

We allow collaboration on homework assignments, and **encourage you to work in study groups of at most 4 students**. You should turn one **assignment** for each study group. We will **not** accept late assignments.

Homework 3 (2 Exercises)
Handed Out on 9/9/2021
Due on 9/16/2021

1. Consider the problem of political parties on the Left-Right spectrum.

Denote the placement of party i as $l_i \in [0, 1]$, where $l_i = 0$ indicates that the party i has chosen to be the to the most left of the left wing and $l_i = 1$ indicates that the party has chosen to be to the most right of the right wing.

The voter political views are uniformly distributed between 0 and 1. There are $M = 1$ voters. So, there is one voter at each location $x \in [0, 1]$.

Voters vote for the political party that is closer to them and, if two parties are equally close, they randomize.

One way to think of it is that the utility that a voter gets from a party is equal to

$$v_i = r_i - T|x - l_i|.$$

T represents the transportation cost.

There are no prices, and r_1 and r_2 are the reservation values ('willingness to pay for that candidate winning the contest') for the candidates. r_1 is the reservation value for the left wing candidate; and r_2 is the reservation value for the right wing candidate.

(The parties have no ideologies, they just place themselves where they get the most votes.)

Suppose that the right wing candidate is located at $7/8$. And the left wing candidate is located at $1/4$.

Assume that the parameters values of T , r_1 , and r_2 are such that the 'market is fully covered' and that both candidates will get some votes.

- a) (5 points) Find the share of votes that each party receives – express it as a function of T , r_1 , and r_2 .

Now, suppose that the utility that a voter gets from a party is equal to $v_i = r_i - T(x - l_i)^2$.

- b) (5 points) Find the share of votes that each party receives. Is it different from a)?

Now, consider again the case where $v_i = r_i - T|x - l_i|$.

- c) (5 points) Suppose that the voter political views are **not** uniformly distributed between 0 and 1 distribution is not uniform, but consider the following scenarios:
- There are $1/8$ of the voters to the left of the point $1/4$; and $1/8$ of the voters to the right of the point $7/8$. Think of these two groups as the “political base” on the left and the political base on the right.
 - The rest of the voters, $3/4$ are uniformly distributed between $1/4$ and $7/8$.

Find the share of votes that each party receives – express it as a function of T , r_1 , and r_2 .

- d) (5 points) Replicate the analysis of c), but now assume that there are $1/3$ of the voters to the left of point $1/4$, $1/3$ of the voters to the right of point $7/8$, and $1/3$ between $1/4$ and $7/8$. How important is to have a large political base?
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2. (5 points each question) The Salop Model. We go over the model seen in class again.

Consider a circle city, around a lake, with a circumference equal to 1.

There is a Market Size of consumer equal to 1.

Consumers are uniformly distributed, so there is a (density of) 1 consumer per location.

There are n firms equidistantly located along the circle city.

Assume that the markets are fully covered. (This will simplify the analysis).

The utility of each consumer at each location x from consuming from firm i is given by $v_i = r - p_i - \tau|x - l_i|$.

- Determine the demand function $Q_i(p_i, p_{i-1}, p_{i+1})$ of firm i , which is located between firm $i-1$ and firm $i+1$.
- Suppose that all firms except firm i are charging the same price p . What does the demand function reduce to?
- Consider the firm i , and consider the consumers that buy from that firm vs buying from firm $i-1$ or $i+1$. What is the total consumer surplus for those consumers?

- d) Consider again question c) above, but now assume that all firms charge the same price p . What would be the total consumer surplus for the consumers who buy from firm i ? And what would be the total consumer surplus in the city?