1 Poisson problem on multipatch

We want to solve the Poisson equation

$$-\operatorname{div}(\nu \operatorname{grad} \phi) = \rho$$

using a the CONGA approach on a multipatch domain.

The CONGA approach has the main advantage that it uses patch-local degrees of freedom and basis functions. The coupling of adjacent patches happens through the use of projection operators onto conforming subspaces. This approach has been studied by Martin Campos Pinto and Yaman Güçlü in the context of FEEC. The projection operators are implemented in Psydac and have been successfully applied to different problems in electromagnetics.

Very short introduction to the CONGA approach

This short section is just a very short introduction of the ideas in our context. For more details see Campos Pinto, Güçlü (2024) - Broken-FEEC discretizations and Hodge Laplace problems and Güçlü, Hadjout, Campos Pinto (2023) - A Broken FEEC Framework for Electromagnetic Problems on Mapped Multipatch Domains

We define finite element spaces V_h and V_h^c. We assume that our domain Ω is given as a disjoint union of patches Ω_k, $k = 1, ..., K$. Let $V_h(\Omega_k)$ be the finite element space on Ω_k. Define the basis of V_h by taking the basis functions of $V_h(\Omega_k)$ and extending them by zero outside of Ω_k. We recognize that the functions of V_h can have jumps across patch boundaries and are thus not continuous. Hence, we call V_h *broken*. Let V be a subspace of $L^2(\Omega)$ with functions fulfilling some regularity requirement e.g. $V = H^1(\Omega)$.

We further define the conforming subspace

$$V_h^c := V_h \cap V$$

i.e. the functions in V_h with appropriate regularity. We then define projection operators

$$P_h : V_h \to V_h^c.$$

and the discrete differential operator

$$\operatorname{grad}_h := \operatorname{grad} P_h.$$

For an operator A defined on a subspace of $L^2(\Omega)$ we denote the adjoint w.r.t. the L^2 inner product as A^*, if it exists. Then we define the CONGA Laplacian operator as $\operatorname{grad}_h^*(\nu \operatorname{grad}_h)$ and introduce a stabilized version via

$$\operatorname{grad}_h^*(\nu \operatorname{grad}_h) + \alpha(I - P_h^*)(I - P_h)$$

where $\alpha > 0$ is some constant and I is the identity. This is discretized weakly by discretizing the bilinear form $a_h : V_h \times V_h \to \mathbb{R}$

$$a_h(\phi_h, \psi_h) = \int_\Omega \nu \operatorname{grad}_h \phi_h \cdot \operatorname{grad}_h \psi_h \, dx + \int_\Omega (I - P_h)\phi_h \, (I - P_h)\psi_h \, dx$$

Applying the CONGA approach

- **Question:** Does the advection field $\mathbf{A}$ have to be continuous? Emily thinks so. Then we need to enforce global C^1-regularity on $\phi \to$ Use CONGA approach with C^1 conforming projection P_h
- The conforming projection is computed by taking suitable averages of spline coefficients close to the boundary of the patch so it only acts on splines close to the edge
- Need the mesh information from neighboring patches to construct the projection matrices
- Spline coefficients of ρ needed to assemble the right hand side
- Poisson solver needs to access all spline coefficients from all patches $\to$ need global managing of the splines, mesh etc to assemble everything
- **Question:** Do we make the assembly of the stiffness matrix completely global?