ECON 4190: Industrial Organization **Fall 2021**

Homework 2

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Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
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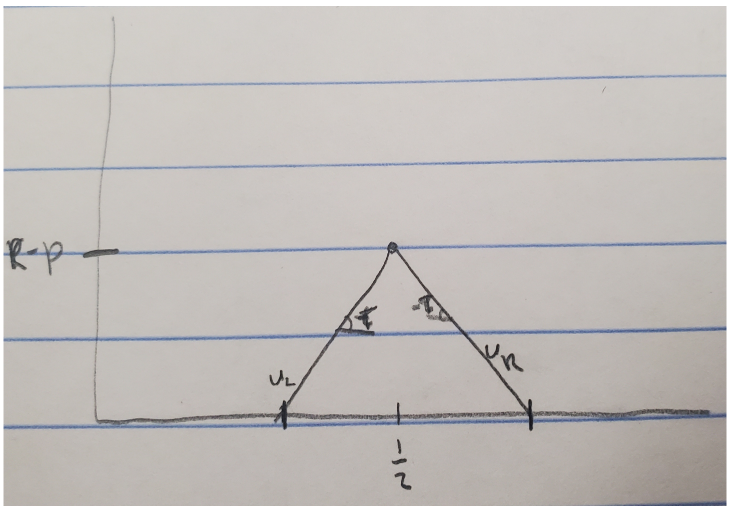
a. A profit-maximizing firm will seek to max $\pi(Q) = PQ - C(Q)Q$, which we can reduce to $\pi(Q) = PQ$ because Firm 1 has no costs. Since Firm 1 has control over its price P, we first attempt to calculate $Q(P)$, or the quantity of consumers that will buy at a given price. To do this we can graph the consumer's utility against location, which yields this:



We represent the utility of all consumers to the "left" of Firm 1 (i.e. $x < \frac{1}{2}$) with v_l and all consumers to the "right" as v_r. They can be represented by the following equations:

$$v_l = R_1 - p + T(x - \frac{1}{2})$$
$$v_r = R_1 - p - T(x - \frac{1}{2})$$

To find Q, we would multiply the fraction of the market that is covered by the total mass M, but since $M = 1$ in this case, we simply have to find the distance between the x-intercepts of v_r and v_l: $\hat{x_r}$ and $\hat{x_l}$.

First solve for $\hat{x_r}$ and $\hat{x_l}$:

$$v_l = 0 = R_1 - p + T(x - \frac{1}{2}) \Leftrightarrow \hat{x_l} = \frac{p - R_1}{T} + \frac{1}{2}$$
$$v_r = R_1 - p - T(x - \frac{1}{2}) \Leftrightarrow \hat{x_r} = \frac{R_1 - p}{T} + \frac{1}{2}$$

Then find the difference:

$$
\begin{aligned}
Q(p) = \hat{x}_r - \hat{x}_l &= \frac{R_1 - p}{T} + \frac{1}{2} - (\frac{p - R_1}{T} + \frac{1}{2}) \\
&= \frac{R_1 - p}{T} - \frac{p - R_1}{T} \\
&= \frac{2R_1 - 2p}{T} \\
Q(p) &= \frac{2}{T}(R_1 - p)
\end{aligned}
$$

Now we return to the profit function and maximize it to find the maximum price:

$$
\begin{aligned}
max_p \pi = p * \frac{2}{T}(R_1 - p) &= \frac{2R_1 p}{T} - \frac{2p^2}{T} \\
\frac{\partial \pi}{\partial p} = \frac{2R_1}{T} - \frac{4p}{T} &= 0 \\
\therefore p^* = \frac{R_1}{2}
\end{aligned}
$$

Knowing price makes it easy to calculate maximum profit:

$$
\begin{aligned}
\pi^* = PQ &= P * \frac{2}{T}(R_1 - P) \\
&= \frac{R_1}{2}\frac{2}{T}(R_1 - \frac{R_1}{2}) \\
&= \frac{R_1}{T} * \frac{R_1}{2} \\
\therefore \pi^* &= \frac{R_1^2}{2T}
\end{aligned}
$$

We also need to address the edge case where the market is completely covered; in that case, instead of using our previous derivations, we can use a much simpler form: since we know that the consumers at 0 and 1 are indifferent, we can use $v_l(0) = 0$ to solve for p^* in this case (which also gives us profit, because Q would be exactly 1):

$$
\begin{aligned}
v_l(0) = R_1 - p + T(-\frac{1}{2}) &= 0 \\
p^* &= R_1 - \frac{T}{2} \\
\pi^* &= R_1 - \frac{T}{2}
\end{aligned}
$$

Then Firm 1 would simply pick whichever scenario (uncovered vs. covered market) has a higher profit based on the parameters.

Solution: $p^* = \frac{R_1}{2}$**,** $\pi^* = \frac{R_1^2}{2T}$ **when** $\frac{R_1^2}{2T} > R_1 - \frac{T}{2}$**, otherwise** $p^* = \pi^* = R_1 - \frac{T}{2}$

b.