We developed lots of theoretical tools in the first chapter. We introduced differential forms, singular homology and Hilbert complexes to prove the existence and uniqueness of the magnetostatic problem on the exterior domain of a toroidal domain where we reformulated the problem using the assumption that the first homology group is generated by the homology class of the curve that we are integrating over. We utilized existence and uniqueness results on the level of homology and applied the de Rham isomorphism to obtain analogous results for differential forms. These were then used together with the Hodge decomposition to prove existence and uniqueness.

In the second part, we looked at the 2D magnetostatic problem with curve integral constraint. We derived a convenient variational formulation and proved well-posedness as well as an a-priori estimate. Numerical experiments confirmed that the method works as expected.

One immediate possibility to extend these results would be to look for solutions on domains with more complicated topologies and different topological constraints. For example, the first Betti number, i.e. the number of generators of the first homology group, could be increased and pose the problem with more curve integrals. This generalization should be straightforward.

We only showed the existence and uniqueness of the homogeneous magnetostatic problem with $\mathbf{J} = 0$. Of course, the uniqueness for $\mathbf{J} \neq 0$ follows from that immediately because the problem is linear. With the existence however, one has to be careful. One issue is the application of the regularity results, which have to formulated and applied with care to make sure that the curve integral constraint is still well-defined.

Another way to go would be to generalize the magnetostatic problem itself and pose it to find a differential form with exterior derivative and codifferential (see [1, Sec. 6.2.6]) equal to zero. This has been done, see e.g. [2, Chap. 3]. A difficulty would then be to obtain the regularity and density results that we relied on which will most likely come down to certain regularity assumptions on the manifold itself. This generalization would certainly not be easy, but would be a way to put this result into the general framework of differential forms together with the exterior derivative and the codifferential. To come back to the very first sentence of this thesis, this would fit in a way to the core of mathematics: The beauty of abstraction.