ECON 4190: Industrial Organization **Fall 2021**

Homework 5

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Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
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3.2)

4.1) 1. We start off by maximizing Firm 1's profit:

$$max_{p_1}\pi_1 = (a - 2p_1 + p_2)(p_1 - 0)$$
$$\frac{\partial\pi_1}{\partial p_1} = a - 4p_1 + p_2 = 0$$
$$\therefore p_1 = \frac{a + p_2}{4}$$

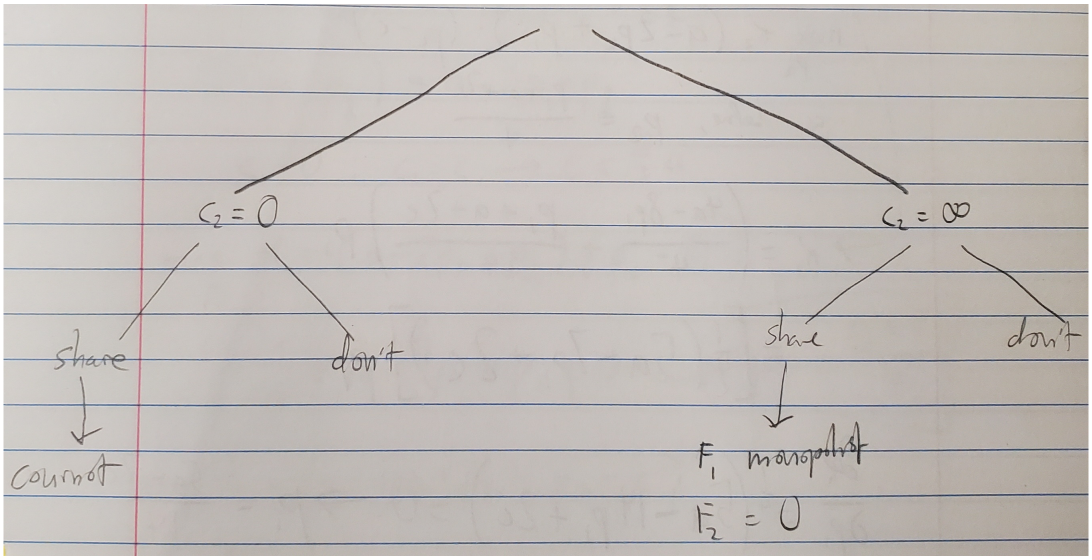
Then, maximize Firm 2's profit to solve for p_2, and then plug in p_1 to solve for both:

$$max_{p_2}\pi_2 = (a - 2p_2 + p_1)(p_2 - c)$$
$$\frac{\partial\pi_2}{\partial p_2} = (a - 2p_2 + p_1) + (p_2 - c)(-2) = 0$$
$$0 = a - 2p_2 + p_1 - 2p_2 + 2c$$
$$\therefore p_2 = \frac{p_1 + a + 2c}{4} = \frac{\frac{a+p_2}{4} + a + 2c}{4}$$
$$16p_2 = a + p_2 + 4a + 8c$$
$$\therefore p_2 = \frac{5a + 8c}{15}$$
$$\therefore p_1 = \frac{5 + 2a}{15}$$

2.

3.

4.3) 1. The game tree here can be (roughly) visualized as follows:



In scenarios where Firm 2 discloses its cost, Firm 1's response is very simple - when $c_2 = 0$, the firms have equal costs and will thus reach the Cournot equilibrium outcome, where $q_1 = q_2 = \frac{1}{3}$, and $\pi_1 = \pi_2 = (1 - q - q) * q = \frac{1}{9}$. If c_2 is prohibitively high, then Firm 1 will simply act as a monopolist, producing $q = \frac{1}{2}$ with a profit of $\frac{1}{4}$, whereas Firm 2 will have a quantity produced and profit of 0.

When Firm 2 doesn't disclose its cost, Firm 1 can only maximize its profit function using an expected value for q_2. Since the probability of both cases is $p = 0.5$, $E(p_2)$ can be determined simply by halving the quantity Firm 2 produces in the Cournot equilibrium, so $E(q_2) = \frac{1-q_1}{2} * \frac{1}{2} = \frac{1-q_1}{4}$. Given this value, we can simply derive Firm 1's best response as follows by maximizing their expected profit:

$$
\begin{aligned}
max_{q_1} E(\pi_1) &= (1 - q_1 - E(q_2)) * (q_1) \\
\frac{\partial E(\pi_1)}{\partial q_1} &= 1 - 2q_1 - E(q_2) = 0 \\
\therefore q_1^* &= \frac{1 - E(q_2)}{2} \\
&= \frac{1}{2} * (1 - \frac{1 - q_1}{4}) \\
8q_1 &= 4 - 1 + q_1 \\
q_1 &= \frac{3}{7} \\
\therefore q_2 &= \frac{2}{7}
\end{aligned}
$$

With these quantities, we can easily determine the profits to be $\pi_1 = (1 - \frac{3}{7} - \frac{2}{2*7}) * \frac{3}{7} = \frac{9}{49}$ (plugging back into our expected value function) and $\pi_2 = (1 - \frac{3}{7} - \frac{2}{7}) * \frac{2}{7} = \frac{4}{49}$ (since Firm 1 is always in the market, Firm 2 doesn't need to use expected values).

Solution: With information sharing, the outcome will either resemble that of Cournot duopoly if $c_2 = 0$ or monopoly by Firm 1 if c_2 is prohibitively high. Without information sharing, the equilibrium is $q_1 = \frac{3}{7}, q_2 = \frac{2}{7}, \pi_1 = \frac{9}{49}, \pi_2 = \frac{4}{49}$

2. If c_2 is prohibitively high, then Firm 2 doesn't care about sharing its information because its profit is 0 either way. Otherwise, if $c_2 = 0$, Firm 2 has an incentive to share its cost as $\frac{1}{9} > \frac{4}{49}$ i.e. their profit is higher in the Cournot duopoly equilibrium than the non-sharing equilibrium.
3. If Firm 2 shares information, then Firm 1's expected profit is $\frac{1}{2}(\frac{1}{4} + \frac{1}{9}) = \frac{13}{72}$. This is less than the expected profit of $\frac{9}{49}$s when information is not shared, so Firm 1 is worse off in information sharing.

4) (a) In general, the profit function for a monopoly (colluding firms) is as follows:

$$\pi^m = P(Q)Q - c(Q)$$

In this case, $P(Q) = a - bQ$ and $c(Q) = cQ$, so:

$$\begin{aligned} \pi^m &= (a - bQ)Q - cQ \\ FOC_{wrt.Q} : 0 &= a - 2bQ^m - c \\ Q^m &= \frac{a - c}{2b} \end{aligned}$$

Firm i's profit function in a 2-firm Cournot model with the given inverse demand and cost function is as follows:

$$\pi_i = (a - b(q_1 + q_2))q_i - cq_i$$

So, the firms' profit functions are as follows:

$$\begin{aligned} \pi_1 &= (a - b(q_1 + q_2))q_1 - cq_1 \\ \pi_2 &= (a - b(q_1 + q_2))q_2 - cq_2 \end{aligned}$$

It is easily seen here that optimizing firm 1's profits will give a similar expression to optimizing firm 2's profits, just with q_1 and q_2 interchanged. Let us optimize firm 1:

$$\begin{aligned} FOC_{wrt.q_1} : 0 &= -bq_1^c + (a - b(q_1^c + q_2)) - c \\ 0 &= a - 2bq_1^c - bq_2 - c \\ q_1^c &= \frac{a - bq_2 - c}{2b} \end{aligned}$$

We may interchange q_1 and q_2 to find q_2^c:

$$q_2^c = \frac{a - bq_1 - c}{2b}$$

To find the Cournot equilibrium, we can plug in firm 2's best response function into firm 1's best response function and solve for q_1^c as a function of constants. Again, we see here that the process for solving for q_1^c as a function of constants is similar to the process for solving for q_2^c as a function of constants, just with q_1^c and q_2^c flipped. Plugging q_2^c into firm 1's best response function, we have:

$$\begin{aligned}
q_1^c &= \frac{a - b(\frac{a-bq_1^c-c}{2b}) - c}{2b} \\
q_1^c &= \frac{a}{2b} - \frac{a}{4b} + \frac{q_1^c}{4} + \frac{c}{4b} - \frac{c}{2b} \\
\frac{3}{4}q_1^c &= \frac{a}{4b} - \frac{c}{4b} \\
q_1^c &= \frac{a-c}{3b}
\end{aligned}$$

We may interchange q_1^c and q_2^c to find q_2^c as a function of constants:

$$\begin{aligned}
q_2^c &= \frac{a-c}{3b} \\
Q &= q_1 + q_2 \\
Q^c &= q_1^c + q_2^c \\
Q^c &= \frac{2(a-c)}{3b} \\
Q^m &= \frac{a-c}{2b} \\
\frac{2(a-c)}{3b} &> \frac{a-c}{2b} \\
Q^c &> Q^m \blacksquare
\end{aligned}$$

(b)

(c) $MC(q_i) = cq_i$, so the profit function for firm i is as follows:

$$\pi_i = (a - b(q_1 + q_2))q_i - \frac{cq_i^2}{2}$$

We can find q_1^c and q_2^c using the same techniques that we used in 4.a:

$$\begin{aligned}
\pi_1 &= (a - b(q_1 + q_2))q_1 - \frac{cq_1^2}{2} \\
FOC_{wrt.q_1} : 0 &= -bq_1^c + (a - b(q_1^c + q_2)) - cq_1^c \\
0 &= a - (2b + c)q_1^c - bq_2 \\
q_1^c &= \frac{a - bq_2}{2b + c}
\end{aligned}$$

By symmetry, we have:

$$q_2^c = \frac{a - bq_1^c}{2b + c}$$

Again, we plug q_2^c into q_1^c to get q_1^c as a function of constants:

$$
\begin{aligned}
q_1^c &= \frac{a - b(\frac{a-bq_1^c}{2b+c})}{2b+c} \\
q_1^c &= \frac{a}{2b+c} - \frac{ab}{(2b+c)^2} + \frac{b^2 q_1^c}{(2b+c)^2} \\
(1 - \frac{b^2}{(2b+c)^2})q_1^c &= \frac{a(b+c)}{(2b+c)^2} \\
((2b+c)^2 - b^2)q_1^c &= a(b+c) \\
q_1^c &= \frac{a(b+c)}{3b^2+4bc+c^2}
\end{aligned}
$$

By symmetry, we have:

$$
\begin{aligned}
q_2^c &= \frac{a(b+c)}{3b^2+4bc+c^2} \\
Q^c &= q_1^c + q_2^c \\
\therefore Q^c &= \frac{2a(b+c)}{3b^2+4bc+c^2}
\end{aligned}
$$

(d) Now that c_1 isn't necessarily equal to c_2, the monopoly (collusion) profit function is as follows:

$$\pi^m = (a - b(q_1+q_2))(q_1+q_2) - (c_1 q_1 + c_2 q_2)$$