MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Requirements for multipatch geometry

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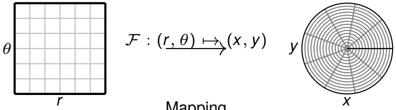
Questions to be answered

What is the most suited way to solve the problems related to multipatch in our case?

What could be missing in the DDC library to implement multipatch?

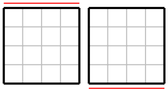
Structure of multi-patch



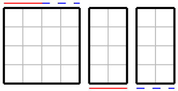
Logical domain

Mapping

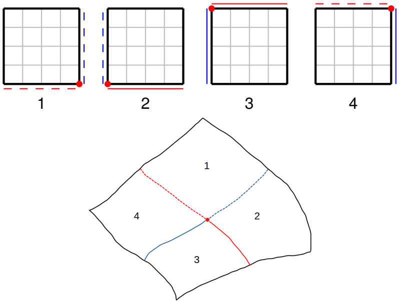
Physical domain



(a) Simple interface.



(b) T-joint.

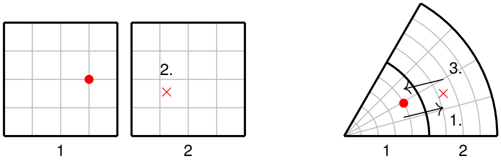


(c) X-point in logical and physical domain.

Patches in the logical domain.

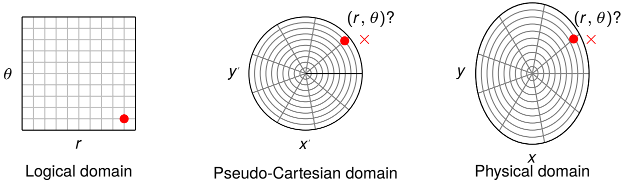
Advection

$$\partial_t \rho + A \cdot \nabla \rho = 0, \qquad \partial_t X(t^n; t^{n+1}, x) = A(t, X(t^n; t^{n+1}, x)). \tag{1}$$



1. compute feet.
2. evaluate function (need to transfer feet to patch 2).
3. transfer value to patch 1.

Example of a characteristic foot outside the patch 1 in the logical and physical domains.



Solving Poisson with CONGA

We want to solve the Poisson equation

$$-\operatorname{div}(\nu \operatorname{grad} \phi) = \rho$$

using a the CONGA approach on a 2D multipatch domain.

- Have finite element space V_h with jump discontinuities across edges
- Subspace $V_h^c \subseteq V_h$ with functions conforming to some global regularity constraint
- Define projection $P_h : V_h \to V_h^c$ and discrete differential operator $\operatorname{grad}_h := \operatorname{grad} P_h$ $\to$ need information on mesh and spline degree of neighboring patches
- Discretize Poisson equation weakly using these operators

Solving Poisson with CONGA

Advantages

- Already implemented in Psydac and successfully applied to several problems.
- Probably easier to generalize to complicated geometries than other approaches like using different splines e.g.

Patch data – Exchange

- Mesh-points,
- Local sums to compute the derivatives at the interfaces

$$\sum_{x_i \in \text{global space}} \alpha_i s(x_i) = \sum_{p \in \text{Patches}} \sum_{x_i \in p} \alpha_i s(x_i),$$

where α_i depend on the mesh points,

- Characteristic feet outside of the patch,
- Interpolated values for $\mathbf{A}$ and ρ.

Patch data – Storage

- Mesh points, dimension (`DimXi`, `DimYi`), mapping, `SplineBuilder`, metadata,
- Boundary condition of global domain if an edge of the patch is on the global boundary,
- Values of functions ρ, ϕ, **A** on mesh points,
- Spline coefficients of functions (ρ, ϕ, **A**),
- Reference to global domain class.

Global domain

- Global domain class
  - References to patches,
  - ’Connectivity’ class which encodes the geometrical information.
- Connectivity class
  - Identify edges and corners of different patches
  - For T-joint, identify sections of edges with sections of other edges, place corners in the middle of edges.

APPENDIX - Structure code

GLOBAL DOMAIN CLASS

Define a global view of the domain.
- Reference to each Patch object.
- Global boundaries (outside boundaries).
- Reference to Interfaces object.
- Computation to find the patch where a given coordinate is?

INTERFACES CLASS

Define the interfaces between each patches.
- Reference to each Patch object.
- Define interfaces between each patches. (simple, T-joint, X-point).

PATCH CLASS

Define a patch.
- Dimensions `DimRi`, `DimPi`.
- Discrete domains and spline domains.
- Local mapping.

APPENDIX - Drift-kinetic equations

$$\begin{cases} \partial_t f + v_{GC} \cdot \nabla_\perp f + v_\| \partial_z f + \dot{v}_\| \partial_{v_\|} f = 0, \\ -\nabla_\perp \cdot (\alpha \nabla_\perp \phi) + \beta(\phi - \langle \phi \rangle) = n \end{cases} \tag{2}$$

Advection

- $\rightarrow$ multi-patch for space $\perp$ domain,
- $\rightarrow$ multi-patch for z domain,
- $\rightarrow$ multi-patch for velocity domain.

Poisson

- $\rightarrow$ similar to 2D0V case.