Physics 111 Section 1
Problem Set 06
September 28, 2016

Energy & Conservative Forces

Help:

i Gravity on Planet X.

4.7 Near to the point where I am standing on the surface of Planet X, the gravitational force on a mass m is vertically down but has magnitude $m\gamma y^2$ where γ is a constant and y is the mass's height above the horizontal ground.

(a) Find the work done by gravity on a mass m moving from $\vec{\mathbf{r}}_1$ to $\vec{\mathbf{r}}_2$, and use your answer to show that gravity on Planet X, although most unusual, is still conservative. find the corresponding potential energy.

(b) Still on the same planet, I thread a bead on a curved, frictionless, rigid wire, which extends from ground level to a height h above the ground. Show clearly in a picture the forces on the bead when it is somewhere on the wire. (Just name the forces so it's clear what they are; don't worry about their magnitude.) Which of the forces are conservative and which are not?

(c) If I release the bead from rest at a height h, how fast will it be going when it reaches the ground?

ii Assume that $\nabla \times \vec{\mathbf{F}} = 0$. With your knowledge of calculus, including Stokes' theorem, show that this statement is equivalent to writing the following:

(a) $\vec{\mathbf{F}} = -\vec{\nabla} U$

(b) $\int_1^2 \vec{\mathbf{F}} \cdot \vec{\mathbf{ds}} =$ path independent

(c) $\oint \vec{\mathbf{F}} \cdot \vec{\mathbf{ds}} = 0$

iii Time-dependent forces and their relationship to conservative forces.

4.27 Suppose that the force $\vec{\mathbf{F}}(\vec{\mathbf{r}}, t)$ depends on the time t but still satisfies $\nabla \times \vec{\mathbf{F}} = 0$. It is a mathematical fact (related to Stokes' theorem as discussed in Problem 4.25) that the work integral $\int_1^2 \vec{\mathbf{F}}(\vec{\mathbf{r}}, t) \cdot d\vec{\mathbf{r}}$ (evaluated at any one time t) is independent of the path taken between the points 1 and 2. Use this to show that the time-dependent PE defined by (4.48), for any fixed time t, has the claimed property that $\vec{\mathbf{F}}(\vec{\mathbf{r}}, t) = -\nabla U(\vec{\mathbf{r}}, t)$. Can you see what goes wrong with the argument leading to Equation (4.19), that is, conservation of energy?