Physics 111 Section 1
Problem Set 13
October 31, 2016

Rotational Motion: Euler's Equations (Reading: Chapter 10.6 - 10.8)

Help:

i Angular momentum and kinetic energy in torque-free rotation

10.40

(a) A rigid body is rotating freely, subject to zero torque. Use Euler's equations (10.88) to prove that the magnitude of the angular momentum $\vec{L}$ is constant. (Multiply the ith equation by $L_i = \lambda_i \omega_i$ and add the three equations.)

(b) In much the same way, show that the kinetic energy of rotation $T_{\text{rot}} = \frac{1}{2}(\lambda_1 {\omega_1}^2 + \lambda_2 {\omega_2}^2 + \lambda_3 {\omega_3}^2)$, as in (10.68), is constant.

ii An accelerating, rotating space station.

10.44 An axially symmetric space station (principal axis $\vec{e}_3$, and $\lambda_1 = \lambda_2$) is floating in free space. It has rockets mounted symmetrically on either side that are firing and exert a constant torque Γ about the symmetry axis. Solve Euler's equations exactly for $\vec{\omega}$ (relative to the body axis) and describe the motion. At $t = 0$ take $\vec{\omega} = (\omega_{10}, 0, \omega_{30})$.

iii Rate of precession in the space frame, Ω_s.

10.46 We saw in Section 10.8 that in the free precession of an axially symmetric body the three vectors $\vec{e}_3$ (the body axis), $\vec{\omega}$, and $\vec{L}$ lie in a plane. As seen in the body frame, $\vec{e}_3$ is fixed, and $\vec{\omega}$ and $\vec{L}$ precess around $\vec{e}_3$ with angular velocity $\Omega_b = \omega_3(\lambda_1 - \lambda_3)/\lambda_1$. As seen in the space frame $\vec{L}$ is fixed and $\vec{\omega}$ and $\vec{e}_3$ precess around $\vec{L}$ with angular velocity Ω_s. In this problem you will find three equivalent expressions for Ω_s.

(a) Argue that $\vec{\Omega}_s = \vec{\Omega}_b + \vec{\omega}$. [Remember that relative angular velocities add like vectors.]

(b) Bearing in mind that $\vec{\Omega}_b$ is parallel to $\vec{e}_3$ prove that $\Omega_s = \omega \sin\alpha / \sin\theta$ where α is the angle between $\vec{e}_3$ and $\vec{\omega}$ and θ is that between $\vec{e}_3$ and $\vec{L}$ (see Figure 10.9).

(c) Thence prove that

$$\Omega_s = \omega \frac{\sin\alpha}{\sin\theta} = \frac{L}{\lambda_1} = \omega \frac{\sqrt{{\lambda_3}^2 + ({\lambda_1}^2 - {\lambda_3}^2)\sin^2\alpha}}{\lambda_1}$$

iv Chandler wobble on a hypothetical Earth.

10.47 Imagine that this world is perfectly rigid, uniform, and spherical and is spinning about sits usual axis at its usual rate. A huge mountain of mass 10^{-8} earth masses is now added at colatitude $60°$, causing the earth to begin the free precession described in Section 10.8. How long will it take the North Pole (defined as the northern end of the diameter along $\vec{\omega}$) to move 100 miles from its current position? [Take the earth's radius to be 4000 miles.]