

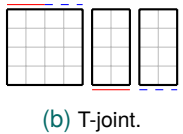
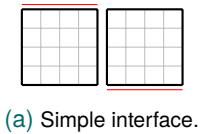
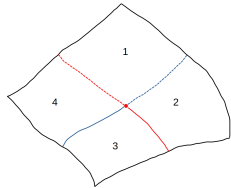
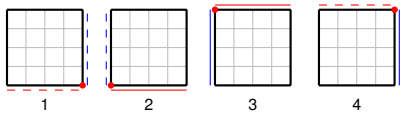
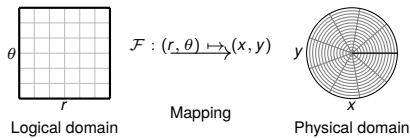


Requirements for multipatch geometry

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Structure of multi-patch



(c) X-point in logical and physical domain.

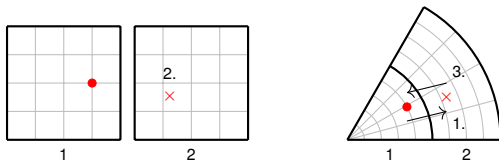
Advection



$$\partial_t \rho + \mathbf{A} \cdot \nabla \rho = 0,$$

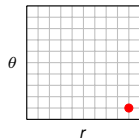
$$\partial_t X(t^n; t^{n+1}, x) = \mathbf{A}(t, X(t^n; t^{n+1}, x)).$$

(1)

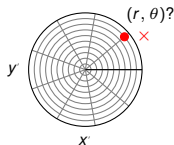


1. compute feet.
2. evaluate function (need to transfer feet to patch 2).
3. transfer value to patch 1.

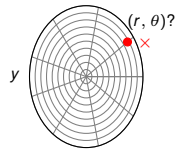
Example of a characteristic foot outside the patch 1 in the logical and physical domains.



Logical domain



Pseudo-Cartesian domain



Physical domain



Solving Poisson with CONGA

We want to solve the Poisson equation

$$-\operatorname{div}(\nu \operatorname{grad} \phi) = \rho$$

using the CONGA approach on a 2D multipatch domain.

- Have finite element space V_h with jump discontinuities across edges
- Subspace $V_h^c \subseteq V_h$ with functions conforming to some global regularity constraint
- Define projection $P_h : V_h \rightarrow V_h^c$ and discrete differential operator $\operatorname{grad}_h := \operatorname{grad} P_h$
- Discretize Poisson equation weakly using these operators

- Already implemented in Psydac and successfully applied to several problems.
- Probably easier to generalize to complicated geometries than other approaches like using different splines e.g.



Patch data – Exchange

- Mesh-points,
- Local sums to compute the derivatives at the interfaces

$$\sum_{x_i \in \text{global space}} \alpha_i \mathbf{s}(x_i) = \sum_{p \in \text{Patches}} \sum_{x_i \in p} \alpha_i \mathbf{s}(x_i),$$

- Characteristic feet outside of the patch,
- Interpolated values for \mathbf{A} and ρ .



Patch data – Storage

- Mesh points, dimension (DimXi , DimYi), mapping, `SplineBuilder`, metadata,
- Boundary condition of global domain if an edge of the patch is on the global boundary,
- Values of functions ρ , ϕ , \mathbf{A} on mesh points,
- Spline coefficients of functions (ρ , ϕ , \mathbf{A}),
- Reference to global domain class.



Global domain

- Global domain class
 - References to patches,
 - 'Connectivity' class which encodes the geometrical information.
- Connectivity class
 - Identify edges and corners of different patches (do we need to identify corners of same patch e.g. when it closes on itself?)
 - For T-joint, identify sections of edges with sections of other edges, place corners in the middle of edges.



APPENDIX - Structure code

GLOBAL DOMAIN CLASS

Define a global view of the domain.

- Reference to each Patch object.
- Global boundaries (outside boundaries).
- Reference to Interfaces object.
- Computation to find the patch where a given coordinate is?

INTERFACES CLASS

Define the interfaces between each patches.

- Reference to each Patch object.
- Define interfaces between each patches. (simple, T-joint, X-point).

PATCH CLASS

Define a patch.

- Dimensions DimRi, DimPi.
- Discrete domains and spline domains.
- Local mapping.



APPENDIX - Drif-kinetic equations

$$\begin{cases} \partial_t f + v_{GC} \cdot \nabla_{\perp} f + v_{\parallel} \partial_z f + \dot{v}_{\parallel} \partial_{v_{\parallel}} f = 0, \\ -\nabla_{\perp} \cdot (\alpha \nabla_{\perp} \phi) + \beta(\phi - \langle \phi \rangle) = n \end{cases} \quad (2)$$

Advection

- multi-patch for space \perp domain,
- multi-patch for z domain,
- multi-patch for velocity domain.

Poisson

- similar to 2D0V case.