

$$\int p(\bar{x}) d\bar{x} = 1 \quad p(\bar{x} \in A) = \int_A p(\bar{x}) d\bar{x}$$

$$p(x) = \int p(x, y) dy \quad \text{marginalization}$$

$$p(x|y) = \frac{p(x, y)}{p(y)}$$

independence

$$p(x, y) = p(x)p(y)$$

$$p(x, y|z) = p(x|z)p(y|z)$$

$$p(x, y) = p(x|y)p(y) = p(y|x)p(x)$$

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

$$p(d|t) = \frac{\overbrace{p(t|d)}^{0.95} \overbrace{p(d)}^{0.01}}{\underbrace{p(t)}_{p(t, d') + p(t, d)}} = \frac{\overbrace{p(t|d)}^{0.95} \overbrace{p(d)}^{0.01}}{\underbrace{p(t|d)}_{0.95} \underbrace{p(d)}_{0.01} + \underbrace{p(t|\bar{d})}_{0.05} \underbrace{p(\bar{d})}_{0.99}} \approx \frac{1}{6}$$

max \uparrow

$$p(\theta|D) = \frac{p(D|\theta) p(\theta)}{p(D)}$$

posterior likelihood prior model evidence

model parameters data

$$\theta_{MAP} = \arg \max_{\theta} p(\theta|D)$$

$$p(\theta|D) \propto p(D|\theta) p(\theta)$$

$$p(y, \bar{x}) = p(y) \cdot \prod p(x_i|y)$$

$$\theta_{wy} = p(x_i = w | y)$$

$$\theta_y = p(y)$$

$$p(\bar{\theta}|D) = p(D|\bar{\theta})$$

$$p(y, \bar{x}|\bar{\theta}) = \left(\prod_a \theta_a^{[y=a]} \right) \cdot \left(\prod_i \prod_w \prod_a \theta_{wa}^{[x_i=w, y=a]} \right)$$

$$p(D|\bar{\theta}) = \prod p(y, \bar{x}|\bar{\theta})$$

ML - max likelihood

$$p(\text{heads}) = \theta$$

$$p(\text{tails}) = 1 - \theta$$

$$D = h h t h t t h h$$

$$p(D|\theta) = \theta^n (1-\theta)^m$$

\downarrow
max

$\underbrace{n}_{\text{heads}} \underbrace{m}_{\text{tails}}$

$$\theta_{ML} = ?$$

$$n \theta^{n-1} (1-\theta)^m - m \theta^n (1-\theta)^{m-1} =$$

$$= \theta^{n-1} (1-\theta)^{m-1} (n(1-\theta) - m\theta) = 0$$

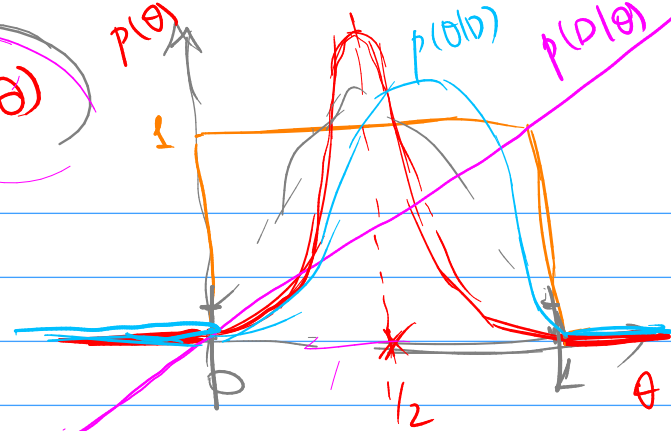
$$D = h$$

$$\theta_{ML} = 1$$

$$p(D|\theta) = \theta \rightarrow \infty$$

$$\theta_{ML} = \frac{n}{n+m}$$

$$p(\theta|D) \propto p(D|\theta)p(\theta)$$



$$p(x=1) = p(x=2) = \frac{1}{6}$$

$$A = \{x \in \{1, 2, 3\}\} \quad p(A) = \frac{1}{2}$$

$$B = \{x \in \{1, 2, 3\}\}$$

$$p(B) = \frac{1}{2}$$

$$p(A, B) = \frac{1}{6}$$

$$p(B|A) = \frac{p(A, B)}{p(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

$$p(A, B) \neq p(A)p(B)$$