Physics 111 Section 1
Problem Set 02
September 7, 2016

Hamilton's Principle; The Lagrangian and Unconstrained Systems (Reading: Chapter 7.1)

Help:

7.4 A mass moving along a horizontal plane held at an angle α with respect to the horizontal. Consider a mass m moving in a frictionless plane that slopes at an angle α with the horizontal. Write down Lagrangian in terms of coordinates x, measured horizontally across the slope, and y, measured down the slope. (Treat the system as two-dimensional, but include the gravitational potential energy.) Find the two Lagrange equations and show that they are what you should have expected.

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7.8 Two masses connected by a spring in 1D. In this problem you'll find that the center-of-mass motion of the system is unimportant (there are no external forces), and that the thing we care about is the relative motion of the two particles. This is a great introduction to the central force problems we'll study in chapter 8.

1. Write down the Lagrangian $\mathcal{L}(x_1, x_2, \dot{x}_1, \dot{x}_2)$ for two particles of equal masses $m_1 = m_2 = m$, confined to the x axis and connected by a spring with potential energy $U = \frac{1}{2}kx^2$. [Here x is the extension of the spring, $x = (x_1 - x_2 - l)$, where l is the spring's unstretched length, and I assume that mass 1 remains to the right of mass 2 at all times.]

2. Rewrite $\mathcal{L}$ in terms of the new variables $X = \frac{1}{2}(x_1 + x_2)$ (the CM postion) and x (the extension),and write down the two Lagrange equations for X and x.

3. Solve for $X(t)$ and $x(t)$ and describe the motion.

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