Chapter 2 — Notes

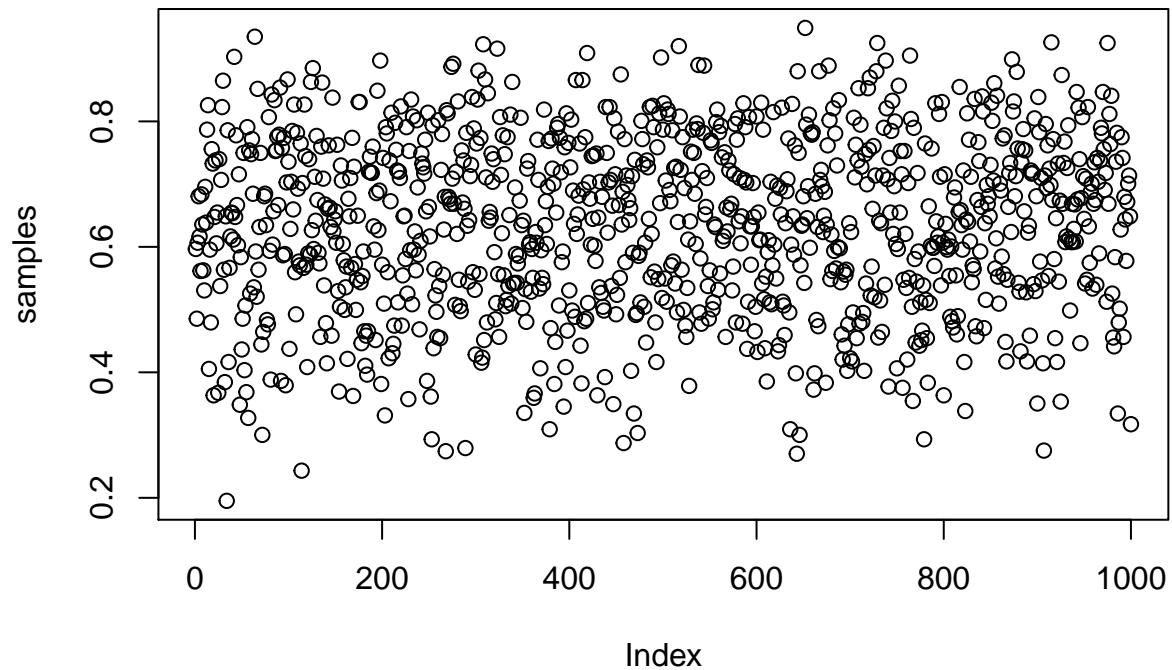## 3.1 Sampling from a grid-approximate posterior

- R Code 3.2:

```
n = 1000
p_grid <- seq(from=0, to=1, length.out=n)
prior <- rep(1, n)
likelihood <- dbinom(x=6, size=9, prob=p_grid)
posterior_notnorm <- likelihood * prior
posterior <- posterior_notnorm / sum(posterior_notnorm)
```

Draw 10,000 samples: * R Code 3.3:

```
samples <- sample(p_grid, prob=posterior, size=n, replace=T)
```
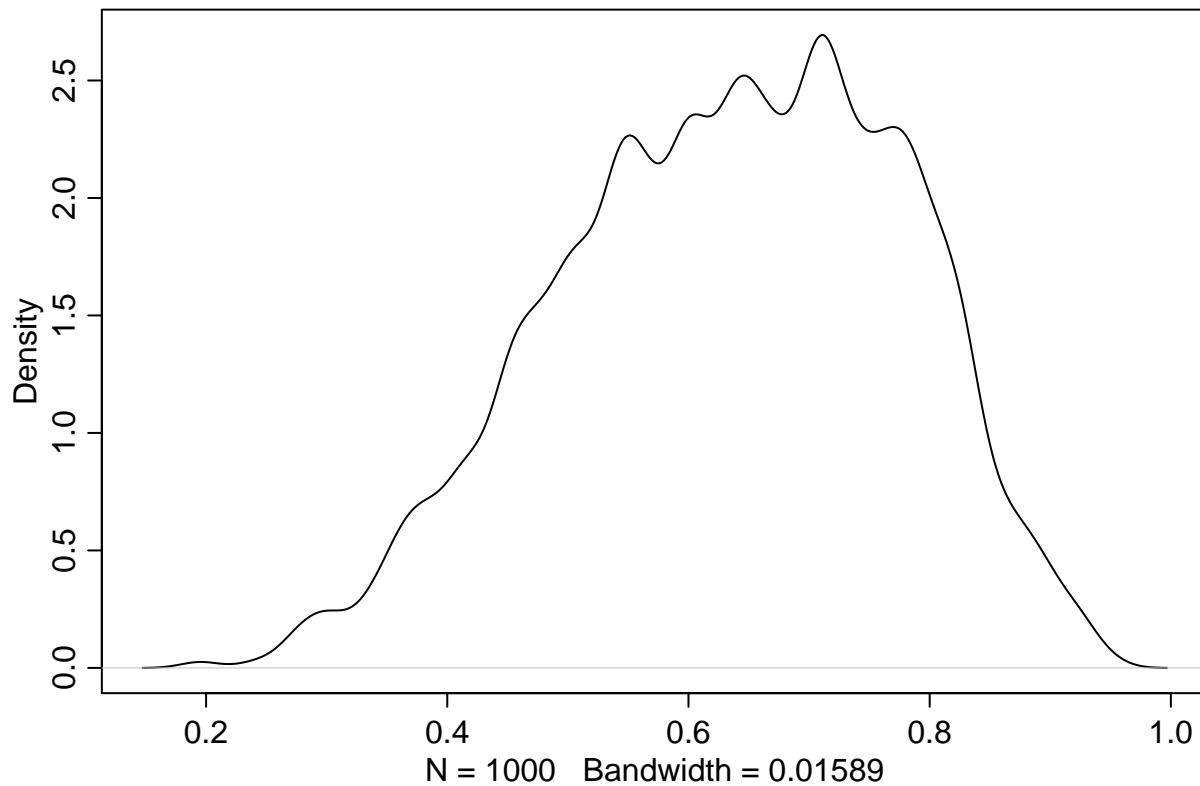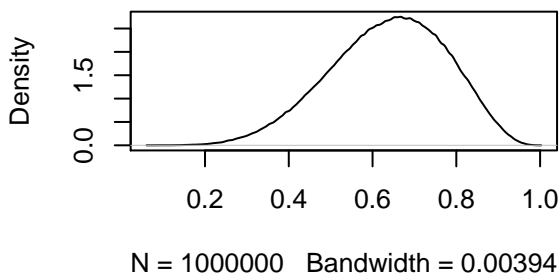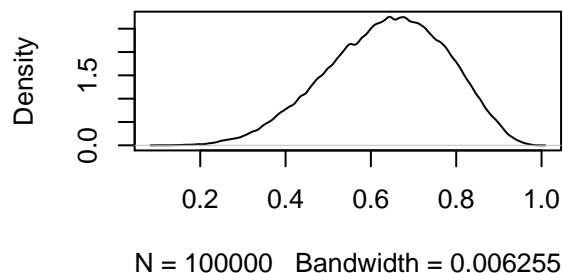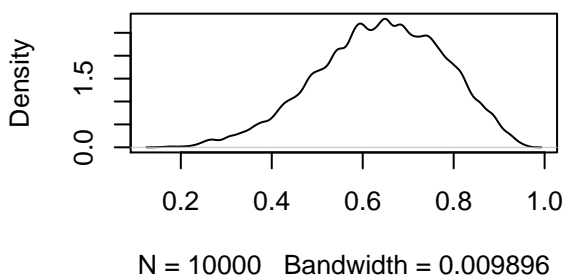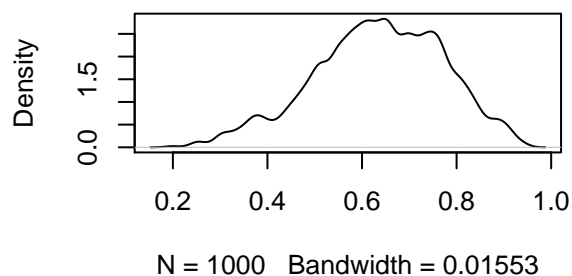
- 3.4:

```
plot(samples)
```


- 3.5:

```
dens(samples)
```

Let's try more samples:

```r
par(mfrow=c(2, 2))
dens(sample(p_grid, prob=posterior, size=1e3, replace=T))
dens(sample(p_grid, prob=posterior, size=1e4, replace=T))
dens(sample(p_grid, prob=posterior, size=1e5, replace=T))
dens(sample(p_grid, prob=posterior, size=1e6, replace=T))
```

N = 1000   Bandwidth = 0.01553

N = 10000   Bandwidth = 0.009896

N = 100000   Bandwidth = 0.006255

N = 1000000   Bandwidth = 0.00394

3.2 Sampling to Summarize

3.2.1. Intervals of defined boundaries.

The posterior probability that the proportion of water is less than 0.5:

- 3.6:

```
p_grid < 0.5
```

```
##    [1]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [12]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [23]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [34]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [45]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [56]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [67]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [78]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##   [89]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [100]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [111]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [122]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [133]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [144]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [155]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
##  [166]   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE   TRUE
```

```
## [177]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [188]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [199]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [210]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [221]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [232]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [243]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [254]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [265]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [276]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [287]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [298]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [309]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [320]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [331]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [342]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [353]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [364]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [375]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [386]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [397]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [408]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [419]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [430]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [441]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [452]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [463]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [474]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [485]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [496]  TRUE  TRUE  TRUE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE
## [507] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [518] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [529] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [540] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [551] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [562] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [573] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [584] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [595] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [606] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [617] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [628] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [639] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [650] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [661] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [672] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [683] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [694] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [705] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [716] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [727] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [738] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [749] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [760] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

```
##  [771] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [782] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [793] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [804] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [815] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [826] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [837] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [848] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [859] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [870] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [881] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [892] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [903] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [914] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [925] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [936] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [947] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [958] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [969] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [980] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##  [991] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

```r
sum(posterior[p_grid < 0.5])
```

```
## [1] 0.1718746
```

Samples array:

```r
head(samples, 100)
```

```
##    [1] 0.5965966 0.4854855 0.6066066 0.6806807 0.6166166 0.5615616 0.6836837
##    [8] 0.6356356 0.5625626 0.5305305 0.6956957 0.6386386 0.7867868 0.8258258
##   [15] 0.4054054 0.5955956 0.4794795 0.7557558 0.7327327 0.3633634 0.6476476
##   [22] 0.7377377 0.6056056 0.6546547 0.3673674 0.7397397 0.5375375 0.7067067
##   [29] 0.8228228 0.8648649 0.5635636 0.3843844 0.6486486 0.1951952 0.7857858
##   [36] 0.4164164 0.5665666 0.6166166 0.6546547 0.6536537 0.6116116 0.9029029
##   [43] 0.6476476 0.7777778 0.6666667 0.6026026 0.7157157 0.3483483 0.5835836
##   [50] 0.4364364 0.4854855 0.7497497 0.4034034 0.5065065 0.3683684 0.7907908
##   [57] 0.3273273 0.7537538 0.5255255 0.7477477 0.7717718 0.6846847 0.5355355
##   [64] 0.9349349 0.5925926 0.5205205 0.8518519 0.5635636 0.6316316 0.7497497
##   [71] 0.4444444 0.3003003 0.4644645 0.6806807 0.6846847 0.6346346 0.4834835
##   [78] 0.4764765 0.8068068 0.5935936 0.3883884 0.8428428 0.6036036 0.6566567
##   [85] 0.8338338 0.7527528 0.7537538 0.7757758 0.6666667 0.7787788 0.8538539
##   [92] 0.3863864 0.7577578 0.5865866 0.7737738 0.5885886 0.3793794 0.7027027
##   [99] 0.8668669 0.7357357
```

The same calculation using samples. Add up all samples that lie in the grid < 0.5, and divide by the total number of samples to get the frequency ~ probability:

- 3.7:

```r
n = 1e4
samples = sample(p_grid, prob=posterior, size=n, replace=T)
sum(samples < 0.5) / n
```

```
## [1] 0.1718
```

How much probability lies between 0.5 and 0.75: * 3.8:

```
sample_points = sum(samples > 0.5 & samples < 0.75)
sample_points
```

```
## [1] 6076
```

```
sample_points / n
```

```
## [1] 0.6076
```

3.2.2. Intervals of defined mass.

Boundaries of the lower 80% posterior probability lies:

- 3.9:

```
quantile(samples, probs = .8)
```

```
##       80%
## 0.7597598
```

Middle 80%, i.e. lying between 10% and 90%:

```
# 3.10
quantile(samples, probs = c(0.1, 0.9))
```

```
##       10%       90%
## 0.4464464 0.8138138
```