Leibniz-type rules in Triebel-Lizorkin and Besov spaces.

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Abstract

Fractional Leibniz rules have been extensively studied due to their connections to partial differential equations that model many real-world situations such as shallow water waves and fluid flow. Broadly speaking, fractional Leibniz rules provide estimates of the size and smoothness of a product of functions in terms of the size and smoothness of the functions involved. In this dissertation, we obtain new Leibniz-type rules associated to bilinear pseudodifferential operators in a variety of function spaces that quantify smoothness and size in appropriate ways. Bilinear pseudodifferential operators combine functions through a symbol and the Fourier transform. When the symbol is identically equal to one, such operators give the product of two functions, and therefore, fractional Leibniz rules are particular cases of the Leibniz-type rules discussed.

The main results of this dissertation concern Leibniz-type rules for operators associated to two classes of symbols: Coifman-Meyer multipliers and symbols in the bilinear Hörmander classes. Leibniz-type rules for Coifman-Meyer multiplier operators are presented in the setting of Triebel-Lizorkin and Besov spaces based on various quasi-Banach spaces that include weighted Lebesgue, weighted Lorentz, weighted Morrey, and variable-exponent Lebesgue spaces. Such results extend and improve previously known fractional Leibniz rules. As applications, we obtain scattering properties of solutions to certain systems of partial differential equations involving fractional powers of the Laplacian. For operators with symbols in the bilinear Hörmander classes, we obtain Leibniz-type rules in the context of Besov and local Hardy spaces. The tools used in the proofs of the main results include Nikol'skiĭ representations of function spaces, pointwise inequalities for maximal functions, and appropriate spectral decompositions of the symbols of the operators.