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ADDICTION

INTERPRETING THE RATIONAL ADDICTION MODEL

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SUMMARY

The rational addiction (RA) model of Becker and Murphy (Becker GS, Murphy KM. A theory of rational addiction. *J Pol Econ* 1988; **96**(4): 675–700) has rapidly become one of the standard models in the literature on addictive behaviour. This paper reviews some theoretical issues surrounding its use, and indicates areas in which caution should be used in applying this model. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS — rational addiction; optimal control

INTRODUCTION

The rational addiction (RA) model of Becker and Murphy [1] has become one of the standard tools in the economic analysis of the markets for drugs, alcohol, tobacco and other potentially addictive goods. Its rapid acceptance has been owing in part to its theoretical rigour, and in part to its empirical success: its key theoretical predictions [2] appear to have been confirmed empirically virtually every time they have been tested. The RA model has been tested on a wide range of addictive goods, with data sets from different countries, at both the level of the individual consumer, and the level of the market, and the only notable published failure of its key predictions to hold has been in its application by Conniffe [3], which may be a result of econometric issues associated with Conniffe's inclusion of a time trend among the explanatory variables.

The purpose of this paper is to reconsider the theoretical predictions of the RA model. We shall argue that success has, perhaps, been more apparent than real, and that its empirical implications must be reconsidered carefully in future empirical applications.

In the next section, we develop a simple, optimal control version of the basic RA model. We shall then use that model to discuss issues which raise some concerns with the standard interpretation of empirical RA results.

THE THEORY OF RA

In the RA model, the consumer's problem is to maximize

$$\int_0^T U(C(t), A(t)), D(t))\,\mathrm{e}^{-rt}\,\mathrm{d}t \tag{1}$$

where $A(t)$ is consumption of the addictive good at time t, $C(t)$ is consumption of non-addictive goods at t, and $D(t)$, which we shall refer to as damage, is the stock of addiction or habit, built up as a result of past consumption of A. The consumer's horizon is from time 0 to time T, and he discounts the future at rate r. The consumer maximizes (1) subject to the equation of motion for D:

$$\dot{D} = A - \delta D \tag{2}$$

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where δ, $0 < \delta < 1$, is the instantaneous rate of decay of the stock of addiction. In this simple version of the RA model, we treat δ as a constant: clearly, it could be made a function of time, or of the accumulated stock of addiction.

The individual's maximization problem is constrained by his budget: in their original formulation, Becker and Murphy [1] introduce a lifetime budget constraint, adding a differential equation describing the consumer's accumulation of assets over time. To make their model more tractable, they then assume that the market interest rate and the subjective discount rate are the same, so the consumer has no inclination to shift his consumption stream either into the future, or into the past, relative to his income stream. The introduction of a differential equation for assets converts the optimal control problem to a two state variable problem. This involves a certain loss of expositional ease, as two state variable control problems generally cannot be analysed by means of phase diagrams. As we wish to make use of phase diagram techniques below, we also simplify the budget constraint, assuming that the consumer is constrained in each period by the instantaneous budget constraint

$$Y(t) = C(t) + pA(t) \tag{3}$$

where $Y(t)$ is current income, and the price of other consumption goods has been normalized to 1. While the consumer must allocate his income between C and A in order to maximize lifetime utility, the assumption that the budget constraint (3) must be satisfied at all times lets us follow standard practice in the literature and substitute out $C(t)$, letting us focus on the time path of A.

The instantaneous utility function $U(C, A, D)$ is assumed to have the following properties:

$$U_C > 0,\ U_{CC} < 0,\ U_C(C = 0) = \infty \tag{4a}$$

$$U_A > 0,\ U_{AA} < 0,\ U_A(A = 0) = \infty \tag{4b}$$

$$U_D \leq 0,\ U_{DD} < 0,\ U_D(D = 0) = 0 \tag{4c}$$

From (4 (a)–(c)), C and A are goods, while D is a bad. In addition, we assume that

$$U_{CA} = 0,\ U_{CD} < 0,\ U_{AD} > 0 \tag{5}$$

The first assumption in (5), setting the cross-partial of C and A in the utility function to zero, is a simplifying assumption. The second and third assumptions get at the nature of addiction. An increase in the stock of addiction reduces the marginal utility the consumer derives from additional C, and increases the marginal utility he derives from additional A, so that, as his stock of addiction accumulates, utility maximization will lead him to tend to shift his consumption expenditure away from C, towards A. The assumption that, as an individual's degree of addiction increases, the marginal utility he derives from the addictive commodity increases, may well seem counter to the usual view of how addiction works. We tend to think of addiction as involving a form of habituation, so that, as the degree of addiction increases, the addict gets less and less subjective benefit from each successive unit of the addictive commodity consumed, and must continually increase his consumption of addictive goods just to get the same level of satisfaction each time. We could modify the model to incorporate this view of addiction by introducing into the utility function a new variable, kicks, where kicks are produced by a production function with the addictive good A as the input. As the degree of addiction increases, the marginal utility of kicks increases, shifting preferences towards kicks, as above, but the marginal productivity of A in the production of kicks declines, so increased quantities of A must be consumed to satisfy the increasing demand for kicks. The general pattern of the results would not be affected by this modification, and kicks would, in general, be unobservable for empirical purposes, so we stick with the simpler version of the model set out above. Clearly the actual behaviour of the optimizing individual will depend on the relative magnitudes of the partial derivatives set out in Equations (4) and (5), above. We will return to this point below.

Using (3) to eliminate $C(t)$ from the problem, we find the current value Hamiltonian for the consumer's problem:

$$\mathscr{H} = U(Y - pA, A, D) + \Psi[A - \delta D] \tag{6}$$

with control variable A, state variable D, costate Ψ and necessary conditions

$$\mathscr{H}_A = \Psi - pU_C + U_A = 0 \tag{7}$$

$$\dot{\Psi} = [r + \delta]\Psi - U_D \tag{8}$$

In addition, we impose the boundary conditions $D(0) = 0$, $D(T) = \text{free}$. This latter condition means that we are dealing with a free endpoint model with a fixed, finite horizon. The transversality condition for such a problem [4,5] requires that the costate variable go to zero at the end of the planning horizon: $\Psi(T) = 0$.

Equation (7) embodies the forward looking rationality in the model. Rewriting (7) gives

$$\Psi = pU_C - U_A \tag{9}$$

As Ψ is the costate for the problem, it is the shadow price of another unit of the state variable, which, in this case, is D, damage. As damage is assumed to be a bad, its shadow price, which is the subjective valuation the agent places on an additional unit of D, is negative. Equation (9) says that a forward looking individual, in deciding on how to allocate his current consumption between C and A, allows for the impact an increase in A today will have on his future utility through its effect on D. A completely myopic individual, who took no account of the future consequences of decisions made today, would have $\Psi = 0$, and would choose A and C so that $pU_C = U_A$. This is just the first order condition for the single period consumer's problem of allocating income between two consumption goods. Rewritten, and remembering that we have set the price of C equal to 1, we have $U_C/U_A = 1/p$: the marginal rate of substitution between C and A equals their price ratio. In the present case, as Ψ is negative, we have $pU_C < U_A$, which means that the forward looking consumer will consume less A and more C than would a completely myopic individual who otherwise had the same preferences over C and A.

As we are ultimately interested in the empirical implications of the RA model, the analysis will be most useful if conducted in state-control rather than state-costate space. Equations (2) and (8) give us a pair of differential equations in D and Ψ, respectively. Differentiating (9) with respect to time gives us a second differential equation for Ψ:

$$\dot{\Psi} = -[p^2U_{CC} + U_{AA}]\dot{A} + [pU_{CD} - U_{AD}]\dot{D} \tag{10}$$

then using (9) to eliminate Ψ from the right hand side of (8), and equating (10) and (8) gives a differential equation in A:

$$\dot{A} = [[pU_{CD} - U_{AD}][A - \delta D] + U_D - [r + \delta][pU_C - U_A]]/[P^2U_{CC} + U_{AA}] \tag{11}$$

Equations (2) and (11) give us a system of two differential equations which can be represented in state-control space, on a phase diagram, with A on the vertical, and D on the horizontal axis. Equation (12) gives us the expression for the stationary locus for D:

$$A = \delta D \tag{12}$$

which has slope

$$\frac{\partial A}{\partial D}(\dot{D} = 0) = \delta \tag{13}$$

Thus, the stationary locus for D is a straight line from the origin, with slope δ. The stationary locus for A is derived from Equation (11). As the denominator in (11) is always negative, the expression in (11) will equal zero when the numerator equals zero. Close to the equilibrium for the system (where $A - \delta D = 0$, so the first part of the numerator equals zero), the stationary locus for A is defined by:

$$U_D - [r + \delta][pU_C - U_A] = 0 \tag{14}$$

The slope of the stationary locus for A, close to the equilibrium, is given by

$$\frac{\partial A}{\partial D}(\dot{A} = 0) = \frac{[[r + \delta][pU_{CD} - U_{AD}] - U_{DD}]}{U_{DA} + [r + \delta][p^2U_{CC} + U_{AA}]} \tag{15}$$

Clearly, the sign of (15) in this case, and, therefore, the slope of the stationary locus for A, will depend on the relative magnitudes of the second partials of the utility function. The terms U_{CD} and U_{AD} determine the strength of the addictive effect, as noted in our discussion of Equation (5), above. In the case where the addictive effect is small, Equation (15) is negative and the stationary locus for A is negatively sloped close to equilibrium.

This model is a variant on Ippolito's [6] model of the optimal consumption of pleasurable but harmful commodities, and of Forster's [7] model of optimal pollution. If there is no addictive effect at all, so that the commodity in question is pleasurable and harmful, but not addictive, we have Ippolito's model. The phase diagram for one version of this model is given in Figure 1 below, where we have assumed that the addictive effect is small enough for the stationary locus for A to be negatively sloped close to equilibrium.

Becker and Murphy [1] assume a utility function that is quadratic in (our) A and D, the budget constraint having been used to substitute out our C variable. A quadratic utility function yields a linear stationary locus for A. Further, in their discussion of their Equation (16), Becker and Murphy assume that the relative magnitudes of the cross partial terms in their utility function are sufficient to give the stationary locus a positive slope. The phase diagram for the RA model under these assumptions is shown in Figure 2, below.

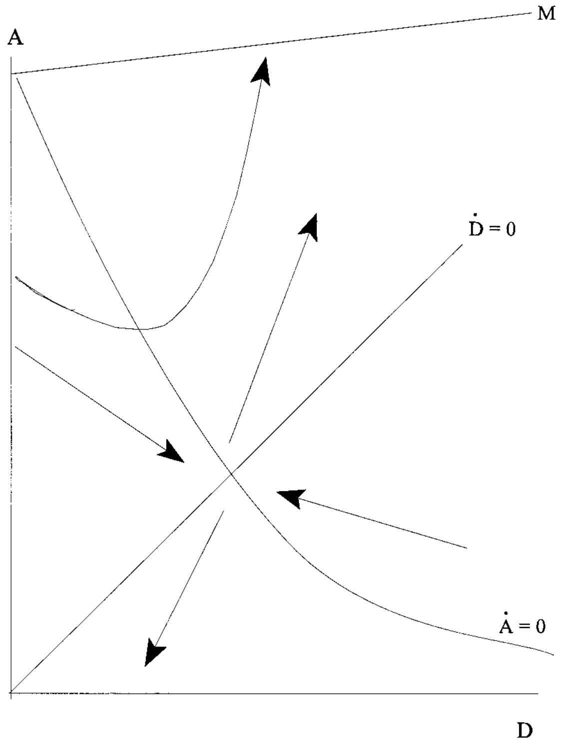

Figure 1. Phase diagram for a weakly addictive commodity

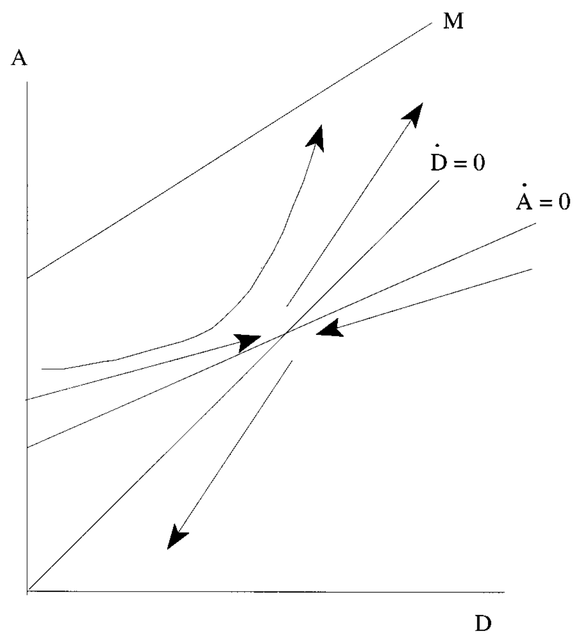

Figure 2. Phase diagram for a strongly addictive commodity

Note that there is only a single equilibrium in the diagram, as distinct from the way this figure is usually drawn [1,8]. Becker and Murphy [1] note explicitly that quadratic utility functions cannot yield multiple stationary states, and treat the assumption of quadratic utility as an approximation to a higher order utility function.

In what follows, we shall refer to the case shown in Figure 1 as that of a mildly addictive substance, while we shall call that shown in Figure 2 that of a highly addictive substance. We should note that, in both cases, the commodities are harmful and consumers are aware of that fact: if that were not the case, the costate, Ψ, would be constant at zero.

In both cases, the equilibrium of the system is a saddlepoint with two roots, one stable and one unstable. The actual trajectory followed by any variable in the system (the solution function for, for example, $A(t)$, showing A explicitly as a function of time) can be written as a linear combination of expressions in those two roots. This kind of equilibrium is termed conditionally stable, in that there are two stable branches, paths leading to the equilibrium, and two unstable branches, paths leading directly away from the equilibrium. The other trajectories are combinations of the stable and unstable branches, and while they may initially appear to be converging on the equilibrium, they must eventually diverge. The only trajectories which actually lead to the equilibrium are the stable branches: if the starting point for the optimizing agent's problem is on one of the stable branches, the system will eventually reach equilibrium, while if its starting point is at any other point on the phase diagram, the system will diverge from the equilibrium. In an optimal control problem such as this one, the initial value of the state variable ($D(0)$) is given, while the initial value of the control variable ($A(0)$) is a choice variable. Given the stock of damage he starts out with, the individual chooses initial consumption of A in order to place himself on the optimal trajectory. The equilibrium point satisfies the usual condition that if the system is at that point, there will be no intrinsic tendency to move away from it. The saddlepoint configuration is the most common form for the solution to an intertemporal optimization problem to adopt: while there are dynamic optimization models whose solutions are not saddlepoints, they are rare.

The phase diagrams for the problem can be used to illustrate the most often tested features of the RA model. Becker and Murphy [1], and the writers who follow them, assume that the system will be, if not at equilibrium, at least on one of the stable branches. In the case of the highly addictive commodity, Figure 2, this means that the trajectory expected to be followed by an individual whose initial level of $D = 0$ is a positively sloped one, with both consumption and stock of addiction rising over time. It should be noted that the phase diagram is drawn for given values of such exogenous variables as price and income: the equilibrium is conditional on unchanging values of the explanatory variables and the trajectory followed represents the approach to long run equilibrium by an individual who is initially not at that equilibrium. As most people can be assumed to start with $D = 0$ (although the phase diagram makes it clear that the model could also be applied to an individual who is born addicted—i.e. born with an initial level of D above the long run equilibrium level), the model's prediction of a steady increase in the levels of A and D until the values at the intersection of the stationary loci are reached describes changes in consumption over time which are owing strictly to the passage of time.

As the phase diagram is drawn for given values of the explanatory variables, we can use it to illustrate the effect of changes in those variables. Consider the effect of a permanent, unanticipated decrease in the price of A in the strongly addictive case. Assume that the consumer was initially at the equilibrium point on the phase diagram, conditional on his income and the old price level. In terms of the phase diagram, the effect of a reduction in price is to shift the stationary locus for A up, leaving that for D unchanged. This shifts the equilibrium point up and to the right along the stationary locus for D. This effect is shown in Figure 3 below.

The consumer's old (A, D) combination, which was his equilibrium at the old price level, is no longer an equilibrium point. In the standard Becker–Murphy analysis, his response to the reduction in the price of A can be broken into two steps. In the long run, in this analysis, he wishes to be at the new saddlepoint equilibrium. Once he reaches it, he will settle in at the new (A, D) combination, so long as price remains unchanged. Thus, the level of A associated with the old saddlepoint is regarded as his old long run consumption level (conditional on the old price level), while the A coordinate of the new saddlepoint is his new long run consumption level. The difference between the vertical coordinates of the old and new equilibria is, therefore, the long run change in consumption due to a permanent drop in price. This long run response is calculated in all empirical applications of the RA model.

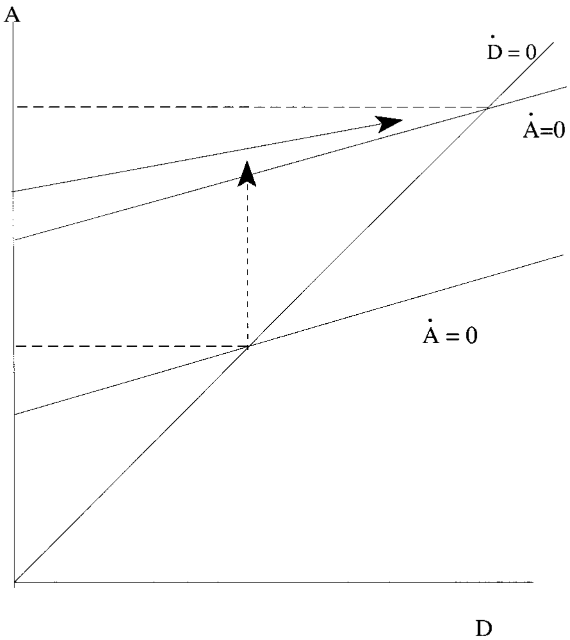

Figure 3. Effects of price decrease on a strongly addictive commodity

Given that the consumer is assumed to want to move from old to new saddlepoints, there is only one way he can accomplish this: once the price increase is announced, he must change his consumption of A by whatever amount is required to place him on the stable branch to the new saddlepoint. He cannot adjust D discretely; D changes only as a result of changes in A over time, so the only variable at his command is A (once A has been chosen, given his level of income, C has also been chosen). In the case shown in Figure 3, he must jump vertically up from his old position to the point on the new stable branch which is directly above the old saddlepoint. This vertical jump in A represents the short run response to a permanent, unanticipated decrease in P. The formula for the short run price elasticity for the case of a discrete time model (which will differ from the formula which applies to a continuous time model, such as we are working with) is given by Becker and Murphy [1]: short run elasticities are less often calculated in the empirical literature.

One implication of this theoretical structure which is often referred to in the RA literature, is that the long run price elasticity of demand for an addictive good is greater than the short run price elasticity. From Figure 3, we see that this is a consequence of the positive slope of the stable branch: the increase in the *A* coordinate of the saddlepoint is greater than the vertical distance from the old saddlepoint to the new stable branch. It is clear from the diagram that the long run elasticity is not necessarily a lot larger than the short run elasticity: that depends on the slope of the stable branch. It is also clear from a consideration of Figure 1 that this result will not hold for a weakly addictive good. In that case, as the stable branch slopes downward to the right, there will be overshooting, with the short run price elasticity being greater than the long run elasticity. The difference between long and short run price responsiveness could presumably be used in assessing whether a commodity is strongly or weakly addictive, conditional on the RA model holding.

We have followed the RA literature in defining the short run price effect in terms of a permanent, but unanticipated, decrease in price. We can also characterize the effects of an announcement (or expectation) that price will be increased permanently at some point in the future. In this case, the consumer can calculate the location of the new saddlepoint, and of the new stable branch. That branch will not actually come into being until the price decrease has occurred, and the consumer cannot follow it until it comes into being. He can, however, calculate where it will be when it comes into being and, therefore, can adjust his current consumption to put him on a trajectory such that, when the new stable branch does appear, he will already be on it. Basically, he can follow a short run trajectory which will take him to a point in the diagram where the new stable branch will appear, and he can pick a trajectory which will result in his reaching that point at precisely the instant at which the new stable branch appears, which, in our example, would mean reaching that point just as the previously announced price reduction comes into effect. Following this strategy means that when the price decrease actually comes into effect, his consumption level will not change (although from then on it will continue to rise along the new, positively sloped stable branch).

It can be shown that, in the standard version of the RA model, the announcement of a price decrease to take effect at an announced time in the future will result in an immediate, relatively small upward jump in consumption, followed by a relatively rapid increase in consumption, until the price decrease actually comes into effect, after which, when the permanently lower price is in effect, consumption will continue to rise, but more slowly, until the new saddlepoint is reached. Similarly, if rational smokers are told that cigarette prices are going to be increased at some precise date in the future, their immediate reaction should be a small reduction in consumption, followed by a steady decline in consumption until the price increase takes effect, after which they should follow the stable branch to the new, lower saddlepoint, which path will involve consumption continuing to fall, but more slowly than it had been falling in the interval between the announcement of the price increase and its actually taking effect. One problem with testing this prediction, of course, is the durable nature of the product. We would expect to see cigarette sales increase in anticipation of a price increase, even if actual smoking declines as the model predicts. Empirical implementation of the RA model must allow for the distinction between sales and consumption, especially in the case of anticipated price increases.

We noted above that it is common in the literature for the phase diagram in (D, A) space to be drawn with the stationary loci intersecting twice. While optimal control problems can have multiple equilibria, with the transversality conditions used to determine to which equilibrium, if any, it is optimal for the decision maker to attempt to converge, the quadratic utility functions cited in most of the empirical literature as the basis for the equations being estimated can only yield a single equilibrium. This can cause a problem for analysts, as in Becker and Murphy's discussion the lower equilibrium is hypothesized to be unstable and to provide the basis for such phenomena as quitting cold turkey. There can also be confusion about the difference between the case of two equilibria, one stable and one unstable, and the case of a single equilibrium with two roots, one stable and one unstable. The latter case is the saddlepoint case, and the Becker–Murphy model predicts that there will be a saddlepoint equilibrium, whether there also exists an unstable equilibrium at a lower (D, A) combination or not.

There are a number of points which should be made in this regard. First, the assumption in the literature that everybody has the same utility function is obviously made only for theoretical convenience. In terms of Figure 2, the greater the disutility that an individual derives from D, the lower will be the stationary locus for A on the diagram and, therefore, the lower will be the equilibrium (D, A) combination. As the stationary locus for D passes through the origin, it is quite possible for the saddlepoint equilibrium to be at the origin, if U_D takes on a large enough negative value at very low values of D. A religious or moral distaste for alcohol or tobacco could well be characterized this way, and individuals with such strong feelings would tend to have equilibria at the origin, and be complete abstainers. Others might have values of U_D which are large enough in absolute value to place their individual saddlepoints at very low (D, A) combinations. Further, we have assumed that the equation of motion for D is the same for all individuals. If, in fact, there are differing degrees of susceptibility to damage in the population, different individuals could have different stationary loci for D.

Cold turkey behaviour can also be included in an RA model with a single, saddlepoint equilibrium. While we have not introduced them formally, the model includes implicit non-negativity constraints on A and C. In terms of Figure 2, this means that while the horizontal axis of the phase diagram is technically in a region in which the phase arrows point down and to the left, in fact, when the horizontal axis is reached, the system will move horizontally to the left along it. Cold turkey behaviour can be characterized as jumping to the horizontal axis, where $A = 0$, and letting D decline at the natural rate, δ.

For the most part, results derived from empirical implementation of the RA model have apparently been consistent with the model's theoretical predictions, at both the individual and the market level. In particular, whenever the roots of the system have been calculated there have turned out to be two, one stable and one unstable, exactly as there should be for saddlepoint dynamics. There are, however, certain caveats which must be considered.

Perhaps the most important of these concerns the role of the equilibrium point. In an intertemporal optimization problem with a saddlepoint equilibrium, the stable branch is the optimal trajectory only for an infinite horizon problem. Becker and Murphy [1] adopt the assumption of an infinitely-lived individual as a simplification in the development of the theoretical model. While this is a common practice in theoretical modelling, we must be careful in translating the predictions of an infinite horizon theoretical model to empirical studies using data from finite-lived individuals. In essence, in assuming that the stable branch is the optimal trajectory for real-world individuals, we are assuming that these individuals are farsighted in all respects, except with regard to their own mortality. It seems questionable to assume that these individuals will take full account of the future consequences of their current actions, while at the same time assuming that they believe that they will live forever.

As we noted earlier, the RA model, as we have set it up here, is a free endpoint model with a fixed, finite horizon. The transversality condition for such a problem [4,5] requires that the costate variable go to zero at the end of the planning horizon: $\Psi(T) = 0$. From Equation (9), this requires that, at time T, $pU_C = U_A$. As we noted earlier, this is the first order condition for the optimal allocation of income between C and A in a one-period consumer choice model. In short, transversality requires that, at the end of the planning horizon, the consumer select A and C as if there were no future consequences to be taken account of. As this is the condition which applies at point T, the end of his life, there are no future consequences. Optimality requires that, at T, he behave as if there is no tomorrow, precisely because there is, in fact, no tomorrow. This condition can be written as

$$pU_C(Y - pA, A, D) - U_A(Y - pA, A, D) = 0 \tag{16}$$

In terms of our phase diagram, expression (16) implicitly defines a relation between A and D giving a locus of (A, D) points, along which Ψ equals zero. The slope of this relation is given by

$$\frac{\partial A}{\partial D}(\dot{\Psi} = 0) = \frac{pU_{CD} - U_{AD}}{p^2 U_{CC} + U_{AA}} \tag{17}$$

On our assumptions, this expression is positive for an addictive good, and zero for a completely non-addictive good. Further, the stronger the degree of addictiveness of A, the steeper the line represented by (16) on the phase diagram. This expression gives the line labelled M in Figures 1 and 2.

Optimality requires that the consumer's trajectory terminate on the M-locus at precisely time T. Exactly where on M the terminal point will lie depends on the length of the planning horizon. In Figure 1, for the weakly addictive commodity, this yields a U-shaped optimal trajectory, with consumption of A relatively high in the early years of the programme, then declining for some years before turning around and rising again. In Figure 2, for the strongly addictive commodity, recognition of the high degree of addictiveness leads to a lower initial level of A, with A rising steadily through the planning horizon, and with the rate of increase in A also increasing. In both figures, D rises steadily over time.

Both figures' trajectories may seem inconsistent with observed lifetime consumption patterns of alcohol, for example, which tend to show consumption declining into older ages [10]. The phase diagrams here are drawn on the assumption that factors such as income and prices are held constant through the individual's life, which is not, of course, true. Declining income in later years would tend to pull the stationary locus for A downward, pulling the optimal trajectory down with it. Presumably, our forward looking individuals would anticipate a future decline in income (into retirement, for example), and would reduce their consumption of A at each point in time, to reduce the degree of addiction they will face at retirement. Further, we have assumed that the depreciation rate, δ ,is constant throughout the consumer's life. It would be more reasonable to have δ decline over time, reflecting the body's diminishing capacity to repair itself as it ages. We could also, as noted earlier, make δ a decreasing function of D, so that the more damage the body suffers, the less its natural capacity to repair itself. All of these factors would tend to pull the optimal consumption path down over time, although whether they could turn the trajectory in the case of a strongly addictive good, in Figure 2, into an inverted-U shape is an empirical question.

The most important implication of the difference in transversality conditions for finite and infinite horizon models is that, while we can calculate the vertical shift in the location of the saddlepoint in response to a reduction in the price of A, that calculation has very little empirical content. It does tell us something about the effect of a reduction in P on the system, but as individuals will not, in fact, tend to be at, or converging on, that point, it cannot be regarded as a good estimate of the long run price elasticity of demand. The short run response to a reduction in price is still the vertical distance between old and new optimal trajectories, but the formula used by Becker and Murphy, based on the formula for the stable branch, must be treated with caution.

EMPIRICAL IMPLICATIONS

The differences between the transversality conditions for the finite and infinite horizon problems have different implications for studies at the individual and market levels. At both micro and macro levels, the usual practice is to estimate an equation of the general form

$$A_t = \beta_0 + \beta_1 A_{t+1} + \beta_2 A_{t-1} + \beta_3 P_t + \beta_4 P_{t+1} + \beta_5 P_{t-1} + \beta_6 X_t \tag{18}$$

where the time subscripts on A and P indicate that future and past, as well as present, values of these variables are entered into the expression, and X_t is a vector of other explanatory variables, including income, which may also be entered in leads and lags form. This expression, which is derived from solving the consumer's optimizing problem on the assumption of a quadratic utility function, is a second order difference equation in A (two differences of P are also included, but they are regarded as exogenous: it is A whose behaviour is determined by this equation). At both the micro [11,12] and macro [8,9,13] levels, when estimated, the fitted coefficients of Equation (18) typically yield two roots, one stable and one unstable, consistent with the RA prediction that the consumer's intertemporal behaviour is characterized by saddlepoint dynamics.

Given this, we might ask whether there is evidence in micro level studies that individuals do, in fact, follow the stable branch. To assess this, we must recall that each trajectory in a phase diagram can be written as a weighted average of the stable and unstable roots of the system. On every trajectory, except a stable branch, the unstable root must eventually dominate, and the system must diverge from its equilibrium in the long run, although along a trajectory on which the stable root is given a very large weight, and the unstable root a very small one, it is possible for the system to tend towards the equilibrium for some considerable length of time. The stable branch, as a trajectory, converges to the saddlepoint only because along it, the unstable root is given a zero weight—only the stable root is operational. While the unstable root is still present in the system, because it has a weight of zero, it does not have any effect on the time path of the variables and, therefore, cannot be detected in their behaviour. In other words, a saddlepoint system which was following the stable branch would behave as though it were a system with a single, stable root.

This conclusion should not be taken to mean that the system is converging on a fully stable equilibrium: the equilibrium is still a saddlepoint. It does, however, mean that if we can obtain a reliable estimate of the unstable root, as has been the case for most micro level estimation of RA models, it must be the case that the system is not on the stable branch. This conclusion is not fatal for RA models—the key prediction, that the system is characterized by saddlepoint dynamics, still holds. It does, however, mean that the usual approach to estimating the long run price response, which in Equation (18) is taken to be

$$\frac{[\beta_3 + \beta_4 + \beta_5]}{[1 - \beta_1 - \beta_2]} \tag{19}$$

is not consistent with the theoretical structure.

At the micro level, then, empirical results tend to be consistent with the general properties of an intertemporal optimization model. At the macro level, for example, at the level of American states, the finding of saddlepoint dynamics is more problematical.

The saddlepoint result applies to the optimal intertemporal consumption path of an individual consumer. In representative agent macro models of consumption behaviour, it is assumed that aggregate consumption will show the same behaviour as does individual consumption, but as Deaton [14] notes, outside representative agent models that there is no good reason for this parallelism to arise.

Consider the U-shaped lifetime consumption profile of individuals consuming a weakly addictive good, as shown in Figure 1 (the arguments that follow apply equally well to the path in Figure 2). At each point in time, the total population of a market will consist of individuals of different ages, each at a different point in their lifetime consumption programme. In terms of Figure 1, at any point in time, if we look at a cross-section of individuals, each individual will be at the point on the optimal trajectory consistent with the stage he has reached in his planning horizon: i.e. consistent with his age. If we think of the optimal trajectory as representing the optimal pattern of consumption through the individual's life, at any point in time, total consumption will be the sum over all ages of the product of age-specific consumption, read from the appropriate point on the optimal trajectory, and the number of individuals at each age. As individual consumption differs with individual age, total consumption will depend on the number of people in each age group. Total consumption can change, even with no change in age-specific individual consumption levels, if the number of individuals in each age group changes.

Consider a population which is demographically stable and stationary, so that its total size and its age distribution both remain unchanged over time. If we follow that population over a number of years, each individual in the population will move through the optimal life cycle consumption profile. In the case of the weakly addictive commodity described in Figure 1, this means consuming relatively large quantities of A in his younger years, with consumption of A falling as he moves into middle age, and then eventually rising again in later years. Because the population is stable and stationary in the demographic sense, while each individual moves through life, the pattern of entry to, and exit from, the population is such that both the total size of the population and the total number of individuals in each age group remain unchanged. In terms of the phase diagram, this means that while the specific individuals at each point on the optimal trajectory change over time, the total number of individuals associated with each point on the trajectory remains unchanged over time. If prices and incomes remain unchanged over time, the optimal trajectory will not shift in the phase diagram, so each new entry cohort of individuals will go through exactly the same age-specific consumption pattern, as did the previous cohort, and as will its successor cohorts. This means that age-specific individual consumption levels will not change over time. As the demographic stability and stationarity of the population means that the number of individuals in each age group also will not change over time, total consumption of A will remain unchanged over time, even as each individual in the population goes through the U-shaped optimal age-specific lifetime consumption pattern. When we look at individual level data from that population, we should observe dynamics consistent with a saddlepoint configuration. When we look at aggregate (or per capita) consumption of A for that population, we should observe no dynamics at all: at the aggregate level, consumption will remain unchanged over time.

Now, consider a population which is growing at a constant rate, but whose age distribution has settled into a stable, unchanging pattern, so that, as the population grows, the total number of individuals in each age group in the population increases at the same rate as does the total population. This situation arises when a population has been growing at a constant rate for a fairly long period of time.

In this case, as the number of individuals associated with each point on the trajectory increases over time at the same rate as does the total population, age-specific individual consumption levels will remain unchanged, but total consumption will increase at the same rate as does the population. In turn, this means that even though the total population and total consumption are growing, overall per capita consumption will remain constant over time.

Finally, consider a population whose age distribution is changing over time. As the population age distribution changes, so will the weight given to each point on the optimal trajectory in the calculation of aggregate consumption. Age-specific consumption levels will remain unchanged, but the changing population proportions in each age group will result in total and overall per capita consumption changing over time. The observed change in per capita consumption will not necessarily mimic the optimal trajectory, however, but rather, will depend on the observed change in the age distribution of the population. If we were to age standardize aggregate consumption data, we would find that age standardized overall per capita consumption, calculated, for example, using a constant standard population, would remain unchanged over time, and that the difference between the actual and age standardized consumption series would depend on how the age distribution changed relative to that of the standard population over time. Again, once the changing age distribution of the population had been properly controlled for, we would observe no intrinsic dynamics in consumption.

At the very least, then, if we are going to investigate the dynamics of the consumption of addictive goods at the market level, we should control for population dynamics. If the population age distribution has remained relatively stable over time, the effect of population dynamics should be fairly small, but we should still attempt allow for it in aggregate empirical work.

As noted above, empirical work at the aggregate level has tended to find saddlepoint type roots in the estimated difference equation for consumption. If we accept the estimated roots, we must accept the associated dynamics, including any unstable behaviour which might result. One approach to testing the reasonableness of an estimated difference equation is to perform dynamic simulations on it, holding the exogenous variables in the equation constant. In terms of Equation (18) above, if we hold the P and X variables constant over time, we are left with a linear, second order difference equation in A. As this difference equation must be satisfied at all time, we can use it to map out the path A must follow over time, with P and X constant. If the data used in estimating the equation were drawn from a system which is on the stable branch to a saddlepoint equilibrium, our simulated values of A should converge to an economically reasonable value. If they do not do so, we have cause to question the validity of our model.

As an example of this, consider the paper by Olekalns and Bardsley [15], in which they apply the rational addiction model to per capita coffee consumption data for the US. Their hypothesis that coffee consumption might display RA behaviour is based on medical evidence suggesting that coffee addicts display behaviour similar to that shown by individuals who are regular consumers of other substances which are generally classed as addictive. Despite what they consider to be imprecision in the estimate of the long run price response term, they conclude that the RA model fits overall per capita coffee consumption well.

Olekalns and Bardsley report the following estimated difference equation for coffee consumption:

$$A_t = -0.353 + 0.523A_{t-1} + 0.475A_{t+1} - 2.887P_t + 1.684P_{t-1} + 1.531P_{t+1} \quad (20)$$

We have not reported their standard errors, as we intend to use Equation (2) for deterministic simulation. Analysed as a second order difference equation in A, this equation yields roots of 1.14 and 0.965 (these are the roots as a theorist would report them: an econometrician would report their inverses as the roots: 0.877 and 1.036, respectively). The conclusion that the second order difference equation for coffee consumption has one stable and one unstable root is certainly consistent with RA behaviour.

The location of the equilibrium value of A for this equation depends on the values of the P terms. The equilibrium point will remain unchanged as long as the value of P remains unchanged. While it is clearly counterfactual to assume that P has remained unchanged over time, as our purpose is to simulate the dynamic behaviour implied by the model it is not unreasonable, for simulation purposes, to make P converge to a real world value, and to hold it constant for the duration of the experiment. Then, as the difference equation must be satisfied at all points in time, we can insert values of A_t and A_{t-1}, and solve for A_{t+1}. In our case, we stabilized the price of coffee at its 1992 level, and simulated the dynamics of consumption on that basis.

In 1991, actual US per capita consumption of coffee was 7.8 pounds per year. Using Equation (20), we obtain the following pattern of per capita coffee consumption for the US (see Table 1).

While most North Americans are familiar with the recent explosive increase in the number of neighbourhood coffee shops, this projected consumption pattern is perhaps a bit extreme.

The coffee data above follows the pattern we suggested would arise whenever an unstable root had sufficient influence on the behaviour of the system to be estimated. The simulation certainly suggests that, despite the model apparently being a well-behaved RA equation, its dynamic implications are unrealistic. This kind of dynamic simulation exercise is quite easy to undertake, and arguably, should be part of the model diagnostics undertaken whenever an RA model is being estimated on aggregate data.

Table 1

Year	Per capita consumption (lb)
1992	14.62
1993	28.87
1994	51.56
1995	83.64
1996	126.20
1997	180.48
1998	247.89
1999	330.04
2000	428.77
2001	546.16
2002	684.59
2003	846.77
2004	1188.22
2005	1576.61
2006	2017.77
2007	2518.89
2008	3088.14
2009	3734.80
2010	4469.42
2011	5303.97
2012	6252.07
2013	7327.19

CONCLUSIONS

While the RA model has become a major tool in the economic analysis of the consumption of addictive commodities, our discussion suggests that conclusions drawn from empirical applications should be treated with caution. At the aggregate level, there is no good reason to expect to see the dynamics predicted by models of individual optimizing behaviour reappear. Whenever the RA model is applied at the aggregate level, standard econometric diagnostic tests should be supplemented by a dynamic simulation exercise. The dynamic behaviour implied by the RA model is one of its most important predictions, so every effort should be made to test its reasonableness.

At the micro level, the results of empirical application of the model are, in general, consistent with RA behaviour. The problem arises in the interpretation of the expression usually used to estimate price responsiveness. While testing the effect of changing prices on the location of the saddlepoint equilibrium is legitimate, interpreting this as a consumer price response is less so, as consumers will not, in general, converge on the equilibrium.

This does not mean that we cannot investigate price responses in an RA model. The best approach would appear to be one derived from standard partial adjustment dynamics. Given the values of the exogenous variables, including past consumption behaviour, an individual of a particular age will have an intertemporally optimal target level for current consumption of the addictive commodity. This target can be represented on the phase diagram as a point on the optimal trajectory. Where exactly the point is on the trajectory will depend on the individual's age—i.e. on how far along he is in his planning horizon. When the price of the addictive good changes, the optimal trajectory will shift. The result will be a new target consumption level, represented by the appropriate point on the new optimal trajectory. The full price response (what the partial adjustment literature would term the long run response) is the vertical distance between the old and new optimal trajectories, measured from the old optimum on the old trajectory. There will be a partial response (what the partial adjustment literature would term the short run response) if it takes time for the individual to calculate the exact magnitude of the shift of the trajectory, and if there are adjustment costs which we have not included in the theoretical model. This suggests that the appropriate way to investigate price dynamics would be to use the RA model to derive the optimal trajectory, and then to use either a partial adjustment model or the more general error-correction [16] model to estimate the dynamic adjustment process.

In general, the RA model is a significant contribution to the theoretical modelling of the consumption of addictive goods, and on that basis, fully deserves the attention it has received. The conclusions drawn from its empirical applications to date must, however, be treated with some caution.

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