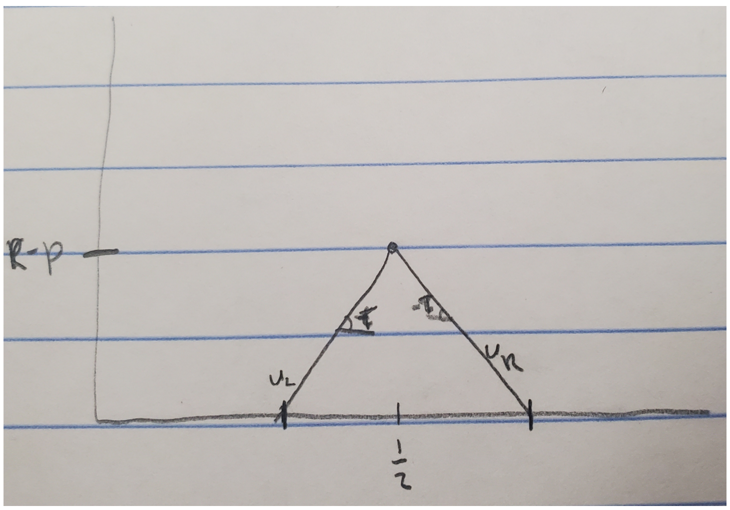
ECON 4190: Industrial Organization	**Fall 2021**

Homework 2

Collaborators: Alex Shen (as5gd), Sean Velhagen (spv5hq), Max Bresticker (mtb9sex)

Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
Signature: *Alex Shen, Sean Velhagen, Max Bresticker*

a. A profit-maximizing firm will seek to max $\pi(Q) = PQ - C(Q)Q$, which we can reduce to $\pi(Q) = PQ$ because Firm 1 has no costs. Since Firm 1 has control over its price P, we first attempt to calculate $Q(P)$, or the quantity of consumers that will buy at a given price. To do this we can graph the consumer's utility against location, which yields this:



We represent the utility of all consumers to the "left" of Firm 1 (i.e. $x < \frac{1}{2}$) with u_l and all consumers to the "right" as u_r. They can be represented by the following equations:

$$u_l = R_1 - p + T(x - \frac{1}{2})$$
$$u_r = R_1 - p - T(x - \frac{1}{2})$$

To find Q, we would multiply the fraction of the market that is covered by the total mass M, but since $M = 1$ in this case, we simply have to find the distance between the x-intercepts of u_r and u_l: x_r and x_l.

First solve for x_r and x_l:

$$u_l = 0 = R_1 - p + T(x - \frac{1}{2}) \Leftrightarrow x_l = \frac{p - R_1}{T} + \frac{1}{2}$$
$$u_r = R_1 - p - T(x - \frac{1}{2}) \Leftrightarrow x_r = \frac{R_1 - p}{T} + \frac{1}{2}$$

Then find the difference:

$$\begin{aligned}
Q(p) = x_r - x_l &= \frac{R_1 - p}{T} + \frac{1}{2} - (\frac{p - R_1}{T} + \frac{1}{2}) \\
&= \frac{R_1 - p}{T} - \frac{p - R_1}{T} \\
&= \frac{2R_1 - 2p}{T} \\
Q(p) &= \frac{2}{T}(R_1 - p)
\end{aligned}$$

Now we return to the profit function and maximize it to find the maximum price:

$$\begin{aligned}
max_p \pi = p * \frac{2}{T}(R_1 - p) &= \frac{2R_1 p}{T} - \frac{2p^2}{T} \\
\frac{\partial \pi}{\partial p} = \frac{2R_1}{T} - \frac{4p}{T} &= 0 \\
\therefore p^* = \frac{R_1}{2}
\end{aligned}$$

Knowing price makes it easy to calculate maximum profit:

$$\begin{aligned}
\pi^* &= PQ = P * \frac{2}{T}(R_1 - P) \\
&= \frac{R_1}{2}\frac{2}{T}(R_1 - \frac{R_1}{2}) \\
&= \frac{R_1}{T} * \frac{R_1}{2} \\
\therefore \pi^* &= \frac{R_1^2}{2T}
\end{aligned}$$

Note on an edge case: our calculations are only valid as long as $x_l > 0$ and $x_r < 1$, as by definition all customers are between 0 and 1. Therefore, we need to determine a constraint that makes sure this is true.

$$\begin{aligned}
x_l &> 0 \\
\frac{p - R_1}{T} + \frac{1}{2} &> 0 \\
\frac{\frac{R_1}{2} - R_1}{T} &> -\frac{1}{2} \\
-\frac{R_1}{2} &> -\frac{T}{2} \\
R_1 &< T
\end{aligned}
\qquad\qquad
\begin{aligned}
x_r &< 1 \\
\frac{R_1 - p}{T} + \frac{1}{2} &< 1 \\
\frac{R_1 - \frac{R_1}{2}}{T} &< \frac{1}{2} \\
\frac{R_1}{2} &< \frac{T}{2} \\
R_1 &< T
\end{aligned}$$

Solution: $p^* = \frac{R_1}{2}$**,** $\pi^* = \frac{R_1^2}{2T}$ **when** $R_1 < T$