Theorem 0.1. *For $D > 0$, let $\{u_j\}_{j\in\mathbb{Z}} \subset \mathcal{S}'(\mathbb{R}^n)$ be a sequence of tempered distributions such that*

$$\operatorname{supp}(\widehat{u_j}) \subset B(0, D\,2^j) \quad \forall j \in \mathbb{Z}.$$

If $w \in A_\infty$, then the following holds:

(i) Let $0 < p < \infty$, $0 < q \leqslant \infty$ and $s > \tau_{p,q}(w)$. If $\|\{2^{js}u_j\}_{j\in\mathbb{Z}}\|_{L^p(w)(\ell^q)} < \infty$, then the series $\sum_{j\in\mathbb{Z}} u_j$ converges in $\dot{F}^s_{p,q}(w)$ (in $\mathcal{S}_0'(\mathbb{R}^n)$ if $q = \infty$) and

$$\Big\|\sum_{j\in\mathbb{Z}} u_j\Big\|_{\dot{F}^s_{p,q}(w)} \lesssim \big\|\{2^{js}u_j\}_{j\in\mathbb{Z}}\big\|_{L^p(w)(\ell^q)},$$

where the implicit constant depends only on n, D, s, p and q. An analogous statement, with $j \in \mathbb{N}_0$, holds true for $F^s_{p,q}(w)$ (when $q = \infty$, the convergence is in $\mathcal{S}'(\mathbb{R}^n)$).

(ii) Let $0 < p, q \leqslant \infty$ and $s > \tau_p(w)$. If $\|\{2^{js}u_j\}_{j\in\mathbb{Z}}\|_{\ell^q(L^p(w))} < \infty$, then the series $\sum_{j\in\mathbb{Z}} u_j$ converges in $\dot{B}^s_{p,q}(w)$ (in $\mathcal{S}_0'(\mathbb{R}^n)$ if $q = \infty$) and

$$\Big\|\sum_{j\in\mathbb{Z}} u_j\Big\|_{\dot{B}^s_{p,q}(w)} \lesssim \big\|\{2^{js}u_j\}_{j\in\mathbb{Z}}\big\|_{\ell^q(L^p(w))},$$

where the implicit constant depends only on n, D, s, p and q. An analogous statement, with $j \in \mathbb{N}_0$, holds true for $B^s_{p,q}(w)$ (when $q = \infty$, the convergence is in $\mathcal{S}'(\mathbb{R}^n)$).

For Part (i)