

Rotational Motion: Euler's Equations

Help:

i Angular momentum and kinetic energy in torque-free rotation

10.40

- (a) A rigid body is rotating freely, subject to zero torque. Use Euler's equations (10.88) to prove that the magnitude of the angular momentum \vec{L} is constant. (Multiply the i th equation by $L_i = \lambda_i \omega_i$ and add the three equations.)
- (b) In much the same way, show that the kinetic energy of rotation $T_{\text{rot}} = \frac{1}{2}(\lambda_1 \omega_1^2 + \lambda_2 \omega_2^2 + \lambda_3 \omega_3^2)$, as in (10.68), is constant.



ii An accelerating, rotating space station.

10.44 An axially symmetric space station (principal axis \vec{e}_3 , and $\lambda_1 = \lambda_2$) is floating in free space. It has rockets mounted symmetrically on either side that are firing and exert a constant torque Γ about the symmetry axis. Solve Euler's equations exactly for $\vec{\omega}$ (relative to the body axis) and describe the motion. At $t = 0$ take $\vec{\omega} = (\omega_{10}, 0, \omega_{30})$.



iii Rate of precession in the space frame, Ω_s .

10.46 We saw in Section 10.8 that in the free precession of an axially symmetric body the three vectors \vec{e}_3 (the body axis), $\vec{\omega}$, and \vec{L} lie in a plane. As seen in the body frame, \vec{e}_3 is fixed, and $\vec{\omega}$ and \vec{L} precess around \vec{e}_3 with angular velocity $\Omega_b = \omega_3(\lambda_1 - \lambda_3)/\lambda_1$. As seen in the space frame \vec{L} is fixed and $\vec{\omega}$ and \vec{e}_3 precess around \vec{L} with angular velocity Ω_s . In this problem you will find three equivalent expressions for Ω_s .

- (a) Argue that $\vec{\Omega}_s = \vec{\Omega}_b + \vec{\omega}$. [Remember that relative angular velocities add like vectors.]
- (b) Bearing in mind that $\vec{\Omega}_b$ is parallel to \vec{e}_3 prove that $\Omega_s = \omega \sin \alpha / \sin \theta$ where α is the angle between \vec{e}_3 and $\vec{\omega}$ and θ is that between \vec{e}_3 and \vec{L} (see Figure 10.9).
- (c) Thence prove that

$$\Omega_s = \omega \frac{\sin \alpha}{\sin \theta} = \frac{L}{\lambda_1} = \omega \frac{\sqrt{\lambda_3^2 + (\lambda_1^2 - \lambda_3^2) \sin^2 \alpha}}{\lambda_1}$$



iv Chandler wobble on a hypothetical Earth.

10.47 Imagine that this world is perfectly rigid, uniform, and spherical and is spinning about its usual axis at its usual rate. A huge mountain of mass 10^{-8} earth masses is now added at colatitude 60° , causing the earth to begin the free precession described in Section 10.8. How long will it take the North Pole (defined as the northern end of the diameter along $\vec{\omega}$) to move 100 miles from its current position? [Take the earth's radius to be 4000 miles.]

