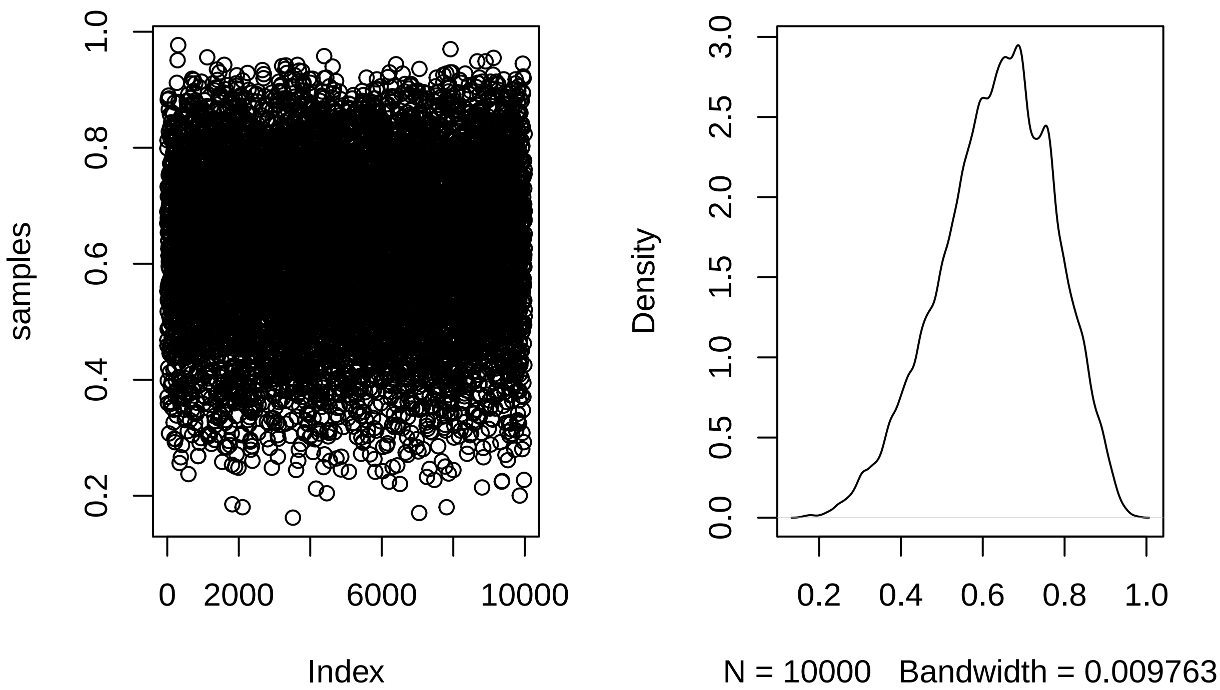
Chapter 3 – Practice

```
p_grid = seq(from=0, to=1, length.out=1000)
prior <- rep(1, 1000)
likelihood <- dbinom(6, size=9, prob=p_grid)
posterior <- likelihood * prior
posterior <- posterior / sum(posterior)
set.seed(100)
samples <- sample(p_grid, prob=posterior, size=1e4, replace=T)
```

```
par(mfrow=c(1, 2))
plot(samples)
plot(density(samples, adjust = 0.5), main="")
```



3E1. How much posterior probability lies below p = 0.2?

```
sum(posterior[p_grid < 0.2])
```

```
## [1] 0.0008560951
```

```
sum(samples < 0.2) / length(samples)
```

```
## [1] 5e-04
```

3E2. How much posterior probability lies above p = 0.8?

```
sum(posterior[p_grid > 0.8])
```

```
## [1] 0.1203449
```

```
sum(samples > 0.8) / length(samples)
```

```
## [1] 0.1117
```

3E3. How much posterior probability lies between p = 0.2 and p = 0.8?

```
sum(posterior[p_grid > 0.2 & p_grid < 0.8])
```

```
## [1] 0.878799
```

```
sum(samples > 0.2 & samples < 0.8) / length(samples)
```

```
## [1] 0.8878
```

3E4. 20% of the posterior probability lies below which value of p?

```
quantile(samples, 0.2)
```

```
##       20%
## 0.5195195
```

3E5. 20% of the posterior probability lies above which value of p?

```
quantile(samples, 0.8)
```

```
##       80%
## 0.7567568
```

3E6. Which values of p contain the narrowest interval equal to 66% of the posterior probability?

```
samples_for_hpdi <- coda::as.mcmc(samples)
# x <- sapply(0.66, function(p) coda::HPDinterval(samples_for_hpdi,  prob = p))
x <- coda::HPDinterval(samples_for_hpdi, prob=0.66)
c(x[1], x[2])
```

```
## [1] 0.5205205 0.7847848
```

```
HPDI(samples, prob=0.66)
```

```
##     |0.66     0.66|
## 0.5205205 0.7847848
```

3E7. Which values of p contain 66% of the posterior probability, assuming equal posterior probability both below and above the interval?

```
low = (1 - 0.66) / 2
up = low + 0.66
interval = c(low, up)
c(interval, interval[2] - interval[1])
```

```
## [1] 0.17 0.83 0.66
```

```
quantile(samples, interval)
```

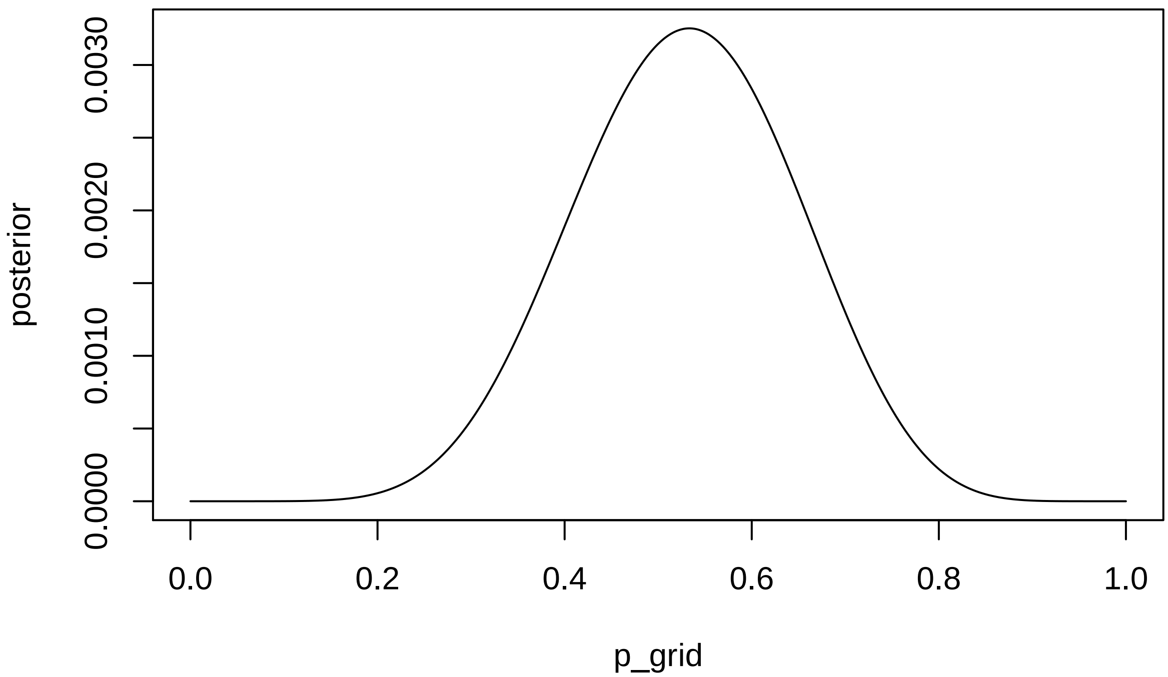
```
##       17%       83%
## 0.5005005 0.7687688
```

If the distribution is not too skewed then the Percentile Interval (PI) will approximately equal to the Highest Posterior Density Interval (HPDI):

- PI: (0.501, 0.769)
- HPDI: (0.521, 0.785)

3M1. Suppose the globe tossing data had turned out to be 8 water in 15 tosses. Construct the posterior distribution, using grid approximation. Use the same flat prior as before.

```
N = 1000
p_grid <- seq(0, 1, length.out = N)
prior <- rep(1, N)
likelihood <- dbinom(8, size=15, prob=p_grid)
unstd_posterior <- likelihood * prior
posterior <- unstd_posterior / sum(unstd_posterior)
plot(posterior ~ p_grid, type="l")
```



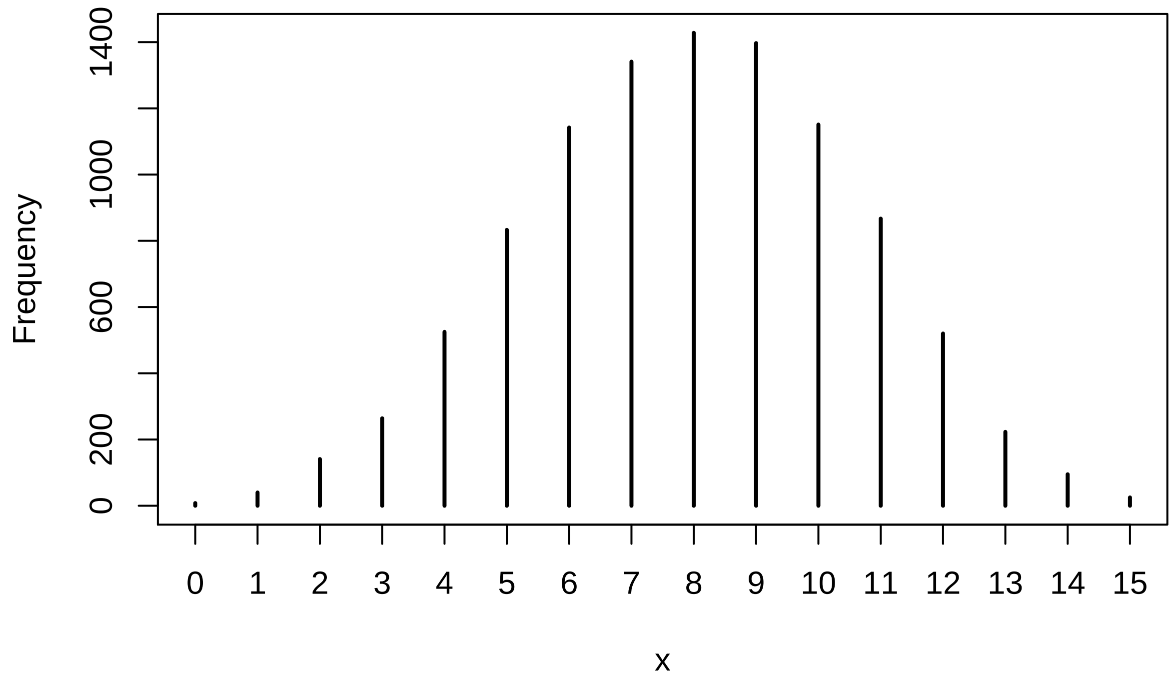
3M2. Draw 10,000 samples from the grid approximation from above. Then use the samples to calculate the 90% HPDI for *p*.

```
samples <- sample(p_grid, size=1e4, replace = T, prob=posterior)
samples_for_hpdi <- coda::as.mcmc(samples)
x <- coda::HPDinterval(samples_for_hpdi, prob=0.90)
c(x[1], x[2])
```

```
## [1] 0.3383383 0.7317317
```

3M3. Construct a posterior predictive check for this model and data. Simulate the distribution of samples, averaging over the posterior uncertainty in *p*. What is the probability of observing 8 water in 15 tosses.

```
w <- rbinom(1e4, size=15, prob=samples)
simplehist(w)
```



Probability of observing 8 water in 15 tosses:

```r
sum(w == 8) / length(w)
```

```
## [1] 0.1428
```

```r
mean(w == 8)
```

```
## [1] 0.1428
```

3M4. Using the posterior distribution constructed from the new (8/15) data, now calculate the probability of observing 6 water in 9 tosses.

```r
w6 <- rbinom(1e4, size=9, prob=samples)
mean(w6 == 6)
```

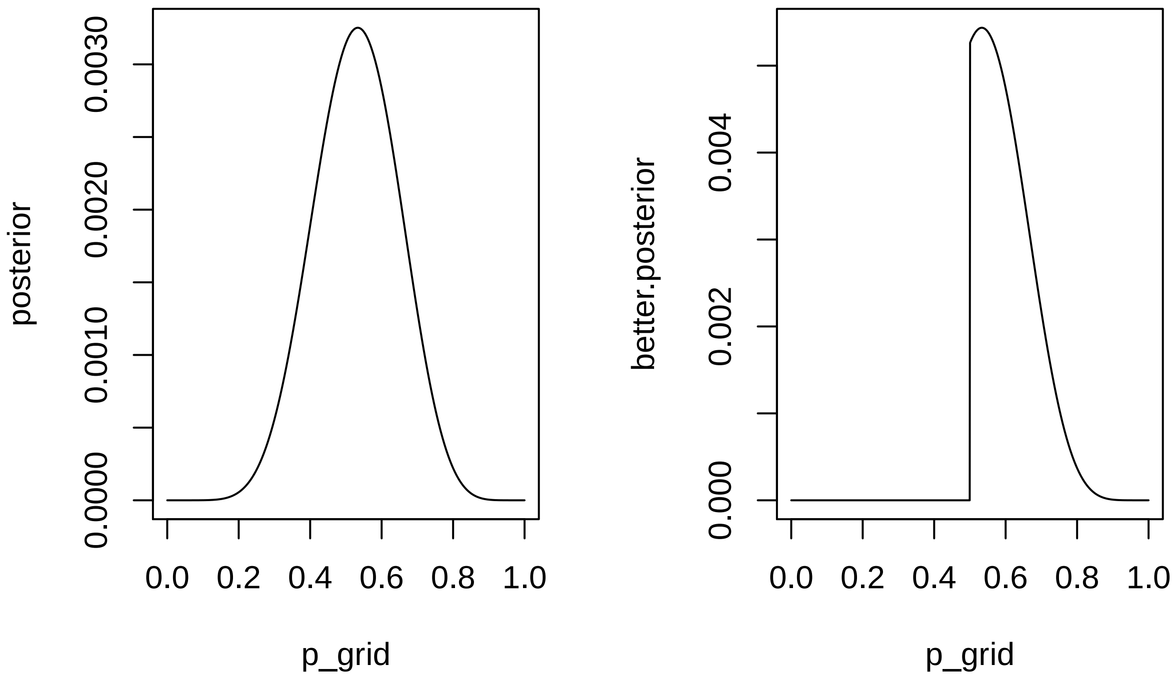
```
## [1] 0.1695
```

```r
sum(w6 == 6) / length(w6)
```

```
## [1] 0.1695
```

3M5. Start over at 3M1, but now use a prior that is zero below $p = 0.5$ and a constant above $p = 0.5$. This corresponds to prior information that a majority of the Earth's surface is water. Repeat each problem above and compare inferences. What difference does the better prior make? If it helps, compare inferences (using both priors) to the true value $p = 0.7$.

```
better.prior <- rep(2, N)
better.prior[p_grid < 0.5] = 0
better.unstd_posterior = likelihood * better.prior
better.posterior = better.unstd_posterior / sum(better.unstd_posterior)
par(mfrow=c(1, 2))
plot(posterior ~ p_grid, type="l")
plot(better.posterior ~ p_grid, type="l")
```



Repeat 3M2. Draw 10,000 samples; calculate the 90% HPDI for p:

```
better.samples <- sample(p_grid, size=1e4, replace = T, prob=better.posterior)
better.samples_for_hpdi <- coda::as.mcmc(better.samples)
better.x <- coda::HPDinterval(better.samples_for_hpdi, prob=0.90)
```

Previous posterior:

```
x
```

```
##          lower     upper
## var1 0.3383383 0.7317317
## attr(,"Probability")
## [1] 0.9
```

Better posterior:

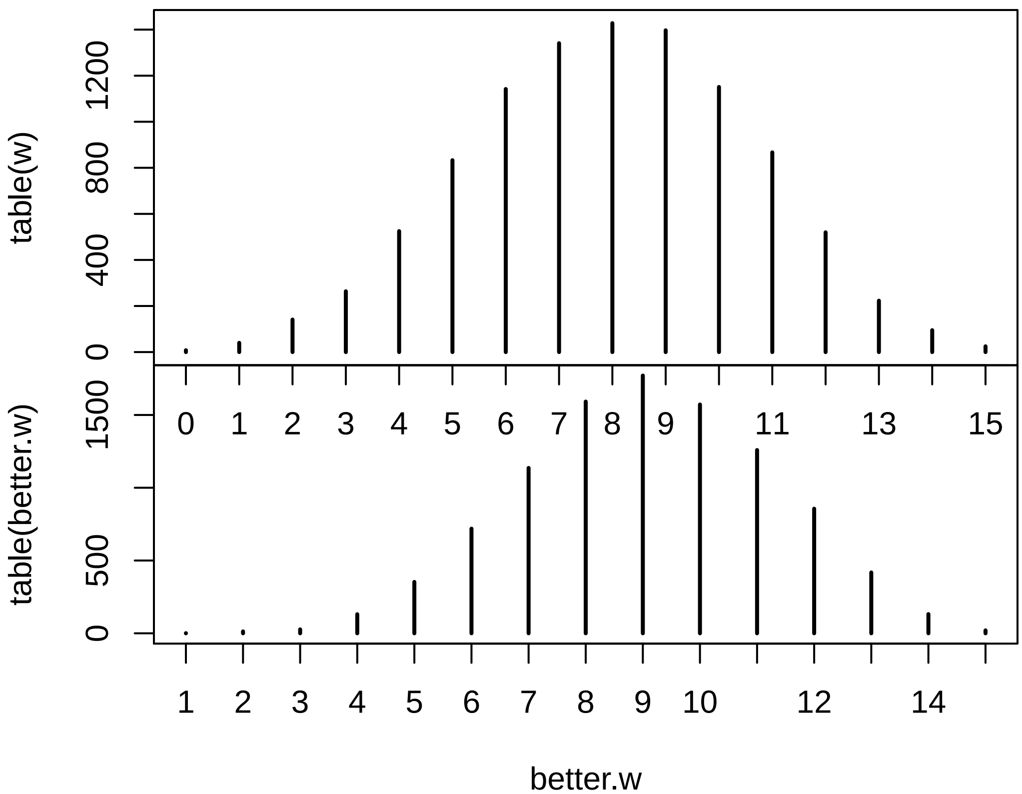
```
better.x
```

```
##          lower     upper
## var1 0.5005005 0.7097097
## attr(,"Probability")
## [1] 0.9
```

Better posterior has a much narrower 90% HPDI that's centered around true value of $p = 0.7$.

Repeat 3M3.

```
better.w <- rbinom(1e4, size=15, prob=better.samples)
layout(rbind(1, 2))
par(mar=c(0,5,2,5))
plot(table(w))
par(mar=c(4,5,0,5))
plot(table(better.w))
```



Probability of observing 8 water in 15 tosses:

```
mean(w == 8)
```

```
## [1] 0.1428
```

```
mean(better.w == 8)
```

```
## [1] 0.1592
```

Repeat 3M4.

```
better.w6 <- rbinom(1e4, size=9, prob=better.samples)
mean(w6 == 6)
```

```
## [1] 0.1695
```

```
mean(better.w6 == 6)
```

```
## [1] 0.2357
```

Hard

```
birth1 <- c(1,0,0,0,1,1,0,1,0,1,0,0,1,1,0,1,1,0,0,0,1,0,0,0,1,0,
0,0,0,1,1,1,0,1,0,1,1,1,0,1,0,1,1,0,1,0,0,1,1,0,1,0,0,0,0,0,0,0,
1,1,0,1,0,0,1,0,0,0,1,0,0,1,1,1,1,0,1,0,1,1,1,1,1,0,0,1,0,1,1,0,
1,0,1,1,1,0,1,1,1,1)

birth2 <- c(0,1,0,1,0,1,1,1,0,0,1,1,1,1,1,0,0,1,1,1,0,0,1,1,1,0,
1,1,1,0,1,1,1,0,1,0,0,1,1,1,1,0,0,1,0,1,1,1,1,1,1,1,1,1,1,1,1,1,
1,1,1,0,1,1,0,1,1,0,1,1,1,0,0,0,0,0,0,1,0,0,0,1,1,0,0,1,0,0,1,1,
0,0,0,1,1,1,0,0,0,0)

sum(birth1) + sum(birth2) # Should be 111
```

```
## [1] 111
```

3H1. Using grid approximation, computer the posterior distribution for the probability of a birth being a boy. Assume a uniform prior. Which parameter value maximizes the posterior probability?