

We allow collaboration on homework assignments, and **encourage you to work in study groups of at most 4 students**. You should turn one **assignment** for each study group. We will **not accept late assignments**.

**Homework 4 (3 Exercises)**  
**Handed Out on 9/17/2021**  
**Due on 9/23/2021**

1) There is one product, say a soup, and is being offered at two locations of a city, which only has one road. The city is represented by the interval  $[0,1]$ . Along the line is distributed uniformly a mass 1 of consumers ( $M=1$ ). Then, the density of consumers at each location is 1 (loosely, there is 1 consumer at each location). Each consumer wishes to buy at most one unit.

FIRM 1 is located at 0. FIRM 2 is located at 1.

The willingness to pay for the good produced at the two restaurants is the same and equal to  $R$ .

The consumer can also not buy any soup, and the utility in that case would be 0.

The transportation cost for a consumer located at  $x$  who travels is  $STx$  per unit of distance if traveling from 0 to  $x$ . The transportation cost for a consumer located at  $x$  who travels is  $ST(1-x)$  per unit of distance if traveling from  $x$  to 1.

The production costs for FIRM 1 is  $c_1$  and  $c_2$  for FIRM 2. The two firms can charge different prices,  $p_1$  and  $p_2$ .

Assume the market is fully covered.

- a) (5 points) (Go over what we did in class. It is also covered at pages 50, 51) Find the Nash equilibrium of the simultaneous move Bertrand game [or: Find the Bertrand-Nash Equilibrium in prices.]
  - b) (10 points) What are the prices that the firms were to choose if they agreed to collude?
  - c) (5 points) Is collusion sustainable?
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2). Apple's iPod has been the portable MP3-player of choice among many gadget enthusiasts. Suppose that Apple has a constant marginal cost of 4 and that market demand is given by  $Q = 200 - 2P$ .

- a) If Apple is a monopolist, find its optimal price and output. What are its profits?
- b) Now suppose there is a competitive fringe of 12 price-taking firms, *each* of which has a total cost function  $TC(q) = 3q^2 + 20q$  with corresponding marginal cost curve  $MC = 6q + 20$ . Find the supply function of the fringe (*Hint: A competitive firm supplies along its marginal cost curve above its shutdown point*).
- c) If Apple operates as the dominant firm facing competition from the fringe in this market, now what is its optimal output? How many units will fringe providers sell? What is the market price, and how much profit does Apple earn?
- d) Graph your answer to part (c).

3). Britney produces pop music albums with the total cost function  $TC(Q) = 8Q$ . Market demand for pop music albums is  $P = 56 - Q$ . Suppose there is a competitive fringe of price-taking pop music artists, with total supply function  $Q_{\text{fringe}} = 2P - y$ , where  $y > 0$  is some positive integer. If Britney behaves like a dominant firm and maximizes her profit by selling at a price of  $P = 16$ , find (i) the value of  $y$ , (ii) Britney's output level, and (iii) the output level of the competitive fringe.