Multi-patch

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March 2024

Contents

1 Structure for multipatch **3**
1.1 Type of global space splitting . . . 3

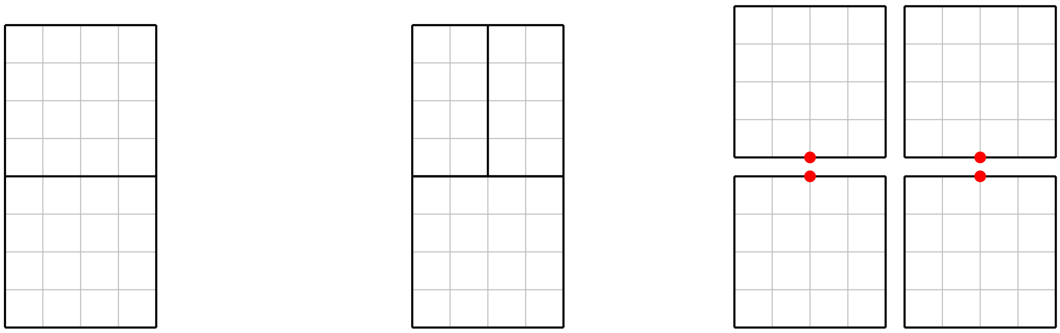
2 Equations system **3**
2.1 Advection equation . . . 3
2.2 Futher questions . . . 5

Contents

1 Structure for multipatch

- What should be done before September?
- What kind of information do we need to store in each patch?
- What kind of information do the patches need to exchange?

1.1 Type of global space splitting



(a) Simple interface. (b) T-joint. (c) X-point.

2 Equations system

2.1 Advection equation

In the guiding center equations example, the advection equation is given by

$$\partial_t \rho + A \cdot \nabla \rho = 0, \tag{1}$$

with A the advection field defined on the logical domain on the physical domain axis.

Computing the advection field. The advection field is computed from the solution of the Poisson equation. In the guiding center equations example,

$$A = -\nabla \phi \wedge e_z, \tag{2}$$

this computation can be done locally.

Store	• Spline representation (and values?) of ϕ. • Spline representation (and values?) of A. • Jacobian matrix.

Solving the characteristic equation. The equation to solve is

$$\begin{cases} \partial_t X(t; s, x) = A(t, X(t; s, x)), \\ X(s; s, x) = x. \end{cases} \tag{3}$$

we compute

$$\begin{cases} \partial_t X(t^n; t^{n+1}, x) = A(t, X(t^n; t^{n+1}, x)), \\ \hat{X}(t^n; t^{n+1}, x) = \bar{\psi}(t^n; t^{n+1}, x) \end{cases} \tag{4}$$

with a time integration method. In the case of Runge-Kutta method, the advection field needs to be evaluated at intermediate feet.

Example of RK2.

$$
\begin{aligned}
X^1 &= X^0 - \frac{dt}{2} A(X^0), \\
X^2 &= X^0 - dt A(X^1).
\end{aligned}
\tag{5}
$$

Outside the patch. In case of X^1 is outside the patch we are working on, we need to communicate with the other patches to evaluate the advection field. The same problem appears when we have solved the characteristic equation and want to evaluate the advected function ρ.

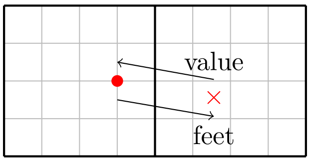
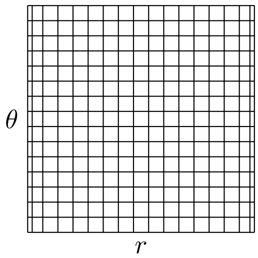


Figure 2: Feet outside the patch.

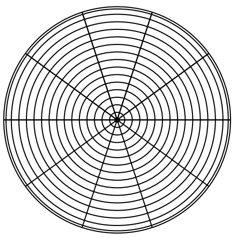
Exchange	• The outside feet (in the physical domain ?). • The evaluated value of the advection field. • The evaluated value of the advected function.
Store	• Spline representation (and values?) of the advected function ρ. • Spline representation (and values?) of the advection field A.

Further questions on outside feet. Problem: the mapping from the logical to the physical domain is not always invertible.

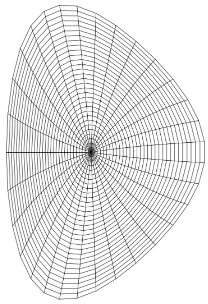
- For an advection in the physical domain, if the feet is outside of the domain, how to get the feet in the logical domain of another patch (could be a not invertible mapping for the other patch)?
- For an advection in the pseudo-Cartesian domain, if the feet is outside of the domain, what are the equivalent coordinate in the physical domain?



(a) Logical domain.



(b) Pseudo-Cartesian domain.



(c) Physical domain.

Build a spline representation of the advected function. To do it, we

- Evaluate the spline representation of ρ at the characteristic feet [**local but need to communicate**];
- Compute the derivatives at the interfaces of each patch. To do it, there are different methods:
  - advect the derivatives [**local method**];
  - compute the derivatives by:
    * using Lagrange polynomials [**global**];

    * using global spline relation [**global method**].

- Build the new spline representation [**local**].

Exchange	• The mesh points around the interfaces for Lagrange interpolation. • The value of ρ around the interfaces for Lagrange interpolation. • The sum of values of the function $\sum_i \alpha_i s(x_i) = \sum_{p \in patches} \sum_{x_i \in p} \alpha_i s(x_i)$.
Store	• Spline representation of the advected function ρ.

2.2 Futher questions

About evaluating funtions.

- How do we know in which patch we are?
    - For a given point in the physical domain, how do we know in which subset it belongs to?
- Let's assume we know in which patch we are, how can we evaluate a function? How can we get $\rho(x, y)$ with (x, y) in the physical domain (especially when the mapping isn't invertible)?

About the space splitting.

- Do we have the splitting of the space from another code?
- Do we have mappings?
- If we are using different mappings between the patches, how to stick correctly the patches at the edge? Are not risks of ill-covering (gaps or covering of two patches)?

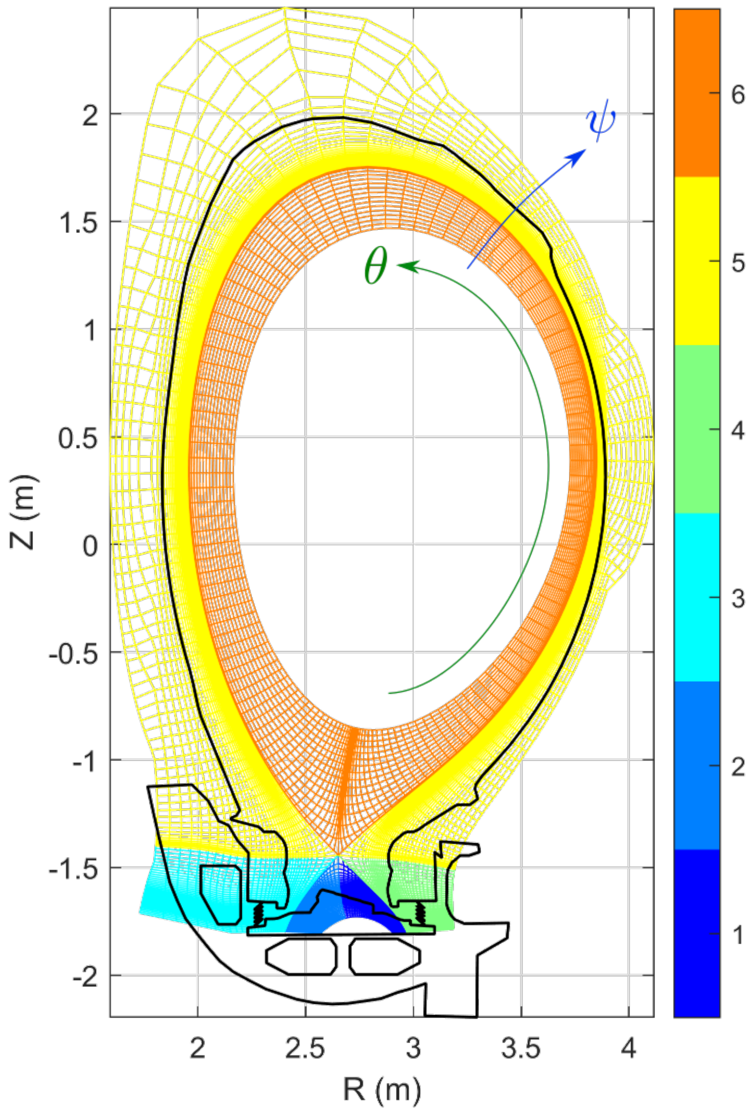


Figure 4: Multi-patch decomposition.

Poisson problem on multipatch

$$-\nabla \cdot \nu \nabla \phi = \rho$$

- **Question:** Does the advection field $\mathbf{A}$ have to be continuous? Emily thinks so. Then we need to enforce global C^1-regularity on ϕ
- Use CONGA approach with C^1 conforming projection $\rightarrow$ need the mesh information and spline coefficients from neighboring mesh to construct
- spline coefficients of ρ needed to assemble rhs
- Poisson solver needs to access all spline coefficients from all patches $\rightarrow$ need global managing of the splines, mesh etc to assemble everything
- **Question:** Do the mappings matter? It should still be local averages of spline coefficients to compute the projections...
- **Question:**Do we make the assembly of the stiffness matrix completely global?

What information do patches exchange?

- Mesh-points (Lagrange polynomials at interfaces, conforming projection)
- Spline coefficients (conforming projection (?))
- Local sums to compute

$$\sum \alpha_i s(x_i) = \sum_{\text{Patches}} \sum_{\text{mesh points}} \alpha_i s(x_i)$$

- Mapping information (?) (Conforming projection)
- Feet of characteristics (Advection)
- Interpolated values for $\mathbf{A}$ and ρ (Advection)
- **Question:** But how do we get those?

What information do patches store?

- mesh points, dimension, mapping (in what form?), `SplineBuilder`, metadata
- Boundary condition
- Values of functions ρ, ϕ, $\mathbf{A}$ on mesh points
- Values of ν on quadrature points (?)
- Spline coefficients of functions (ρ, ϕ, $\mathbf{A}$)
- Reference to global domain class

Global domain

- Global domain class (more details!)
- References to patches
- **Question:** What about the mappings? How are they represented? How regular are they? Where are they defined?
- **Question:** What patches should we use? Should the upper patch be split?
- identify edges and corners of different edges (Do we need to identify corners of same patch?)