Physics 111 Section 1
Problem Set 15
November 7, 2016

Oscillations: Damped Systems (Reading: Chapter 5.1 - 5.4)

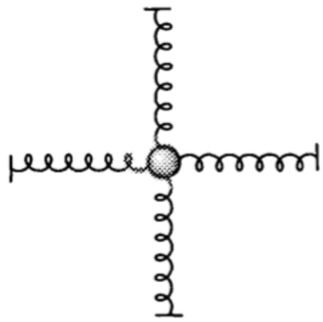
Help:

5.2 The potential energy of two atoms in a molecule can sometimes be approximated by the Morse function,

$$U(r) = A\left[\left(e^{(R-r)/S} - 1\right)^2 - 1\right]$$

where r is the distance between the two atoms and A, R, and S are positive constants with $S \ll R$. Sketch this function for $0 < r < \infty$. Find the equilibrium separation r_0, at which $U(r)$ is minimum. Now write $r = r_0 + x$ so that x is the displacement from equilibrium, and show that, for small displacements, U has the approximate form U = const + $\frac{1}{2}kx^2$. That is, Hooke's law applies. What is the force constant k?

5.19 Consider the mass attached to four identical springs, as shown in Figure 5.7(b). Each spring has force constant k and unstretched length l_0, and the length of each spring when the mass is at its equilibrium at the origin is a (not necessarily the same as l_0). When the mass is displaced a small distance to the point (x, y), show that its potential energy has the form $\frac{1}{2}k'r^2$ appropriate to an isotropic harmonic oscillator. What is the constant k' in terms of k? Give an expression for the corresponding force.



5.25 Consider a damped oscillator with $\beta < \omega_0$. There is a little difficulty defining the "period" τ_1 since the motion (5.38) is not periodic. However, a definition that makes sense is that τ_1 is the time between successive maxima of $x(t)$.

(a) Make a sketch of $x(t)$ against t and indicate this definition of τ on your graph. Show that $\tau_1 = 2\pi/\omega_1$.

(b) Show that an equivalent definition is that τ_1 is twice the time between successive zeros of $x(t)$. Show this one on your sketch.

(c) If $\beta = \omega_0/2$, by what factor does the amplitude shrink in one period?

5.27 As the damping on an oscillator is increased there comes a point when the name ”oscillator” seems barely appropriate.

(a) To illustrate this, prove that a critically damped oscillator can never pass through the origin $x = 0$ more than once.

(b) Prove the same for an overdamped oscillator.