MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Requirements for multipatch geometry

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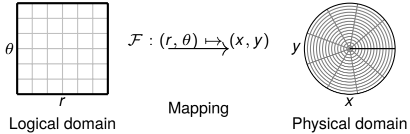
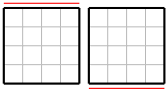
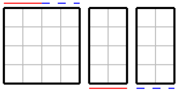
Structure of multi-patch

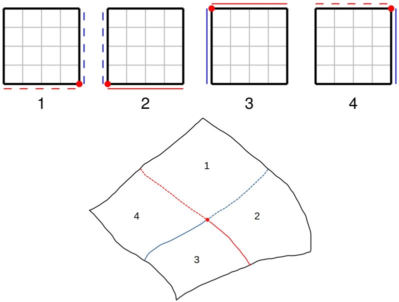




(a) Simple interface.



(b) T-joint.

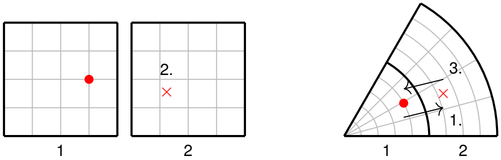


(c) X-point in logical and physical domain.

Patches in the logical domain.

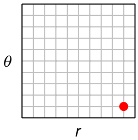
Advection

$$\partial_t \rho + A \cdot \nabla \rho = 0, \qquad \partial_t X(t^n; t^{n+1}, x) = A(t, X(t^n; t^{n+1}, x)). \tag{1}$$

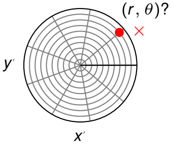


1. compute feet.
2. evaluate function (need to transfer feet to patch 2).
3. transfer value to patch 1.

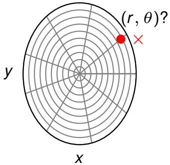
Example of a characteristic foot outside the patch 1 in the logical and physical domains.



Logical domain



Pseudo-Cartesian domain



Physical domain

Solving Poisson with CONGA

We want to solve the Poisson equation

$$-\operatorname{div}(\nu \operatorname{grad} \phi) = \rho$$

using a the CONGA approach on a 2D multipatch domain.

- Have finite element space V_h with jump discontinuities across edges
- Subspace $V_h^c \subseteq V_h$ with functions conforming to some global regularity constraint
- Define projection $P_h : V_h \to V_h^c$ and discrete differential operator $\operatorname{grad}_h := \operatorname{grad} P_h$ $\to$ need information on mesh and spline degree of neighboring patches
- Discretize Poisson equation weakly using these operators

Solving Poisson with CONGA

Advantages

- Already implemented in Psydac and successfully applied to several problems.
- Probably easier to generalize to complicated geometries than other approaches like using different splines e.g.

Patch data – Exchange

- Mesh-points,
- Local sums to compute the derivatives at the interfaces

$$\sum_{x_i \in \text{global space}} \alpha_i s(x_i) = \sum_{p \in \text{Patches}} \sum_{x_i \in p} \alpha_i s(x_i),$$

  where α_i depend on the mesh points,
- Characteristic feet outside of the patch,
- Interpolated values for **A** and ρ.

Patch data – Storage

- Mesh points, dimension (`DimXi`, `DimYi`), mapping, `SplineBuilder`, metadata,
- Boundary condition of global domain if an edge of the patch is on the global boundary,
- Values of functions ρ, ϕ, $\mathbf{A}$ on mesh points,
- Spline coefficients of functions (ρ, ϕ, $\mathbf{A}$),
- Reference to global domain class.

Global domain

- Global domain class
  - References to patches,
  - ’Connectivity’ class which encodes the geometrical information.
- Connectivity class
  - Identify edges and corners of different patches (do we need to identify corners of same patch e.g. when it closes on itself?)
  - For T-joint, identify sections of edges with sections of other edges, place corners in the middle of edges.

APPENDIX - Structure code

Global domain class

Define a global view of the domain.

- Reference to each Patch object.
- Global boundaries (outside boundaries).
- Reference to Interfaces object.
- Computation to find the patch where a given coordinate is?

Interfaces class

Define the interfaces between each patches.

- Reference to each Patch object.
- Define interfaces between each patches. (simple, T-joint, X-point).

Patch class

Define a patch.

- Dimensions `DimRi`, `DimPi`.
- Discrete domains and spline domains.
- Local mapping.

APPENDIX - Drift-kinetic equations

$$\begin{cases} \partial_t f + v_{GC} \cdot \nabla_\perp f + v_\| \partial_z f + \dot{v}_\| \partial_{v_\|} f = 0, \\ -\nabla_\perp \cdot (\alpha \nabla_\perp \phi) + \beta(\phi - \langle \phi \rangle) = n \end{cases} \tag{2}$$

Advection

$\rightarrow$ multi-patch for space $\perp$ domain,

$\rightarrow$ multi-patch for z domain,

$\rightarrow$ multi-patch for velocity domain.

Poisson

$\rightarrow$ similar to 2D0V case.