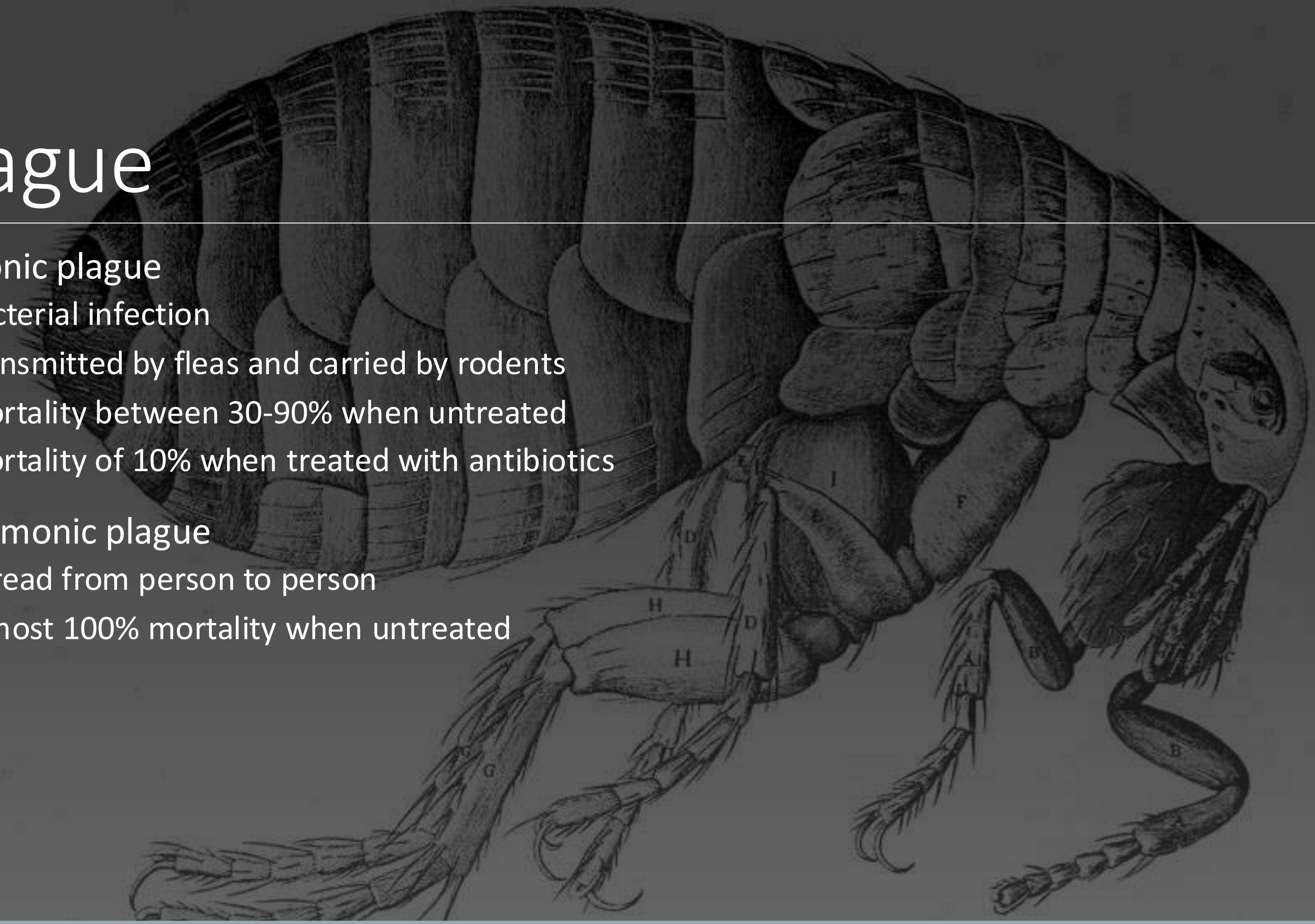
Modeling Bubonic Plague in Eyam

WENXIN DU, CHENZE LI, AND ALAN GAN

Plague

- Bubonic plague
  - Bacterial infection
  - Transmitted by fleas and carried by rodents
  - Mortality between 30-90% when untreated
  - Mortality of 10% when treated with antibiotics
- Pneumonic plague
  - Spread from person to person
  - Almost 100% mortality when untreated



The Black Plague

- Brought to Europe around 1347 and spread quickly by ship.
- Killed an estimated 40-60% of the European population between 1346-1353.
- Killed an estimated 75-200 million people in Europe and Asia.
- Initial epidemic was during the mid 1300s.
- Afterwards, smaller plague epidemics continued to pop up for the 500 years.
- 1665-66 Great Plague of London:
  - Killed 100,000 people (25% of population)

Eyam, England

- Small village in England.
  - In 1660s population was around 800.

Eyam and the Plague

- Plague arrived in 1665:
  - A bundle of flea-infested cloth was shipped from London to the local tailor.
  - They imposed quarantine.
- Killed at least 260 people

Eyam and the Plague

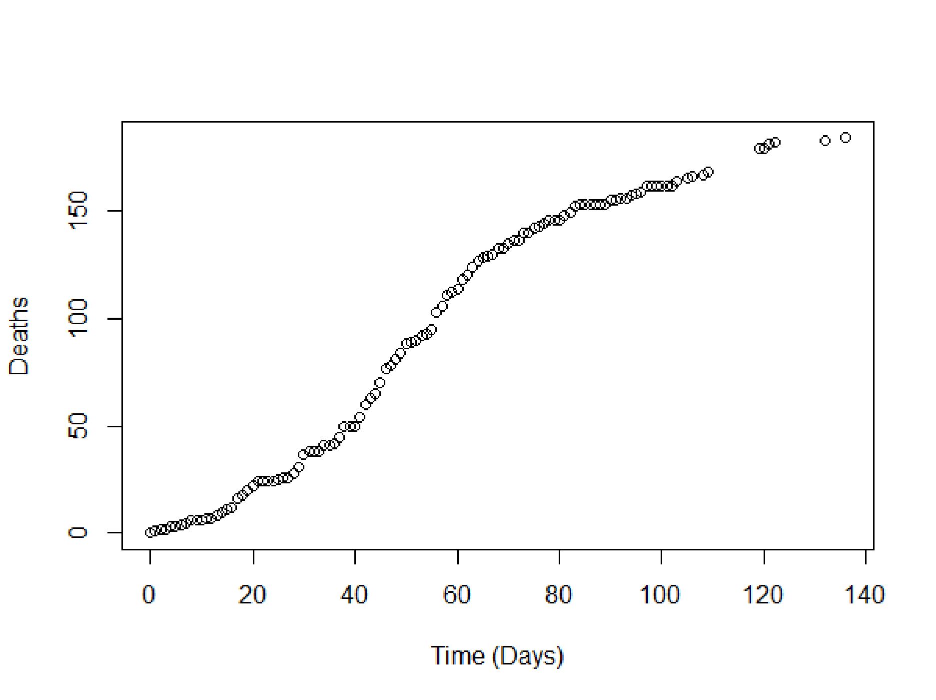
- Eyam kept very detailed burial records.

- Thomas Alleyn 6 Apr 1666
- Joan Blackwall 6 Apr 1666
- Alice Thorpe 15 Apr 1666
- Edward Barnsley 16 Apr 1666
- Margaret Blackwell 16 Apr 1666
- Samuel Hadfeild 18 Apr 1666
- Margaret Gregory 21 Apr 1666
- Alleyn (an infant) 28 Apr 1666
- Emmott Sydall 29 Apr 1666
- Robert Thorpe 2 May 1666
- William Thorpe 2 May 1666
- James Teylour 11 May 1666

Eyam and the Plague

- Eyam kept very detailed burial records.

- Jane Townend 25 Jun 1666
- Emnett Heald 26 Jun 1666
- John Swanne 29 Jun 1666
- Elizabeth Heald 1 Jul 1666
- William Lowe 2 Jul 1666
- Ellenor Lowe (his wife) 2 Jul 1666
- Deborah Yealott 3 Jul 1666
- George Darbye 4 Jul 1666
- Anne Coyle 5 Jul 1666
- Bridget Talbot 5 Jul 1666
- Mary Talbot 5 Jul 1666
- John Danyell 5 Jul 1666



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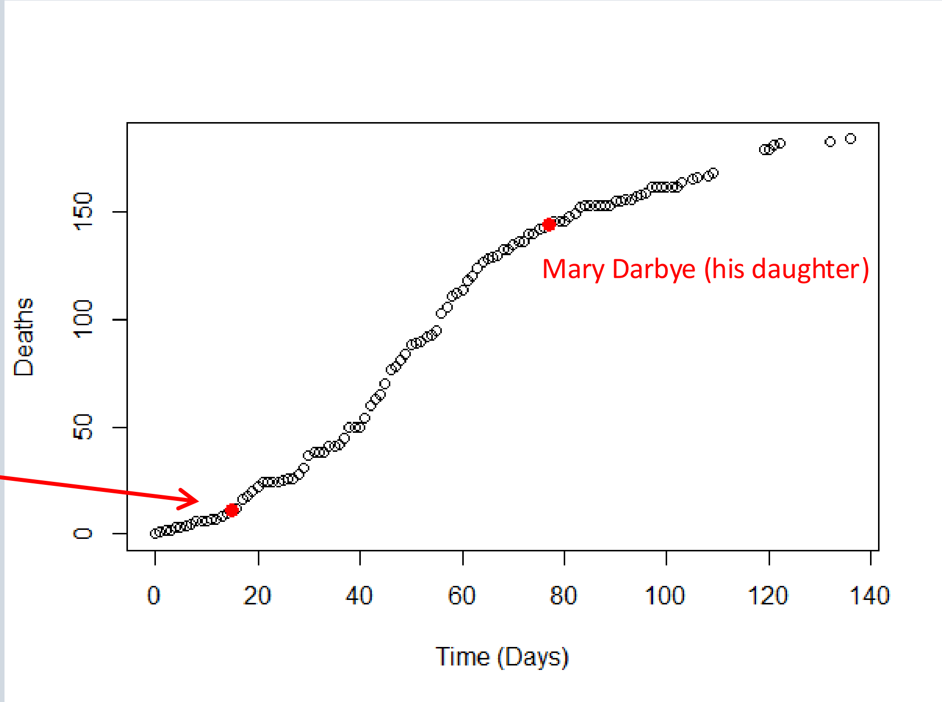

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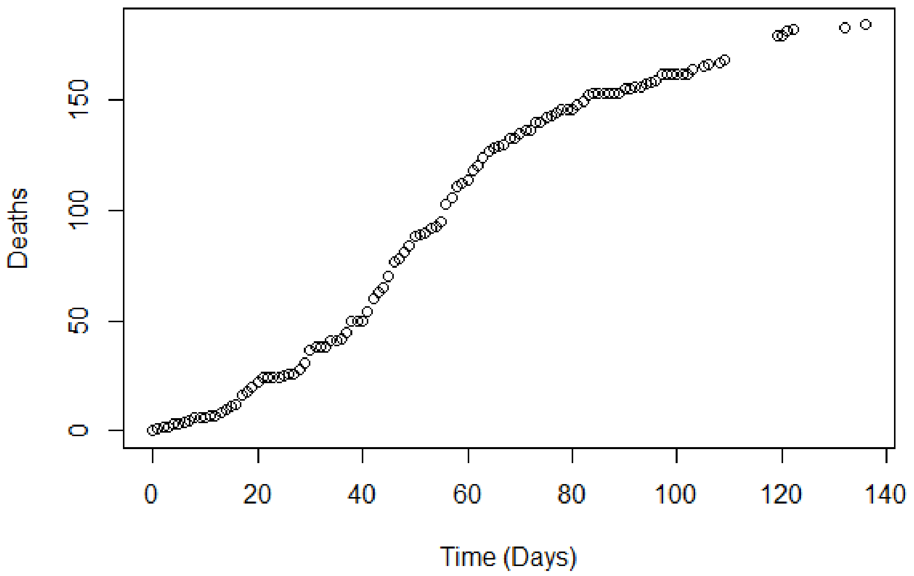
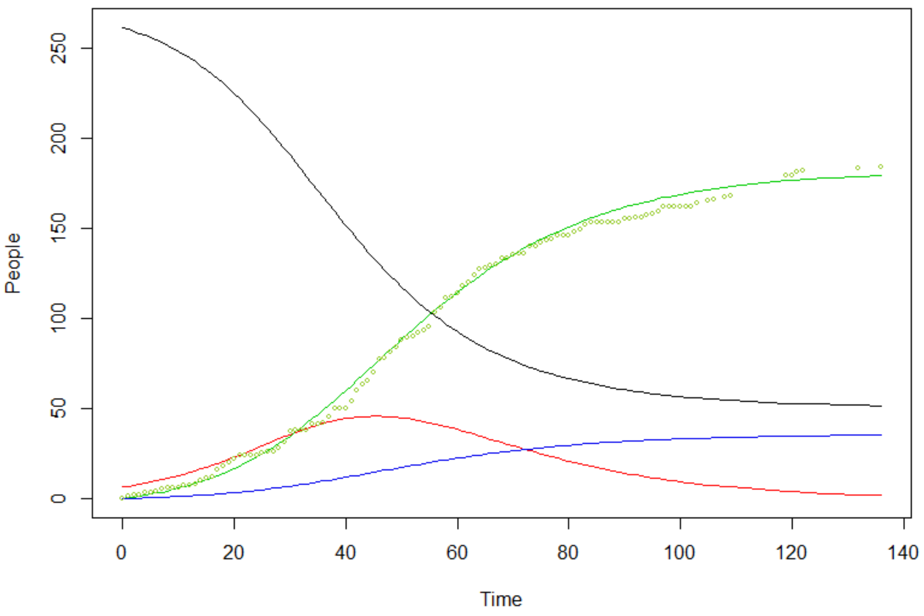
Mary Darbye (his daughter)

Objective

- Model the plague epidemic in Eyam.
  - And fit to burial data.





SIR Modeling

SIR model

- Ordinary Differential Equations

$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

where S, I, R means the number of people who are susceptible, infected and dead. The initial values of ODE equations are $S(0)=S_0$, $I(0)= I_0$ and $R(0)=0$.

Non-linear model

- Non linear model

$$y(t_i) = R(t_i, \theta) + \epsilon_i$$

where y_i is the observation and ϵ_i is the error term.

- Non linear least squares method
  - $\mathrm{RSS} = \sum_i^n (y_i - R_i(\theta))^2$ with $\theta = argmin\ RSS$
    - use L-BFGS-B to minimize the target function RSS.
      - Parameters: $\theta = (\beta, \gamma, I_0)$

Bayesian methods

- Bayesian methods
  - Assume prior distributions for parameters
  - Markov chain Monte Carlo (Hamiltonian Monte Carlo algorithm) to sample from the posterior distribution.

Bayesian methods

- Negative Binomial Assumption
  - Assume $y(t_i) \sim NegBinom(R(t_i), \phi)$ with ϕ unknown;
  - Parameters: $\theta = (\beta, \gamma, I_0, S_0)$ and ϕ;
  - Priors:
    - $\beta \sim Beta(0.5,0.5)$; (Jeffreys prior)
    - $\gamma \sim Beta(0.5,0.5)$; (Jeffreys prior)
    - $S_0 \sim Normal(261, 100)$;
    - $I_0 \sim TruncNormal(1, 9), I_0 > 0$ (Truncated Normal);
    - $\frac{1}{\phi} \sim Exponential(2)$
  - Package used: Rstan

SIR Model Results

NLS Model Results

Parameter	β	γ	I_0
Fitted Value	0.176	0.107	4

$R_0 = \beta/\gamma$
1.65

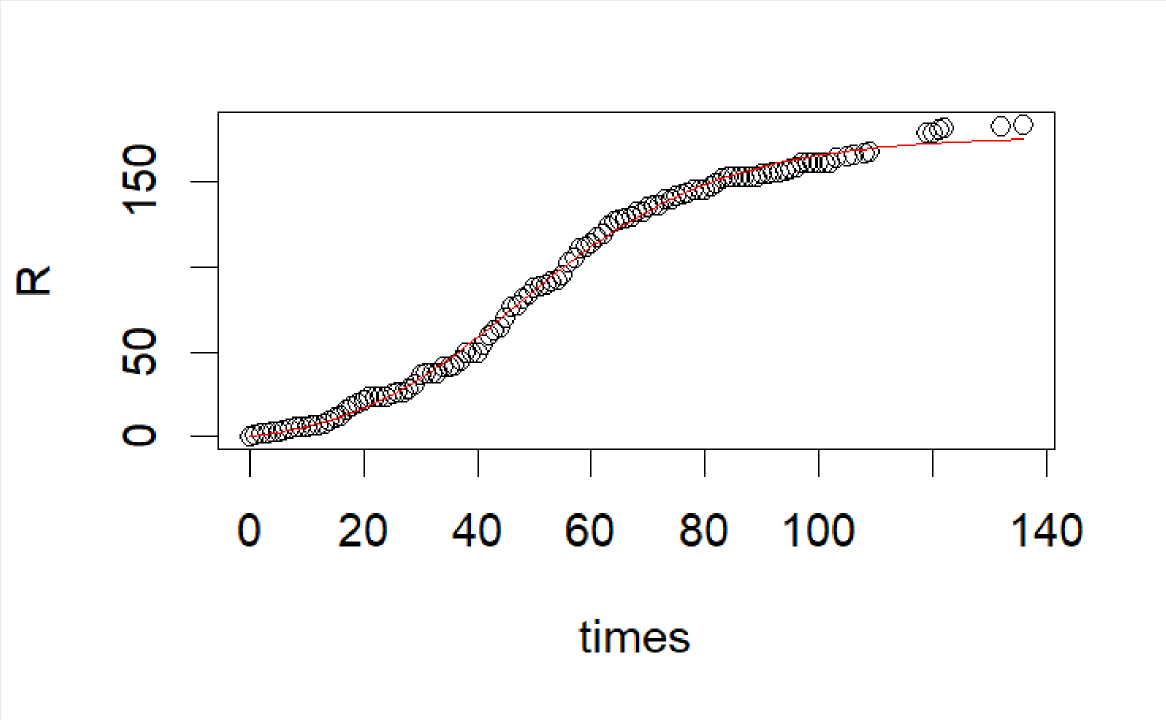
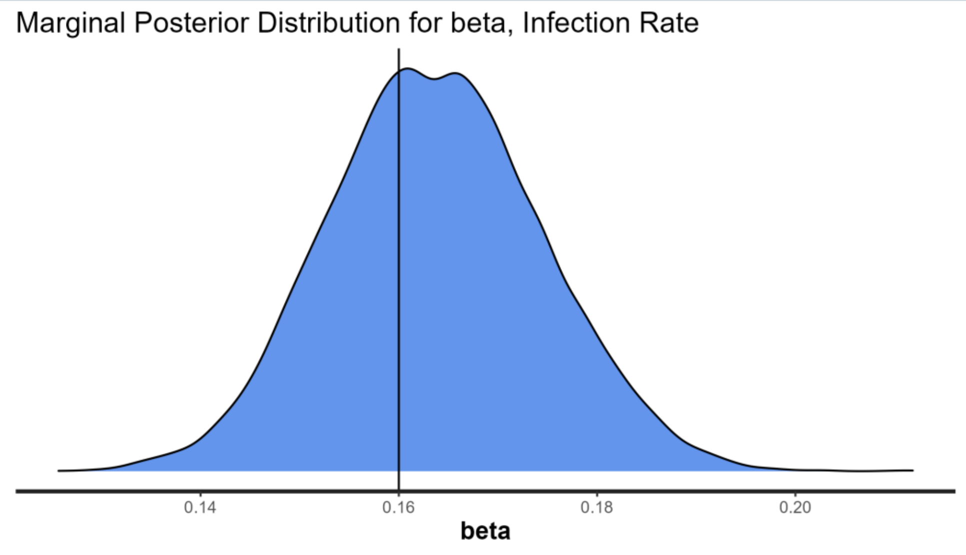


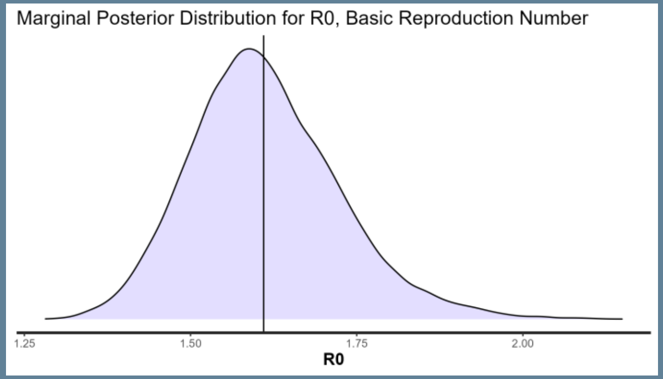
Figure: Fitted Plot using NLS.

Bayesian Method Results



Posterior mean for β
0.16

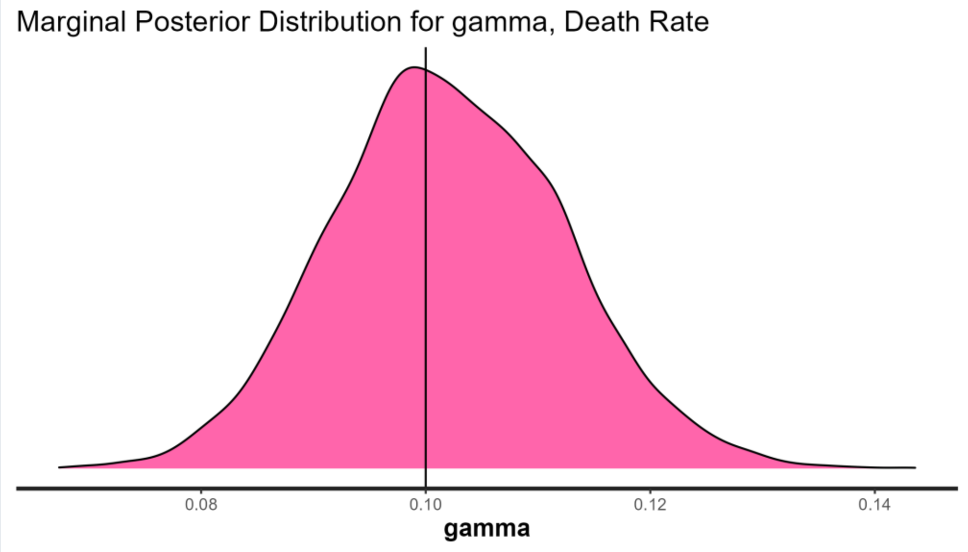
2.5% C.I.	97.5% C.I.
0.14	0.19



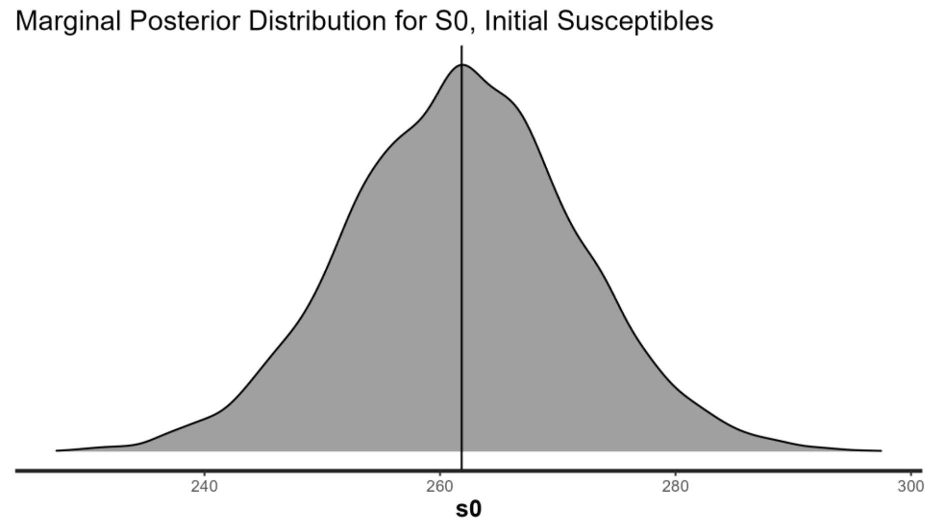
Posterior mean for R_0	2.5% C.I.	97.5% C.I.
1.61	1.42	1.86

Posterior mean for γ
0.1

2.5% C.I.	97.5% C.I.
0.08	0.12

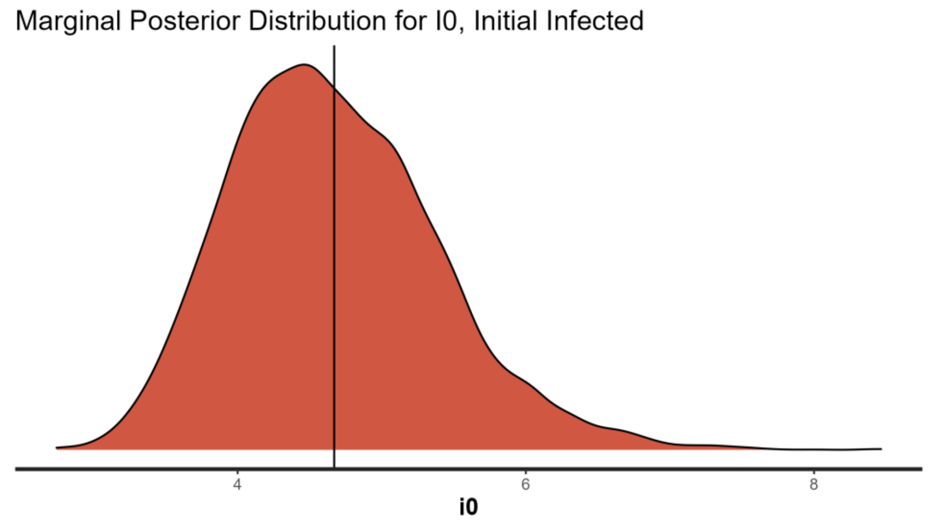


Bayesian Method Results



Posterior mean for S_0
261.8

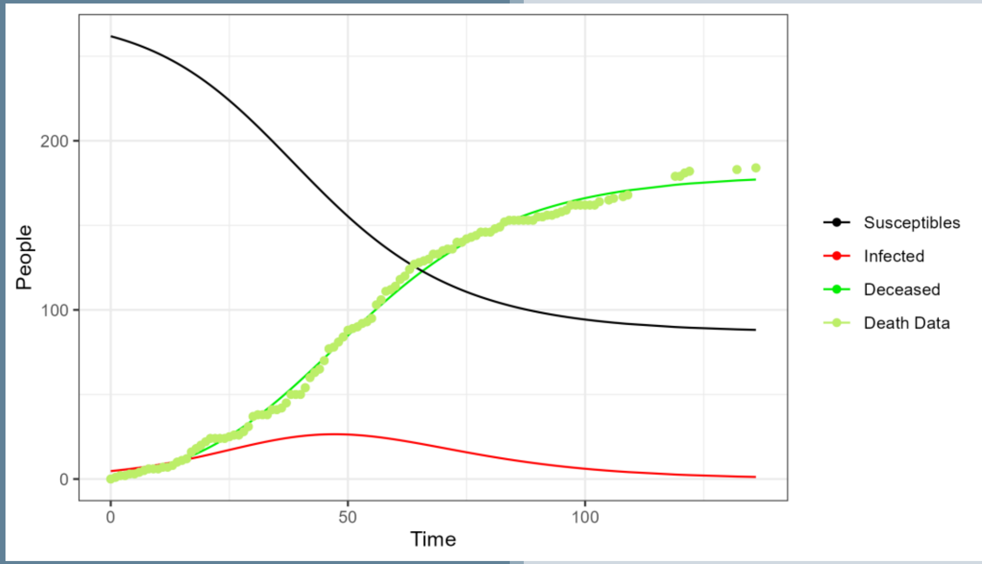
2.5% C.I.	97.5% C.I.
242.5	281.2

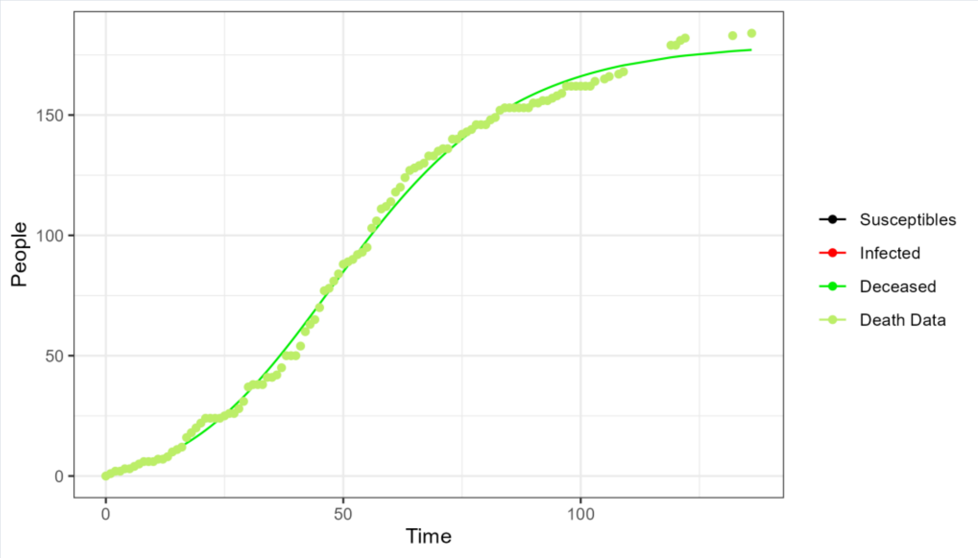


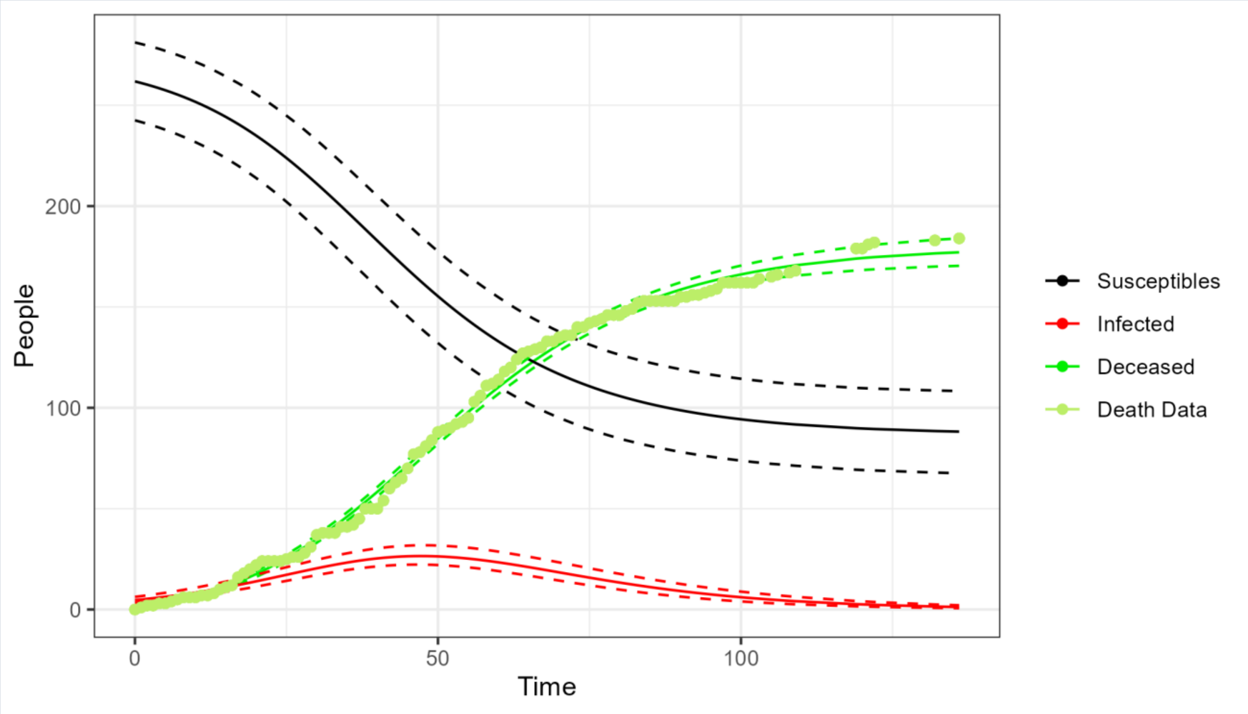
Posterior mean for I_0
4.67

2.5% C.I.	97.5% C.I.
3.45	6.26

Bayesian Method Results







SIRD model

- Ordinary Differential Equations

$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I - \delta I$$
$$\frac{dR}{dt} = \delta I$$
$$\frac{dD}{dt} = \gamma I$$

where S, I, R, D are the number of people who are susceptible, infected, recovered and dead.
$S(0) = S_0, I(0) = I_0$, $R(0) = 0$ and $D(0) = 0$

Non-linear Least Squares Methods

- Non-linear least squares (Non-linear model)
  - use L-BFGS-B minimize the target function RSS.
    - Parameters: $\theta = (\beta, \gamma, \delta, I_0)$

Bayesian Method

- Normal Assumption
  - Assume $y(t_i) = R(t_i, \theta) + \epsilon_i$ with $\epsilon_i \sim N(0, \sigma^2)$
  - parameters: $\theta = (\beta, \gamma, \delta, I_0, S_0)$ and σ
  - Priors: $\beta \sim Unif(0,100), \gamma \sim \mathrm{Unif}(0,100), \delta \sim Unif\ (0,100), \mathrm{S}_0 \sim N(261,100), I_0 \sim Unif(0,10); \sigma^2 \sim Inv - Gamma(1,1).$

SIRD Model Results

NLS Model Results

Parameter	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\delta}$	$\boldsymbol{I_0}$
Fitted Value	0.0673	0.0000	0.0113	130

$R_0 = \dfrac{\beta}{\gamma + \delta}$
5.96

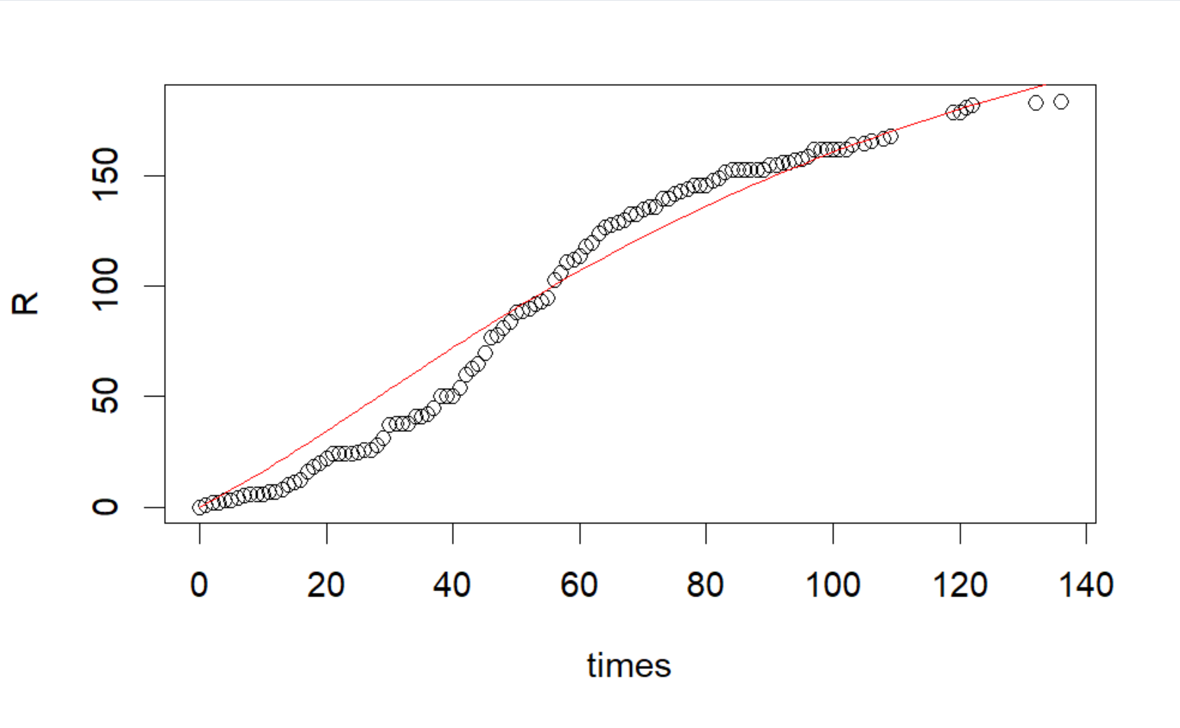
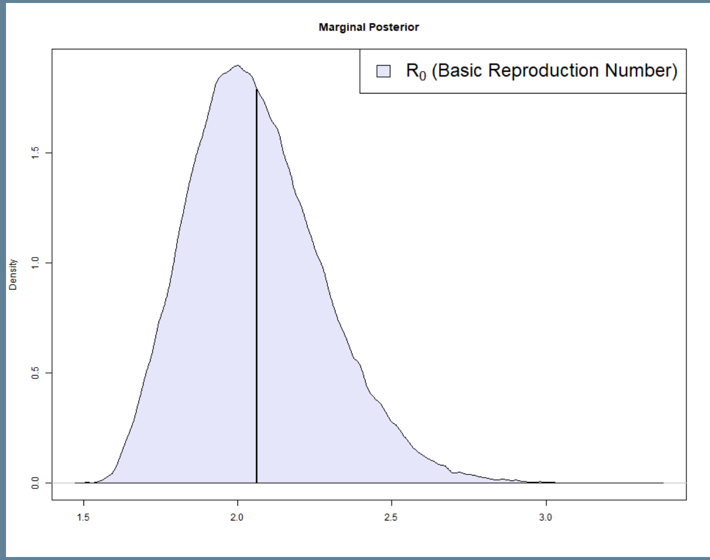
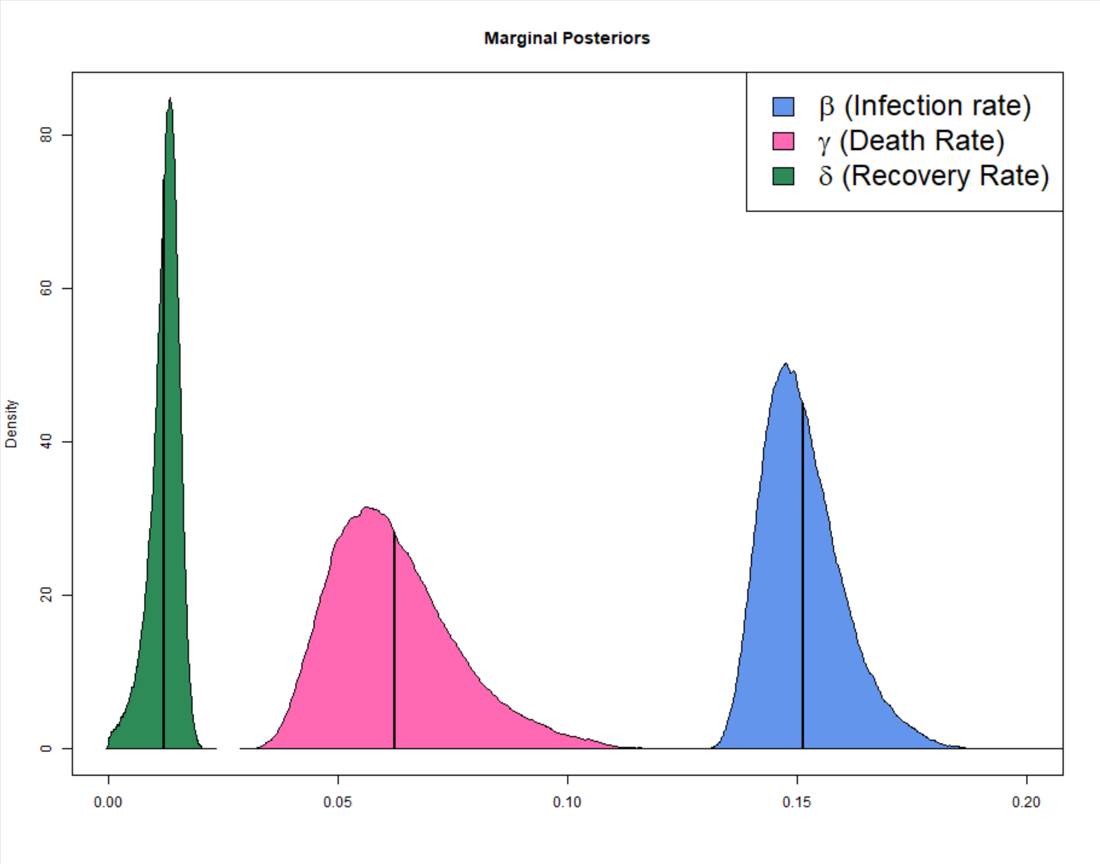


Figure: Fitted Plot using NLS.

Bayesian Method Results



Posterior mean for R_0	2.5% C.I.	97.5% C.I.
2.06	1.698	2.544

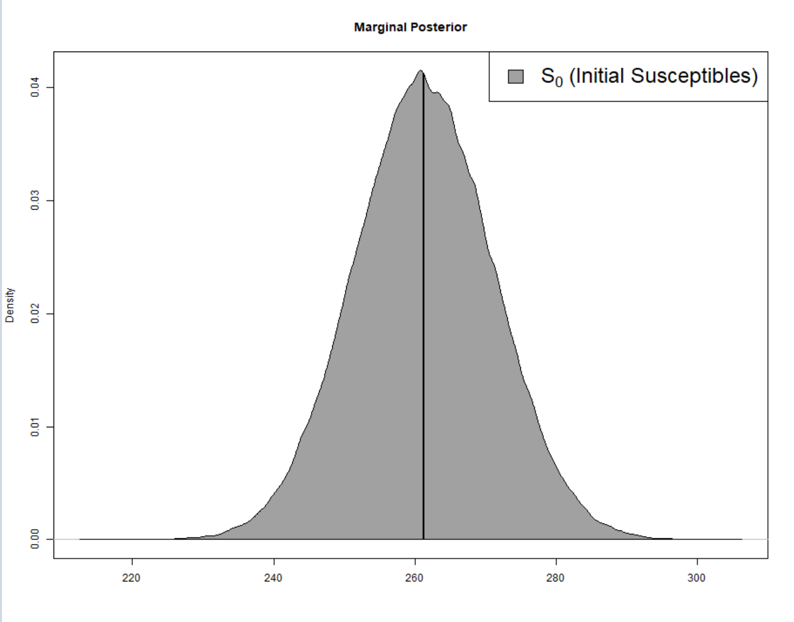


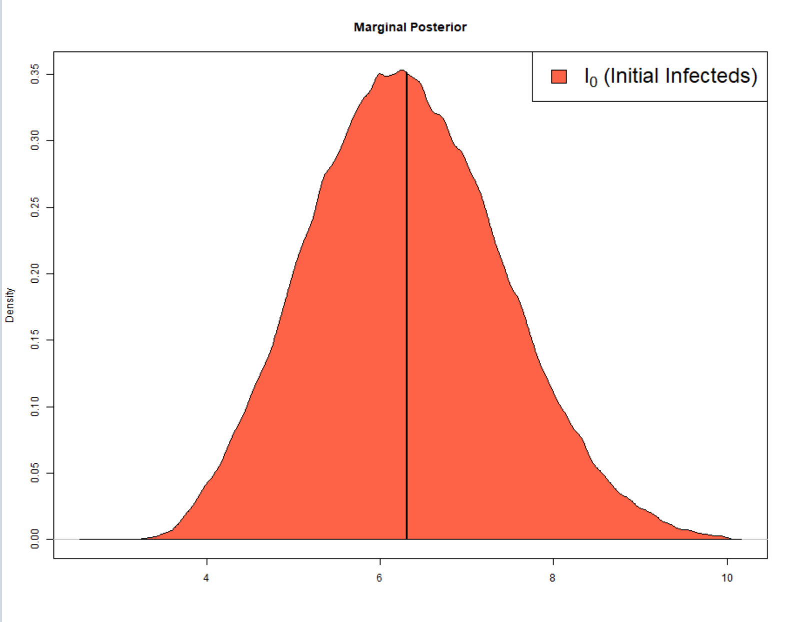
Posterior mean for δ	
0.012	
2.5% C.I.	97.5% C.I.
0.0041	0.0172

Posterior mean for γ	
0.062	
2.5% C.I.	97.5% C.I.
0.041	0.095

Posterior mean for β	
0.15	
2.5% C.I.	97.5% C.I.
0.138	0.172

Bayesian Method Results





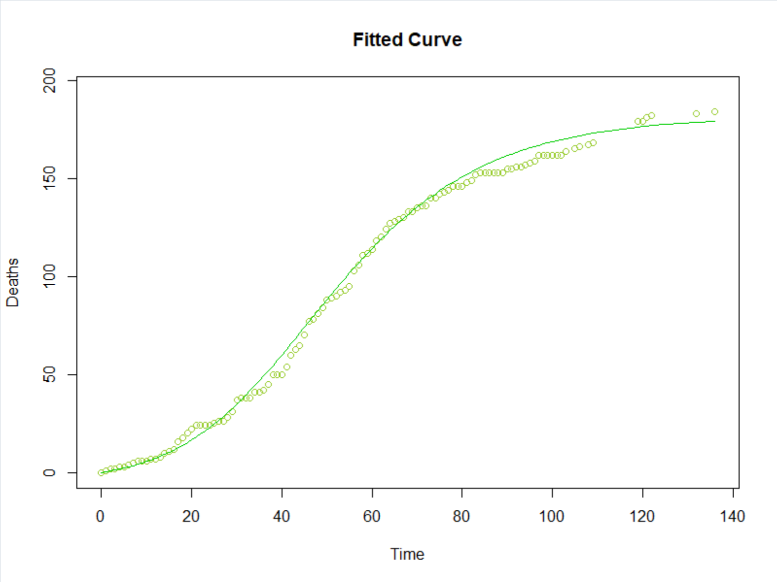
Posterior mean for S_0
261.2

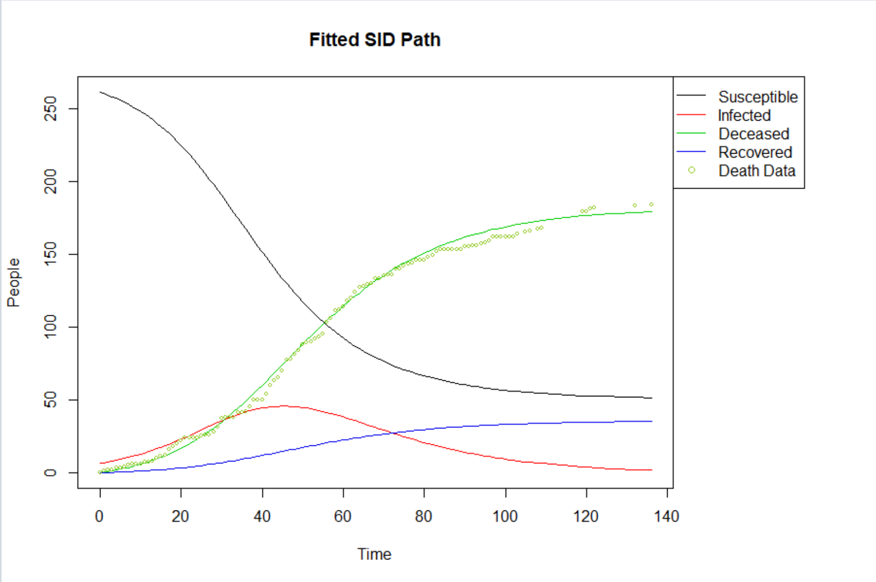
2.5% C.I.	97.5% C.I.
242.0	280.5

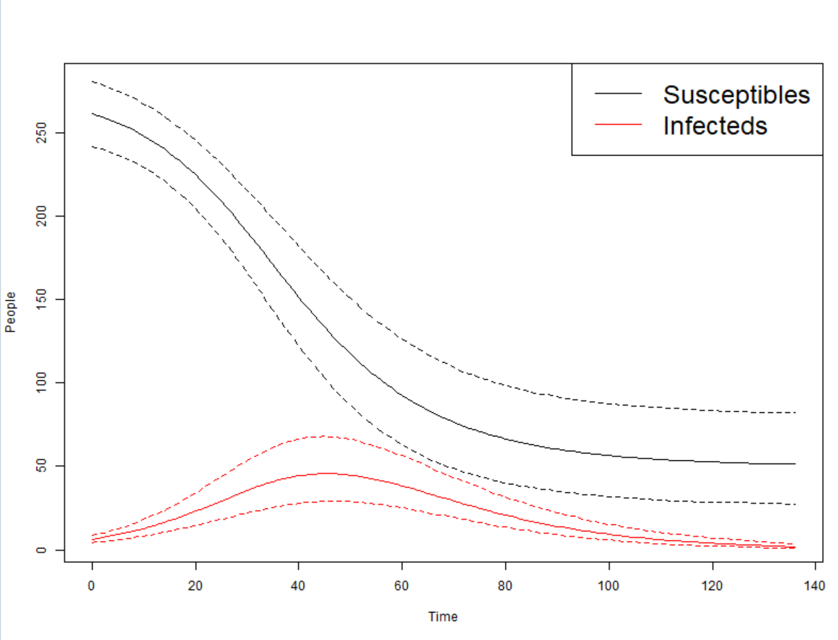
Posterior mean for I_0
6.3

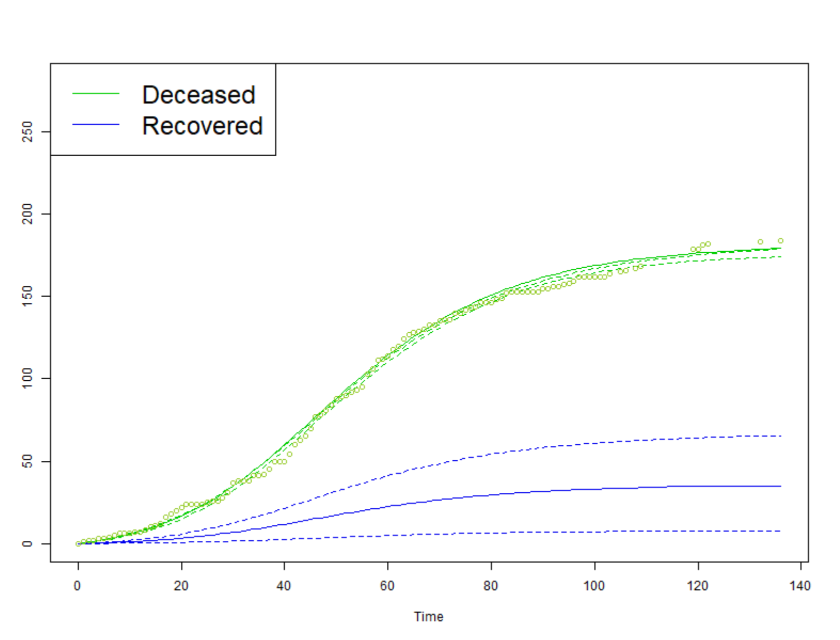
2.5% C.I.	97.5% C.I.
4.26	8.57

Bayesian Method Results









Conclusion

- We fit 2 ODE models (SIR and SIRD) using 2 different techniques (NLS and Bayesian modeling).
  - There is evidence that the Eyam plague did not have 100% mortality.
  - SIRD is likely more appropriate here.
  - Bayesian method gave better results.
- We estimate the epidemic progression.
- We estimate important parameters.
  - $\beta \in (0.138,\ 0.172)$
  - $\gamma \in (0.041,\ 0.095)$
  - $\delta \in (0.004,\ 0.017)$
  - $R_0 \in (1.70,\ 2.54)$
  - Mortality Rate is between 73% and 96%.