

# Turing Machines

# Alan Turing

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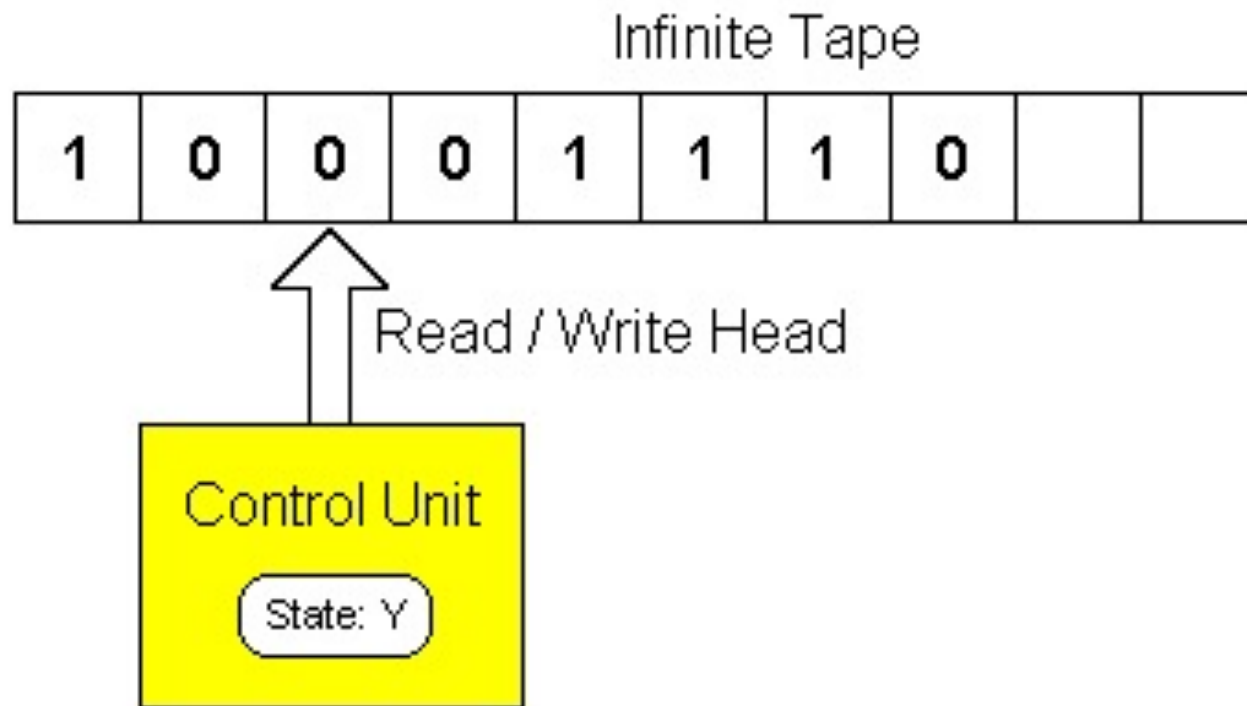
- Enigma (criptography)
- Turing test
- Turing machine (1937)



# Turing Machine

- Mathematical model for computation
- Abstract machine
- Can simulate any algorithm

# Turing Machine



- Input band (infinite)
- Reading head
- Control Unit: states
- Transitions / moves

# Turing machine – definition

7-tuple  $M = (Q, \Gamma, b, \Sigma, \delta, q_0, F)$  where:

- $Q$  – finite set of states
- $\Gamma$  - alphabet (finite set of band symbols)
- $b \in \Gamma$  - blank (symbol)
- $\Sigma \subseteq \Gamma \setminus \{b\}$  – input alphabet
- $\delta : (Q \setminus F) \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$  – transition function
- $q_0 \in Q$  – initial state
- $F \subseteq Q$  – set of final states

L = left  
R = right

# Example – palindrome over $\{0,1\}$

- 001100, 00100, 101101 a.s.o. accepted
- 00110, 1011 a.s.o. not accepted

001100

# Example – palindrome over $\{0,1\}$

	0	1	b
$q_0$	$(p_1, b, R)$	$(p_2, b, R)$	$(q_f, b, R)$
$p_1$	$(p_1, 0, R)$	$(p_1, 1, R)$	$(q_1, b, L)$
$p_2$	$(p_2, 0, R)$	$(p_2, 1, R)$	$(q_2, b, L)$
$q_1$	$(q_r, b, L)$		$(q_f, b, R)$
$q_2$		$(q_r, b, L)$	$(q_f, b, R)$
$q_r$	$(q_r, 0, L)$	$(q_r, 1, L)$	$(q_0, b, R)$
$q_f$			

Delete 0 in left side;  
search 0 in right side

Delete 1 in left side;  
search 1 in right side

On right is 0 or 1?

Shift right

$q_1$  and  $q_2$  – process 0 and  
1 on the right

$q_f$  – final state

0110

0	1	1	0	
	1	1	0	
	1	1	0	
	1	1	0	
	1	1	0	
	1	1	0	
	1	1	0	

	1	1		
	1	1		
	1	1		
	1	1		
	1	1		
		1		

...



$(q_0, \underline{0}11\underline{0}) \mid - (p_1, \underline{1}1\underline{0}) \mid - (p_1, 1\underline{1}\underline{0})$

$\mid - (p_1, 11\underline{0}) \mid - (p_1, 11\underline{0}\underline{b}) \mid - (q_1, 11\underline{0})$

$\mid - (q_r, 1\underline{1}) \mid - (q_r, \underline{1}1) \mid - (q_r, \underline{b}11)$

$\mid - (q_0, \underline{1}1) \mid - \dots$

	0	1	b
$q_0$	$(p_1, b, R)$	$(p_2, b, R)$	$(q_f, b, R)$
$p_1$	$(p_1, 0, R)$	$(p_1, 1, R)$	$(q_1, b, L)$
$p_2$	$(p_2, 0, R)$	$(p_2, 1, R)$	$(q_2, b, L)$
$q_1$	$(q_r, b, L)$		$(q_f, b, R)$
$q_2$		$(q_r, b, L)$	$(q_f, b, R)$
$q_r$	$(q_r, 0, L)$	$(q_r, 1, L)$	$(q_0, b, R)$
$q_f$			

<https://turingmachinesimulator.com>