Mathematics is the art of abstraction, of putting basic theories into more general frameworks to gain insights that would otherwise be hard or impossible to see.

This thesis is about the magnetostatic problem which reads. It is a fascinating PDE for the reason that it combines the differential equations with this curve integral constraint which gives this problem a strong topological flavour. For a student familiar with PDEs it begs the question how these notions can be combined to investigate this problem.

James Clerk Maxwell recognized how many years ago that some vector fields are to be understood in relation to curves i.e. the natural relation is the curve integral and some are to be understood as flux through a surface. This thought paves the way to a new paradigm which presents itself in differential forms which are a classic subject of differential geometry. Differential geometry especially on Riemannian manifolds gives a much more general framework in which basic vector calculus can be embedded. Differential forms turn out to be the more suited objects to study in fields like electromagnetism [**<empty citation>**] and fluid dynamics.

Differential forms become especially useful when we start integrating. Whitney derives in his classic book [**t**]hat differential forms are the actual objects suited to be integrated over manifolds like curves and surfaces which comes back to Maxwell. Another nice exposition about this topic is Terence Tao's paper [**<empty citation>**].

This integration of differential forms over manifolds turns out to provide a beautiful synergy between three fields, calculus, differential geometry and topology. De Rham's famous theorem which relies on Stokes theorem for gives a very simple and yet deep relationship between the singular homology – a popular tool in algebraic topology to study topological quanities – and the integration of differential forms.

For a mathematician interested in analysis and motivated by the beauty of abstraction these reasons should already be sufficient to start learning and investigating differential forms and as a necessary fundament differential geometry. We will give a short exposition of these topics in the first sections of this book.

Applying these ideas to the magnetostatic problem above has been done before, but here another challenge arises. Usually this problem is either posed on bounded domains. Traditionally, the theory of PDEs on unbounded domains is sparse in comparison to bounded ones. But unbounded domains can change the situation drastically. Even one of the simplest most basic problems in the topic of PDEs, the Laplace equation with Dirichlet boundary conditions, does not have a unique solution anymore when posed on an unbounded domains.

Combining the tools from singular homology, differential geometry and analysis we will the existsnce and uniqueness of the magnetostatic problem under suitable assumptions.

The second part of this thesis will then be concerned with the numerical approximation. As usual for the numerical analysis – especially in a master's thesis – we will simplify the problem. to bounded domains in 2D, but we keep the curve integral constraint and the non-trivial topology of the domain. As explained above, the natural way to study an equation like this is in the language of differential forms. In the last decade, there has been a lot of research done about the comination of that which has been put in abstraction in the Acta Numerica paper from Arnold, Falk and Winther. [**<empty citation>**] who coined the term Finite Element Exterior Calculus (FEEC). This is the perfect tool for our purposes. We will replace the curve integral using the integration by parts in 2D to obtain a new variational formulation to study. We will prove an inf-sup condition and a a-priori error estimate greatly relying on the general framework provided be the FEEC theory.

The outline is as follows. We will first spend a lot of time introducing the necessary tools for the proof of well-posedness of the magnetostatic problelm in 3D. We start with differential forms in Sec. **??** from the direction of multilinear algebra. Afterwards we will give a short introduction on singular homology in Sec. which concludes in the de Rham isomorphism and then we will talk about unbounded operators and the special situation of Hilbert complexes which provide us with the good tools to deal with taht. We will put all this together in Sec. to prove the existence and uniqueness of the magnetostatic problem.

In Chapter **??** we will focus on numerics. We derive a variational formulation of the 2D problem of the magnetostatic problem in Sec. . Then we give a short overview of the discrete Hilbert complexes which are fundamental to FEEC theory before applying it to our new variational formulation to obtain the well-posedness and a-priori estimate. In the end we will explain how this problem is actually implemented and provide some numerical examples.