ECON 4190: Industrial Organization **Fall 2021**

Homework 6

Collaborators: Alex Shen (as5gd), Sean Velhagen (spv5hq), Max Bresticker (mtb9sex)

Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
Signature: *Alex Shen, Sean Velhagen, Max Bresticker*

1. (a) Firms on the competitive fringe are price-takers, so they take P as given and produce a quantity where $MC = P$. In this case, $P = MC = 10 + 50q_i$, so an individual fringe firm will produce $q_i = \frac{1}{50}P - \frac{1}{10}$. Since there are 100 firms, the total supply for the competitive fringe $Q_f = 100q_i = 2P - 20$. Note that this only holds if $P > 10$, as otherwise this model implies that firms would produce a negative quantity (when in reality they would shut down and produce nothing).

   Solution: The supply curve for the competitive fringe is $Q_f = 2P - 20$ where $P > 10$, otherwise 0.

   (b) To solve for residual demand, all we need to do is take market demand $Q_m = 400 - 8P$ and subtract Q_f from it. Since the residual demand is the demand for the dominant firm, we will call it $Q_d = 400 - 8P - (2P - 20) = 420 - 10P$.
   Solution: Residual demand faced by the dominant firm is $Q_d = 420 - 10P$

   (c) Given the residual demand, we can simply solve this like any other monopolist problem, where the objective is to maximize the dominant firm's profit π_d by setting price:

$$
\begin{aligned}
maxp\pi_d &= (420 - 10P)(P - 10) \\
\frac{\partial \pi}{\partial p} &= (420 - 10P) + (-10)(P - 10) \\
0 &= 420 - 10P - 10P + 100 \\
P &= 520/20 = 26
\end{aligned}
$$

   Now that we know P, we can solve for everything else quite trivially; $Q_d = 420 - 10(26) = 160, Q_f = 2(26) - 20 = 32$, and the market shares for the dominant firm and the fringe are $\frac{160}{160+32} = \frac{5}{6}$ and $\frac{1}{6}$ respectively.

   Solution: $P = 26, Q_d = 160, Q_f = 32$, and market shares for dominant firm and fringe are $\frac{5}{6}$ and $\frac{1}{6}$ respectively.

   (d) Recall that $Q_d = 420 - 10P$, or equivalently, $P = 42 - \frac{1}{10}Q$. Considering that now Q is equal to $q_1 + q_2$ (the combined total that firms 1 and 2 produce), we can solve for both firms' profit-maximizing quantity via symmetry:

$$
\begin{aligned}
max_{q_1}\pi_1 &= (P-c)(q_1)\\
&= (42-\frac{1}{10}q_1-\frac{1}{10}q_2-10)(q_1)\\
\frac{\partial \pi_1}{\partial q_1} &= 32-\frac{2}{10}q_1-\frac{1}{10}q_2=0\\
q_1 &= \frac{10}{2}(32-\frac{1}{10}q_2)\\
&= 160-\frac{1}{2}q_2\\
\therefore q_2 &= 160-\frac{1}{2}q_1\\
\therefore q_1 &= 160-\frac{1}{2}(160-\frac{1}{2}q_1)\\
&= 80+\frac{1}{4}q_1\\
q_1=q_2 &= \frac{4}{3}*80=\frac{320}{3}\\
\therefore Q_d &= \frac{640}{3}\approx 213.33
\end{aligned}
$$

Plugging into the inverse residual demand function, we get $P=42-\frac{1}{10}*\frac{640}{3}=\frac{126-64}{3}=\frac{62}{3}\approx 20.667$. Recalling the fringe demand $Q_f=2P-20$, we can see that $Q_f=\frac{124}{3}-\frac{60}{3}=\frac{64}{3}\approx 21.333$. Fringe market share is $\frac{64}{3}/\frac{64+640}{3}=\frac{1}{11}$.

NOTE: posted solutions technically don't line up with this but I'm pretty sure there's a typo. Waiting on response from prof

Solution: In Cournot, $q_1=q_2=\frac{320}{3}\therefore Q_d=\frac{640}{3}\approx 213.333$ setting market price to $P=\frac{62}{3}\approx 20.667$, fringe produces $Q_f=\frac{64}{3}\approx 21.333$ achieving a market share of $\frac{1}{11}$ to the duopoly's market share of $\frac{10}{11}$.

2.