? What is the value of doing that?

Solving the Model: Representative FOC

Since we have no Lagrangian, we can just consider the unconstrained optimum. We need the FOCs relative to the two choice variables (m_a, c_a). First, consider the marginal value of c_a (using the product rule):

$$\frac{u_c^S}{u_x^S} = p_c + \beta \cdot \frac{dc}{U_x^S}\left[\sum_{j=1}^{2}\left(\frac{\partial Pr(S_{a+1}=j)}{\partial H_{a+1}}\mathbb{EV}^j(H_{a+1}, a+1) + Pr(s_{a+1}=j)\frac{\partial \mathbb{EV}^j(a+1)}{\partial H_{a+1}}\right)\right]$$

On the left-hand side, we have the marginal benefit of smoking (relative to changes in other consumption). On the right, we have the marginal cost in smoking, decomposed into present-value terms involving how choices affect future state/health and continuation values.

Next, consider the second FOC with respect to m_a:

$$p_m = \beta \cdot \frac{dm}{U_x^S}\left[\sum_{j=1}^{2}\left(\frac{\partial Pr(S_{a+1}=j)}{\partial H_{a+1}}\mathbb{EV}^j(H_{a+1}, a+1) + Pr(s_{a+1}=j)\frac{\partial \mathbb{EV}^j(a+1)}{\partial H_{a+1}}\right)\right]$$

What do we learn?

1. As c_a goes up, H_{a+1} goes down, but this *increases* the probability of needing inpatient care in time $a+1$
2. On the other hand, increasing c_a may also *lower* m_{a+1} if lower income means reduced x_a, thereby decreasing the incentive to invest in health and increasing the incentive to smoke. Hence, the effect of c_a on medical expenditures over all is **theoretically ambiguous**.

Model Implications and Empirical Research Design

A) **What is the problem?** Dynamic Selection. We have both non-random smoking cessation and non-random mortality in our (observational) data (endogeneity). A couple of facts are important here:

- First, if $Pr(S_a = 2)$ decreases with H_a, then the marginal cost of smoking is higher if H_a is already lower. This means that as health deteriorates, optimal smoking may go down even further (reasonable?). But since we have cross-sectional data, this means that if we observe $c_a = 0$, it may be for someone with *worse* health, not better.
- Also, if smokers die more than non-smokers, then the highest-cost smokers are **missing** from the data. Conditional on survival, smokers at age a are less representative of all smokers than non-smokers at age a are of all non-smokers.

B) **What are the implications?** Show the paper directly (Section 3.3).

C) **What is the design? Retrospective comparisons rather than prospective ones**.

- See Section 5 (Equation 8 and page 269 in detail)
- Look at Figures 4 and 5