PREP

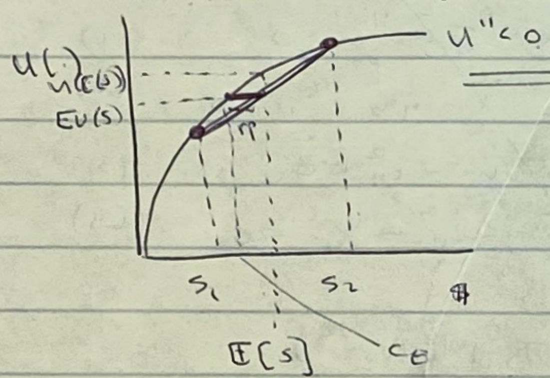
Lecture 2 - Moral hazard

(0) * Start w/ sample discussion of RR's

- obviously no decision for journal
- Think: contribution, flaws, assumptions, presentation, usefulness, theoretical/empirical extensions.

(1) SIDENOTE 1: RISK AVERSION + EXPECTED UTILITY

- Utility is concave ($u'' < 0$)
- This means $u(\text{Average}) > u(\text{Expected})$



⟹ difference is risk premium, amount ~~to pay someone to take gamble.~~ someone would pay to avoid bet

So $EU = p_1 u(s_1) + p_2 u(s_2) \Rightarrow \sum_{i \text{ states}} p_i u(x_i) \Rightarrow \int u(x_i)\, dF(x_i)$

(2) Effort model

- Principal wants to maximize π ⟶ observes profits, picks wage $w(\pi)$
- Agent wants to maximize $V \sim v(w(\pi|e)) - g(e)$

What is expected utility?

Problem: $\min_{w(\cdot)} \int w(\pi)\, dF(\pi|e_H)$ s.t. $\int v(w(\pi))\, dF(\pi|e_H) - g(e_H) \geq 0$

and $\int v(w(\pi))\, dF(\pi|e_H) - g(e_H) \geq \int v(w(\pi))\, dF(\pi|e_L) - g(e_L)$

EXAMPLE: Let $v = \sqrt{w}$ (risk averse)

~~$w = \frac{1}{4}\pi$ (more profits ⟹ more wages)~~

$g = e^2$ (costly effort)

jump to PA

~~Only~~ undefined: Two states: $\pi \in (0, 100)$ $p|e_H = (.25, .75)$ $p|e_L = (.75, .25)$

NOTES ON PA PROBLEM

- Grossman + Hart (1983) Fudenberg + Tirole (1990), both ETCA
- Basically, this is any contract. 2 problems arise
  - Hidden Action (e) → on focus
  - Hidden Type/Ability (AS)

PROBLEM: principal: $u_P = \cancel{Q(e(c))(1-c)}$ endogenous $Q(e)-w$ [risk neutral]

$U_A = \sqrt{\cancel{cQ(e(c))}} \cancel{-3e}$ [risk ~~agn~~ averse]

$\sqrt{w}-e$ exogenous, could be $g(e)$

Assume $\underline{U_A} = 10$ (reservation wage)

Let $Q = \begin{cases} 400 & p \\ 100 & 1-p \end{cases}$ $p = 2/3$ if $e=1$, $1/3$ if $e=0$

Problem: want to solve principal's problem (what is c?)

s.t. (1) participation ($U_A \geq 0$)

(2) incentive compatability ($U_A(e=1) \geq U_A(e=0)$)

Solution (3 steps):

① set of participation contracts ~~...~~ when $e^* = \bar{e}$

② find element of (1) @ least cost to P ⇒ here, PA will bind

③ Choose $\bar{e}$ to MAX π_P / U_P

Hence $U_A = \underline{U} + \varepsilon$

CASE 1: If e is obs.

then c^* can be function of e

① $\bar{e} = 0$. Then $EU_A = \frac{1}{3}[\sqrt{c \cdot 400} - 0] + \frac{2}{3}[\sqrt{c \cdot 100} - 0] \geq 10$

$$\frac{20}{3}c^{1/2} + \frac{20}{3}c^{1/2} \geq 10$$

$$\frac{40}{3}c^{1/2} \geq 10$$

$$c^{1/2} \geq \frac{30}{40}$$

$$c \geq \frac{900}{1600} = 56.25\%$$

(this binds as cheapest op)

Now if $\bar{e}=1$, then

$$EU_A = \frac{2}{3}\left[\sqrt{c\cdot 400}-3\right] + \frac{1}{3}\left[\sqrt{c\cdot 100}-3\right] \geq 10$$

$$\frac{40}{3}c^{1/2} + \frac{10}{3}c^{1/2} - 3 \geq 10$$

$$\frac{50}{3}c^{1/2} \geq 13$$

$$c^{1/2} \geq \frac{33}{50}$$

$$\boxed{c \approx 43.56\%} \qquad \boxed{c^* = .6084}$$

~~contract is then~~

(3) What is better for P?

$$EU_P(\bar{e}=0) = \frac{1}{3}\left(400\left(1-\frac{9}{16}\right)\right) + \frac{2}{3}\left(100\left(1-\frac{9}{16}\right)\right) = 175/2 = 87.5$$

$$EU_P(\bar{e}=1) = \frac{1}{3}(100(.40)) + \frac{2}{3}(400(.4)) = 65.26$$

Prefers to implement low effort!

CASE 1: Effort is Observed:

(1, 2) Let $\bar{e}=0$: then $EU_A = \underset{a=400}{\frac{1}{3}(\sqrt{w}-0)} + \underset{a=100}{\frac{2}{3}(\sqrt{w}-0)} \geq 10$

$$\sqrt{w} \geq 10$$

$$\boxed{w^* = 100}$$

Let $\bar{e}=1$: then $EU_A = \frac{1}{3}(\sqrt{w}-1) + \frac{2}{3}(\sqrt{w}-1) \geq 10$

$$\sqrt{w} \geq 11$$

$$\boxed{w^* = 121}$$

"rent" from effort

(3) Which is better?

$$EU_P(\bar{e}=0) = \frac{1}{3}(400-100) + \frac{2}{3}(400-100) = 100$$

$$EU_P(\bar{e}=1) = \frac{1}{3}(100-121) + \frac{2}{3}(400-121) = 179$$

$$\boxed{\text{optimal contract: } e=(1,0) \text{ then } c=(121,-\infty)}$$

CASE 2: What if e is unobserved?

- Contract must be tied to output
- Hence, two wages: w_L, w_H
- Agent's risk aversion makes this tricky.

(1,2) $\bar{e} = 0$: $EU_A = \frac{1}{3}(\sqrt{w_H} - 0) + \frac{2}{3}(\sqrt{w_L} - 0) \geq 10$

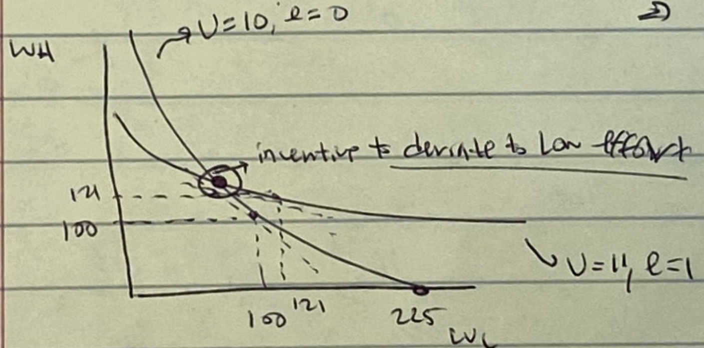
two variables!

could set up a minimization problem:

$$\min_{w_L, w_H} \frac{1}{3} w_H + \frac{2}{3} w_L \quad \text{s.t.} \quad \frac{1}{3} w_H^{1/2} + \frac{2}{3} w_L^{1/2} = 10$$

Alternatively, let's graph: $\frac{1}{3} w_H^{1/2} + \frac{2}{3} w_L^{1/2} = 10$

$\Rightarrow w_H = 4(w_L - 30\sqrt{w_L} + 225)$



Now $\bar{e} = 1$: $EU_A = \frac{1}{3}(\sqrt{w_L} - 1) + \frac{2}{3}(\sqrt{w_H} - 1) \geq 10$

$\frac{1}{3} w_L^{1/2} + \frac{2}{3} w_H^{1/2} = 11$

* Lagrangian won't work! Will give us $w_L^* = w_H^* = 121$.
  - Why is this a problem?
  - Solution: must add a second constraint:

~~$EU_A(e=1) \geq EU_A(e=0)$~~ (binding in eqbm(?))

incentive compat: $\frac{1}{3}(\sqrt{w_H}) + \frac{2}{3}(\sqrt{w_L}) < 10$ ⇐ no chance for (just) giving low effort

Now ~~can solve~~: set the two constraints equal or see graphically!

$\boxed{w_H^* = 144, \ w_L^* = 81}$

Zeckhauser (1970) JET

- Consumers are either (sick or healthy)

$p = Pr(sick)$, $1-p = Pr(healthy)$

$$EU_c = pU(w-s) + (1-p)U(w)$$

↳ measure in wealth? Good or bad?

Now suppose a treatment exists:

$$EU_c|\text{treatment} = (1-p)U(w) + pU(w-s+f(h)-h)$$

↳ assume $f'>0$, $f''<0$

→ ex-post, what would you do?

$$U = U(\underbrace{w-s+f(h)-h})$$

$\max_h$: $w-s+f(h)-h$

choose h^* s.t. $\underline{f'(h^*)=1}$

ex-post optimal trmt.

→ What about ex-ante? If consumers are risk averse, they want insurance:

$$EU_c = (1-p)U(w-\pi) + pU(w-\pi-s+\tau+f(h^*)-h^*)$$

(this is a 2PC, we'll return to that) ← - Contract pays you to get treated: $\tau = \underbrace{\cancel{s}-f(h^*)+h^*}_{\text{full recuperation}}$

- premium is actuarily fair: $\pi = p \cdot \tau$

$\boxed{\text{* What do consumers do in optimum? Would you get treatment if } s \leq 0\text{?}}$

Now trust sickness is unverifiable.

Okay, so what are parallels to moral hazard?

- illness is unverifiable
- So is illness like effort? Grossman model thinks so
- Also ex-post treatment decisions: do you want h^* if you don't pay as much for it? of course!

UNVERIFIABLE ILLNESS

- Need to satisfy ~~$u(\text{sick, treatment}) \geq 0$~~

PARTICIPATION $EU(\text{insurance}) \geq EU(\text{no insurance})$

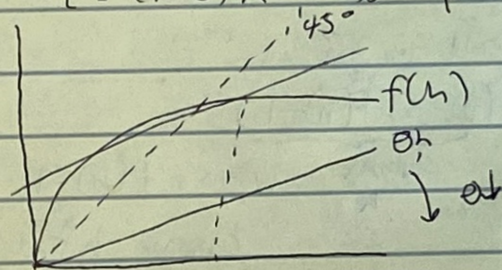
IC: $u(\text{sick, treatment}) \geq u(\text{sick, no treatment})$ [trivial given ↺)]

$u(\text{healthy, treatment}) < u(\text{healthy, no t})$

- In US, cost-sharing, in Canada?

- suppose $\tau = (1-\theta)h$ so you pay θh.

EX-POST:



Now if you're sick:

$w - s + f(h) - \theta h$

$\boxed{f'(h) = \theta}$

→ decreases ~~increases~~ w/ θ: $\partial h^* / \partial \theta \leq 0$

or $\partial h^* / \partial \tau > 0$ Law of demand (ex-post MH)

So what's the eqlbm?

$$\max_{\pi, \theta} (1-p)U(w-\pi) + pU(w-s-\pi+f(h(\theta))-\theta h(\theta)) \quad \text{s.t. } \underbrace{\pi = p\cdot(1-\theta)h(\theta)}_{\text{FAIR insurance}}$$

FOC's:

(1) $$\frac{\partial \mathcal{L}}{\partial \pi} = \underbrace{-(1-p)U'(w-\pi) - pU'(w-s-\pi+f(h(\theta))-\theta h(\theta))}_{*} - \lambda = 0$$

(2) $$\mathcal{L}_\theta = pU'(w-s-\pi+f(h(\theta))-\theta h(\theta))[f'(h(\theta))\cdot h'(\theta) - h(\theta) - \theta h'(\theta)] + p\lambda[-h(\theta)+(1-\theta)h'(\theta)] = 0$$

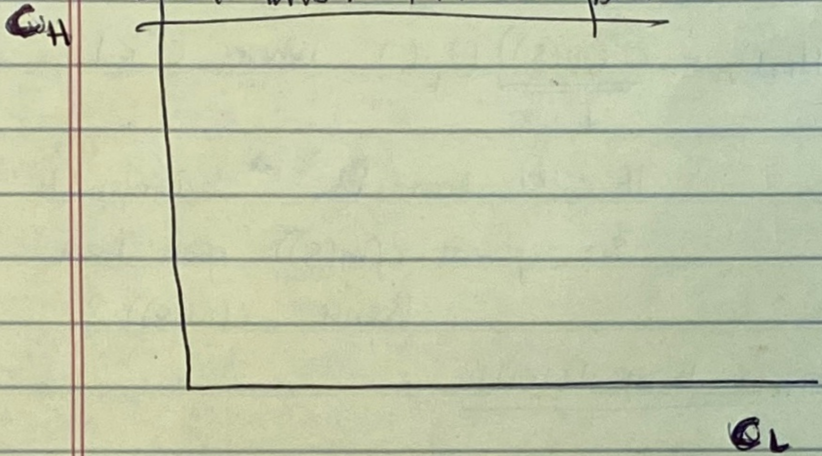
$$\mathcal{L}_\lambda = \pi = p(1-\theta)h(\theta)$$

Interpreting: (1) Loss in marginal utility from premium equal to λ (shadow price)

[more interesting] (2) - drop p's

$$\underbrace{u'(\text{sick, treatment})[(f'(h(\theta))-\theta)h'(\theta) - h(\theta)]}_{\text{marginal utility of bearing when sick}} = \underbrace{-\lambda[(1-\theta)h'(\theta) - h(\theta)]}_{\text{utility gains from insurance (better risk)}}$$

Zeckhauser 1970 eqbm



OVERCONSUMPTION: mathematically

one more insight: if $\theta < 1$ and ~~h'(θ)~~ $f'(h(\theta)) = \theta$, then

(2) becomes

$$u'(\text{sick, treatment})\left[-h(\theta)\right] = -\lambda\left[\underbrace{(1-\theta)h'(\theta)}_{\text{where}} - h(\theta)\right]$$

$(1-\theta) > 0$ and $h'(\theta) < 0$

★ given that $(1-\theta)h'(\theta) < 0$,
the FOC will have to compensate by
$u'(\text{sick, treatment}) \downarrow$, which means $h^* \uparrow$ (since $u' > 0$ and $u'' < 0$)
– relies on concavity of $u(\cdot)$ ⇒ risk aversion.

Can we achieve first best? $h = h^*$?

First best (s's verifiable): $\max\limits_{m(s)} \int U[\cancel{w} - \pi - c(m(s)), H(s,m)]\, f(s)\,ds$

not y

$m(s)$ ↳ consumers choose function

H ↓ health; $f(s)$ ↓ sickness "state"

FOC: $u_i(w-\pi-c(m(s)), H(s,m))(-c'(m(s))) + u_h(w-\pi-c(m(s)), H_m(s,m))$

or $\underline{H_m u_h(\cdot)} = \underline{u_i(\cdot)\,c'(m(s))}$

$H_m u_h$ ↓ consumption gain from health

u_i ↓ ~~u gain from H on health~~ exp cons. across states

Should be in slides, but in full eqbm, $c' = 1$!

★ if $c' < 1$, then $u_h \uparrow$