ECON 4190: Industrial Organization **Fall 2021**

Homework 4

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Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
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1.

2. (a) Given $Q = 200 - 2P$, we know that $P = 100 - \frac{1}{2}Q$. If Apple is a monopolist with $MC = 4$, its profit can be determined by $\pi(Q) = (P - C)Q = (100 - \frac{1}{2}Q - 4)(Q)$. Thus, the first order condition for this equation is $\frac{d\pi}{dP} = 96 - Q = 0$, meaning that $Q^M = 96$. Plugging this value back into our previous equations gives us $P^M = 52$ and $\pi^M = 96 * (52 - 4) = 4608$.

   Solution: $Q^M = 96,\ P^M = 52,\ \pi^M = 4608$

   (b) Since a competitive firm supplies along its MC curve (meaning that $p = MC$, which we know to be true in perfect competition), to solve for a given firm's quantity produced, q, we simply substitute in price and invert their MC function to get $q = \frac{p-20}{6}$. Since there are 12 identical firms in the fringe, the fringe supply is in total $12 * (\frac{p-20}{6}) = 2p - 40$. Importantly, we have to also add the constraint that this curve only holds when $p \geq 20$, as otherwise the firm would be producing a negative quantity which is impossible.

   Solution: $Q_{fringe} = 2p - 40$ if $p \geq 20$, otherwise 0

   (c) We first find the residual demand for Apple's product (Q_{Apple}) by subtracting the quantity the fringe will produce (Q_{fringe}) at a given price from the total market demand (Q_{market}) at a given price.

$$\begin{aligned} Q_{Apple} &= Q_{market} - Q_{fringe} \\ &= 200 - 2P - (2P - 40) \\ &= 240 - 4P \\ \therefore P &= 60 - \frac{1}{4}Q_{Apple} \end{aligned}$$

   We then maximize the resulting profit function to determine Q_{Apple}:

$$\begin{aligned} max_{Q_{Apple}}\pi &= (60 - \frac{1}{4}Q_{Apple} - 4) * Q_{Apple} \\ \frac{d\pi}{dQ_{Apple}} &= 56 - \frac{1}{2}Q_{Apple} = 0 \\ \therefore Q_{Apple} &= 56 * 2 = 112 \end{aligned}$$

   From there, everything else is straightforward - plug in Q_{Apple} to find $P = 60 - \frac{112}{4} = 32$, which then gives us $Q_{fringe} = 2(32) - 40 = 24$ and $\pi_{Apple} = (56 - \frac{112}{4}) * 112 = 3136$.

   Solution: $Q_{Apple} = 112,\ P = 32,\ Q_{fringe} = 24,\ \pi_{Apple} = 3136$

(d) TODO: graph 2C

3. From $TC(Q) = 8Q$, we know that $MC = 8$. Total quantity demanded by the market is $Q = 56 - P$, so subtracting Q_{fringe} to find Britney's residual demand gives us $Q_B = 56 - P - (2P - y) = 56 - 3P + y$. We then maximize her derived profit function as follows to find the relationship between y and P^*:

$$\begin{aligned}
max_p \pi_B &= (56 - 3P + y) * (p - 8) \\
FOC : \frac{d\pi}{dp} &= (56 - 3P + y) - (-3)(p - 8) = 0 \\
0 &= 56 - 3P + y - 3P + 24 \\
0 &= 80 - 6P + y \\
y &= 6P - 80
\end{aligned}$$

Since we know that $P = 16$, simple substitution yields that $y = 6 * 16 - 80 = 16$, which gives us $Q_B = 56 - 3 * 16 + 16 = 24$ and $Q_{fringe} = 2 * 16 - 16 = 16$.

Solution: $y = 16$, $Q_B = 24$, $Q_{fringe} = 16$