ECON 4190: Industrial Organization **Fall 2021**

Homework 4

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Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
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1. (a) As usual, we find $\hat{x}$, the indifferent consumer between Firms 1 and 2. Since $M = 1$, this will also give us Q_1, the quantity produced by Firm 1, and therefore $1 - \hat{x} = Q_2$, the quantity produced by Firm 2. Since the value for a consumer at $\hat{x}$ must be the same for both, assuming that the market is fully covered we can simply use the utility equation to solve for $\hat{x}$ as such:

$$\begin{aligned}
R - p_1 - \tau\hat{x} &= R - p_2 - \tau(1 - \hat{x}) \\
p_2 - p_1 + 1 &= 2\tau\hat{x} \\
\hat{x} &= \frac{p_2 - p_1}{2\tau} + \frac{1}{2}
\end{aligned}$$

Using $\hat{x}$ as Q_1, we maximize Firm 1's profit equation to solve for its reaction function for p_1, which will then give us Firm 2's reaction function for p_2 by symmetry:

$$\begin{aligned}
max_{p_1}\pi_1 &= (\frac{p_2 - p_1}{2\tau} + \frac{1}{2})(p_1 - c_1) \\
\frac{\partial \pi}{\partial p_1} &= (-\frac{1}{2\tau})(p_1 - c) + \frac{p_2 - p_1}{2\tau} + \frac{1}{2} = 0 \\
0 &= \frac{p_2 - p_1 - (p_1 - c_1)}{2\tau} + \frac{1}{2} \\
&= \frac{p_2 - 2p_1 + c}{2\tau} + \frac{1}{2} \\
\therefore p_1^* &= \frac{p_2 + c_1 + \tau}{2} \\
\therefore p_2^* &= \frac{p_1 + c_2 + \tau}{2}
\end{aligned}$$

Plugging Firm 2's reaction function into Firm 1's allows us to solve for p_1^* and then p_2^*:

$$\begin{aligned}
p_1 &= \frac{\frac{p_1 + c_2 + \tau}{2} + c_1 + \tau}{2} \\
4p_1 &= p_1 + c_2 + \tau + 2c_1 + 2\tau \\
p_1^* &= \tau + \frac{2c_1 + c_2}{3} \\
\therefore p_2^* &= \tau + \frac{2c_2 + c_1}{3}
\end{aligned}$$

Solution: In Bertrand-Nash equilibrium, $p_1^* = \tau + \frac{2c_1+c_2}{3}$, $p_2^* = \tau + \frac{2c_2+c_1}{3}$

(b) When colluding, the firms will set prices such that the indifferent consumer receives a utility of 0, which minimizes total consumer surplus (thus maximizing producer surplus/profit). Plugging back the calculated $\hat{x}$ into our value functions gives us the following:

$$R - p_1 - \tau(\frac{p_2 - p_1}{2\tau} + \frac{1}{2}) = 0$$

$$R - p_2 - \tau(1 - ((\frac{p_2 - p_1}{2\tau} + \frac{1}{2}))) = 0$$
$$R - p_2 - \tau(\frac{p_1 - p_2}{2\tau} + \frac{1}{2}) = 0$$

Given the symmetry we see in these equations, we can conclude that $p_1 = p_2$. This also makes sense intuitively both because this sets the indifferent consumer point exactly between them, and means they split the profit evenly. In any case, redoing our equation with $p_1 = p_2$ gives us $p_1 = R - \frac{\tau}{2} = p_2$.

Solution: If colluding, Firms 1 and 2 will both set their price to $R - \frac{\tau}{2}$.

(c) To verify sustainability, we simply assume that a given firm (say, Firm 2) is using that price, use the reaction function for the other firm (Firm 1 in this case) to see what its best response is, and see if the value matches the collusion price.

$$\begin{aligned} p_1^*(p_2) &= \frac{p_2 + c_1 + \tau}{2} \\ p_1^*(R - \frac{\tau}{2}) &= \frac{R - \frac{\tau}{2} + c_1 + \tau}{2} \\ &= \frac{R + c_1}{2} + \frac{\tau}{4} \end{aligned}$$

This is obviously not the same thing as the collusion price, so Firm 1 will deviate from colluding. Because of symmetry, we also know that Firm 2 will deviate for similar reasons. Thus, collusion is not sustainable.

Solution: No, collusion is not sustainable.

2. (a) Given $Q = 200 - 2P$, we know that $P = 100 - \frac{1}{2}Q$. If Apple is a monopolist with $MC = 4$, its profit can be determined by $\pi(Q) = (P - C)Q = (100 - \frac{1}{2}Q - 4)(Q)$. Thus, the first order condition for this equation is $\frac{d\pi}{dP} = 96 - Q = 0$, meaning that $Q^M = 96$. Plugging this value back into our previous equations gives us $P^M = 52$ and $\pi^M = 96 * (52 - 4) = 4608$.

Solution: $Q^M = 96$, $P^M = 52$, $\pi^M = 4608$

(b) Since a competitive firm supplies along its MC curve (meaning that $p = MC$, which we know to be true in perfect competition), to solve for a given firm's quantity produced, q, we simply substitute in price and invert their MC function to get $q = \frac{p-20}{6}$. Since there are 12 identical firms in the fringe, the fringe supply is in total $12 * (\frac{p-20}{6}) = 2p - 40$. Importantly, we have to also add the constraint that this curve only holds when $p \geq 20$, as otherwise the firm would be producing a negative quantity which is impossible.

Solution: $Q_{fringe} = 2p - 40$ if $p \geq 20$, otherwise 0

(c) We first find the residual demand for Apple's product (Q_{Apple}) by subtracting the quantity the fringe will produce (Q_{fringe}) at a given price from the total market demand (Q_{market}) at a given price.

$$\begin{aligned} Q_{Apple} &= Q_{market} - Q_{fringe} \\ &= 200 - 2P - (2P - 40) \\ &= 240 - 4P \\ \therefore P &= 60 - \frac{1}{4} Q_{Apple} \end{aligned}$$

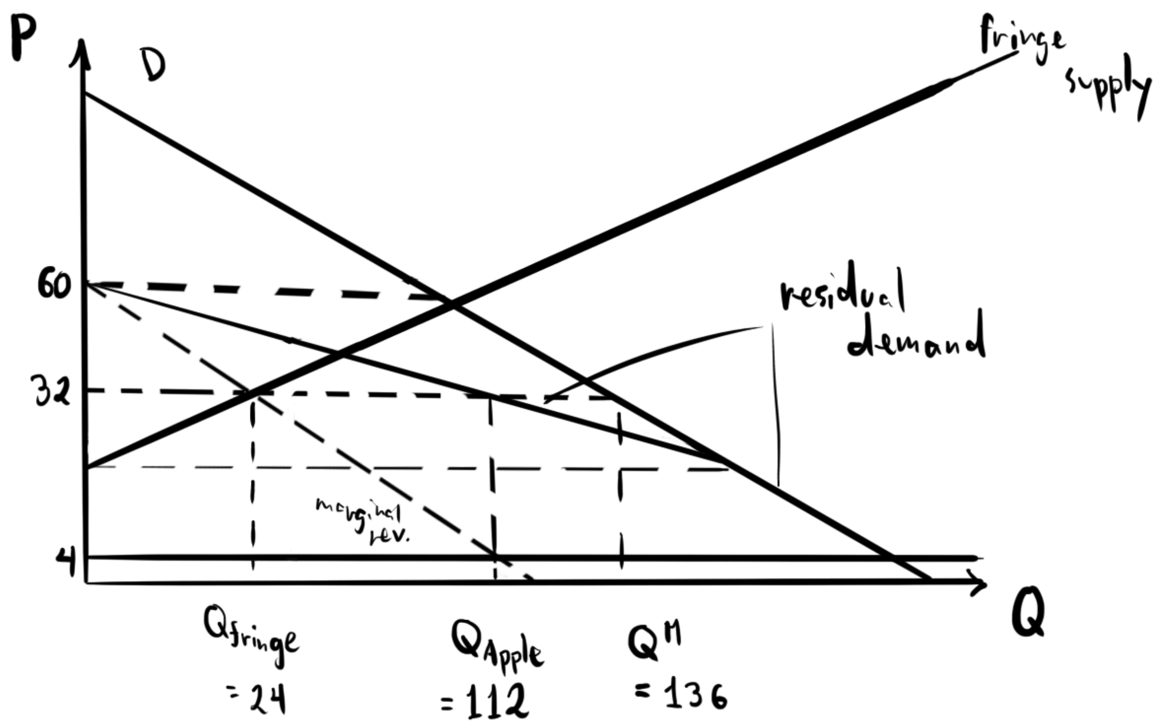
We then maximize the resulting profit function to determine Q_{Apple}:

$$\begin{aligned} max_{Q_{Apple}} \pi &= (60 - \frac{1}{4} Q_{Apple} - 4) * Q_{Apple} \\ \frac{d\pi}{dQ_{Apple}} &= 56 - \frac{1}{2} Q_{Apple} = 0 \\ \therefore Q_{Apple} &= 56 * 2 = 112 \end{aligned}$$

From there, everything else is straightforward - plug in Q_{Apple} to find $P = 60 - \frac{112}{4} = 32$, which then gives us $Q_{fringe} = 2(32) - 40 = 24$ and $\pi_{Apple} = (56 - \frac{112}{4}) * 112 = 3136$.

Solution: $Q_{Apple} = 112$, $P = 32$, $Q_{fringe} = 24$, $\pi_{Apple} = 3136$

(d) Graph as shown:



3. From $TC(Q) = 8Q$, we know that $MC = 8$. Total quantity demanded by the market is $Q = 56 - P$, so subtracting Q_{fringe} to find Britney's residual demand gives us $Q_B = 56 - P - (2P - y) = 56 - 3P + y$. We then maximize her derived profit function as follows to find the relationship between y and P^*:

$$
\begin{aligned}
max_p \pi_B &= (56 - 3P + y) * (p - 8) \\
FOC : \frac{d\pi}{dp} &= (56 - 3P + y) - (-3)(p - 8) = 0 \\
0 &= 56 - 3P + y - 3P + 24 \\
0 &= 80 - 6P + y \\
y &= 6P - 80
\end{aligned}
$$

Since we know that $P = 16$, simple substitution yields that $y = 6 * 16 - 80 = 16$, which gives us $Q_B = 56 - 3 * 16 + 16 = 24$ and $Q_{fringe} = 2 * 16 - 16 = 16$.

Solution: $y = 16$, $Q_B = 24$, $Q_{fringe} = 16$