DEPARTMENT OF FINANCE

TECHNICAL UNIVERSITY OF MUNICH

Interdisciplinary Project in Finance and Informatics

Leads and Lags of Corporate Bonds and Stocks
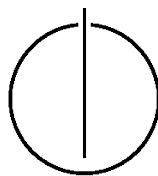
Leads und Lags von Unternehmensanleihen und Aktien

Author:	Alex Kulikov
Supervisor:	Prof. Dr. Sebastian Müller
Advisor:	Zihan Gong
Date:	March 31, 2021

Abstract

Contents

1. Introduction

In the scope of an Interdisciplinary Project (*IDP* in the following) at the Technical University of Munich, the lead and lag relationship between corporate bond and stock returns had to be analyzed. With the needed stock data already provided, the first step of the IDP was to develop a tool which would be able to automatically extract static and time series data from the Thomson Reuters Datastream financial database. In the second step of the IDP, the extracted bond data had to be cleaned and prepared for further analysis by various transformation techniques. Additionally, a matching approach to join the stock and bond datasets by issuing company had to be developed. In the third and final step of the IDP, the resulting bond-stock database had to be checked for any sort of lead and/or lag relationships within bond and stock return pairs.

1.1. Motivation

The main goal of this IDP is to investigate the relation between corporate bond returns and stock returns by analyzing the underlying historical and current financial data. In the first part of this IDP, an international, extensive corporate bond database will be built up for further research. The initial data will be taken from the financial market information provider Refinitiv. To this end, some code for automated data extraction in an appropriate programming language will be developed, tested, and applied to extract the data in the most convenient manner. The code should be flexible enough to be used for data updates and/or further data extensions. As financial market data tends to be error-prone, the received information might need to be scanned for outliers and cleaned in an appropriate way before conducting further analysis based on it. At this point, if there is some spare time in the schedule, the data received from the Refinitiv database may be compared to financial data extracted from a similar database provided by Bloomberg through a so-called Bloomberg Terminal. This step would allow for more clean and reliable corporate bond information as a starting point for the next project step. In the second part of this IDP, I will be provided with a comprehensive data sample of international stock returns. By merging this data with the acquired corporate bond dataset, the so-called lead-lag-relation between stock and bond returns will be tested by applying suited analysis techniques. This will allow us to identify which market is faster in incorporating new information. In addition to the data science part of the IDP, upon completion of this project, I will have acquired substantial knowledge about capital market databases, fixed income research, empirical data analysis, and the functioning of financial markets in general. Among others, these skills are of high practical relevance for jobs in Banking, Asset Management, and Fintech.

1.2. Methods

The present work is organized as following. At first, the necessary preliminaries with focus on the skyline operator are given. A brief overview of related work and the existing skyline algorithms takes place, with special emphasis on Naive- and Block-Nested-Loops, Divide-and-Conquer and ST-S algorithms. Hereafter, this paper briefly reviews the main advantages of the ART tree in the context of databases, and proposes a novel skyline algorithm called SARTS, which makes efficient use of the tree for dominance checks. The new algorithm and its implementation approaches are presented in greater detail. Afterwards, some of the modern parallelization techniques are first explained and then applied to the given skyline algorithms. At this point, two different query pipelining models are introduced: volcano and produce/consume. Then, an evaluation of this work's results takes place. For this purpose, various performance tests are conducted and their outcomes discussed. In the conclusion to this work, the main results and proposals of the paper are once again summarized, and an outlook to possible future work is given.

2. Data Extraction

In order to draw any conclusions regarding the relationship of bond and stock returns, the respective static and time series data needs to be acquired first. Since equity data is already available from the beginning of the IDP, the bond data is the only one which had to be acquired. For bond data extraction, the financial database product Datastream, provided by Thomson Reuters, can be used, since it is licensed for usage by TUM students and employees.

2.1. Download Solution

As Thomson Reuters has a wide range of products which can be used for different types of data, the first thing that needed to be done, was to determine the most suitable product to download both static and time series data for corporate bonds. After some time spent reading up and gathering information on the Thomson Reuters product portfolio, it became apparent that some of the products, such as the TR Python API, are only suitable for equity data download, and not for corporate bonds, and only have a very limited number of parameters available for download. On the other hand, it was found that other Thomson Reuters products, which would normally be suitable for automated download of bond data, such as e.g. DataScope Select (DSS) or Thomson Reuters Tick History (TRTH), are not included in the existing academic license. Other products – noticeably the Datastream Web Service (DSWS) API, which is most suited for such requests – are generally not available for academic clients.

  These findings were a significant setback for the bond data extraction, since the only option left to acquire large amounts of corporate bond data, was over the Datastream Add-In for Microsoft Excel. While this add-in is rather convenient for small-scale manual requests with the help of so-called request tables, it is not optimized for large data extraction queries. It does not provide an API for customizable requests. Instead, communication with the Datastream server is handled over a single API call available in VBA. This one and only callable function is implemented in C++, and can only be invoked in a black-box manner, since the provider does not give out its implementation. This leads to only one possible solution to automatically extract corporate bond data from Datastream. It can be described with the following steps:

1. Acquire Datastream codes / identifiers for all financial instruments which need to be downloaded.

2. Split these identifiers into batches small enough to be processed in a single Datastream request.

3. Fill a request table with as many requests as needed to include all the batches.

4. Launch the Datastream requests for all the batches one after the other.

5. Monitor the download process to ensure that the data is being consistently downloaded.

The programmatic development of the download tool will be based exactly on these five steps. The last step (monitoring the execution) is especially crucial and complex to implement. The reason for this is, as previously mentioned, that the Datastream Excel Add-In is not well-fit for large data downloads. Therefore, the following problems continuously arise during the download process:

- Datastream add-in eventually signs out for no obvious reason.

- Data download hangs, without any notification stating the reason or the hanging fact itself.

- Excel suspends the add-in and places it into a blacklist for repeated faulty behavior.

Since VBA is single threaded and cannot detect or react to erroneous behavior when the download is running, it is impossible to do the monitoring in the VBA/Excel environment. For this purpose, a Python wrapper was developed as will be explained in .

2.2. Bond Identifiers Acquisition

Since for both static and time series requests Datastream requires unique financial instrument codes to be provided, it is first necessary to obtain a list of identifiers for the securities for which the data needs to be downloaded. The most commonly used unique security identifier in Datastream is the so-called Datastream Code (short *dscd*). In the scope of the project two different approaches have been developed for this task, and will be introduced in the following.

2.2.1. Programmatic Identifier Extraction

For the purpose of this work, we are interested in corporate bonds from all possible jurisdictions, and with any possible coupon an currency parameters. The only restriction is that we only concentrate on the issue date range between Dec 31, 1999 and June 30, 2020 (date of extraction).

Datastream allows to filter its financial security dataset by these parameters, e.g. when clicking on *Find Series* in a request table. After the securities have been filtered for the desired corporate bonds, these can be selected by repeatedly checking the box to select all bonds on the current page, and then clicking on *Next* to switch to next page. This is due to Datastream not providing an option to select all filtered securities at once if there are more than 4,000. Hence, if there are for instance 60,000 corporate bonds in the database in total, one would not be able to select them all at once. Instead, one would have to select the 15 bonds on the current page and then switch to next page $60,000/15 = 4,000$ times. This is of course very cumbersome for the user, and the repeated clicking sounds like a good process to automate programmatically.

There are multiple tools and scripting languages which enable fast and easy click automation. Specifically for this project I decided to go with Python 3 for this purpose, since it was already part of the environment. One of the packages which enable GUI automation in Python is *pyautogui*. With build-in methods like $click()$ and $hotkey()$ it enables the user to simulate mouse clicks on the computer screen by giving the functions the screen coordinates of the buttons. Placing the commands into a loop in the right order makes it possible to simulate the entire process of selecting corporate bonds in Datastream. For a possible Python implementation see Note that the screen coordinates can significantly differ depending on the screen resolution and window settings.

While the described approach solves the problem of selecting all the needed corporate bonds from Datastream, there are two downsides to it. The first one is that it is cumbersome for the developer to determine and to enter the screen coordinates of all the buttons involved. The second is that, even when fully automated, the tool needs a lot of time to select and return all of the chosen securities if there are many of them. At this point, the second approach, even though it is manual, is both faster and easier to apply.

2.2.2. Manual Identifier Extraction

To extract the needed corporate bond identifiers manually, we can make use of the fact that Datastream allows to select all filtered securities at once when there are less than 4,000. Because of this, we can simply split our entire data into multiple chunks that are all smaller than 4,000 bonds in total. This can be done by selecting one or more parameters (the number depends on the size of the dataset) according to which the bonds can be filtered even further. For example, an entire bond dataset with 60,000 bonds in total can be split by coupon size first, and then additionally by currency to produce bond batches of maximum 4,000 bonds each. For a visualization of this approach, see Fig. If the split has been done properly, it will include all of the desired securities, since each bond belongs to one particular coupon size as well as currency group. In most cases, there will be only few groups that need to be extracted from the interface. In the case of 60,000 bonds, one would only have $60,000/4,000 = 15$ groups in total. In reality, for this concrete use case, 19 different bond groups had to be created. This is because not all splits are perfect, and some of them just consist of 3,500 bonds instead of 4,000 for example. After the splitting work has been done, the resulting bond groups can be extracted manually with just a few clicks.

2.3. Automating the Request Table

After the bond identifiers in form of *dscd* codes have been extracted, they can be used to retrieve both static and time series data from Datastream. For this purpose, I wrote a VBA program which fully automates the download process. The only input needed from the user is the *dscd* identifiers, the desired variable codes (such as price, issued volume, etc.) as well as the desired time frames in the case of a time series request. Since the program code is rather complex, I will only cover the main approach briefly. A more in-depth description can be found in the appendix to this work.

At the beginning, the VBA tool retrieves the user-provided identifiers and datatypes

from the respective Excel files. Then, based on the input size, multiple calculations take place. For static information requests not much needs to be done, since these are usually relatively small and only depend on the number of securities for which the data is requested. For time series requests though, the tool estimates how large the entire request will get, depending on dates window, frequency of time points (e.g. daily or quarterly), the number of datatypes, and the number of identifiers. If the request is too large to be processed by Datastream in one run, it gets split into multiple smaller requests of equal size. Since there is no particular metric to estimate in advance whether a particular request will be executed by Datastream, or whether it is too large for that, the tool only computes an approximation based on an empirically measured *Bytes per Field* metric. The single requests then get entered into the request table one below the other, and each receive an own Excel file as destination to store the data.

As soon as the request table has been filled (which does not take long), the command to process the first request is issued to Datastream. This happens via a call to the single available function, which tells Datastream to process the current request table in a black-box manner. After the request ends, the tool checks whether the requested data has arrived to the destination file. If not, it checks the connection of the Datastream add-in and issues a warning to the user if the add-in unexpectedly disconnected. In both cases, the result of the request is logged, in order for the user to be able to read up on the proceedings later. To prevent the computer from sleeping or going in idle mode, the tool moves the computer mouse pointer after each request with a dedicated VBA function. When the first request of the request table has been processed, the other ones get executed in the same manner one after the other. Note that while it is possible to submit the execution for all requests at once, it is not advisable, since Datastream might issue an error due to the data being too large, or might otherwise simply hang during execution. This is exactly the reason why we had to split up the original request in multiple parts in the first place.

While the entire Datastream Extraction Tool is much more complex than what has been described here, the given explanation covers the most crucial parts of the download process. At this point, note that the error monitoring step, which was previously mentioned as essential, cannot be completed in VBA due to its single-threaded execution engine. Section 2.6 will cover the required workaround for this functionality.

2.4. User Interface

In order to provide a graphical user interface as well as an error monitoring capacity (section 2.6) for the created VBA tool, a Python 3 wrapper program has been created. It's architecture can be seen in Fig. 2.1. At this point, note that a detailed usage manual for the Datastream Extraction Tool – including its user interface – can be found in the appendix to this work.

As shown in the visualization, the Python program has a *GUI* component, which runs on its main thread (Thread 1). The layout of the user interface is depicted in Fig. 2.2.

Its features include:

- Request type selection (static or time series)

- Time frames and frequency for time series requests
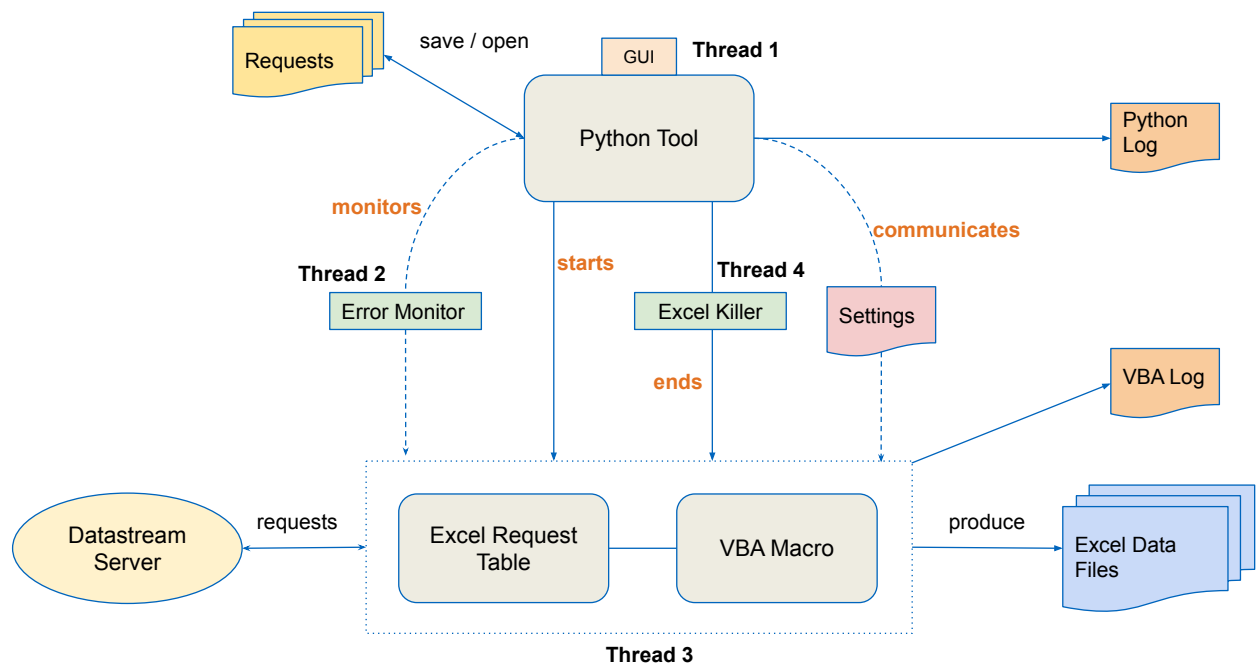
Figure 2.1.: Architecture of the Datastream Extraction Tool

- Request datatypes, which can be entered either in a manual list or via an Excel file

- Data identifiers, which can also be entered either manually or via Excel file

- Request format with one or more Datastream request options (e.g. header row, currency, etc.)

- Destination choice for downloaded data

- Saving and reusing frequently entered requests

The user interface has been programmed using the *Tkinter*[1] package, which enables rudimentary container-based gui creation.

2.5. Other Functionality

Besides the user interface, the Python wrapper offers a logging facility for the actions taken, as well as a functionality to exchange settings and messages with the VBA component. For the latter purpose, a *settings.txt* file is provided which encompasses user-provided request details to forward the request to the VBA program. The entries are made in a key-value manner, which enables a fast and easy information exchange between the Python and the VBA parts. Besides request forwarding, the settings file is used by the Python tool to receive regular updates from the VBA on the current download status. This

---

[1] https://docs.python.org/3/library/tkinter.html
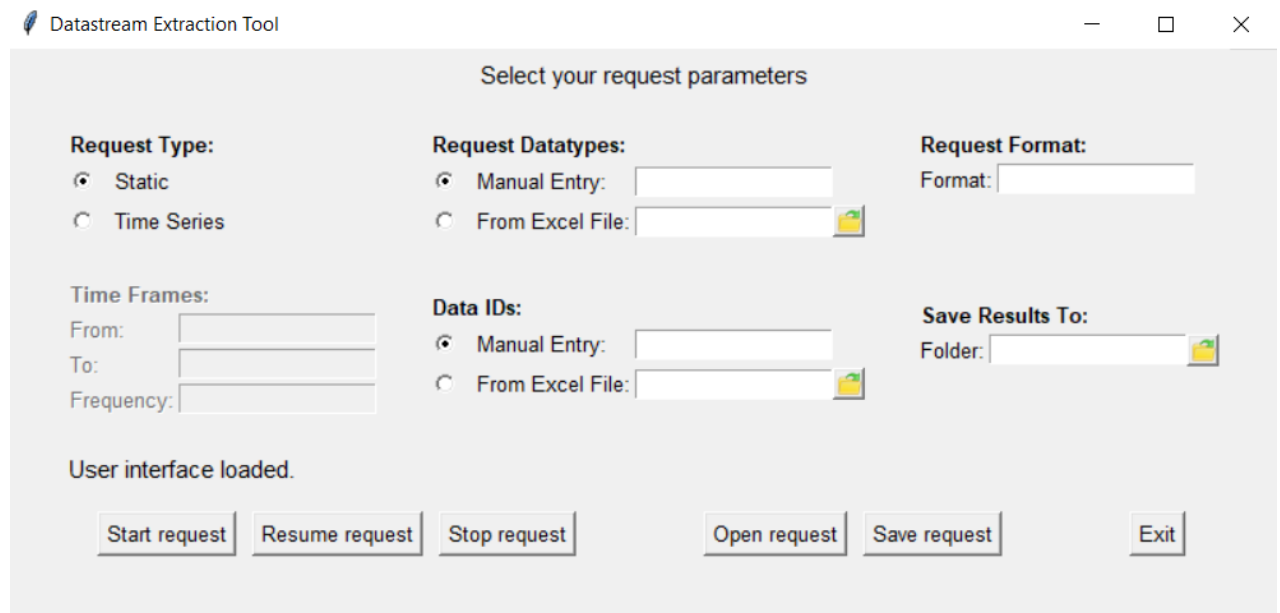
Figure 2.2.: Graphical User Interface of the Datastream Extraction Tool

information is used for both status updates within the gui as well as for the error monitoring functionality which will be explained in section 2.6.

2.6. Error Monitor

As shown in the visualization, the *Error Monitor* module runs in a separate thread (Thread 2), since Python allows execution on multiple threads simultaneously. It is started from the main thread, which controls the graphical user interface, whenever a new download request (Thread 3) has been issued to Datastream. The module maintains an internal counter which signifies the waiting time for the current request in Datastream to process. The counter gets increased each time the Python tool notices that the currently processed request in VBA has not yet changed to the next one. VBA, in its turn, keeps posting updates on the number of the request that is currently being downloaded to the settings file. The ping frequency of the error monitor can be adjusted to the user needs, and is currently set to 1 minute cycle time.

Whenever the counter of the error monitor reaches a programmer-defined threshold (currently 25 minutes), another component of the Python wrapper, the *Excel Killer* (Thread 4) comes into play. It issues an operating-system-level command to (violently) terminate the currently running Excel process. The reason why this has to be done violently is that Excel becomes entirely non-responsive whenever the Datastream download is hanging. Therefore, asking Excel to exit nicely does not result in Excel shutting down. The terminate request needs to run on a separate thread so as to not interfere with the Python gui and status updater.

After the current Excel process and thus also the running Datastream download have been shut down, the Python tool restarts the download request from the same point where

it finished its download before it started hanging. In other words, the download request gets automatically resumed.

Another functionality which the error monitor provides is the ability to detect when all needed data has been downloaded. This happens in a manner similar to the error monitoring itself. The tool simply reads in the number of the last executed download request from the settings file. Based on this information and on the total number of requests to be executed, it determines when the last data batch has been downloaded and ends the download. A corresponding status message is delivered to the user interface to notify the user that the download has finished.

The entire Datastream Extraction Tool consisting of the VBA and Python parts, and including the required folder structure, is currently hosted on Google Drive under `https: //bit.ly/3rB3lqg` -¿ *Datastream Extraction Tool*.

3. Data Preparation

As it is often the case with data science projects, the data preparation part is rather tedious. After the static and time series bond data has been extracted from Datastream – which will likely take several days – it is available in form of multiple data parts in Excel format. Since it is more convenient to perform further statistical analysis in a dedicated statistical environment, such as Stata or MatLab, the data needs to be brought into 'long' format, and additionally to be cleaned from null entries and outliers. The undertaken procedures are described in the following.

3.1. Data Formatting

For the static bond data, there is not much to be done in terms of formatting. Its original format, as downloaded from Datastream, is mostly suitable for further analysis and can be directly imported into Stata.

The downloaded time series data is initially in 'wide' format and has multiple bonds in one row. It looks like shown in Fig. 3.1.

Start	31.12.1999														
End	30.06.2020														
Frequency	D														
Name	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA	LEGRAND SA
Code	233CX0(AC)	233CX0(YA)	233CX0(LF)	233CX0(MV)	233CX0(DM)	233CX0(CMP)	233CX0(MPD)	233CX0(CP)	233CX0(GP)	233CX0(RI)	233CX0(IY)	233CX0(RY)	6063KG(AC)	6063KG(YA)	6063KG(LF)
CURRENCY	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
31.12.1999	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
03.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
04.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
05.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
06.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
07.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
10.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
11.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
12.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
13.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
14.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
17.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
18.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
19.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
20.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL

Figure 3.1.: Sample of downloaded raw time series bond data

The goal is to transform this time series data into 'long' format, as can be seen in Fig. 3.2, by saving the bonds one below the other. Additionally, the header has to be removed, and the *dscd* identifier of each bond as well as its currency have to be entered as a separate column for each date instead of being at the top.

I wrote a VBA macro with a function called *ToLongFormat()* which accomplishes the described task. While Stata might have also been able to do the formatting, I decided to work with VBA at this point, since it is native to the MS Excel environment. While the entire code can be found in the appendix, the main procedure is as follows:

1. Define data layout constants depending on the initial format: header height, number of time stamps, number of datatypes, bonds per block, and number of blocks. A

Date	AC	YA	LF	MV	DM	CP	CMPM	MPD	GP	RI	IY	RY	Code(dscd)	Currency
31.12.1999	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
03.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
04.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
05.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
06.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
07.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
10.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
11.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
12.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
13.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
14.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
17.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
18.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
19.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E
20.01.2000	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	6086YF	E

Figure 3.2.: Sample of downloaded time series bond data in 'long' format

block is defined as all time stamps for multiple bonds which are located row-wise next to each other. An example can be seen in Fig. 3.1 where two bonds from the same firm, but with different *dscd* codes, are next to each other. There can be multiple such blocks one below the other in one Excel file, depending on how the data was downloaded.

2. For all data files (as there will be multiple for larger requests) and for all blocks within each file, remove the header rows, place bonds one below the other, and create columns for *dscd* and currency.

3. Add a newly created header row once at the beginning of the file. This header row can later be used to define variable names in statistical software.

Note that if the original Excel files were very large, i.e. with a high amount of securities or dates, Excel might reach its sheet length limit when running this macro. If you notice such behavior, there is another function shipped with this macro, called *SplitInSubfiles()*. You can use this function on your initial downloaded Excel data to reshape it into smaller-sized files, before transforming it to 'long' format.

3.2. Stata Import

As soon as the data has been formatted, it can be cleaned conveniently within statistical software. Since I am working with Stata, the Excel files with static as well as reshaped time series data can be simply imported to Stata with the *import excel* Stata command. It is possible to save frequently used Stata scripts in form of *.do* files to reuse them later. You can find all the do-files in the appendix attached to this project. The file do_import_excel can be found in .

For the static bond data, it merely needs to be checked for duplicates, e.g. by using the ... Stata command. Empirically, the static bond data extracted from Datastream is significantly cleaner than historical pricing data. Therefore, all the following cleaning procedures only need to be applied to time series data.

Because Stata can generally work with larger files than Excel, it makes sense to merge the imported time series files – now already in Stata format – to files of larger size. For this purpose, the do file ... can be used. After the data has been imported to Stata, it can now be cleaned with the help of standard Stata procedures.

3.3. Data Cleaning

For cleaning, multiple different procedures need to be applied. Since they are all commutative, it does not make a difference in which order these are executed.

- Null values can be cleaned with the *drop* command, e.g. with drop if MPD=="NULL" | MPD=="" (). Be prepared for a lot of values to be deleted when working with historical bond data. This is due to many bonds having been issued recently and thus not having older price entries.

- Erroneous values which sometimes occur in Datastream typically have high length. Remove these with e.g. drop if length(var) > 40, with var being the variable names ().

- Cast date stamps from string to date format. This can be done by generating a new variable for the date first (gen date = date(Date, "MDY")). Then the new variable should be formatted to be well-readable (format date %tdnn/dd/CCYY). After that the old variable *Date* can be dropped, so that the newly created *date* takes its place.

- Cast integer and double values that are coded as strings back to numeric, e.g. with the destring command ().

- Remove duplicates based on the variables *date* and *dscd*. There should not be two or more different entries for the same security on the same date.

Keep in mind to manually check the resulting data. No matter how thorough the cleaning procedure, there might still be some erroneous data which needs to be cleaned up or removed manually. Besides the listed cleaning methods, the data should additionally be searched for outliers that can have a negative impact on the further analysis. However, different values and tuples can be considered outliers depending on the analysis scenario. Therefore, I decided to leave the (not necessarily erroneous) outliers in the dataset at this step of the project. They will be filtered out as shown in chapter ... later on.

After the data has been cleaned, many tuples will have been deleted. To make further analysis more convenient, it therefore makes sense to merge all the single data files into one. While depending on the size of the entire dataset, this should should be possible in most cases. Otherwise e.g. two or three files can be produced in total. Files can be merged easily in Stata, e.g. by using the append command (). The resulting cleaned and compact data is now ready for future statistical analysis.

4. Matching

In order to analyze the lead-lag relationship of corporate bonds and stocks, we need a large, survivorship bias free database with both stock and bond returns for any given point in time. So far, we only have two separate databases – one with historical corporate bonds data, and one with historical equity data. Therefore, the two databases have to be joined into one, based on the company that issued both. The task is not as trivial as it might seem, since there is no unique company identifier available in both databases. In the following, the available matching options will be discussed, and the most suitable approach chosen.

4.1. Available Options

To begin with, the following extracted bond and equity parameters were considered for the matching:

- SEDOL code
- WKN code
- CUSIP-9 code
- ISIN code
- Worldscope identifier
- Company name

4.1.1. SEDOL

The SEDOL is a unique 7-character identification code which stands for 'Stock Exchange Daily Official List'. It is issued for securities registered in the United Kingdom and Ireland by the London Stock Exchange. Despite being used to uniquely identify securities, it does not, in general, contain a unique issuing company identifier, because the codes are simply issued sequentially. For example, two bonds, which were both issued by Apple Inc., can have the SEDOL codes *BF43J24* and *BK9WPP6*, respectively. The only similarity between the two is that these were issued only two years apart, and thus have the *B* at the beginning in common. Besides, the identifier is not available in our stocks database, and only exists for securities of companies listed on the LSE.

4.1.2. WKN

The WKN is a German 6-digit alphanumeric security identification code and stands for 'Wertpapierkennnummer'. Since 2004, it is possible for companies to obtain a WKN with a

unique company identifier included. A WKN includes a company identifier if it starts with at least two characters before proceeding with digits. However, not all companies make use of this opportunity when ordering a WKN for their securities. Taking into account that there are also multiple exceptions from the rule base of WKN identifiers, it is hard to use these as unique company identifiers. This is especially the case because WKN are generally only available for German securities. Also, the parameter is not available in our existing equities database.

4.1.3. CUSIP-9

The CUSIP number is a unique identification number assigned to all equities and bonds that are registered in the United States and Canada. The CUSIP consists of 9 alphanumeric characters, of which the first 6 comprise the unique issuing company identifier. The code is often used in one of its shorter forms, i.e. as CUSIP-8 and CUSIP-6. However, in our case, only the CUSIP-6 variant is of interest. It can be derived from CUSIP-9 by simply dropping the last three characters. The CUSIP-9 code is directly available in our equities database. In the bonds database, it can only be found directly for some of the securities in the so-called *local code* variable (LOC), which can be found in Datastream. Unfortunately, the CUSIP values entered in this variable are not very reliable. A workaround can be achieved by using the security ISIN, as will be explained in 4.3.

4.1.4. ISIN

The ISIN stands for 'International Securities Identification Number' and is an international standard way to uniquely identify securities. The ISIN by itself is not a unique company identifier. However, it sometimes contains a company identifier as part of it. In particular, for U.S. and Canadian securities, the ISIN usually contains the Cusip-9 code, which, in its turn, contains a 6-digit unique company identifier. For U.K. and Irish securities, the ISIN usually contains the SEDOL code. And for German securities, the ISIN contains the WKN. Therefore, while the ISIN itself cannot be directly used for the matching, it can nevertheless be used to obtain missing matching code values by extracting them from the ISIN.

4.1.5. Worldscope identifier

The Worldscope Identifier is a 9-digit code issued by Worldscope, a Thomson Reuters' fundamentals product. It is used to uniquely identify both issuing companies and securities. For U.S. companies, the Worldscope Identifier is identical with the CUSIP-9 code. For non-U.S. companies, a derived identifier is used, based on the country where the issuing company is domiciled, and also includes a unique company code. A more detailed explanation of the mechanics can be found in the Datastream database or the appendix to this work. Unfortunately, the Worldscope Identifier is only available in the equities database, and not for bonds. Therefore, it cannot be used for the matching.

4.1.6. Company name

As the 'method of last resort', the company name itself can be used to join the bond and equity databases. The problem with company names as identifiers is though that these are

not necessarily unique on the one hand, and also tend to have heterogeneous spelling – i.e. one and the same company can be spelled in multiple different ways across the database. To provide an example, the company names *THE WILLIAMS COMPANIES INCO* and *WILLIAMS PARTNERS L.P.* refer to the same company, but are written differently, which makes the join between the two databases ambiguous. Nevertheless, the approach can be a good starting point when other options are not available, and will be introduced in greater detail in the next section.

4.2. Fuzzy String Matching

Having considered the different options to perform the matching, it becomes apparent that the only unique identifier which can be reliably used for the task is the CUSIP-9 code. Additionally, the company name can be used to produce a solid baseline to start with. In this section, a matching approach called Fuzzy String Matching will be introduced as a means to join the datasets via company name. In the next section, a matching approach based on the CUSIP code will be explained.

The term Fuzzy String Matching – also called Approximate String Matching – refers to use cases when there are two or more strings which have the same meaning, but are spelled somewhat differently. There exist multiple approaches to measure the extent of 'difference' between strings. The formula which is most commonly used is the so-called Levenshtein distance. It measures the minimum number of single-character edits required to change one given sequence into the other. An *edit*, in its turn, is defined as one of the three operations performed on a string:

- insertion

- deletion

- substitution

Depending on the implementation, a substitution of a character can count as either one or two edits. This is because a substitution technically consists of both an insertion and a deletion. For simplicity reasons it can be assumed to count as one edit just like the other two operations. To give an example, consider a company that is called *Apple Inc.* in one dataset and *Apple Incorp.* in the other. The Levenshtein distance between the two company names would be 3, because exactly 3 insertions need to be performed in order to produce *Apple Incorp.* from *Apple Inc.* These insertions are the 3 characters *o*, *r* and *p*. Alternatively, we can also start the other way around, from the company name *Apple Incorp.* In this case, we would need 3 deletions to produce *Apple Inc.* In particular, we would have to delete the 3 characters *o*, *r* and *p*.

While there exists a concrete formula which defines the Levenshtein distance, it is not very relevant in this context, since we will not be implementing the Levenshtein distance ourselves. Instead, we will make use of dedicated Python packages, which compute the Levenshtein distance between two strings behind the scenes in order to produce a similarity score. In this work, we consider the two packages *fuzzywuzzy*[1] and *rapidfuzz*[2] for

---

[1]https://pypi.org/project/fuzzywuzzy
[2]https://pypi.org/project/rapidfuzz

the task. In reality, these packages do slightly more than just computing the edit distance, depending on the particular function called. A detailed description of these packages' capabilities can be found in their respective documentation.

The concept will be used in our case to select for each company name from the bonds database the one from the equities database which has the smallest Levenshtein distance to it. This way, matching company names from the two datasets will be connected to each other to produce a join.

4.2.1. Fuzzy-Wuzzy

Fuzzywuzzy is the most commonly used package for fuzzy string matching in the Python community. Therefore, my first approach to perform the matching was with the fuzzy-wuzzy package. It requires additionally the package *python-Levenshtein* to be installed, so it can use its faster C implementation of the Levenshtein distance. The rest of the *fuzzy-wuzzy* package is programmed in Python.

To start with the implementation, the static bond and equity data needs to be read into *pandas*[3] dataframes to perform further operations on it. For this purpose, it is advisable to export the static data from Stata to the CSV[4] format, and then to import it in Python from CSV files. I explicitly discourage exporting data from Excel to CSV, because Excel has its own understanding of the CSV format, which might not be compatible with *pandas*.

After the data has been loaded into dataframes, one for bond company names, and one for stock company names, it can be fed into one of the predefined *fuzzywuzzy* functions. To start simple, two company names can be compared to each other with the built-in function ratio(), which computes the standard Levenshtein distance similarity ratio between the two sequences. Note that this function also takes into account whether the characters are capitalized or not. Thus, *Apple Inc.* and *apple Inc.* would produce a similarity ratio lower than 100% due to the difference in the capitalization of the first letter. This is a not very desired behavior for our use case, because we do not care much whether a company's name is written in upper or lower case letters. There exist modifications to this function, such as partial_ratio(), which can also detect similarities within substrings, or token_sort_ratio, which can detect similarities between substrings which are differently positioned. The function which is best-fitted to our use case is extractOne(). For each bond company name, it evaluates the Levenshtein similarity score with all stock company names. The one with the highest similarity score is considered a match.

In practice, this approach can be implemented with two nested for-loops, which is not very efficient, but necessary, because we have no index structure on our datasets. The resulting algorithm is thus somewhat similar to a standard Nested-Loop-Join. Faster approaches for unstructured data like a Sort-Merge-Join or a Hash-Join would not work, since the company names are not unique. For the resulting matching, it suffices to store the *dscd* pairs of the bonds and equities which were determined to be most likely join partners. By the *dscd* codes, any other static or time series data can be joined in later, because

---

[3]https://pandas.pydata.org

[4]CSV means comma-separated-values and is a frequently used data storage format. In the first row of a CSV file there are usually column headers, all separated by commas. In all further rows the single column values are stored, also separated by commas. The format is frequently used in data science applications due it being lightweight and information-dense.

the Datastream code is unique security identifier and present in all tuples from both static and time series databases. Keep in mind that if the Datastream code is called *dscd* for both stocks and bonds, you will first have to rename it in e.g. *bond_dscd* and *stock_dscd* first to avoid ambiguity. The results of the matching can be exported from a *pandas* dataframe to CSV format again. This CSV file can then be imported e.g. into Stata for further processing.

Despite its convenience of rapid prototyping, the *fuzzywuzzy* package has turned out to be rather slow in practice. Based on time measurements for 1,000 bonds, it would need around 150 hours of computation time to complete the matching, if scaled up to all the bonds in our database. On the one hand, it is not surprising, since we have around 60,000 bonds and 80,000 equities in our respective datasets, and are running a nested loop join of a sort on them. On the other hand, a faster approach would be preferred to avoid long waiting times. A better approach to this task is provided by the *rapidfuzz* package.

4.2.2. Rapidfuzz

Rapidfuzz is a (rather unknown) Python package which takes *fuzzywuzzy* as a base, and improves it by not only implementing the Levenshtein distance in C, but also the rest of the package in C++. This and also some algorithmic improvements make it significantly faster than original *fuzzywuzzy*. It has a somewhat more complex interface, but provides a very noticeable boost to the matching program. The general approach is very similar to matching with *fuzzywuzzy*: The static data is imported to pandas dataframes in CSV format and gets joined with some of the functions the package provides. Since *rapidfuzz* is based on *fuzzywuzzy*, the author has kept some of the function definitions similar to the original package. Therefore, extractOne() is still the best-fitting matching function for our use case, because it returns the most similar matching partner to the given bond company name.

To additionally improve the matching quality, another optimization should be performed on both bond and stock company names before the matching is done. To reduce spelling differences à-priori, all company names should be cast to lower case, and all punctuation signs and special symbols should be removed beforehand. This will make sure that when the data is fed into the matching algorithm, it will only need to compare the similarity based on the semantics of the sequences, without taking into account 'unnecessary' symbols. The resulting performance is significantly improved compared to the *fuzzywuzzy* approach. The updated matching algorithm only needs around 7 hours to compute the matching, compared to the 150 hours from before, which is a running time reduction of 95%.

Just like before, the results should be stored in dataframe format first, and then exported to CSV for future needs. It is sensible to not only export *dscd* pairs, but also the similarity score between the two. This makes it possible to cut off insufficiently matched data for certain analyses depending on the needs. I set the original score cut-off at runtime to 90% similarity to avoid having many false positives. More on this in section 4.4. The entire matching program code can be found in the appendix.

4.3. CUSIP Matching

Having accomplished a baseline matching with the Fuzzy String Matching approach, it can be further refined by using the CUSIP-9 identifier. As mentioned in section 4.1, the identifier is already available in the equities database. However, in the bonds dataset, it is only given as part of the *local code* (LOC), where it is not reliably entered, and thus cannot be used for matching. At this point, we can make use of the fact that the ISIN of U.S. and Canadian securities includes the CUSIP-9 identifier as part of it. This means that in order to obtain the CUSIP code for the bonds we simply have to take the alphanumeric characters 3 to 11 of the ISIN. Taking into account that we are actually interested in the CUSIP-6 instead of CUSIP-9, because only its first 6 digits represent a unique company identifier, it is enough to take the characters 3 to 8 of the ISIN.

As there is no complex data science involved here, the procedure can be done directly in Stata. At first, the static bond and equity data needs to be loaded into Stata, if not already done so during data preparation as explained in section 3. Since the CUSIP identifier can only be used to match U.S. and Canadian securities, both bonds and equities need to be filtered by these two countries. The rest of the countries can be dropped from the dataset for this purpose. Remember to always work on a local copy of the original dataset so as to not irreversibly lose data. The next step is to generate new variables for the CUSIP-6 identifier. For equities, this can be done with the Stata command gen cusip_6 = substr(cusip_9, 1, 6). For bonds – with gen cusip_6 = substr(ISIN, 3, 6). Finally, the two datasets need to be merged. This can be done with the merge command on a N:M relation. Herewith, the matching over the CUSIP-9 identifier is accomplished and the results can be saved.

4.4. Evaluation

The results of the two matching approaches are as following:

- With Fuzzy String Matching, around 26,032 bonds have found an equity matching partner. This equals appx. 42.89% of the total 60,688 bonds.

- With the CUSIP-9 approach, 6,460 (North American) bonds have found an equity match. This equals appx. 10.64% of the total 60,688 bonds.

While the Fuzzy String Matching approach has a significantly higher matching ratio, one should keep in mind that the CUSIP-9 results are more reliable, because it is a unique key attribute and not a fuzzy one. For further analysis, it makes sense to merge the two matching tables to come up with one single matching database of highest possible accuracy. For this, we can use the *append* command from Stata to simply concatenate the two datasets. Remember to make sure that both datasets have the same variables and same variable names in order for the union to work. For example, one might need to create an empty similarity_score variable in the CUSIP matching first, because it is present in the fuzzy string matching. As soon as the matching tables have been concatenated, the resulting dataset will contain duplicates in respect to bond_dscd and stock_dscd parameters. You can check this by running the command duplicates report bond_dscd stock_dscd. This is due to the two matchings having overlaps. While the duplicates should be removed, it is important

to keep the CUSIP matching pair and not the fuzzy matching one for each encountered duplicate. This is because for the CUSIP matching we can be 100% sure that it is accurate. The information that a bond and an equity are perfect matches, and not just based on a similarity score of e.g. 92%, might be useful in later analyses. After the duplicates have been removed, the total matching ratio increases from 42.89% (26,032 bonds with fuzzy matching only) to 44.91% (28,254 bonds with both approaches combined). While the improvement is only minor, keep in mind that some of the matching pairs are now more reliable than with fuzzy string matching alone. It is hard to give an concise measure of how many matches are perfect matches without manually checking all of them. But since the similarity score was already over 90% for the fuzzy string matching, my estimation is that around 40% of the pairs are perfect matches.

5. Statistical Analysis

Having prepared the bond data and matched it with the stocks, some statistical analysis can now be performed. In particular, we are interested in summary statistics of corporate bond returns, as well as in the analysis of the lead-lag relationship of corporate bond and stock returns. For this, the returns themselves need to be calculated from daily prices first. Additionally, as the matching has so far only been done for static data, it needs to be extended to historical return data as well. All in all, the following procedure for the statistical analysis arises:

1. calculate monthly bond returns

2. match bond and equity returns

3. calculate summary statistics

4. analyze lead-lag relationship

5.1. Monthly Bond Returns

In order to calculate the monthly bond returns, the following formula will be used:
$$R_n = \frac{P_n}{P_{n-1}},$$
where R_n stands for bond return for month n, and P_n for the bond price on last day of month n. To implement the formula in Stata, in the time series bond data only the last price of each bond for each month needs to be kept. The rest of the data can be dropped, as it is not relevant for the current analysis. The pricing parameter which will be used to calculate the returns is *MPD*. It represents the Datastream Selected Default Price, and is the most reliable price parameter in the extracted database. Other price parameters, such as e.g. *CP* (Clean Price), have significantly more missing values and are thus less suited for the purpose. Having kept the last monthly prices, a variable group consisting of the variables *dscd* and *month* has to be generated (*egen group* command). This group can then be set as a Stata time-series (*tsset*). Based on the time-series, the monthly return variable can be generated with the introduced formula. The Stata do file for the task can be found in.

5.2. Matching Bond and Equity Returns

To match the bond and equity returns with each other, we can make use of the matching that we accomplished in chapter 4. For this, we load the corporate bond returns, which we just calculated, into Stata. These bond returns can then be merged with the existing matching over the $bond_dscd$ parameter, as shown in Fig. 5.1. After that, the resulting dataset has
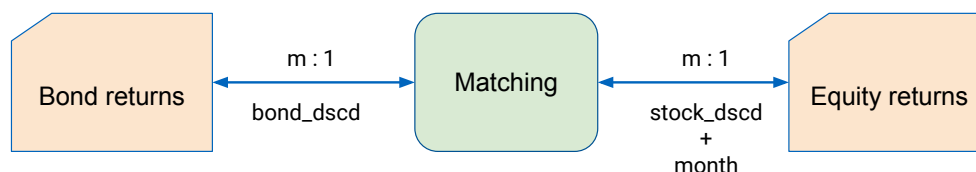
Figure 5.1.: Matching of Bond and Equity Returns

to be merged with the provided stock time series data, which also includes the required monthly stock returns. This merge should take place over the two parameters $stock\_dscd$ and $month$. This is because we want the bond and stock returns to be comparable for one and the same month later on. When executing the merges in Stata, keep in mind to make sure that the Datastream code parameter is named the same in the bond dataset and the matching (i.e. $bond\_dscd$), and similarly for the stocks and the matching (i.e. $stock\_dscd$). Otherwise, the matching will fail. The bonds-matching merge is performed on a M-to-1 relation, because multiple bonds can be mapped to the same equity over the issuing company. The merge with the equities is performed on a M-to-1 relation as well for the same reason. The resulting database builds the foundation for further statistical analysis.

5.3. Summary Statistics

Stata offers very useful tools to determine the underlying distribution of the data, as well as to perform correlation and regression calculations. In order to obtain summary statistics separately for different bond groups, I split the bond returns dataset by geography and by credit rating.

Group	World	USA / CA	EUW	EUE	South America	China
Mean (%)	0.03%	0.06%	0.01%	0.00%	0.00%	-0.02%
Std. deviation (%)	2.82%	2.96%	2.72%	4.82%	3.68%	1.06%
Correlation with equity returns	0.1892	0.2298	0.1851	0.1010	0.1213	0.0290
Correlation with equity returns (1-m. lead)	0.0460	0.0557	0.0323	0.0494	0.0859	-0.0451
Correlation with equity returns (1-m. lag)	0.0404	0.0515	0.0366	0.0029	0.0397	0.0336
No. bond-month observations	683,086	403,967	99,928	4,837	24,258	29,202

Figure 5.2.: Monthly Return Statistics by Bond Geography

Fig. 5.2 features monthly return statistics for:

- The entire world

- North America region (NA), represented by USA and Canada

- Western Europe region (EUW), represented by Germany, France, UK, Italy, Ireland, Luxembourg, Netherlands, Spain, Portugal, Belgium, Austria, Switzerland, and Scandinavian countries

- Eastern Europe region (EUE), represented by Poland, Czech Republic, Bulgaria, Greece, Serbia, Croatia, Hungary, Montenegro, Ukraine, Turkey, and Georgia

- South America region, represented by Brazil, Argentina, Chile, Colombia, Uruguay, Paraguay, Panama, Peru, and Mexico (though NA)

- China

Note that the list is neither regionally exhaustive, nor is it 100% geographically accurate. Its sole purpose is to gain a general idea of the distribution of corporate bond returns for countries with a similar financial structure.

Fig. 5.3 features monthly return statistics for different credit ratings of corporate bonds. Please note that the number of bond-month observations for the single credit ratings does not sum up to to the number in the *All* column. This is due to the procedure which was used to split the bonds up by their rating. To give an example, bonds listed in rating category *BBB* are those bonds which have either a *BBB* rating as given by the S&P agency, or a *Baa* rating as given by the Moody's agency. In some cases, it occurs that e.g. the Moody's rating is given as *A*, and the S&P rating as *BBB*. In this case, the bond would be listed in both the *A* and the *BBB* categories, which results in intersections between the single categories. The procedure has been chosen this way to receive a larger amount of data for analysis, as e.g. entries for the Moody's rating alone were somewhat scarce. If needed, further analysis can be done based on ratings of a single agency only.

Group	All	AAA	AA	A	BBB	BB	B	CCC	Other / Unk.
Mean (%)	0.03%	0.14%	0.08%	0.09%	0.10%	0.08%	-0.03%	-0.05%	-0.05%
Std. deviation (%)	2.82%	2.36%	1.56%	2.02%	2.36%	2.91%	4.09%	4.62%	4.64%
Correlation with equity returns	0.1892	0.0815	0.0160	0.0360	0.0986	0.2920	0.3716	0.3671	0.1882
Correlation with equity returns (1-m. lead)	0.0460	-0.0546	-0.0435	-0.0105	0.0351	0.0430	0.0472	0.1214	0.042
Correlation with equity returns (1-m. lag)	0.0404	0.1132	0.0479	0.0638	0.0631	0.0240	0.0099	0.0455	0.0163
No. bond-month observations	683,086	4,853	40,457	136,515	308,298	81,106	32,656	13,166	123,768

Figure 5.3.: Monthly Return Statistics by Bond Rating

The distribution mean and standard deviation of the bond returns are largely in-line with my expectations. Bonds in regions with a majority of developed countries generally have a higher mean return and a lower standard deviation than bonds in regions with more developing countries. Similarly, investment grade bonds have higher mean return and lower volatility than non-investment grade bonds.

The correlation with equity returns looks higher for developed regions, but at the same time lower for investment grade bonds. Additionally, the correlation of monthly bond returns with lagged stock returns strictly monotonically increases with falling credit rating. As such, the lowest bond-lead correlation can be seen for corporate bonds rated *AAA*, while the highest correlation is listed for *CCC* rated bonds. This already explains quite well why existing research on the topic of the lead-lag relation of corporate bonds and stocks mostly focuses on high yield bonds, as these have the highest lead correlation with stocks. The correlation results of stock returns with lagged bond returns are less conclusive, but tend to be higher for investment grade bonds. From a geographical point of view, there seems to be no significant insight for the lead and lag correlations with equity returns.

5.4. Lead-Lag Relationship

In order to estimate the existence of a potential lead or lag relationship between corporate bonds and stocks, multiple regression analyses have been run. In particular, the calculations were conducted for the following bond groups:

- all corporate bonds

- split by geographical region: North America, South America, Western Europe, Eastern Europe, and China

- split by bond credit rating: AAA, AA, A, BBB, BB, B, CCC, other

- split by aggregated bond credit rating: investment grade and non-investment grade bonds

- two rating-based approaches to high-yield bonds

All regression analyses have been run as a duo of:

1. regression with monthly bond returns as dependent variable and 1-month lagged stock returns as independent variable

2. regression with monthly stock returns as dependent variable and 1-month lagged bond returns as independent variable

For shortness, not all the regression results will be introduced here in detail. This section will mostly focus on high yield bonds, as these have shown the most likely lead correlation with equities in section 5.3, and also because the leads and lags of high yield bonds have seen the highest attention in the existing literature on this subject so far. All the other regression results can be found either in the appendix to this work, or as part of the project files.

Source	SS	df	MS			
				Number of obs	=	2,120
				F(1, 2118)	=	89.30
Model	.23193612	1	.23193612	Prob > F	=	0.0000
Residual	5.50074596	2,118	.002597142	R-squared	=	0.0405
				Adj R-squared	=	0.0400
Total	5.73268208	2,119	.002705371	Root MSE	=	.05096

bond_return_w	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
lagged_stock_return_w	.0583903	.0061788	9.45	0.000	.0462732	.0705075
_cons	-.0013249	.0011069	-1.20	0.231	-.0034957	.0008459

Figure 5.4.: Results: regression with high-yield corporate bonds (as by Moody's) as dependent variable, and stocks as independent variable

Fig. 5.4 features the regression results for high-yield corporate bond return as dependent variable, and stock return as independent variable. Specifically, high-yield bonds have

been taken as those with either a *Ca* or a *C* current rating provided by Moody's. From a statistical perspective, the following interpretation of the results can be given:

- The *model sum of squares* (*MSS*) is not very close to the *total sum of squares* (*TSS*), which means that the fit of the model function is not very tight.

- The *p-value* (here *Prob > F*) is noticeably lower than 0.05. This means that the results of the regression are statistically significant.

- The r^2 is the amount of variance of bond returns that is explained by the lagged equity returns. The number is rather low, though it is the highest achieved in all the regression analyses run. Apparently, equities are best in explaining the variance of high-yield bonds, compared to other bond groups.

- The *root mean-squared-error* is the standard deviation of the regression model. It is close to 0, which means that the single bond return values do not deviate much from the distribution mean on average. On the other hand, the bond returns themselves are not very high, so even an absolutely small standard deviation can have a significant impact on the spread.

- The modeled regression function itself has the shape $bond\_return_n = -0.0013 + 0.0584 * stock\_return_{n-1}$, with n being the month, at the end of which the return occurred. The magnitude of the steepness factor and the intercept seems in-line with existing literature on the subject.

- The *t-values* signify the importance of the variable in the regression model. An absolute *t-value* greater than 1.96 is usually considered as good, which is here the case for the steepness coefficient.

- The *two-tail p-value* (here $P > |t|$)is lower than 0.05 for the growth coefficient, which means that it is statistically significant. The value for the intercept is not very significant at 0.231.

Source	SS	df	MS			
				Number of obs	=	2,124
				F(1, 2122)	=	11.43
Model	.3648643	1	.3648643	Prob > F	=	0.0007
Residual	67.7542046	2,122	.031929408	R-squared	=	0.0054
				Adj R-squared	=	0.0049
Total	68.1190689	2,123	.032086231	Root MSE	=	.17869

stock_return_w	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
lagged_bond_return_w	.2345471	.0693841	3.38	0.001	.0984792	.3706149
_cons	-.0012946	.0038796	-0.33	0.739	-.0089028	.0063136

Figure 5.5.: Results: regression with high-yield corporate bonds (as by Moody's) as independent variable, and stocks as dependent variable

Fig. 5.5 shows the regression results for high-yield corporate bond return as independent variable, and stock return as dependent variable. The same definition of 'high-yield' applies as for the first regression.

6. Conclusion

6.1. Summary

6.2. Outlook

Appendix

A. Performance Measurements

B. C++ Code

B.1. Dominates Operation for NNL, BNL and DNC

```cpp
/**
* Checks whether one tuple dominates the other and returns true/false
* @param dominator the tuple to check for dominating
* @param dominated the tuple to check for being dominated
*/
bool dominates(const std::vector<int> &dominator, const std::vector<int>
→   &dominated){
    bool flag = true;
    for(std::vector<int>::size_type i = 0; i< dominated.size(); i++){
        if(dominator[i] > dominated[i]) return false;
        if(dominated[i] > dominator[i]) flag = false;
    }
    if(flag) return false;
    return true;
}
```

B.2. Naive-Nested-Loops

```cpp
void computeSkylineProduce(){
    for(std::size_t i = 0; i < storage.size(); i++){
        bool not_dominated = true;
        for(std::size_t j = 0; j < storage.size(); j++){
            if(i != j){
                if(dominates(storage[j], storage[i])){
                    not_dominated = false;
                    break;
                }
            }
        }
        if(not_dominated){
            parent->consume(storage[i]);
        }
    }
}
```

B.3. Naive-Nested-Loops Parallelized

```cpp
void computeSkylineProduceParallel(){
    const std::vector<std::vector<int>> storage = this->storage;
```

```cpp
    CatOperator *parent = this->parent;
    parallel_for(std::size_t(0), storage.size(), [this, storage, parent](
    ↪  std::size_t i ) {
        bool not_dominated = true;
        bool flag = false;
        for(std::size_t j = 0; j < storage.size() && !flag; j++){
            if(i != j){
                if(dominates(storage[j], storage[i])){
                    not_dominated = false;
                    flag = true;
                }
            }
        }
        // The following mutex slows down the parallelization, but
        ↪  provides an easy way to avoid race conditions
        // In case no mutex is used, the programmer needs to make sure
        ↪  that no race condition occurs in the following if-block
        static spin_mutex mtx;
        spin_mutex::scoped_lock lock(mtx);
        if(not_dominated){
            parent->consume(storage[i]);
        }
    } );
}
```

B.4. Block-Nested-Loops Volcano Model

```cpp
void computeSkyline(){
    // storage is the window here
    storage.push_back(child->getNext());
    while(true){
        std::vector<int> tuple = child->getNext();
        if(!tuple.empty()){
            storage.push_back(tuple);
            for(std::size_t j = 0; j < storage.size()-1; j++){
                if(dominates(storage.back(), storage[j])){
                    storage.erase(storage.begin() + j);
                    j--;
                }
                else if (dominates(storage[j], storage.back())){
                    storage.erase(storage.begin() + storage.size()-1);
                    break;
                }
            }
        }
        else break;
    }
}
```

B.5. Block-Nested-Loops Produce/Consume

```cpp
void computeSkylineProduce(){
   // storage contains tuples produced by the generator
   std::vector<std::vector<int>> window;
   window.push_back(storage[0]);
   for(std::size_t i = 1; i < storage.size(); i++){
      std::vector<int> tuple = storage[i];
      window.push_back(tuple);
      for(std::size_t j = 0; j < window.size()-1; j++){
         if(dominates(window.back(), window[j])){
            window.erase(window.begin() + j);
            j--;
         }
         else if (dominates(window[j], window.back())){
            window.erase(window.begin() + window.size()-1);
            break;
         }
      }
   }
   for(std::size_t i = 0; i < window.size(); i++){
      parent->consume(window[i]);
   }
}
```

B.6. Divide-and-Conquer

Algorithm

```cpp
std::vector<std::vector<double>> computeSkyline(const
↪  std::vector<std::vector<double>> &M, const int &dimension){
   if(M.size() == 1) return M;

   std::vector<double> pivot = median(M, dimension-1); // dimension-1
   ↪  because we need the last index
   std::pair<std::vector<std::vector<double>>,
   ↪  std::vector<std::vector<double>>> P = partition(M, dimension-1,
   ↪  pivot);

   std::vector<std::vector<double>> S_1, S_2;
   S_1 = computeSkyline(P.first, dimension);
   S_2 = computeSkyline(P.second, dimension);

   std::vector<std::vector<double>> result;
   std::vector<std::vector<double>> merge_result = mergeBasic(S_1, S_2,
   ↪  dimension);

   // Union S_1 and merge_result
   for(std::vector<std::vector<double>>::size_type i = 0; i <
   ↪  S_1.size(); i++){
      result.push_back(S_1[i]);
```

```
  }
  for(std::vector<std::vector<double>>::size_type i = 0; i <
  ↪  merge_result.size(); i++){
     result.push_back(merge_result[i]);
  }

  return result;
}
```

Partition Operation

```
std::pair<std::vector<std::vector<double>>,
↪  std::vector<std::vector<double>>> partition(const
↪  std::vector<std::vector<double>> &tuples, const int &dimension,
↪  const std::vector<double> &pivot){
  std::vector<std::vector<double>> P_1, P_2;
  std::pair<std::vector<std::vector<double>>,
  ↪  std::vector<std::vector<double>>> partitions;

  for(std::vector<std::vector<double>>::size_type i = 0; i <
  ↪  tuples.size(); i++){
     if(tuples[i][dimension] < pivot[dimension])
        P_1.push_back(tuples[i]);
     else
        P_2.push_back(tuples[i]);
  }

  partitions.first = P_1;
  partitions.second = P_2;

  return partitions;
}
```

Merge Operation

```
std::vector<std::vector<double>>
↪  mergeBasic(std::vector<std::vector<double>> S_1, const
↪  std::vector<std::vector<double>> &S_2, const int &dimension){
  std::vector<std::vector<double>> result;

  if(S_2.size() == 0) return result;

  if(S_1.size() == 1){ // trivial case - S_1 has only 1 tuple
     for(std::vector<std::vector<double>>::size_type i = 0; i <
     ↪  S_2.size(); i++){
        if(!dominates(S_1[0], S_2[i]))
           result.push_back(S_2[i]);
     }
  }
  else if(S_2.size() == 1){ // trivial case - S_2 has only 1 tuple
     result.push_back(S_2[0]);
```

```cpp
      for(std::vector<std::vector<double>>::size_type i = 0; i <
      ↪  S_1.size(); i++){
         if(dominates(S_1[i], S_2[0])){
             result.erase(result.begin());
             break;
         }
      }
   }
   else if(S_1[0].size() == 2){ // low dimension
      // Min from S_1 according to dimension 1
      std::sort(S_1.begin(), S_1.end(), [](const std::vector<double> &a,
      ↪  const std::vector<double> &b){
         return a[0] < b[0];
      });
      std::vector<double> min = S_1[0];
      // Compare S_2 to Min according to dimension 1; in dimension 2 S_1
      ↪  is always better
      for(std::vector<std::vector<double>>::size_type i = 0; i <
      ↪  S_2.size(); i++){
         if(S_2[i][0] < min[0]) result.push_back(S_2[i]);
      }
   }
   else{ // general case
      std::vector<double> pivot = median(S_1, dimension-1-1);
      std::pair<std::vector<std::vector<double>>,
      ↪  std::vector<std::vector<double>>> partitions_dim_1 =
      ↪  partition(S_1, dimension-1-1, pivot);
      std::pair<std::vector<std::vector<double>>,
      ↪  std::vector<std::vector<double>>> partitions_dim_2 =
      ↪  partition(S_2, dimension-1-1, pivot);
      std::vector<std::vector<double>> result_1, result_2, result_3;

      result_1 = mergeBasic(partitions_dim_1.first,
      ↪  partitions_dim_2.first, dimension);
      result_2 = mergeBasic(partitions_dim_1.second,
      ↪  partitions_dim_2.second, dimension);
      result_3 = mergeBasic(partitions_dim_1.first, result_2,
      ↪  dimension-1);

      // Union result_1 and result_3
      for(std::vector<std::vector<double>>::size_type i = 0; i <
      ↪  result_1.size(); i++){
         result.push_back(result_1[i]);
      }
      for(std::vector<std::vector<double>>::size_type i = 0; i <
      ↪  result_3.size(); i++){
         result.push_back(result_3[i]);
      }
   }

   return result;
}
```

B.7. ST-S/ SARTS

```cpp
void computeSkylineProduce(){
   // pre-sort tuples in place
   sort(storage);
   std::vector<int> t_stop = storage[0];
   tree.insert(storage[0], tree.root, 0);
   parent->consume(storage[0]);
   for(std::size_t i = 1; i < storage.size(); i++){
      // stop if all the tuples left are dominated à –priori
      if((max(t_stop) <= min(storage[i])) && (t_stop != storage[i])){
         return;
      }
      // check for dominance
      if(!tree.is_dominated(storage[i], tree.root, 0,
      ↪  tree.score(storage[i]))){
         parent->consume(storage[i]);
         tree.insert(storage[i], tree.root, 0);
         if(max(storage[i]) < max(t_stop)){
            t_stop = storage[i];
         }
      }
   }
}
```

B.8. ST-S/ SARTS Parallelized

Algorithm

```cpp
void computeSkylineProduce(){
   const std::vector<std::vector<int>> storage = this->storage;
   const std::size_t number_of_threads = NUMBER_OF_THREADS;
   std::vector<Tree*> subtrees;
   for(std::size_t i = 0; i < NUMBER_OF_THREADS; i++){
      Tree* subtree = new Tree(tree.get_attributes());
      subtrees.push_back(subtree);
   }
   std::future<void> futures[NUMBER_OF_THREADS];
   // compute subquery skylines
   for(std::size_t i = 0; i < number_of_threads; i++){
      std::vector<std::vector<int>> subset;
      if(i == number_of_threads-1){
         subset.resize(storage.size() / number_of_threads +
         ↪  storage.size() % number_of_threads);
      }
      else{
         subset.resize(storage.size() / number_of_threads);
      }
      for(std::size_t j = 0; j < subset.size(); j++){
         subset[j] = storage[i*(storage.size()/number_of_threads) + j];
      }
```

```
    // Replace &ParallelSTS by &ParallelSARTS to receive SARTS
    futures[i] = std::async(std::launch::async,
    ↪   &ParallelSTS::computeSkylineSubset, this, i, subset,
    ↪   subtrees[i]);
  }
  for(std::size_t i = 0; i < NUMBER_OF_THREADS; i++){
    futures[i].get();
  }
  // compute final skyline
  std::vector<std::vector<int>> input;
  for(std::size_t i = 0; i < subset_results.size(); i++){
    if(!subset_results[i].empty()){
      input.push_back(subset_results[i]);
    }
  }
  sort(input);
  std::vector<int> t_stop = input[0];
  tree.insert(input[0], tree.root, 0);
  parent->consume(input[0]);
  for(std::size_t i = 1; i < input.size(); i++){
    if((max(t_stop) <= min(input[i])) && (t_stop != input[i])){
      return;
    }
    if(!ntree.is_dominated(input[i], tree.root, 0,
    ↪   tree.score(input[i]))){
      parent->consume(input[i]);
      tree.insert(input[i], tree.root, 0);
      if(max(input[i]) < max(t_stop)){
        t_stop = input[i];
      }
    }
  }
  // free memory
  for(std::size_t i = 0; i < subtrees.size(); i++){
    if(subtrees[i] != nullptr) delete subtrees[i];
  }
}
```

ComputeSkylineSubset Operation

```
void computeSkylineSubset(unsigned threadNumber,
↪   std::vector<std::vector<int>> tuples, Tree* tree){
  // pre-sort tuples in place
  sort(tuples);
  std::vector<int> t_stop = tuples[0];
  tree->insert(tuples[0], tree->root, 0);
  subset_results[threadNumber * (subset_results.size() /
  ↪   NUMBER_OF_THREADS) + 0] = tuples[0];
  for(std::size_t k = 1; k < tuples.size(); k++){
    // stop if all tuples left are dominated a-priori
    if((max(t_stop) <= min(tuples[k])) && (t_stop != tuples[k])){
```

```
        return;
    }
    // check for dominance
    if(!tree->is_dominated(tuples[k], tree->root, 0,
    ↪  tree->score(tuples[k]))){
        subset_results[threadNumber * (subset_results.size() /
        ↪  NUMBER_OF_THREADS) + k] = tuples[k];
        tree->insert(tuples[k], tree->root, 0);
        if(max(tuples[k]) < max(t_stop)){
            t_stop = tuples[k];
        }
    }
  }
}
}
```

B.9. N-Tree

Insert Operation

```
void NTree::insert(const std::vector<int> &tuple, node* p, unsigned int
↪  level){
  if(level == 0){
    p->minScore = 0;
    p->maxScore = 0;
    for(std::size_t i = 0; i < tuple.size(); i++){
        p->maxScore += (int) (pow(2.0, (double) (tuple.size()-i)) *
        ↪  attributes[attributes.size()-1]);
    }
  }
  else{
    p->minScore = 0;
    for(std::size_t i = 0; i < level; i++){
        p->minScore += (int) (pow(2.0, (double) (tuple.size()-i)) *
        ↪  tuple[i]);
    }
    p->maxScore = p->minScore;
    for(std::size_t i = level; i < tuple.size(); i++){
        p->maxScore += (int) (pow(2.0, (double) (tuple.size()-i)) *
        ↪  attributes[attributes.size()-1]);
    }
  }
  if(level == tuple.size()){
    p->tupleIDs.push_back(tupleID++);
  }
  else{
    if(p->children.empty()){
        p->children.resize(attributes.size());
    }
    if(!p->children[tuple[level]]){
        p->children[tuple[level]]=new node();
    }
```

```
            insert(tuple, p->children[tuple[level]], level+1);
    }
}
```

Is_Dominated Operation

```cpp
bool NTree::is_dominated(const std::vector<int> &tuple, node* p,
↪  unsigned int level, unsigned int currentScore){
    if(p==nullptr || (currentScore < p->minScore)){
        return false;
    }
    if((level == tuple.size()) && (score(tuple) != p->minScore)){
        return true;
    }
    if((level == tuple.size()) && (score(tuple) == p->minScore)){
        return false;
    }
    // search the subtrees from left to right
    unsigned int weight = (int) (pow(2.0, (double) (tuple.size()-level))
    ↪  * tuple[level]);
    for(int i = 0; i < tuple[level]; i++){
        if(is_dominated(tuple, p->children[i], level+1, currentScore +
        ↪  weight)){
            return true;
        }
    }
    if(is_dominated(tuple, p->children[tuple[level]], level+1,
    ↪  currentScore)){
        return true;
    }
    return false;
}
```

B.10. ART

Insert Operation

```cpp
void ART::insert(const std::vector<int> &tuple, Node *&parent, Node
↪  *&current, unsigned int level){
    if(level == 0){
        current->minScore = 0;
        current->maxScore = 0;
        for(std::size_t i = 0; i < tuple.size(); i++){
            current->maxScore += (int) (pow(2.0, (double) (tuple.size()-i))
            ↪  * attributes[attributes.size()-1]);
        }
    }
    else{
        current->minScore = 0;
        for(std::size_t i = 0; i < level; i++){
```

```cpp
                current->minScore += (int) (pow(2.0, (double) (tuple.size()-i))
                ↪    * tuple[i]);
            }
            current->maxScore = current->minScore;
            for(std::size_t i = level; i < tuple.size(); i++){
                current->maxScore += (int) (pow(2.0, (double) (tuple.size()-i))
                ↪    * attributes[attributes.size()-1]);
            }
        }
    }
    if(level == tuple.size()){
        current->tupleIDs.push_back(tupleID++);
    }
    else{
        Node* child = findChild(current, tuple[level]);
        if(!child){
            switch(current->type){
                case NodeType4:
                    if(current->count == 4)
                        grow(parent, current, tuple[level-1]);
                    break;
                case NodeType16:
                    if(current->count == 16)
                        grow(parent, current, tuple[level-1]);
                    break;
                case NodeType48:
                    if(current->count == 48)
                        grow(parent, current, tuple[level-1]);
                    break;
                case NodeType256:
                default:
                    break;
            }
            child = newChild(current, tuple[level]);
        }
        insert(tuple, current, child, level+1);
    }
}
```

Is_Dominated Operation

```cpp
bool ART::is_dominated(const std::vector<int> &tuple, Node* p, unsigned
↪   int level, unsigned int currentScore){
    if(p==NULL || (currentScore < p->minScore)){
        return false;
    }
    if((level == tuple.size()) && (score(tuple) != p->minScore)){
        return true;
    }
    if((level == tuple.size()) && (score(tuple) == p->minScore)){
        return false;
    }
```

```
   // search the subtrees from left to right
   unsigned int weight = (int) (pow(2.0, (double) (tuple.size()-level))
   ↪ * tuple[level]);
   for(int i = 0; i < tuple[level]; i++){
      Node* child = findChild(p, i);
      if(child){ // if child not null
         if(is_dominated(tuple, child, level+1, currentScore + weight)){
            return true;
         }
      }
   }
   Node* child = findChild(p, tuple[level]);
   if(child){
      if(is_dominated(tuple, child, level+1, currentScore)){
         return true;
      }
   }
   return false;
}
```

Find_Child Operation

```
Node* ART::findChild(Node* parent, const int &attribute){
   switch(parent->type){
      case NodeType4: {
         Node4* node = static_cast<Node4*>(parent);
         for (unsigned i = 0; i < node->count; i++){
            if (node->key[i] == attribute){
               return node->children[i];
            }
         }
         return NULL;
      }
      case NodeType16: {
         Node16* node = static_cast<Node16*>(parent);
         for (unsigned i = 0; i < node->count; i++){
            if (node->key[i] == attribute){
               return node->children[i];
            }
         }
         return NULL;
      }
      case NodeType48: {
         Node48* node = static_cast<Node48*>(parent);
         if (node->childIndex[attribute] != emptyMarker){
            return node->children[node->childIndex[attribute]];
         }
         else
            return NULL;
      }
      case NodeType256: {
```

```cpp
        Node256* node = static_cast<Node256*>(parent);
        return node->children[attribute];
    }
    default: {
        return NULL;
    }
    }
}
```

New_Child Operation

```cpp
Node* ART::newChild(Node *&node, const int &attribute){
    Node4* child = new Node4();
    switch(node->type){
    case NodeType4: {
        Node4* parent = static_cast<Node4*>(node);
        unsigned pos;
        // make free space for the new child entry
        for (pos = 0; (pos < parent->count) && (parent->key[pos] <
        ↪   attribute); pos++);
        memmove(parent->key+pos+1, parent->key+pos, parent->count-pos);
        memmove(parent->children+pos+1, parent->children+pos,
        ↪   (parent->count-pos)*sizeof(Node*));
        parent->key[pos] = attribute;
        parent->children[pos] = child;
        parent->count++;
        break;
    }
    case NodeType16: {
        Node16* parent = static_cast<Node16*>(node);
        unsigned pos;
        // make free space for the new child entry
        for (pos = 0; (pos < parent->count) && (parent->key[pos] <
        ↪   attribute); pos++);
        memmove(parent->key+pos+1, parent->key+pos, parent->count-pos);
        memmove(parent->children+pos+1, parent->children+pos,
        ↪   (parent->count-pos)*sizeof(Node*));
        parent->key[pos] = attribute;
        parent->children[pos] = child;
        parent->count++;
        break;
    }
    case NodeType48: {
        Node48* parent = static_cast<Node48*>(node);
        unsigned pos = parent->count;
        // if there are empty slots inbetween, use them instead of
        ↪   appending the child pointer at the end
        if(parent->children[pos]){
            for(pos = 0; parent->children[pos] != NULL; pos++);
        }
        parent->children[pos] = child;
```

```
        parent->childIndex[attribute] = pos;
        parent->count++;
        break;
    }
    case NodeType256: {
        Node256* parent = static_cast<Node256*>(node);
        parent->children[attribute] = child;
        parent->count++;
        break;
    }
    default:
        break;
    }
    return child;
}
```

Grow Operation

```
void ART::grow(Node *&parent, Node *&node, const int &indexOfCurrent){
    Node* newNode;
    switch(node->type){
    case NodeType4:
        newNode = new Node16();
        newNode->count = 4;
        for(std::size_t i = 0; i < 4; i++){
            static_cast<Node16*>(newNode)->key[i] =
            ↪   static_cast<Node4*>(node)->key[i];
        }
        for(std::size_t i = 0; i < 4; i++){
            static_cast<Node16*>(newNode)->children[i] =
            ↪   static_cast<Node4*>(node)->children[i];
        }
        break;
    case NodeType16:
        newNode = new Node48();
        newNode->count = 16;
        for(std::size_t i = 0; i < 16; i++){
            static_cast<Node48*>(newNode)->children[i] =
            ↪   static_cast<Node16*>(node)->children[i];
        }
        for (unsigned i = 0; i < node->count; i++){
            static_cast<Node48*>(newNode)->childIndex
            ↪   [static_cast<Node16*>(node)->key[i]] = i;
        }
        break;
    case NodeType48:
        newNode = new Node256();
        newNode->count = 48;
        for (unsigned i = 0; i < 256; i++){
            if (static_cast<Node48*>(node)->childIndex[i] != 48){ //
            ↪   slot not empty
```

```cpp
                    static_cast<Node256*>(newNode)->children[i] =
                ↪   static_cast<Node48*>(node)->children
                ↪   [static_cast<Node48*>(node)->childIndex[i]];
                }
            }
            break;
        default:
            break;
    }
    newNode->minScore = node->minScore;
    newNode->maxScore = node->maxScore;
    for(std::size_t i = 0; i < node->tupleIDs.size(); i++){
        newNode->tupleIDs[i] = node->tupleIDs[i];
    }

    if(node != root){
        // Code to updateParent() is not given for space reasons. Contact
        ↪   the author if needed.
        updateParent(parent, newNode, indexOfCurrent);
    }
    delete node;
    node = newNode;
}
```

Bibliography

[1] *Volcano Model.* `http://dbms-arch.wikia.com/wiki/Volcano_Model.` Last Accessed on Sept. 25, 2018.

[2] Ilaria Bartolini, Paolo Ciaccia, and Marco Patella. *Efficient Sort-Based Skyline Evaluation.* 33(4), 11 2008.

[3] OpenMP Architecture Review Board. *OpenMP application programming interface.* `https://www.openmp.org/.` Last Accessed on Sept. 25, 2018.

[4] Kenneth S. Bøgh, Sean Chester, and Ira Assent. *Work-Efficient Parallel Skyline Computation for the GPU.* volume 8, pages 962–973, 2015.

[5] S. Borzsony, D. Kossmann, and K. Stocker. *The Skyline operator.* In *Proceedings 17th International Conference on Data Engineering*, pages 421–430, April 2001.

[6] Sean Chester, Darius Šidlauskas, Ira Assent, and Kenneth S. Bøgh. *Scalable Parallelization of Skyline Computation for Multi-core Processors.* In *IEEE 31st International Conference on Data Engineering*, Seoul, South Korea, April 2015. IEEE.

[7] J. Chomicki, P. Godfrey, J. Gryz, and D. Liang. *Skyline with presorting.* In *Proceedings 19th International Conference on Data Engineering*, pages 717–719, March 2003.

[8] Intel Corporation. *parallel_for construct, part of Threading Building Blocks.* `https://software.intel.com/en-us/node/506057.` Last Accessed on Sept. 25, 2018.

[9] Intel Corporation. *parallel_sort Template Function, part of Threading Building Blocks.* `https://software.intel.com/en-us/node/506167.` Last Accessed on Sept. 25, 2018.

[10] Christos Kalyvas and Theodoros Tzouramanis. *A Survey of Skyline Query Processing.* April 2017.

[11] Alfons Kemper and André Eickler. *Datenbanksysteme: Eine Einfuehrung.* De Gruyter Oldenbourg, 2015.

[12] Alfons Kemper and Thomas Neumann. *HyPer: A Hybrid OLTP&OLAP Main Memory Database System Based on Virtual Memory Snapshots.* In *IEEE 27th International Conference on Data Engineering*, pages 195–206, Hannover, Germany, April 2011. IEEE.

[13] Donald Kossmann, Frank Ramsak, and Steffen Rost. *Shooting Stars in the Sky: An Online Algorithm for Skyline Queries.* In *Proceedings of the 28th VLDB Conference*, pages 275–286, Hong Kong, China, August 2002.

[14] H. T. Kung, F. Luccio, and F. P. Preparata. *On finding the Maxima of a Set of Vectors*. In *Journal of the Association for Computing Machinery*, volume 22, pages 469–476, October 1975.

[15] Viktor Leis, Alfons Kemper, and Thomas Neumann. *The Adaptive Radix Tree: ARTful Indexing for Main-memory Databases*. In *Proceedings of the 2013 IEEE International Conference on Data Engineering (ICDE 2013)*, ICDE '13, pages 38–49, Washington, DC, USA, 2013. IEEE Computer Society.

[16] Stian Liknes, Akrivi Vlachou, Christos Doulkeridis, and Kjetil Nørvåg. *APSkyline: Improved Skyline Computation for Multicore Architectures*, volume 8421 of *Lecture Notes in Computer Science*, pages 312–326. Springer International Publishing, Switzerland, March 2014.

[17] Kasper Mullesgaard, Jens Laurits Pedersen, Hua Lu, and Yongluan Zhou. *Efficient Skyline Computation in MapReduce*. In *Proc. 17th International Conference on Extending Database Technology*, pages 37–48, Athens, Greece, March 2014.

[18] Dimitris Papadias, Yufei Tao, Greg Fu, and Bernhard Seeger. *An Optimal and Progressive Algorithm for Skyline Queries*. In *ACM Proceedings*, San Diego, CA, USA, June 2003.

[19] Sungwoo Park, Taekyung Kim, Jonghyun Park, Jinha Kim, and Hyeonseung Im. *Parallel Skyline Computation on Multicore Architectures*. In *2009 IEEE 25th International Conference on Data Engineering*, pages 760–771, Shanghai, China, March 2009.

[20] Md Farhadur Rahman, Abolfazl Asudeh, Nick Koudas, and Gautam Das. *Efficient Computation of Subspace Skyline over Categorical Domains*. In *ACM Proceedings*, pages 407–416, Singapur, November 2017. CIKM. Session 2E: Skyline Queries.

[21] Kian-Lee Tan, Pin-Kwang Eng, and Beng Chin Ooi. *Efficient Progressive Skyline Computation*. In *Proc. 27th International Conference on Very Large Data Bases*, Roma, Italy, September 2001.