

1. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither

(a) Neither

Smoke	Fire	$\text{Smoke} \Rightarrow \text{Fire}$	$\sim \text{Smoke} \Rightarrow \sim \text{Fire}$	$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\sim \text{Smoke} \Rightarrow \sim \text{Fire})$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(b) Neither

Smoke	Fire	Heat	$\text{Smoke} \Rightarrow \text{Fire}$	$\text{Smoke} \vee \text{Heat}$	$(\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire}$	$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

(c) Valid

Smoke	Fire	Heat	$\text{Smoke} \wedge \text{Heat}$	$(\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}$	$\text{Smoke} \Rightarrow \text{Fire}$	$\text{Heat} \Rightarrow \text{Fire}$	$(\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire})$	$((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

2. (a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

If the unicorn is mythical, then it is immortal

Mythical $\Rightarrow \sim$ Mortal

if it is not mythical, then it is mortal and it is a mammal

\sim Mythical \Rightarrow (Mortal \wedge Mammal)

If the unicorn is either immortal or a mammal, then it is horned

(\sim Mortal \vee Mammal) \Rightarrow Horned

The unicorn is magical if it is horned

Horned \Rightarrow Magical

2. (b) Convert the knowledge base into CNF.

Propositional Logic

(1) Mythical $\Rightarrow \sim$ Mortal

CNF

(5) \sim Mythical $\vee \sim$ Mortal

(2) \sim Mythical \Rightarrow (Mortal \wedge Mammal)

(Mortal \vee Mythical) \wedge (Mammal \vee Mythical)

(Can be broken down as)

(6) (Mortal \vee Mythical)

(7) (Mammal \vee Mythical)

(3) (\sim Mortal \vee Mammal) \Rightarrow Horned

(Mortal \vee Horned) \wedge (\sim Mammal \vee Horned)

(Can be broken down as)

(8) (Mortal \vee Horned)

(9) (\sim Mammal \vee Horned)

(4) Horned \Rightarrow Magical

(10) \sim Horned \vee Magical

2. (c) (i) Is it possible to derive from the knowledge base that the unicorn is mythical? (ii) How about magical? (iii) Horned? Justify your answers using resolution.

Assume the unicorn is not mythical

(11) \sim Mythical

Assumption

(12) (Mortal \wedge Mammal)

Modus Ponens (2) and (11)

(13) Mortal

And Elimination (12)

(14) Mammal

And Elimination (12)

(15) $\sim\text{Mortal} \vee \text{Mammal}$

Resolution (5) and (7)

(16) Horned

Modus Ponens (3) and (15)

(17) Magical

Modus Ponens (4) and (16)

(18) $\sim\text{Mythical}$

Disjunctive syllogism (5) and (13)

We cannot find a contradiction, which means $\text{KB} \wedge \neg\alpha$ is satisfiable. Thus, we cannot prove that the unicorn is mythical from the knowledge base

Assume the unicorn is not magical

(19) $\sim\text{Magical}$

Assumption

(20) $\sim\text{Horned}$

Disjunctive syllogism (10) and (19)

(21) $\sim\text{Mortal} \vee \text{Mammal}$

Resolution (5) and (7)

(22) Horned

Modus Ponens (3) and (21)

We have found the contradiction between (20) and (22), which means $\text{KB} \wedge \neg\alpha$ is unsatisfiable. Thus, we have proved that the unicorn is magical

Assume the unicorn is not horned

(23) $\sim\text{Horned}$

Assumption

(24) Mortal

Disjunctive syllogism (8) and (23)

(25) $\sim\text{Mammal}$

Disjunctive syllogism (9) and (23)

(26) Mythical

Disjunctive syllogism (7) and (25)

(27) $\sim\text{Mortal}$

Disjunctive syllogism (5) and (26)

We have found the contradiction between (24) and (27), which means $\text{KB} \wedge \neg\alpha$ is unsatisfiable. Thus, we have proved that the unicorn is horned

(3) Suppose the test comes back positive. What's the probability that oil is present?

$$P(\text{oil}) = 0.5$$

$$P(\text{gas}) = 0.2$$

$$P(\text{neither}) = 0.3$$

$$P(\text{positive} \mid \text{oil}) = 0.9$$

$$P(\text{positive} \mid \text{gas}) = 0.3$$

$$P(\text{positive} \mid \text{neither}) = 0.1$$

We want to have $P(\text{oil} \mid \text{positive})$

$$P(\text{positive})$$

$$= P(\text{oil}) * P(\text{positive} \mid \text{oil}) + P(\text{gas}) * P(\text{positive} \mid \text{gas}) + P(\text{neither}) * P(\text{positive} \mid \text{neither})$$

$$= 0.5 * 0.9 + 0.2 * 0.3 + 0.3 * 0.1 = 0.45 + 0.06 + 0.03$$

$$= 0.54$$

$$P(\text{oil} \mid \text{positive})$$

$$= P(\text{oil}) * P(\text{positive} \mid \text{oil}) / P(\text{positive})$$

$$= (0.5 * 0.9) / (0.54)$$

$$= 5/6$$

$$\approx 0.833$$