**4.1**

(a)

In a prediction process using Gaussian process with N training input and output pairs (X, Y), and T test inputs XT, the joint training and test marginal likelihood:

Condition on training outputs:

When N is large, the inverse of the covariance matrix will be computationally expensive .)

(b)

We can reduce the rank of the covariance using following approximation:

*Given a positive definite matrix K of order n × n and a randomly generated Johnson -Lindenstrauss matrix Ω of order r × n, we find the projection matrix Φ of order m × n which approximates the range and compute the approximate SVD decomposition via Nystrom approximation with Φ.[[1]](#footnote-1)*

Consider the Gaussian Process with (Gaussian) noisy observations:

Using the Sherman-Morrison-Woodbury matrix inversion lemma,

In the above is a diagonal matrix and the matrix inversion is avoided.

1. Banerjee, Anjishnu, David B. Dunson, and Surya T. Tokdar. "Efficient Gaussian process regression for large datasets." Biometrika 100.1 (2012): 75-89. [↑](#footnote-ref-1)