ECON 4190: Industrial Organization **Fall 2021**

Homework 6

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Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
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1. (a) Firms on the competitive fringe are price-takers, so they take P as given and produce a quantity where $MC = P$. In this case, $P = MC = 10 + 50q_i$, so an individual fringe firm will produce $q_i = \frac{1}{50}P - \frac{1}{10}$. Since there are 100 firms, the total supply for the competitive fringe $Q_f = 100q_i = 2P - 20$. Note that this only holds if $P > 10$, as otherwise this model implies that firms would produce a negative quantity (when in reality they would shut down and produce nothing).

   Solution: The supply curve for the competitive fringe is $Q_f = 2P - 20$ where $P > 10$, otherwise 0.

   (b) To solve for residual demand, all we need to do is take market demand $Q_m = 400 - 8P$ and subtract Q_f from it. Since the residual demand is the demand for the dominant firm, we will call it $Q_d = 400 - 8P - (2P - 20) = 420 - 10P$.
   Solution: Residual demand faced by the dominant firm is $Q_d = 420 - 10P$

   (c) Given the residual demand, we can simply solve this like any other monopolist problem, where the objective is to maximize the dominant firm's profit π_d by setting price:

$$
\begin{aligned}
max p \pi_d &= (420 - 10P)(P - 10) \\
\frac{\partial \pi}{\partial p} &= (420 - 10P) + (-10)(P - 10) \\
0 &= 420 - 10P - 10P + 100 \\
P &= 520/20 = 26
\end{aligned}
$$

   Now that we know P, we can solve for everything else quite trivially; $Q_d = 420 - 10(26) = 160, Q_f = 2(26) - 20 = 32$, and the market shares for the dominant firm and the fringe are $\frac{160}{160+32} = \frac{5}{6}$ and $\frac{1}{6}$ respectively.
   Solution: $P = 26, Q_d = 160, Q_f = 32$, and market shares for dominant firm and fringe are $\frac{5}{6}$ and $\frac{1}{6}$ respectively.

   (d) Recall that $Q_d = 420 - 10P$, or equivalently, $P = 42 - \frac{1}{10}Q$. Considering that now Q is equal to $q_1 + q_2$ (the combined total that firms 1 and 2 produce), we can solve for both firms' profit-maximizing quantity via symmetry:

$$\begin{aligned}
max_{q_1}\pi_1 &= (P-c)(q_1) \\
&= (42 - \frac{1}{10}q_1 - \frac{1}{10}q_2 - 10)(q_1) \\
\frac{\partial \pi_1}{\partial q_1} &= 32 - \frac{2}{10}q_1 - \frac{1}{10}q_2 = 0 \\
q_1 &= \frac{10}{2}(32 - \frac{1}{10}q_2) \\
&= 160 - \frac{1}{2}q_2 \\
\therefore q_2 &= 160 - \frac{1}{2}q_1 \\
\therefore q_1 &= 160 - \frac{1}{2}(160 - \frac{1}{2}q_1) \\
&= 80 + \frac{1}{4}q_1 \\
q_1 = q_2 &= \frac{4}{3} * 80 = \frac{320}{3} \\
\therefore Q_d &= \frac{640}{3} \approx 213.33
\end{aligned}$$

Plugging into the inverse residual demand function, we get $P = 42 - \frac{1}{10} * \frac{640}{3} = \frac{126-64}{3} = \frac{62}{3} \approx 20.667$. Recalling the fringe demand $Q_f = 2P - 20$, we can see that $Q_f = \frac{124}{3} - \frac{60}{3} = \frac{64}{3} \approx 21.333$. Fringe market share is $\frac{64}{3}/\frac{64+640}{3} = \frac{1}{11}$.

NOTE: posted solutions technically don't line up with this but I'm pretty sure there's a typo. Waiting on response from prof

Solution: In Cournot, $q_1 = q_2 = \frac{320}{3} \therefore Q_d = \frac{640}{3} \approx 213.333$ setting market price to $P = \frac{62}{3} \approx 20.667$, fringe produces $Q_f = \frac{64}{3} \approx 21.333$ achieving a market share of $\frac{1}{11}$ to the duopoly's market share of $\frac{10}{11}$.

2. (a) Let q_1 and q_2 refer to X and Y as defined in the problem, and Q to represent the combined total produced. We then solve for the quantities produced by maximiz-

ing firm 1's profit function (which will give us both quantities via symmetry):

$$\begin{aligned}
max_{q_1}\pi_1 &= (280 - 2q_1 - 2q_2 - 40) * q_1 \\
\frac{\partial \pi_1}{\partial q_1} &= 240 - 4q_1 - 2q_2 = 0 \\
q_1 &= 60 - \frac{1}{2}q_2 \\
\therefore q_2 &= 60 - \frac{1}{2}q_1 \\
\therefore q_1 &= 60 - \frac{1}{2}(60 - \frac{1}{2}q_1) \\
&= 30 + \frac{1}{4}q_1 \\
\therefore q_1 &= 40 \\
\therefore q_2 &= 60 - 40/2 = 40
\end{aligned}$$

Solving for P and then profits is simply a matter of plugging in the solved quantities: $P = 280 - 2(40 + 40) = 120, \pi_1 = \pi_2 = (120 - 40)(40) = 3200$.

Solution: In Cournot equilibrium, $q_1 = q_2 = 40, P = 120, \pi_1 = \pi_2 = 3200$.

(b) From part (a), we already know firm 2's reaction function $q_2 = 60 - \frac{1}{2}q_2$. Only firm 1's reaction function changes here, and we solve for it as such:

$$\begin{aligned}
max_{q_1}\pi_1 &= (280 - 2q_1 - 2(60 - \frac{1}{2}q_2) - 40) * q_1 \\
&= (120 - q_1)(q_1) \\
\frac{\partial \pi_1}{\partial q_1} &= 120 - 2q_1 = 0 \\
\therefore q_1 &= 60 \\
\therefore q_2 &= 60 - 60/2 = 30
\end{aligned}$$

We then use these quantities to solve for P and profits: $P = 280 - 2(60 + 30) = 100, \pi_1 = (100 - 40)(60) = 3600, \pi_2 = (100 - 40)(30) = 1800$

Solution: In Stackelberg equilibrium with firm 1 choosing first, $q_1 = 60, q_2 = 30, P = 100, \pi_1 = 3600, \pi_2 = 1800$