RepnDecomp

Decompose representations of finite groups into irreducibles

0.1

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Chapter 1

Block diagonalizing representations

1.1 Finding the correct basis

Given a representation $\rho : G \to GL(V)$, it is often desirable to find a basis for V that block diagonalizes each $\rho(g)$ with the block sizes being as small as possible.

1.1.1 BlockDiagonalBasis (for IsGroupHomomorphism)

▷ `BlockDiagonalBasis(rho)` (attribute)

Returns: Basis for V that block diagonalizes ρ.

Let G have irreducible representations ρ_i, with dimension d_i and multiplicity m_i. The basis returned by this operation gives each $\rho(g)$ as a block diagonal matrix which has m_i blocks of size $d_i \times d_i$ for each i.

1.1.2 BlockDiagonalRepresentation (for IsGroupHomomorphism)

▷ `BlockDiagonalRepresentation(rho)` (attribute)

Returns: Representation of G isomorphic to ρ where the images $\rho(g)$ are block diagonalized.

This is just a convenience operation that uses `BlockDiagonalBasis` (**??**) to calculate the basis change matrix and put ρ into a nice form.

Chapter 2

Calculating centralizer rings

2.1 Centralizer (commutant) of a representation

2.1.1 RepresentationCentralizerBlocks

▷ `RepresentationCentralizerBlocks(rho)` (function)

Returns: List of standard generators (as a vector space) for the centralizer ring of $\rho(G)$, written in the basis given by `BlockDiagonalBasis` (**??**). The matrices are given as a list of blocks.

Let G have irreducible representations ρ_i with multiplicities m_i. The centralizer has dimension $\sum_i m_i^2$ as a $\mathbb{C}$-vector space. This function gives the minimal number of generators required.

2.2 Useful convenience functions

2.2.1 RepresentationCentralizer

▷ `RepresentationCentralizer(rho)` (function)

Returns: List of standard generators (as a vector space) for the centralizer ring of $\rho(G)$.

This gives the same result as `RepresentationCentralizerBlocks` (2.1.1), but with the matrices given in their entirety: not as lists of blocks, but as full matrices.

2.2.2 RepresentationCentralizerDecomposed

▷ `RepresentationCentralizerDecomposed(rho)` (function)

Returns: List of generators (as a vector space) for the centralizer ring of $\rho(G)$, under the map taking each identity matrix block to a 1 by 1 block.

This function is here to demonstrate the reduction in dimension of the centralizer C by writing it in the basis given by `BlockDiagonalBasis` (**??**). The matrices given are as reduced as possible.

Chapter 3

Useful predicates

3.1 Types of group representations

3.1.1 IsFiniteGroupLinearRepresentation (for IsGroupHomomorphism)

▷ `IsFiniteGroupLinearRepresentation(`*rho*`)` (attribute)

Returns: true or false

Tells you if *rho* is a linear representation of a finite group. This is important since Serre's algorithms only work on these.

3.1.2 IsFiniteGroupPermutationRepresentation (for IsGroupHomomorphism)

▷ `IsFiniteGroupPermutationRepresentation(`*rho*`)` (attribute)

Returns: true or false

Tells you if *rho* is a homomorphism from finite group to a permutation group. Such homomorphisms occur often in applications.

Chapter 4

Computing decompositions of representations

4.1 Algorithms due to Serre

These operations compute various decompositions of a representation $\rho : G \to GL(V)$ where G is finite and V is a finite-dimensional $\mathbb{C}$-vector space. The terms used here are taken from Serre's Linear Representations of Finite Groups.

4.1.1 CanonicalDecomposition (for IsGroupHomomorphism)

▷ `CanonicalDecomposition(rho)` (attribute)

Returns: List of vector spaces V_i, each G-invariant and a direct sum of isomorphic irreducibles. That is, for each i, $V_i \cong \oplus_j W_i$ (as representations) where W_i is an irreducible G-invariant vector space.

Computes the canonical decomposition of V into $\oplus_i V_i$ using the formulas for projections $V \to V_i$ due to Serre.

4.1.2 IrreducibleDecomposition (for IsGroupHomomorphism)

▷ `IrreducibleDecomposition(rho)` (attribute)

Returns: List of vector spaces W_j such that $V = \oplus_j W_j$ and each W_j is an irreducible G-invariant vector space.

Computes the decomposition of V into irreducible subprepresentations.

4.1.3 IrreducibleDecompositionCollected (for IsGroupHomomorphism)

▷ `IrreducibleDecompositionCollected(rho)` (attribute)

Returns: List of lists V_i of vector spaces V_{ij} such that $V = \oplus_i \oplus_j V_{ij}$ and $V_{ik} \cong V_{il}$ for all i, k and l (as representations).

Computes the decomposition of V into irreducible subrepresentations, grouping together the isomorphic subrepresentations.

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