Kangcheng Li
Com Sci 161
Assignment 6

1.

(a) List the Markovian assumptions (also known as topological semantics) encoded in the Bayesian network structure.

Markovian assumptions states that a node is conditionally independent of its non-descendants given its parents

$A \perp B$ $A \perp E$

$B \perp A$ $B \perp C$

$C \perp B \mid A$ $C \perp D \mid A$ $C \perp E \mid A$

$D \perp C \mid A, B$ $D \perp E \mid A, B$

$E \perp A \mid B$ $E \perp C \mid B$ $E \perp D \mid B$ $E \perp F \mid B$ $E \perp G \mid B$

$F \perp A \mid C, D$ $F \perp B \mid C, D$ $F \perp E \mid C, D$

$G \perp A \mid F$ $G \perp B \mid F$ $G \perp C \mid F$ $G \perp D \mid F$ $G \perp E \mid F$ $G \perp H \mid F$

$H \perp A \mid E, F$ $H \perp B \mid E, F$ $H \perp C \mid E, F$ $H \perp D \mid E, F$ $H \perp G \mid E, F$

(b) Provide the Markov blanket for variable D.

Markov blanket: parents + children + and children's parents

Markov blanket for variable D is {A, B, C, F}

(c) Express Pr(A,B,C,D,E,F,G,H) as a multiplication of conditional and marginal probabilities, using the chain rule for Bayesian networks.

Pr(A,B,C,D,E,F,G,H) = Pr(A) * Pr(B) * Pr(C|A) * Pr(D|A, B) * Pr(E|B) * Pr(F|C, D) * Pr(G|F) * Pr(H|E, F)

(d) Derive Pr(E,F,G,H) from the result of Pr(A,B,C,D,E,F,G,H) computed above. Express it using factors.

(e) Multiply the factors (tables) of Pr(D|AB) and Pr(E|B). Show the new factor.

(f) Sum out D from the factor computed above. Show the new factor.

(g) Express Pr(a, ¬b, c, d, ¬e, f, ¬g, h) in terms of the parameters in the Conditional Probability Table(CPT)s in Figure 1 (here a denotes A = 1 and ¬a denotes A = 0). Use placeholder symbols for the parameters that are not shown in the CPTs.

Pr(a, ¬b, c, d, ¬e, f, ¬g, h)

= Pr(a) * Pr(¬b) * Pr(c | a) * Pr(d | a,¬b) * Pr(¬e | ¬b) * Pr(f | c, d) * Pr(¬g | f) * Pr(h | ¬e, f)

= 0.2 * 0.3 * Pr(c | a) * 0.6 * 0.1 * Pr(f | c, d) * Pr(¬g | f) * Pr(h | ¬e, f)

= 0.0036 * Pr(c | a) * Pr(f | c, d) * Pr(¬g | f) * Pr(h | ¬e, f)

(h) Compute Pr(¬a, b).

Pr(¬a, b) = Pr(¬a) * Pr(b) = 0.8 * 0.7 = 0.56

(i) Compute Pr(¬e | a).

Pr(¬e | a) = Pr(¬e) = Pr(b) * Pr(¬e | b) + Pr(¬b) * Pr(¬e | ¬b) = 0.7 * 0.9 + 0.3 * 0.1 = 0.66

2. Consider the following sentences
i. John likes all kinds of food.
$\forall x(Food(x) \Rightarrow Likes(John, x))$

ii. Apples are food.
Food(apple)

iii. Chicken is food.
Food(chicken)

iv. Anything anyone eats and isn't made sick by is food.
$\forall y \forall z((Eats(z, y) \wedge \sim Sicks(y, z)) \Rightarrow Food(y))$

v. If you are made sick by something, you are not well.
$\forall a \forall b(Sicks(a, b) \Rightarrow \sim Well(b))$

vi. Bill eats peanuts and is well.
Eats(Bill, Peanuts) /\ Well(Bill)

vii. Sue eats everything Bill eats.
$\forall c(Eats(Bill, c) \Rightarrow Eats(Sue, c))$