Physics 111 Section 1
Problem Set 17
November 21, 2016

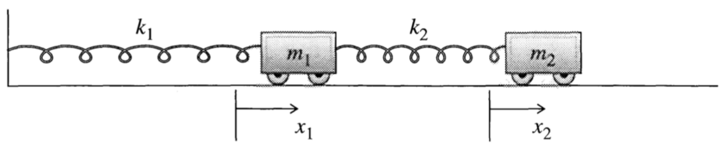
Oscillations: Springs (Reading: Chapter 11.1 - 11.2, 11.7)

Help:

i Spring-mass system with only one wall.

(a) Find the normal frequencies, ω_1 and ω_2, for the carts shown in Figure 11.15, assuming that $m_1 = m_2$ and $k_1 = k_2$.

(b) Find and describe the motion for each of the normal modes in turn.



ii A little practice with normal coordinates. In part (b), the general result you arrive at should match the result expressed in (11.21).

(a) Write down the equations of motion (11.2) for the equal mass carts of Section 11.2 with three identical springs. Show that the change of variables to the normal coordinates $\xi_1 = \frac{1}{2}(x_1 + x_2)$ and $\xi_2 = \frac{1}{2}(x_1 - x_2)$ leads to uncoupled equations for ξ_1 and ξ_2.

(b) Solve for ξ_1 and ξ_2 and hence write down the general solution for x_1 and x_2. (Notice how very simple this procedure is, once you have guessed what the normal coordinates are. For a symmetric system like this, you can sometimes guess the form of ξ_1 and ξ_2 by considering the symmetry - especially once you have some experience working with normal modes.)

iii Damped carts and normal coordinates. In part (c), don't forget to add the homogeneous solution, and in part (e) don't take the "prove" statement literally - strong argument is fine.

(a) Write down the equations of motion corresponding to (11.2) for the equal-mass carts of Section 11.2 with three identical springs, but with each cart subject to a linear resistive force $-b\vec{v}$.(same coefficient b for both carts) and with a driving force $F(t) = F_0 \cos \omega t$ applied to cart 1.

(b) Show that if you change variables to the normal coordinates $\xi_1 = \frac{1}{2}(x_1 + x_2)$ and $\xi_2 = \frac{1}{2}(x_1 - x_2)$, the equations of motion for ξ_1 and ξ_2 are uncoupled.

(c) Using the methods of Section 5.5, write down the general solutions.

(d) Assuming $\beta = b/2m \ll \omega_0$, show that ξ_1 resonates when $\omega \approx \omega_0 = \sqrt{k/m}$ and likewise ξ_2 when $\omega \approx \sqrt{3}\omega_0$.

(e) Prove, on the other hand, that if both carts are driven in phase with the same force $F_0 \cos \omega t$, only ξ_1 shows resonance. Explain.

ii Carts coupled by molasses.

Here is a different way to couple two oscillators. The two carts in Figure 11.16 have equal masses m (though different shapes). They are joined by identical but separate springs (force constant k) to separate walls. Cart 2 rides in cart 1, as shown, and cart 1 is filled with molasses, whose viscous drag supplies the coupling between the carts.

(a) Assuming that the drag force has magnitude $\beta m v$ where $\vec{v}$ is the relative velocity of the two carts, write down the equations of motion of the two carts using as coordinates x_1 and x_2, the displacements of the carts from their equilibrium positions. Show that they can be written in matrix form as $\ddot{\vec{x}} + \beta \underline{\vec{D}} \dot{\vec{x}} + {\omega_0}^2 x = 0$, where $\vec{x}$ is the 2×1 column made up of x_1 and x_2, $\omega_0 = \sqrt{k/m}$ and $\underline{\vec{D}}$ is a certain 2×2 square matrix.

(b) There is nothing to stop you from seeking a solution for the form $\vec{x}(t) = Re\vec{z}(t) = \vec{a}e^{rt}$. Show that you do indeed get two solutions of this form with $r = i\omega_0$ or $r = -\beta + i\omega_1$ where $\omega_1 = \sqrt{{\omega_0}^2 - \beta^2}$. (Assume that the viscous force is weak, so that $\beta < \omega_0$.)

(c) Describe the corresponding motions. Explain why one of these modes is damped but the other is not.