ECON 4190: Industrial Organization **Fall 2021**

Homework 3

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Pledge: On my honor, I pledge that I have neither given nor received help on this assignment
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1. (a) Let L refer to the left-wing candidate and R refer to the right-wing candidate. Simply as a matter of their positions, L will always receive the $\frac{1}{4}$ to their left, and R will always receive the $\frac{1}{8}$ to their right. However, given a uniform distribution we only have to figure out $\hat{x}$, the location of the voter that is indifferent between the candidates, to determine the share each candidate receives.

   Given the general utility function $v_i = r_i - T|x - l_i|$, utilities for a voter for L and R can be determined to equal $v_L = r_1 - T(x - \frac{1}{4})$ and $v_R = r_2 - T(\frac{7}{8} - x)$ respectively. Now we simply solve for x when $v_L = v_R$:

$$
\begin{aligned}
v_L &= v_R \\
r_1 - T(\hat{x} - \frac{1}{4}) &= r_2 - T(\frac{7}{8} - \hat{x}) \\
2T(\hat{x}) &= r_1 - r_2 + \frac{9}{8}T \\
\hat{x} &= \frac{r_1 - r_2}{2T} + \frac{9}{16}
\end{aligned}
$$

   Therefore, L will receive $\hat{x}$ of the vote, and R will receive $1 - \hat{x}$ of the vote.

   Note: I have numbers written for some reason? Don't think it's possible to have simple numbers

   (b) Very similar process to before, simply change the function slightly.

$$
\begin{aligned}
r_1 - T(\hat{x} - \frac{1}{4})^2 &= r_2 - T(\frac{7}{8} - \hat{x})^2 \\
tl; dr & \\
\hat{x} &= \frac{4}{5}(\frac{r_1 - r_2}{T}) + \frac{9}{16}
\end{aligned}
$$

   TODO: derive pls

   (c) Once again, we know that at the very least, L will always receive the voters spanning the $\frac{1}{4}$ to their left, and R will always receive the voters spannihg the $\frac{1}{8}$ to their right. Given the changed distribution, we know that they will each receive $\frac{1}{8}$ of the votes at least.

To determine the rest, we first need the location of the indifferent voter $\hat{x}$, which we already calculated for this equation to be $\frac{r_1 - r_2}{2T} + \frac{9}{16}$. From this, we can then calculate how many votes from the "middle" each candidate gets (represented by M_L and M_R respectively) by multiplying the "distance" to the indifferent voter by the amount of voters between the candidates:

$$M_L = \frac{\hat{x} - \frac{1}{4}}{\frac{7}{8} - \frac{1}{4}} * \frac{3}{4} \qquad M_R = \frac{\frac{7}{8} - \hat{x}}{\frac{7}{8} - \frac{1}{4}} * \frac{3}{4}$$

$$tl; dr$$

$$\hat{x} = \frac{3}{5} * \frac{r_1 - r_2}{T} + \frac{1}{2}$$

derive pls