Output Feedback Adaptive Inverse OptimalSecurity Control for Stochastic NonlinearCyber-Physical Systems under Sensor andActuator Attacks

Zebin Chen
East China University of Science and Technology
Zhaoxu Yu
yyzx@ecust.edu.cn
East China University of Science and Technology
Shugang Li
Shanghai University

Research Article

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Posted Date: March 13th, 2024

DOI: https://doi.org/10.21203/rs.3.rs-4010166/v1

Additional Declarations: No competing interests reported.

Output Feedback Adaptive Inverse Optimal Security Control for Stochastic Nonlinear Cyber-Physical Systems under Sensor and Actuator Attacks

Zebin Chen[1], Zhaoxu Yu[1*], Shugang Li[2]

[1*]Key Laboratory of Smart Manufacturing in Energy Chemical Process of Ministry of Education, Department of Automation, East China University of Science and Technology, Shanghai, 200237, China.
[2]Department of Information Management, Shanghai University, Shanghai, 200444, China.

*Corresponding author(s). E-mail(s): yyzx@ecust.edu.cn;

Abstract

This paper addresses the inverse optimal security control problems for a class of stochastic non-strict feedback nonlinear Cyber-Physical Systems under sensor and actuator attacks. The concerned system model includes both stochastic disturbances and more general nonlinearity. First, to make the control design feasible, a linear state transformation is applied to the attacked system. Furthermore, in the process of backstepping design, based on the Nussbaum gain function formula, fuzzy logic system approximation method, and inverse optimal control theory, combing the available output signal, an output feedback inverse optimal controller is proposed. Specifically, the designed controller not only ensures that the system is secure under network attacks but also optimal in terms of the cost function. Finally, two physical examples are given to verify the effectiveness of the proposed control scheme in various network attacks.

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1 Introduction

As one of the most common phenomena, stochastic disturbances often unavoidably occur in practical systems. In recent years, this issue has attracted widespread attention, and a large number of meaningful adaptive control schemes for stochastic nonlinear systems were proposed. On one hand, some researchers have focused on stochastic nonlinear systems represented in the strict-feedback form [1–3]. Especially, considering unknown control gain functions and unmeasurable states, an adaptive fuzzy output-feedback control methodology was proposed in [3]. On the other hand, References [4–7] directed their focus towards non-strict feedback stochastic nonlinear systems. For example, by constructing a novel observer and combining some techniques, the problem for a class of non-strict feedback nonlinear systems with input and output quantization as well as sensor faults was solved in [7].

The Cyber-Physical System (CPS) is an engineering system that integrates computing, networking, and physical processes [8], which can adapt to more complex environments and complete more complex tasks. Due to CPS's increasingly obvious social and economic significance, research in various fields [9, 10] has increased significantly. From a practical perspective, there is no doubt that CPS will be affected by stochastic disturbances whenever and wherever possible. In addition, due to the openness of CPS, data collected from sensors and actuators may be tampered without security protection, which is called actuator attacks and sensor attacks. These two aspects inspire us to study the stochastic CPS security control problem under actuator and sensor attacks.

Fortunately, vast research results on CPS subject to actuator and/or sensor attacks have been reported. For linear CPS under sensor attacks, the adaptive security control strategy was first proposed in [11]. Afterward, researchers began to study the security control problems of nonlinear CPS [12–18]. In [15], both deception attacks and injection attacks were considered for a second-order nonlinear system. In [18], a novel coordinate transformation technology was proposed, aiming to achieve tracking control for the attacked CPS. There is also a part of the literature that considers CPS subject to stochastic disturbances and network attacks [19–24]. For example, an innovative distributed adaptive control strategy is developed in [19] for networked multi-agent systems against sensor and actuator attacks, as well as stochastic disturbances. The authors in [23] address the design problems of switching threshold event-triggered adaptive resilient controllers for uncertain nonlinear stochastic CPS affected by unknown deception attacks. Especially, by employing the Nussbaum function, reference [24] solves the issue of unknown control directions stemming from sensor and actuator attacks. However, it should be noted that the control schemes mentioned in [12–18, 21–24] require that the model be strict-feedback nonlinear systems; namely, these control schemes could not be directly applied to non-strict feedback nonlinear stochastic systems subjected to sensor and actuator attacks.

Due to the fact that the high energy consumption of CPS is unacceptable, optimal control has attracted the attention from researchers. Inverse optimal control (IOC) theory, which can easily resolve optimal control challenges of nonlinear systems, was first proposed by Krstic [25]. Several research groups have completed some preliminary work. The inverse optimal output-feedback control problem was first studied in [26]. For stochastic nonlinear systems, reference [27] is the first paper to study the IOC problem. Subsequently, [28] addressed the output feedback IOC problem for stochastic nonlinear systems. The work in [29] achieves the goals of prescribed-time mean-square stabilization and optimality for stochastic strict-feedback nonlinear systems. For more works of the IOC, one can further refer to [30–35] and some references therein. To the authors' knowledge, up to now, little attention is payed to dealing with the optimal control problem in nonlinear attacked CPS, which raises our interest.

Motivated by the above considerations, it is worthy to design an output-feedback IOC law for non-strict feedback nonlinear stochastic CPS under network attacks. The main works of this paper are summarized as follows:

1. This paper first focuses on the optimal control problem of non-strict feedback nonlinear stochastic CPS under network attacks, and an adaptive IOC strategy based on output feedback is proposed. Both theoretical analysis and simulation show that the proposed controller can ensure the attacked system security under stochastic disturbances and various network attacks;
2. Different from the IOC design process in [33–35], we do not construct an auxiliary system and the proof of optimality is simplified by selecting a special function;
3. Unlike the CPS deterministic/stochastic systems models in [12–18, 22–24], the considered model is in a non-strict feedback form. Besides, some challenges caused by unknown network attacks and control directions are overcome by using only one Nussbaum gain function in controller design.

The rest of this paper is organized as follows. Section 2 introduces some preliminaries, including some definitions and useful lemmas, and the system model. Section 3 mainly includes the design of the observer and controller. Section 4 is devoted to stability analysis and optimal control performance. Two physical examples are provided in Section 5 and finally, Section 6 is the summary.

2 Preliminaries and problem statement

2.1 Preliminaries

Firstly, the following notations are used throughout this paper. For a given matrix A, $\lambda_{\min}(A)\big(\text{or } \lambda_{\max}(A)\big)$ means its minimum (or maximum) eigenvalues, A^T stands for its transpose, Tr(A) denotes its trace, $|A|$ is the absolute value, and $\|A\|$ represents the Euclidean norm. Additionally, I_n is the $n \times n$ unit matrix, $E[B]$ denotes the expectation of stochastic variable B, and $\mathbb{P}\{C\}$ is the probability of event C. For simplicity, variables are occasionally omitted from the corresponding functions, e.g., F_i means $F_i(\cdot)$.

To proceed, we introduce the following definitions and lemmas.

Consider the following stochastic nonlinear system:

$$\mathrm{d}\chi = f(\chi, t)\mathrm{d}x + h^T(\chi, t)\mathrm{d}w, \tag{1}$$

where χ is the system state; f and h are locally Lipschitz continuous functions with $f(\mathbf{0}, 0) = h(\mathbf{0}, 0) = 0$; w is an independent standard Wiener process defined on the complete probability space.

Definition 1. *([22]) Consider a function $V(\chi, t)$ and system (1), the differential operator $\mathcal{L}$ is defined as*

$$\mathcal{L}V(\chi, t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \chi} f + \frac{1}{2}\mathrm{Tr}\left\{h^T \frac{\partial^2 V}{\partial \chi^2} h\right\}. \tag{2}$$

Definition 2. *([21]) System (1) under network attacks is secure with a probability of $1 - \epsilon$, if the system state $\chi(t)$ satisfy $\mathbb{P}\{\|\chi(t)\| \leq \sigma\} \geq 1 - \epsilon$ for a security parameter $\sigma > 0$ and a positive scalar ϵ.*

Definition 3. *([24]) For any initial state $\chi_0 = \chi(t_0)$ and two positive constants a_0, a_1, such that*

$$E[\|\chi(t)\|^p] < a_1$$

for $\forall t \geq t_0$ and $\|\chi(t)\|^p < a_0$, then the trajectory $\chi(t)$ of system (1) is bounded in p-th moment.

Definition 4. *A differentiable function $\mathcal{N}(\zeta)$ is called as Nussbaum function if it satisfies the following two conditions:*

$$\lim_{\eta\to\infty} \sup\left(\frac{1}{\eta}\int_0^\eta \mathcal{N}(\zeta)\mathrm{d}\zeta\right) = +\infty,$$
$$\lim_{\eta\to\infty} \inf\left(\frac{1}{\eta}\int_0^\eta \mathcal{N}(\zeta)\mathrm{d}\zeta\right) = -\infty.$$

According to Definition 4, $\mathcal{N}(\zeta) = \zeta^2\cos(\zeta)$ is selected as a Nussbaum function in this article.

Lemma 1. *([1]) Let $\mathcal{N}(\zeta)$ be a Nussbaum function, $V(x,t)$ and $\zeta(t)$ be the smooth functions, and function r^* satisfy $0 < |r^*(x,t)| \leq C_1$. If the following inequality holds*

$$V(x,t) \leq C_2 + e^{-C_0 t}\int_0^t (r^*(x,t)\mathcal{N}(\zeta)+1)\dot{\zeta}e^{C_0\tau}\mathrm{d}\tau + C_3(t), \tag{3}$$

where $C_0, C_1 > 0$ are constants, C_2 is a nonnegative random variable and $C_3(t)$ is a real-valued continuous local martingale, then $\zeta(t)$, $V(x,t)$, and $\int_0^t(r^*(x,t)\mathcal{N}(\zeta)+1)\dot{\zeta}\mathrm{d}\tau$ are bounded in probability.

Lemma 2. *([5]) If $m > 1$, $n > 1$, $\pi > 0$, and $(m-1)(n-1) = 1$, the inequality $pq \leq \frac{\pi^m}{m}|p|^m + \frac{1}{n\pi^n}|q|^n$ holds for any $p \in R$ and $q \in R$.*

Lemma 3. *([4]) For any $\delta > 0$ and $\rho \in R$, the inequality $0 \leq |\rho| - \rho\tanh(\frac{\rho}{\delta}) \leq 0.2785\delta$ holds.*

Lemma 4. *([28]) For two vectors X and Y, the following inequality holds*

$$X^TY \leq \gamma(\|X\|) + \ell\gamma(\|Y\|)$$

and it is achieved if and only if $X = (\gamma)^{-1}(\|Y\|)\frac{Y}{\|Y\|}$.

Lemma 5. *([28]) For $r \in [0,e) \to [0,\infty)$, a continuous scalar function $\gamma(r)$ belongs to class $\mathcal{K}$ function if it is strictly increasing and $\gamma(0) = 0$, and it is a member of class*

$\mathcal{K}_\infty$ function if $e = \infty$ and $\gamma(r) \to \infty$ as $r \to \infty$. For a $\mathcal{K}_\infty$ function $\gamma(r)$ and its derivative γ', the following properties hold:

1. *$\ell\gamma(r) = r(\gamma')^{-1}(r) - \gamma\left((\gamma')^{-1}(r)\right) = \int_0^v (\gamma')^{-1}\mathrm{d}s$ is a class $\mathcal{K}_\infty$ function;*
2. *$\ell\ell\gamma = \gamma$;*
3. *$\ell\gamma\left(\gamma'(r)\right) = r\gamma'(r) - \gamma(r)$;*

where $(\gamma')^{-1}(r)$ represents the inverse function of $\frac{\mathrm{d}\gamma(r)}{\mathrm{d}r}$.

This paper uses fuzzy logic system (FLS) to approximate unknown nonlinear functions, and the properties of FLS are expressed by the following lemma.

Lemma 6. *([26]) For the continuous function $f(v)$ defined on a compact set Ω, there exists a positive constant ε and an FLS such that*

$$\sup_{v\in\Omega}\left|f(v) - \Theta^T S(v)\right| \leq \varepsilon,$$

where $\Theta = [\Theta_1, \Theta_2, \cdots, \Theta_N]^T$ is the weight vector, $S(v)=[s_1(v), s_2(v), \cdots, s_N(v)]^T$ denotes the kernel vector with the fuzzy basis functions $s_i(v)$ chosen as Gaussian functions, and N is the number of fuzzy rules. Furthermore, the inequality $0 < \|S(v)\| \leq 1$ holds according to definition of $S(v)$.

Based on Lemma 6, any unknown continuous function $f(v)$ can be represented as

$$f(v) = \Theta^T S(v) + \mathcal{E}(v), |\mathcal{E}(v)| \leq \varepsilon \tag{4}$$

where $\mathcal{E}(v)$ denotes the approximation error.

2.2 System description and control objective

Consider the following stochastic non-strict feedback nonlinear CPS affected by unknown sensor and actuator attacks:

$$\begin{cases} \mathrm{d}x_i = (g_i x_{i+1} + f_i(x))\mathrm{d}t + h_i^T(x)\mathrm{d}w, i = 1, 2, \cdots, n-1 \\ \mathrm{d}x_n = (g_n u_a + f_n(x))\mathrm{d}t + h_n^T(x)\mathrm{d}w, \\ y = x_1, \end{cases} \tag{5}$$

where $x_i \in R(i = 1, 2, \cdots, n)$ and $y \in R$ denote state variables and system output, respectively; $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the state vector; $g_i \neq 0$ are unknown constants; For $i = 1, 2, \cdots, n$, f_i and h_i are unknown local Lipschitz nonlinear functions which satisfy $f_i(\mathbf{0}) = 0$ and $h_i(\mathbf{0}) = 0$. Especially, $g_i x_{i+1}$ cannot be canceled by $f_i(x)$, otherwise, it is impossible to design a controller by backstepping technique. The definition of w is the same as in (1). In addition, u_a represents the attacked system input signal, which can be described as

$$u_a = \varrho u + \psi_a(t), \tag{6}$$

where ϱ denotes the unknown multiplicative actuator attack, u represents the true control input produced by controller, and $\psi_a(t)$ means the unknown additive actuator attack signal.

Due to existence of unknown sensor attacks modeled as $\psi_s(t, y)$, the sensor output y_s can be expressed as

$$y_s = y + \psi_s(t, y). \tag{7}$$

Remark 1. *When $f_i(x) = f_i(\bar{x}_i)$ and $h_i(x) = h_i(\bar{x}_i)$ with $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T$, system (5) reduces to a usual stochastic system, and the cyber-security problems have been researched in [21–24]. However, due to the fact that the above existing security* *control methods are based on strict-feedback systems, these methods cannot be applied to the attacked system (5). Meanwhile, the control methods mentioned above do not consider the optimal control problem, which may cause high energy loss.*

Remark 2. *Fig. 1 gives the block diagram description of the overall system. It can be seen from Fig. 1 that network attacks have two main impacts on CPS (5). On the one hand, all the system state variables x_i, including output y, become unavailable and only y_s is measurable. On the other hand, the attacker can change the control signal, thus u is tampered to u_a, which will lead to the controller unavailable.*

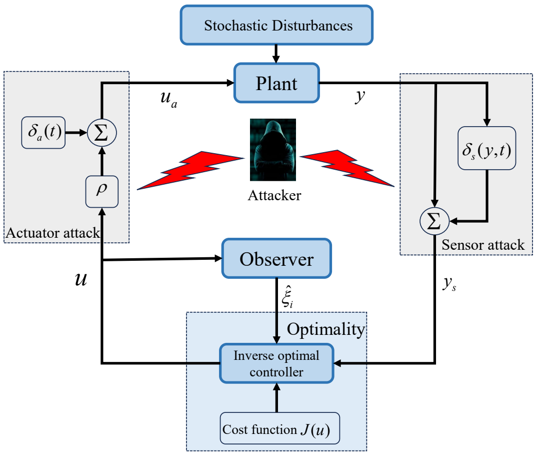

Fig. 1 The block diagram of the system under network attacks

Control Objective: The control objective of this article is to design an adaptive output feedback controller based on IOC for CPS (5) affected by unknown network attacks (6) and (7), which make sure that all the closed-loop signals are bound in probability, and the attacked system is secure. At the same time, the inverse optimal stabilization problem in probability for system (5) is solvable and attain the minimum of the cost function. To accomplish these objectives, it is necessary to make the following two assumptions.

Assumption 1. *([24]) Concerning the actuator attack (6), it is assumed that* $\varrho \neq 0$ *is a constant, and the condition* $|\psi_a(t)| \leq \psi_0$ *is satisfied, where* ψ_0 *is a constant with* $\psi_0 \geq 0$.

Assumption 2. *([35]) Assume that the unknown sensor attack signal* $\psi_s(t, y)$ *is related to* y*, and can be parameterized as* $\psi_s(t, y) = \hbar(t)y$*, where* $\hbar(t)$ *is an unknown time-varying function. Define* $\varpi(t) = \hbar(t) + 1$*, then (7) is expressed as* $y_s = y\varpi(t)$*. In addition, it is assumed that* $\varpi(t)$ *and its first derivative* $\dot{\varpi}(t)$ *satisfy* $0 < \varpi_m \leq |\varpi(t)| \leq \varpi_M < +\infty$ *and* $|\dot{\varpi}(t)| \leq \varpi_{dM}$*, where* ϖ_m*,* ϖ_M*, and* ϖ_{dM} *are unknown positive constants.*

Remark 3. *Some supplementary explanations on the two assumptions are given as follows:*

1. *The types of actuator attacks are shown in Table 1. Under different network attacks,* ϱ *may be positive or negative. Therefore, it is important to consider the sign of* ϱ *in advance. Moreover, when* $\varrho = 0$*, it means that the control signal is ineffective, so this situation is not taken into consideration.*
2. *Assumption 2 is rational because if* $\hbar(t) = -1$*, indicating* $\varpi(t) = 0$*, there is no available signal for output feedback control, not to mention achieving Control Objective. Furthermore, similar assumptions are presented in [16, 18] and [23]. However, it's worth noting that the sensor attacks considered in these references are state-dependent rather than output-dependent.*
3. *Due to limited energy, the attacker cannot carry out attacks arbitrarily, so* ψ_a *and* ϖ *are bounded.*

Table 1 TYPES OF ACTUATOR ATTACKS (6)

Types	ϱ	δ_a
Normal	$\varrho = 1$	0
(6) in [12, 13, 23]	$\varrho = 1$	time-varying
(6) in [15, 19]	ϱ is time-varying and satisfies $0 < \varrho_{\min} \leq \varrho \leq \varrho_{\max}$	time-varying
(6)	$\varrho \neq 0$ is a constant	time-varying

2.3 Background of IOC problem

This section briefly introduces the definition and lemma of the IOC problem which will be used in the next section.

Definition 5. *([28]) The inverse optimal stabilization in probability problem of system (5) can be solved if there exists positive definite functions $l(x)$ and $\mathbb{S}(x)$, a matrix-valued function $\mathcal{R}(x)$ satisfying $\mathcal{R}(x) = \mathcal{R}^T(x) > 0$ for $\forall x$, a class $\mathcal{K}_\infty$ function γ, and a control strategy $u(x)$, which minimize the cost functional*

$$J(u) = E\left[\mathbb{S}(x) + \int_0^t \left(l(x) - \gamma(\left|\mathcal{R}^{\frac{1}{2}}(x)u\right|)\right)\mathrm{d}\tau\right]. \tag{8}$$

Lemma 7. *([28][29]) Suppose the control strategy*

$$u = -\mathcal{R}(x)z_n \tag{9}$$

with $\mathcal{R}(x)$ being defined in Definition 5 can ensure system (5) is bounded in probability, then the control input

$$u_{op} = \tfrac{2}{3}\alpha u, \alpha \geq 2 \tag{10}$$

can solve the problem of inverse optimal stabilization in probability.

3 Adaptive backstepping control design

3.1 Observer design

Please note that control coefficients in (5) are unknown. Additionally, the system is also affected by uncertain multiplicative actuator attacks, so it is imperative to introduce a transformation to make the output feedback control design easier.

Firstly define $\xi_i = x_i\delta_i$ with $\delta_i = 1\Big/(\varrho \prod\limits_{j=i}^{n} g_j)$ for $i = 1, 2, \cdots, n$, and then take network attacks (6) and (7) into consideration. The attacked system (5) can be written as

$$\begin{cases} \mathrm{d}\xi_i = (\xi_{i+1} + \delta_i \bar{f}_i(\xi))\mathrm{d}t + \delta_i \bar{h}_i^T(\xi)\mathrm{d}w, i = 1, 2, \cdots, n-1 \\ \mathrm{d}\xi_n = (u + \psi_a/\varrho + \delta_n \bar{f}_n(\xi))\mathrm{d}t + \delta_n \bar{h}_n^T(\xi)\mathrm{d}w, \\ y_s = \delta_1^{-1}\varpi(t)\xi_1, \end{cases} \tag{11}$$

where $\xi = [\xi_1, \xi_2, \cdots, \xi_n]^T$, $\bar{f}_i(\xi) = f_i(N\xi)$, $\bar{h}_i(\xi) = h_i(N\xi)$, and $N = \mathrm{diag}[1/\delta_1, 1/\delta_2, \cdots, 1/\delta_n]$.

Obviously, since ϱ and g_i are unknown, all the states ξ_i cannot be measured, thus the following state observer is formulated to estimate these states

$$\begin{cases} \dot{\hat{\xi}}_i = \hat{\xi}_{i+1} - c_i\hat{\xi}_1, i = 1, 2, \cdots, n-1 \\ \dot{\hat{\xi}}_n = u - c_n\hat{\xi}_1, \end{cases} \tag{12}$$

where $c_i > 0$ are the design parameters, and $\hat{\xi}_i$ denotes the estimate value of ξ_i, $i = 1, 2, \cdots, n$.

The observer error is given as $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$ ($i = 1, 2, \cdots, n$), then the dynamic system of error signals $\tilde{\xi} = [\tilde{\xi}_1, \tilde{\xi}_2, \cdots, \tilde{\xi}_n]^T$ can be obtained as

$$\mathrm{d}\tilde{\xi} = (\mathcal{A}\tilde{\xi} + \mathcal{B}\psi_a/\varrho + \mathcal{C}\xi_1 + \mathcal{F}(\xi))\mathrm{d}t + \mathcal{H}(\xi)\mathrm{d}w, \tag{13}$$

where $\mathcal{B} = [0, 0, \cdots, 1]^T$, $\mathcal{C} = [c_1, c_2, \cdots, c_n]^T$, $\mathcal{F}(\xi) = [\delta_1\bar{f}_1, \delta_2\bar{f}_2, \cdots, \delta_n\bar{f}_n]^T$, $\mathcal{H}(\xi) = \left[\delta_1\bar{h}_1^T, \delta_2\bar{h}_2^T, \cdots, \delta_n\bar{h}_n^T\right]^T$, and $\mathcal{A} = \begin{bmatrix} -c_1 & & \\ \vdots & & I_{n-1} \\ & & \\ -c_n & 0 & \cdots & 0 \end{bmatrix}$.

By choosing the parameters $c_i (i = 1, 2, \cdots, n)$ properly, $\mathcal{A}$ becomes a Hurwitz matrix. For a given positive definite matrix $Q = Q^T$, there is a positive definite matrix $P = P^T$,we have $\mathcal{A}^T P + P\mathcal{A} = -Q$.

Define $\hat{\xi} = [\hat{\xi}_1, \hat{\xi}_2, \cdots, \hat{\xi}_n]^T$ and $X = [\hat{\xi}^T, \tilde{\xi}^T]^T$, and then combining (12) and (13), one has

$$\mathrm{d}X = F(X)\mathrm{d}t + G_1 u\mathrm{d}t + G_2(X)\mathrm{d}w. \tag{14}$$

where $F(X) = \begin{bmatrix} \mathcal{A}\hat{\xi} \\ F_1 \end{bmatrix}$, $F_1 = \mathcal{A}\tilde{\xi} + \mathcal{B}\psi_a/\varrho + \mathcal{C}\xi_1 + \mathcal{F}(\xi)$, $G_1 = [\underbrace{0, \cdots, 0}_{n-1}, 1, \underbrace{0, \cdots, 0}_{n}]^T$, and $G_2(X) = [\underbrace{0, \cdots, 0}_{n}, \mathcal{H}^T(\xi)]^T$.

Remark 4. *For $i = 1, 2, \cdots, n$, z_i and V_i will be defined in the following subsection, and all of them are the functions of $\tilde{\xi}_i$ and $\hat{\xi}_i$. It is necessary to construct a new system (14) that includes $\hat{\xi}_i$ and $\tilde{\xi}_i$ for optimality analysis.*

3.2 Adaptive output-feedback inverse optimal design

Before the backstepping control process, the unknown constant θ is defined as

$$\theta = \max\{\theta_i, i = 0, 1, \cdots, n\},$$

with $\theta_i > 0$ to be given later.

The coordinate transformation is defined below:

$$\begin{cases} z_1 = y \\ z_i = \hat{\xi}_i - \beta_{i-1}, i = 2, 3, \cdots, n \end{cases} \tag{15}$$

where β_{i-1} stands for the virtual control signal.

As usual, the control design procedure begins with the definition of θ and the aforementioned coordinate changes:

Step 0: Choose a Lyapunov function in the form of $V_0 = \frac{1}{2}(\tilde{\xi}^T P\tilde{\xi})^2$, from (2), it can be easily calculated:

$$\begin{aligned}\mathcal{L}V_0 =&(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T(\mathcal{A}^T P + P\mathcal{A})\tilde{\xi} + 2(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T P(\mathcal{F} + \mathcal{B}\psi_a/\varrho + \mathcal{C}\xi_1)\\ &+ 2\mathrm{Tr}\{\mathcal{H}^T(2P\tilde{\xi}\tilde{\xi}^T + \tilde{\xi}^T P\tilde{\xi}P)\mathcal{H}\}.\end{aligned} \tag{16}$$

Utilizing Lemma 3, and Assumptions 1-2, we have

$$\begin{aligned}2(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T P\mathcal{F} =&2(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T P\mathcal{F}(\xi)\\ \le&\tfrac{3}{2}\kappa_1^{\frac{4}{3}}\left\|\tilde{\xi}\right\|^4 + \tfrac{1}{2\kappa_1^4}\|P\|^8\|\mathcal{F}(\xi)\|^4,\end{aligned} \tag{17}$$

$$\begin{aligned}2(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T P\mathcal{B}\psi_a/\varrho \le&2\left\|\tilde{\xi}\right\|^3\|P\|^2\|\mathcal{B}\|\,|\psi_a/\varrho|\\ \le&\tfrac{3}{2}\kappa_2^{\frac{4}{3}}\left\|\tilde{\xi}\right\|^4 + \tfrac{1}{2\kappa_2^4\varrho^4}\|P\|^8\psi_0^4,\end{aligned} \tag{18}$$

$$\begin{aligned}2(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T P\mathcal{C}\xi_1 =&2(\tilde{\xi}^T P\tilde{\xi})\tilde{\xi}^T P\mathcal{C}\delta_1\tfrac{y_s}{\varpi}\\ \le&2\left|\tfrac{\delta_1}{\varpi_m}\right|\left\|\tilde{\xi}\right\|^3\|P\|^2\|\mathcal{C}\|\,|y_s|\\ \le&\tfrac{3}{2}\kappa_3^{\frac{4}{3}}\left\|\tilde{\xi}\right\|^4 + \tfrac{\delta_1^4}{2\kappa_3^4\varpi_m^4}\|P\|^8\|\mathcal{C}\|^4 y_s^4,\end{aligned} \tag{19}$$

$$\begin{aligned}2\mathrm{Tr}\{\mathcal{H}^T(2P\tilde{\xi}\tilde{\xi}^T + \tilde{\xi}^T P\tilde{\xi}P)\mathcal{H}\} \le&6n\sqrt{n}\|\mathcal{H}(\xi)\|^2\|P\|^2\left\|\tilde{\xi}\right\|^2\\ \le&3\kappa_4 n\sqrt{n}\left\|\tilde{\xi}\right\|^4 + \tfrac{3n\sqrt{n}}{\kappa_4}\|P\|^4\|\mathcal{H}(\xi)\|^4,\end{aligned} \tag{20}$$

where $\kappa_i > 0 (i = 1, 2, 3, 4)$ are constants.

To handle some nonlinear terms, the nonlinear function $\Gamma_0(\xi)$ is defined as $\Gamma_0(\xi) = \frac{1}{2\kappa_1^4}\|P\|^8\|\mathcal{F}(\xi)\|^4 + \frac{3n\sqrt{n}}{\kappa_4}\|P\|^4\|\mathcal{H}(\xi)\|^4$, then an FLS is utilized to reconstruct $\Gamma_0(\xi)$. One has $\Gamma_0(\xi) = \Theta_0^T S_0(\xi) + \mathcal{E}_0(\xi)$. Futhermore, by using the definition of θ, we obtain

$$\begin{aligned}\Gamma_0(\xi) &\le \|\Theta_0\|\|S_0(\xi)\| + \|\mathcal{E}_0(\xi)\|\\ &\le \theta_0 + \varepsilon_0\\ &\le \theta + \varepsilon_0,\end{aligned} \tag{21}$$

where $\theta_0 = \|\Theta_0\|$, and approximation error $\mathcal{E}_0(\xi)$ satisfies $\|\mathcal{E}_0(\xi)\| \le \varepsilon_0$ for a constant $\varepsilon_0 > 0$.

Consequently, combining (17)-(21), $\mathcal{L}V_0$ in (16) is given by

$$\mathcal{L}V_0 \le -\Lambda_0 \left\|\tilde{\xi}\right\|^4 + \frac{\delta_1^4}{2\kappa_3^4 \varpi_m^4} \|P\|^8 \|\mathcal{C}\|^4 y_s^4 + \Delta_0, \tag{22}$$

where $\Lambda_0 = \lambda_{\min}(P)\lambda_{\min}(Q) - \frac{3}{2}(\kappa_1^{\frac{4}{3}} + \kappa_2^{\frac{4}{3}} + \kappa_3^{\frac{4}{3}}) - 3\kappa_4 n\sqrt{n}$ and $\Delta_0 = \frac{1}{2\kappa_2^4 \varrho^4}\|P\|^8 \psi_0^4 + \theta + \varepsilon_0$.

Remark 5. *It can be seen that $\bar{f}_i$ and $\bar{h}_i$ are the functions of the whole state variables, thus a main difficulty for output-feedback security control design of this system comes from the functions $\mathcal{F}$ and $\mathcal{H}$. Previous design methods for non-strict feedback systems, e.g. in [2, 3], require assumptions to be made about these functions. Inspired by [4], we use the characteristics of the FLS and algebraic operation skills instead of imposing restrictions on the system, algebraic loop problems have been avoided.*

Step 1: Based on the definition of z_1, we have

$$\mathrm{d}z_1 = [\delta_1^{-1}(\tilde{\xi}_2 + z_2 + \beta_1) + \bar{f}_1(\xi)]\mathrm{d}t + \bar{h}_1^T(\xi)\mathrm{d}w. \tag{23}$$

The Lyapunov function is designed as $V_1 = V_0 + \frac{1}{4}z_1^4 + \frac{1}{2}\tilde{\theta}^2$, where $\tilde{\theta} = \theta - \hat{\theta}$ with $\hat{\theta}$ being the estimate value of θ. Using (2) and (23) yields

$$\mathcal{L}V_1 = \mathcal{L}V_0 + z_1^3[\delta_1^{-1}(\tilde{\xi}_2 + z_2 + \beta_1) + \bar{f}_1(\xi)] + \frac{3}{2}z_1^2 \mathrm{Tr}\{\bar{h}_1(\xi)\bar{h}_1^T(\xi)\} - \tilde{\theta}\dot{\hat{\theta}}. \tag{24}$$

By using Assumption 2, Lemmas 2 and 3, we have

$$
\begin{aligned}
\tfrac{3}{2} z_1^2 \mathrm{Tr}\{\bar{h}_1(\xi)\bar{h}_1^T(\xi)\} =& \tfrac{3}{2} z_1^2 \left\|\bar{h}_1(\xi)\right\|^2 \\
\leq& \tfrac{3}{4} z_1^4 \left\|\bar{h}_1(\xi)\right\|^4 + \tfrac{3}{4} \\
\leq& \tfrac{3}{4} \varpi_m^{-4} y_s^4 \left\|\bar{h}_1(\xi)\right\|^4 + \tfrac{3}{4},
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
z_1^3[\bar{f}_1(\xi) + \delta_1^{-1}(\tilde{\xi}_2 + z_2)] =& \varpi^{-3} y_s^3 \bar{f}_1(\xi) + \varpi^{-3} y_s^3 [\delta_1^{-1}(\tilde{\xi}_2 + z_2)] \\
\leq& \varpi_m^{-3} |y_s^3| |\bar{f}_1(\xi)| + \varpi_m^{-3} |y_s^3 \delta_1^{-1}| (|\tilde{\xi}_2| + |z_2|) \\
\leq& \varpi_m^{-3} y_s^3 \bar{f}_1 \tanh(\frac{y_s^3 \bar{f}_1}{\varsigma_{11}}) + \tfrac{3}{2} \varpi_m^{-4} \delta_1^{-\frac{4}{3}} y_s^4 + \tfrac{1}{4} \left\|\tilde{\xi}\right\|^4 + \tfrac{1}{4} z_2^4 \\
&+ 0.2785 \varpi_m^{-3} \varsigma_{11},
\end{aligned} \tag{26}
$$

where ς_{11} is a positive constant.

Designate a unknown nonlinear function $\Gamma_1(Z_1)$ as $\Gamma_1(Z_1) = \frac{3}{4}\varpi_m^{-4} y_s \left\|\bar{h}_1(\xi)\right\|^4 + \frac{3}{2} y_s \varpi_m^{-4} \delta_1^{-\frac{4}{3}} + \varpi_m^{-3} \bar{f}_1 \tanh(\frac{y_s^3 \bar{f}_1}{\varsigma_{11}}) + \frac{\delta_1^4}{2\kappa_3^4 \varpi_m^4} y_s \|P\|^8 \|\mathcal{C}\|^4$, where $Z_1 = [y_s, \xi]^T$. Then FLS is employed to approximate $\Gamma_1(Z_1)$, thus one has $\Gamma_1(Z_1) = \Theta_1^T S_1(Z_1) + \mathcal{E}_1(Z_1)$ with $\|\mathcal{E}_1(Z_1)\| \leq \varepsilon_1$, where ε_1 is a positive constant. By using the property of $S_1(Z_1)$ and Lemma 3, one has

$$
\begin{aligned}
y_s^3 \Gamma_1(Z_1) =& y_s^3 (\Theta_1^T S_1(Z_1) + \mathcal{E}_1(Z_1)) \\
\leq& |y_s^3| \, \|\Theta_1\| \, \|S_1(Z_1)\| + |y_s^3| \varepsilon_1 \\
\leq& \tfrac{3}{4} \theta_1 y_s^4 + \tfrac{1}{4} + \tfrac{3}{4} y_s^4 + \tfrac{1}{4} \varepsilon_1^4 \\
\leq& \tfrac{3}{4} \theta y_s^4 + \tfrac{1}{4} + \tfrac{3}{4} y_s^4 + \tfrac{1}{4} \varepsilon_1^4,
\end{aligned} \tag{27}
$$

where $\theta_1 = \|\Theta_1\|^{\frac{4}{3}}$.

Choose the virtual control signal β_1 as

$$
\beta_1 = \mathcal{N}(\zeta) \Psi(y_s, \hat{\theta}), \tag{28}
$$

with

$$\dot{\zeta} = y_s^3 \Psi(y_s, \hat{\theta}), \tag{29}$$

where $\Psi(y_s, \hat{\theta}) = k_1 y_s + \frac{3}{4}\hat{\theta} y_s$, and $k_1 > \frac{3}{4}$ represents a design parameter.

Substituting (25)-(29) into (24), it can be deduced that

$$\begin{aligned} \mathcal{L}V_1 \leq & -\Lambda_1 \left\|\tilde{\xi}\right\|^4 - (k_1 - \tfrac{3}{4})\varpi_m^4 z_1^4 + \left(\tfrac{1}{\varpi^3 \delta_1}\mathcal{N}(\zeta) + 1\right)\dot{\zeta} + \tfrac{1}{4} z_2^4 \\ & -\tilde{\theta}(\dot{\hat{\theta}} - \tfrac{3}{4} y_s^4) + \Delta_1, \end{aligned} \tag{30}$$

where $\Lambda_1 = \Lambda_0 - \frac{1}{4}$, and $\Delta_1 = \Delta_0 + \frac{1}{4}\varepsilon_1^4 + 1$.

Step i ($i = 2, 3, \cdots, n-1$): From (15), $\mathrm{d}z_i$ can be derived as

$$\mathrm{d}z_i = \mathrm{d}(\hat{\xi}_i - \beta_{i-1}) = (z_{i+1} + \beta_i - c_i \hat{\xi}_1 - \mathbb{L}\beta_{i-1})\mathrm{d}t - \tfrac{\partial \beta_{i-1}}{\partial y_s} \varpi \bar{h}_1^T \mathrm{d}w, \tag{31}$$

where $\mathbb{L}\beta_{i-1} = \sum\limits_{j=1}^{i-1} \frac{\partial \beta_{i-1}}{\partial \hat{\xi}_j}\dot{\hat{\xi}}_j + \frac{\partial \beta_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{\partial \beta_{i-1}}{\partial \zeta}\dot{\zeta} + \frac{\partial \beta_{i-1}}{\partial y_s}(\dot{\varpi} y + \varpi \delta_1^{-1}(\hat{\xi}_2 + \tilde{\xi}_2) + \varpi \bar{f}_1) + \frac{1}{2}\frac{\partial^2 \beta_{i-1}}{\partial y_s^2}\varpi \bar{h}_1^T \varpi \bar{h}_1$.

The Lyapunov function is defined as $V_i = V_{i-1} + \frac{1}{4} z_i^4$, by utilizing (2) and (31), we obtain

$$\mathcal{L}V_i = \mathcal{L}V_{i-1} + z_i^3(z_{i+1} + \beta_i - c_i \hat{\xi}_1 - \mathbb{L}\beta_{i-1}) + \tfrac{3}{2} z_i^2 \mathrm{Tr}\{(\tfrac{\partial \beta_{i-1}}{\partial y_s})^2 \varpi^2 \bar{h}_1 \bar{h}_1^T\}. \tag{32}$$

With the help of Lemmas 2-3, and Assumption 2, the following inequalities hold:

$$z_i^3 z_{i+1} \leq \tfrac{3}{4} z_i^4 + \tfrac{1}{4} z_{i+1}^4, \tag{33}$$

$$
\begin{aligned}
-z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\dot{\varpi}y =& -z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\frac{\dot{\varpi}}{\varpi}y_s \\
\leq& \frac{\varpi_{dM}}{\varpi_m}\left|z_i^3\right|\left|\frac{\partial\beta_{i-1}}{\partial y_s}\right||y_s| \\
\leq& \frac{\varpi_{dM}}{\varpi_m}z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}y_s\tanh(\frac{z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}y_s}{\varsigma_{i1}}) + 0.2785\frac{\varpi_{dM}}{\varpi_m}\varsigma_{i1},
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
-z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\varpi\delta_1^{-1}(\hat{\xi}_2+\tilde{\xi}_2) \leq& \varpi_M\left|\delta_1^{-1}z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\hat{\xi}_2\right| + \varpi_M\left|\delta_1^{-1}z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\tilde{\xi}_2\right| \\
\leq& \varpi_M\delta_1^{-1}z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\hat{\xi}_2\tanh(\frac{z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\hat{\xi}_2}{\delta_1\varsigma_{i2}}) + 0.2785\varpi_M\varsigma_{i2} \\
&+ \tfrac{3}{4}(\varpi_M\delta_1^{-1})^{\frac{4}{3}}(\frac{\partial\beta_{i-1}}{\partial y_s})^{\frac{4}{3}}z_i^4 + \tfrac{1}{4}\left\|\tilde{\xi}\right\|^4,
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
-z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\varpi\bar{f}_1 \leq& \varpi_M\left|z_i^3\right|\left|\frac{\partial\beta_{i-1}}{\partial y_s}\right||\bar{f}_1| \\
\leq& \varpi_M z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\bar{f}_1\tanh(\frac{z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\bar{f}_1}{\varsigma_{i3}}) + 0.2785\varpi_M\varsigma_{i3},
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
-\tfrac{1}{2}z_i^3\tfrac{\partial^2\beta_{i-1}}{\partial y_s^2}\varpi h_1^T\varpi h_1 \leq& \tfrac{1}{2}\varpi_M^2\left|z_i^3\tfrac{\partial^2\beta_{i-1}}{\partial y_s^2}\right|\left\|\bar{h}_1\right\|^2 \\
\leq& \tfrac{1}{2}\varpi_M^2 z_i^3\tfrac{\partial^2\beta_{i-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2\tanh(\frac{z_i^3\frac{\partial^2\beta_{i-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2}{\varsigma_{i4}}) + \tfrac{0.2785}{2}\varpi_M^2\varsigma_{i4},
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
\tfrac{3}{2}z_i^2\mathrm{Tr}\{(\tfrac{\partial\beta_{i-1}}{\partial y_s})^2\varpi^2\bar{h}_1\bar{h}_1^T\} =& \tfrac{3}{2}z_i^2(\tfrac{\partial\beta_{i-1}}{\partial y_s})^2\varpi^2\left\|\bar{h}_1\right\|^2 \\
\leq& \tfrac{3}{4}z_i^4(\tfrac{\partial\beta_{i-1}}{\partial y_s})^4\varpi_M^4\left\|\bar{h}_1\right\|^4 + \tfrac{3}{4},
\end{aligned} \tag{38}
$$

where $\varsigma_{ij} > 0 (j = 1, 2, 3, 4)$ are constants.

Let $Z_i = [y_s, \hat{\theta}, \zeta, \hat{\xi}_1, \hat{\xi}_2, \cdots, \hat{\xi}_i, \xi]^T$ and define a nonlinear function $\Gamma_i(Z_i)$ as $\Gamma_i(Z_i) = \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}}{\partial\hat{\xi}_j}\dot{\hat{\xi}}_j + \frac{\partial\beta_{i-1}}{\partial\zeta}\dot{\zeta} + \frac{\varpi_{dM}}{\varpi_m}\frac{\partial\beta_{i-1}}{\partial y_s}y_s\tanh(\frac{z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}y_s}{\varsigma_{i1}}) + \varpi_M\delta_1^{-1}\frac{\partial\beta_{i-1}}{\partial y_s}\hat{\xi}_2\tanh(\frac{z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\hat{\xi}_2}{\delta_1\varsigma_{i2}}) + \frac{3}{4}(\varpi_M\delta_1^{-1})^{\frac{4}{3}}(\frac{\partial\beta_{i-1}}{\partial y_s})^{\frac{4}{3}}z_i + \varpi_M\frac{\partial\beta_{i-1}}{\partial y_s}\bar{f}_1\tanh(\frac{z_i^3\frac{\partial\beta_{i-1}}{\partial y_s}\bar{f}_1}{\varsigma_{i3}}) + \frac{1}{2}\varpi_M^2 z_i^3\frac{\partial^2\beta_{i-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2\tanh(\frac{z_i^3\frac{\partial^2\beta_{i-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2}{\varsigma_{i4}}) + \frac{3}{4}z_i(\frac{\partial\beta_{i-1}}{\partial y_s})^4\varpi_M^4\left\|\bar{h}_1\right\|^4 - M_i(Z_i)$, where $M_i(Z_i)$ is a continuous function to be specified later.

Again, an FLS is utilized to approximate $\Gamma_i(Z_i)$, thus $\Gamma_i(Z_i) = \Theta_i^T S_i(Z_i) + \mathcal{E}_i(Z_i)$ with $\|\mathcal{E}_i(Z_i)\| \leq \varepsilon_i$ with $\varepsilon_i > 0$. Similar to (27), we have

$$\begin{aligned} z_i^3\Gamma_i(Z_i) =& z_i^3(\Theta_i^T S_i(Z_i) + \mathcal{E}_i(Z_i)) \\ \leq& \tfrac{3}{4}\theta_i z_i^4 + \tfrac{1}{4} + \tfrac{3}{4}z_i^4 + \tfrac{1}{4}\varepsilon_i^4 \\ \leq& \tfrac{3}{4}\theta z_i^4 + \tfrac{1}{4} + \tfrac{3}{4}z_i^4 + \tfrac{1}{4}\varepsilon_i^4, \end{aligned} \tag{39}$$

where $\theta_i = \|\Theta_i\|^{\frac{4}{3}}$.

Now, construct the virtual control β_i as

$$\beta_i = -k_i z_i - \tfrac{3}{4}\hat{\theta} z_i + c_i\hat{\xi}_1, \tag{40}$$

where $k_i > \frac{7}{4}$ denote design parameters.

Then, $\mathcal{L}V_i$ in (32) can be rewritten as

$$\begin{aligned} \mathcal{L}V_i \leq& -\Lambda_i\left\|\tilde{\xi}\right\|^4 - \sum_{j=2}^{i}(k_i - \tfrac{7}{4})z_j^4 - (k_1 - \tfrac{3}{4})\varpi_m^4 z_1^4 + \left(\tfrac{1}{\varpi^3\delta_1}\mathcal{N}(\zeta) + 1\right)\dot{\zeta} + \tfrac{1}{4}z_{i+1}^4 \\ & - \tilde{\theta}\left(\dot{\hat{\theta}} - \tfrac{3}{4}y_s^4 - \tfrac{3}{4}\sum_{j=2}^{i} z_j^4\right) + \sum_{j=2}^{i} z_j(M_j - \frac{\partial\beta_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}) + \Delta_i, \end{aligned} \tag{41}$$

where $\Lambda_i = \Lambda_{i-1} - \frac{1}{4}$, and $\Delta_i = \Delta_{i-1} + 0.2785(\frac{\varpi_{dM}}{\varpi_m}\varsigma_{i1} + \varpi_M\varsigma_{i2} + \varpi_M\varsigma_{i3} + \frac{1}{2}\varpi_M^2\varsigma_{i4}) + \frac{1}{4}\varepsilon_i^4 + 1$.

Step n: $\mathrm{d}z_n$ is given as follows

$$\mathrm{d}z_n = (u - c_n\hat{\xi}_1 - \mathbb{L}\beta_{n-1})\mathrm{d}t - \tfrac{\partial\beta_{n-1}}{\partial y_s}\varpi\bar{h}_1^T\mathrm{d}w, \tag{42}$$

where $\mathbb{L}\beta_{n-1} = \sum\limits_{j=1}^{n-1}\frac{\partial\beta_{n-1}}{\partial\hat{\xi}_j}\dot{\hat{\xi}}_j + \frac{\partial\beta_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\beta_{n-1}}{\partial\zeta}\dot{\zeta} + \frac{\partial\beta_{n-1}}{\partial y_s}(\dot{\varpi}y + \varpi{\delta_1}^{-1}(\hat{\xi}_2 + \tilde{\xi}_2) + \varpi\bar{f}_1) + \frac{1}{2}\frac{\partial^2\beta_{n-1}}{\partial y_s^2}\varpi\bar{h}_1^T\varpi\bar{h}_1$.

Choose the Lyapunov candidate function as $V_n = V_{n-1} + \frac{1}{4}z_n^4$, then taking (2) on V_n gives

$$\mathcal{L}V_n = \mathcal{L}V_{n-1} + z_n^3(u - c_n\hat{\xi}_1 - \mathbb{L}\beta_{n-1} - c_n\xi_1 + c_n\xi_1) + \tfrac{3}{2}z_n^2\mathrm{Tr}\{(\tfrac{\partial\beta_{n-1}}{\partial y_s})^2\varpi^2\bar{h}_1\bar{h}_1^T\}. \quad (43)$$

Similar to **Step** i, one has

$$z_n^3 c_n\tilde{\xi}_1 \le \tfrac{1}{4}\left\|\tilde{\xi}\right\|^4 + \tfrac{3}{4}c_n^{\frac{4}{3}}z_n^4, \quad (44)$$

$$\begin{aligned} -z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\dot{\varpi}y \le& \frac{\varpi_{dM}}{\varpi_m}\left|z_n^3\right|\left|\frac{\partial\beta_{n-1}}{\partial y_s}\right||y_s| \\ \le& \frac{\varpi_{dM}}{\varpi_m}z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}y_s\tanh(\frac{z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}y_s}{\varsigma_{n1}}) + 0.2785\frac{\varpi_{dM}}{\varpi_m}\varsigma_{n1}, \end{aligned} \quad (45)$$

$$\begin{aligned} -z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\varpi\delta_1^{-1}(\hat{\xi}_2+\tilde{\xi}_2) \le& \varpi_M\left|\delta_1^{-1}z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\hat{\xi}_2\right| + \varpi_M\left|\delta_1^{-1}z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\tilde{\xi}_2\right| \\ \le& \varpi_M\delta_1^{-1}z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\hat{\xi}_2\tanh(\frac{z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\hat{\xi}_2}{\delta_1\varsigma_{n2}}) + 0.2785\varpi_M\varsigma_{n2} \\ &+\tfrac{3}{4}(\varpi_M\delta_1^{-1})^{\frac{4}{3}}(\frac{\partial\beta_{n-1}}{\partial y_s})^{\frac{4}{3}}z_n^4 + \tfrac{1}{4}\left\|\tilde{\xi}\right\|^4, \end{aligned} \quad (46)$$

$$\begin{aligned} -z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\varpi\bar{f}_1 \le& \varpi_M\left|z_n^3\right|\left|\frac{\partial\beta_{n-1}}{\partial y_s}\right||\bar{f}_1| \\ \le& \varpi_M z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\bar{f}_1\tanh(\frac{z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\bar{f}_1}{\varsigma_{n3}}) + 0.2785\varpi_M\varsigma_{n3}, \end{aligned} \quad (47)$$

$$\begin{aligned} -\tfrac{1}{2}z_n^3\tfrac{\partial^2\beta_{n-1}}{\partial y_s^2}\varpi\bar{h}_1^T\varpi\bar{h}_1 \le& \tfrac{1}{2}\varpi_M^2\left|z_n^3\tfrac{\partial^2\beta_{n-1}}{\partial y_s^2}\right|\left\|\bar{h}_1\right\|^2 \\ \le& \tfrac{1}{2}\varpi_M^2 z_n^3\tfrac{\partial^2\beta_{n-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2\tanh(\frac{z_n^3\frac{\partial^2\beta_{n-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2}{\varsigma_{n4}}) + \tfrac{0.2785}{2}\varpi_M^2\varsigma_{n4}, \end{aligned} \quad (48)$$

$$\tfrac{3}{2}z_n^2\mathrm{Tr}\{(\tfrac{\partial\beta_{n-1}}{\partial y_s})^2\varpi^2\bar{h}_1\bar{h}_1^T\} \le \tfrac{3}{4}z_n^4(\tfrac{\partial\beta_{n-1}}{\partial y_s})^4\varpi_M^4\left\|\bar{h}_1\right\|^4 + \tfrac{3}{4}, \quad (49)$$

$$\begin{aligned} -z_n^3c_n\xi_1 \le& \left|z_n^3c_n\delta_1\tfrac{y_s}{\varpi_m}\right| \\ \le& \tfrac{c_n}{\varpi_m}z_n^3\delta_1 y_s\tanh(\frac{z_n^3\delta_1 y_s}{\varsigma_{n5}}) + 0.2785\tfrac{c_n\varsigma_{n5}}{\varpi_m}, \end{aligned} \quad (50)$$

where $\varsigma_{nj} > 0 (j = 1, 2, 3, 4, 5)$ are constants.

Define a unknown nonlinear function $\Gamma_n(Z_n) = \sum_{j=1}^{n-1} \frac{\partial\beta_{n-1}}{\partial\hat{\xi}_j}\hat{\xi}_j + \frac{\partial\beta_{n-1}}{\partial\zeta}\dot{\zeta} + \frac{\varpi_{dM}}{\varpi_m}\frac{\partial\beta_{n-1}}{\partial y_s} y_s \tanh(\frac{z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}y_s}{\varsigma_{n1}}) + \varpi_M\delta_1^{-1}\frac{\partial\beta_{n-1}}{\partial y_s}\hat{\xi}_2 \tanh(\frac{z_n^3\frac{\partial\beta_{n-1}}{\partial y_s}\hat{\xi}_2}{\delta_1\varsigma_{n2}}) + \frac{3}{4}(\varpi_M\delta_1^{-1})^{\frac{4}{3}}(\frac{\partial\beta_{n-1}}{\partial y_s})^{\frac{4}{3}} z_n + \frac{1}{2}\varpi_M^2 z_n^3 \frac{\partial^2\beta_{n-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2 \tanh(\frac{z_n^3\frac{\partial^2\beta_{n-1}}{\partial y_s^2}\left\|\bar{h}_1\right\|^2}{\varsigma_{n4}}) + \frac{3}{4}z_n(\frac{\partial\beta_{n-1}}{\partial y_s})^4\varpi_M^4\left\|\bar{h}_1\right\|^4 + \frac{c_n}{\varpi_m}y_s\delta_1\tanh(\frac{z_n^3\delta_1 y_s}{\varsigma_{n5}}) - M_n(Z_n)$, where $Z_n = [y_s, \hat{\theta}, \zeta, \hat{\xi_1}, \hat{\xi_2}, \cdots, \hat{\xi_n}, \xi]^T$ and $M_n(Z_n)$ is the continuous function to be defined later.

Like (27) and (39), we also have

$$
\begin{aligned}
z_n^3\Gamma_n\left(Z_n\right) =& z_n^3(\Theta_n^T S_n(Z_n) + \mathcal{E}_n(Z_n)) \\
\leq& \tfrac{3}{4}\theta_n z_n^4 + \tfrac{1}{4} + \tfrac{3}{4}z_n^4 + \tfrac{1}{4}\varepsilon_n^4 \\
\leq& \tfrac{3}{4}\theta z_n^4 + \tfrac{1}{4} + \tfrac{3}{4}z_n^4 + \tfrac{1}{4}\varepsilon_n^4,
\end{aligned}
\tag{51}
$$

where $\theta_n = \|\Theta_n\|^{\frac{4}{3}}$. Besides, $\mathcal{E}_n(Z_n)$ represents the approximation error satisfying $\|\mathcal{E}_n(Z_n)\| \leq \varepsilon_n$ with $\varepsilon_n > 0$.

The control signal u and the adaptation law $\dot{\hat{\theta}}$ are chosen in the following form:

$$
u = -\mathcal{R}(\hat{\theta})z_n = -(k_n + \tfrac{3}{4}c_n^{\frac{4}{3}} + \tfrac{3}{4}\hat{\theta})z_n, \tag{52}
$$

with

$$
\mathcal{R}(\hat{\theta}) = k_n + \tfrac{3}{4}c_n^{\frac{4}{3}} + \tfrac{3}{4}\hat{\theta} \tag{53}
$$

and

$$
\dot{\hat{\theta}} = -\iota\hat{\theta} + \tfrac{3}{4}y_s^4 + \tfrac{3}{4}\sum_{i=2}^{n} z_i^4, \tag{54}
$$

where $k_n > 1$ and $\iota > 0$ are design parameters.

Similar to [35], choose $M_i (i = 2, 3, \cdots, n)$ as

$$
M_i(Z_i) = -\iota\frac{\partial\beta_{i-1}}{\partial\hat{\theta}}\hat{\theta} + \tfrac{3}{4}\frac{\partial\beta_{i-1}}{\partial\hat{\theta}}y_s^4 + \tfrac{3}{4}\frac{\partial\beta_{i-1}}{\partial\hat{\theta}}\sum_{j=2}^{i-1} z_i^4 + \tfrac{3}{4}z_i^4\sum_{j=2}^{i}\frac{\partial\beta_{j-1}}{\partial\hat{\theta}}, \tag{55}
$$

then

$$\sum_{i=2}^{n} z_i^3 (M_i - \frac{\partial \beta_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}}) = 0. \tag{56}$$

Substituting (44)-(56) into (43) and applying the inequality $-\tilde{\theta}\hat{\theta} \leq -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2$, $\mathcal{L}V_n$ is derived as

$$\begin{aligned} \mathcal{L}V_n \leq & -\Lambda_n \left\|\tilde{\xi}\right\|^4 - \sum_{i=2}^{n-1} (k_i - \tfrac{7}{4}) z_i^4 - (k_n - 1) z_n^4 - (k_1 - \tfrac{3}{4}) \varpi_m^4 z_1^4 + \left(\tfrac{1}{\varpi^3 \delta_1} \mathcal{N}(\zeta) + 1\right) \dot{\zeta} \\ & - \tfrac{\iota}{2} \tilde{\theta}^2 + \Delta_n, \end{aligned} \tag{57}$$

where $\Lambda_n = \Lambda_{n-1} - \frac{1}{2}$, and $\Delta_n = \Delta_{n-1} + 0.2785(\frac{\varpi_{dM}}{\varpi_m} \varsigma_{n1} + \varpi_M \varsigma_{n2} + \varpi_M \varsigma_{n3} + \varpi_M^2 \varsigma_{n4} + \frac{c_n \varsigma_{n5}}{\varpi_m}) + \frac{1}{4}\varepsilon_n^4 + \frac{\iota}{2}\theta^2 + 1$.

Finally, by selecting appropriate value of $\hat{\theta}(0)$, we can deduce that $\mathcal{R}$ in (53) is positive, thus by Lemma 7 the inverse optimal controller u_{op} is

$$u_{op} = \tfrac{2}{3}\alpha u, \alpha \geq 2. \tag{58}$$

4 Stability and performance analysis

The main result of this paper can be generalized as the following theorem.

Theorem 1. *For the attacked stochastic nonlinear CPS (5) that satisfies Assumption 1 and Assumption 2, under virtual control signals (28), (29), (40), the adaptation law (54), and inverse optimal controller (58), the following two conclusions hold:*

1. *all the signals are bounded in probability, and the system is secure with probability* $1 - \epsilon$*;*
2. *the inverse optimal controller (58) can solve the inverse optimal stabilization problem and minimize the following cost function*

$$J(u) = E\left[2\alpha V + \int_0^t \left(l(X, \hat{\theta}) + \frac{27}{16\alpha^2} \mathcal{R}^{-3} u^4\right) \mathrm{d}\tau\right], \tag{59}$$

where

$$l(X,\hat{\theta}) = 2\alpha\left[\mathcal{R}z_n^4 - L_F V + \frac{\partial V}{\partial\tilde{\theta}}\dot{\hat{\theta}} - \frac{1}{2}\mathrm{Tr}\left\{G_2^T\frac{\partial^2 V}{\partial X^2}G_2\right\}\right] + \alpha(\alpha-2)\mathcal{R}z_n^4, \tag{60}$$

with $L_F V = \frac{\partial V}{\partial X}F$ *being Lie derivative, and* X, F, G_2 *being defined in (14).*

Proof: The proof of **Theorem 1** consists of two parts, detailed as follows:

1) Let $V = V_n$, it can be easily obtained that

$$\mathcal{L}V \leq -\mu_1 V + \left(\frac{1}{\varpi^3\delta_1}\mathcal{N}(\zeta) + 1\right)\dot{\zeta} + \mu_2, \tag{61}$$

where $\mu_1 = \min\{2\Lambda_n/\lambda_{\max}^2(P), (4k_1-3)\varpi_m^4, 4k_2-7, \cdots, 4k_n-4, \iota\}$, and $\mu_2 = \Delta_n$.

Multiplying V by $e^{\mu_1 t}$ and by Itô formula yields

$$\mathrm{d}(e^{\mu_1 t}V) = e^{\mu_1 t}(\mu_1 V + \mathcal{L}V)\mathrm{d}t + e^{\mu_1 t}\varPhi(t)\mathrm{d}w \tag{62}$$

where $\varPhi(t) = \frac{\partial V}{\partial z_1}\bar{h}_1^T + \sum_{i=2}^{n}\frac{\partial V}{\partial z_i}\frac{\partial\beta_{i-1}}{\partial y_s}\varpi\bar{h}_1^T$.

Substituting (61) into (62) and integrating on both sides, it yields

$$V(t) \leq e^{-\mu_1 t}V(0) + \int_0^t e^{\mu_1(\tau-t)}\left(\frac{1}{\varpi^3\delta_1}\mathcal{N}(\zeta) + 1\right)\dot{\zeta}\mathrm{d}\tau + \int_0^t e^{\mu_1(\tau-t)}\varPhi(\tau)\mathrm{d}\omega + \frac{\mu_2}{\mu_1}. \tag{63}$$

By using the conclusions in [1], $\int_0^t e^{\mu_1(\tau-t)}\varPhi(\tau)\mathrm{d}\omega$ is a real-valued continuous local martingale. Note that $0 < |\frac{1}{\varpi^3\delta_1}| \leq |\frac{1}{\varpi_m^3\delta_1}|$, then by Lemma 1, it can be deduced that $V(t)$ and $\zeta(t)$ are bounded in probability. Furthermore, combining the definition of V, we can draw the conclusion that z_i, $\tilde{\theta}$ and $\tilde{\xi}$ are bounded in probability. According to the definition of θ, (15) and (52), it is easily proved that $\hat{\theta}$, y, $\hat{\xi}_i$, β_i and u are also bounded in probability. Thus, all the signals in the closed-loop system are bounded in probability.

Then, there exists a positive constant μ_3 such that $\int_0^t(\frac{1}{\varpi^3\delta_1}\mathcal{N}(\zeta)+1)\dot{\zeta}\mathrm{d}\tau \leq \mu_3$. Consider $\frac{d}{dt}[EV] = E[\mathcal{L}V]$, taking expectations and integrating for (61), we can get

$$0 \leq \tfrac{1}{4}E\Big[\sum_{i=1}^{n} z_i^4\Big] \leq E[V] \leq V(0)e^{-\mu_1 t} + \tfrac{\mu_2+\mu_3}{\mu_1} \leq V(0) + \tfrac{\mu_2+\mu_3}{\mu_1} := U,$$

which means $E\Big[\sum\limits_{i=1}^{n} z_i^4\Big] \leq 4U$, i.e., the error signals z_i are all bounded according to Definition 3.

Furthermore by choosing suitable initial values and design parameters, we can deduce that all the signals in the closed-loop system including x_i converge to an enough small σ-neighborhood with a probability of $1-\epsilon$ for a sufficiently small constant ϵ, which means

$$\mathbb{P}\{\|x_i(t)\| \leq \sigma\} \geq 1-\epsilon.$$

Thus from Definition 2, the attacked system (5) is secure with a probability $1-\epsilon$.

2) From (2) and (14), we can obtain that there exists a continuous positive function $\mathcal{W}$ satisfying

$$\frac{\partial V}{\partial X}F - \mathcal{R}z_n^4 - \frac{\partial V}{\partial \tilde{\theta}}\dot{\hat{\theta}} + \frac{1}{2}\mathrm{Tr}\left\{G_2^T\frac{\partial^2 V}{\partial X^2}G_2\right\} \leq -\mathcal{W}. \tag{64}$$

Substituting (64) into (60), we can get that

$$l(X,\hat{\theta}) \geq 2\alpha\mathcal{W} + \alpha(\alpha-2)\mathcal{R}z_n^4. \tag{65}$$

Considering that $\alpha \geq 2$, $\mathcal{R}$ and $\mathcal{W}$ are positive functions, so $l(X,\hat{\theta})$ is also positive, thereby $J(u)$ is a meaningful cost function.

Due to $E\Big[V(0)+\int_0^t \mathcal{L}V\mathrm{d}\tau - V(t)\Big]=0$, we obtain

$$
\begin{aligned}
J(u) &= 2\alpha E\Big[V(x(0),\hat{\xi}(0),\hat{\theta}(0))\Big] + E\left[\int_0^t \left(2\alpha\mathcal{L}V + l + \frac{27}{16\alpha^2}\mathcal{R}^{-3}u^4\right)\mathrm{d}\tau\right] \\
&= 2\alpha E\Big[V(x(0),\hat{\xi}(0),\hat{\theta}(0))\Big] + E\left[\int_0^t \left(2\alpha z_n^3 u + \alpha^2\mathcal{R}z_n^4 + \frac{27}{16\alpha^2}\mathcal{R}^{-3}u^4\right)\mathrm{d}\tau\right].
\end{aligned} \tag{66}
$$

By choosing $\gamma = v^{\frac{4}{3}}$ and using Lemma 5, we have

$$
\begin{aligned}
-2\alpha z_n^3 u =& \alpha^2\left(-z_n^3\mathcal{R}^{\frac{3}{4}}\right)\left(\frac{2}{\alpha}\mathcal{R}^{-\frac{3}{4}}u\right) \\
\leq& \alpha^2\mathcal{R}z_n^4 + \frac{27}{16\alpha^2}\mathcal{R}^{-3}u^4,
\end{aligned} \tag{67}
$$

and the equality holds if and only if $u=-\frac{2}{3}\alpha\mathcal{R}z_n$, i.e. $u=u_{op}$.

Then substituting (67) into (66) results in

$$
J(u) \geq 2\alpha E\Big[V(x(0),\hat{\xi}(0),\hat{\theta}(0))\Big]. \tag{68}
$$

The above inequality means that for any $\alpha \geq 2$, the smaller the value of α, the smaller the value of $J(u)$, thus the minimum value of $J(u)$ is $J(u)_{\min} = 4E\Big[V(x(0),\hat{\xi}(0),\hat{\theta}(0))\Big]$ and $u_{op}=-\frac{4}{3}\mathcal{R}z_n$ right now.

This completes the proof.

5 Simulation examples

In this section, we present two physical examples affected by diverse unknown sensor and actuator attacks to verify the effectiveness of our proposed method.

Example 1: As a common nonlinear system, a mass-spring-damper system with unmodelled dynamics in [24] is given by

$$\begin{cases} \dot{x}_1 = x_2 + \wp_1(x_1, x_2), \\ \dot{x}_2 = \dfrac{1}{m}u - \dfrac{1}{m}\phi_1(x_1) - \dfrac{1}{m}\phi_2(x_2) + \wp_2(x_1, x_2), \\ y = x_1, \end{cases} \tag{69}$$

where x_1, x_2 and u are the displacement, velocity and control signal, respectively; m means a unknown positive constant; $\wp_1(x_1, x_2)$ and $\wp_2(x_1, x_2)$ are unmodelled dynamics; $\phi_1(x_1)$ and $\phi_2(x_2)$ are nonlinear functions .

Case 1: To demonstrate the optimal properties of our proposed security control scheme, a comparison is made with [24]. Similar to [24], we select $m = 3$, $\phi_1(x_1) = 2x_1^2$, and $\phi_2(x_2) = x_2^2\cos(x_2)$. Furthermore, the unmodelled dynamics are chosen as $\wp_1(x_1, x_2) = 0.1x_1\sin(x_1)$, and $\wp_2(x_1, x_2) = 0.2\sin(x_1x_2)$.

Considering the actual situations, we assume that the system (69) is also affected by stochastic disturbances $\Xi_1(x_1, x_2) = x_1\sin(x_1)$, and $\Xi_2(x_1, x_2) = x_2\cos(x_1)$. The actuator attack is $u_a = 6u - e^{-0.1t}$, the sensor attack is $\psi_s = (-5 + 0.8\cos(t))y$, and both attacks occur simultaneously at 5s. Then system (69) can be rewritten as

$$\begin{cases} \mathrm{d}x_1 = (x_2 + \wp_1(x_1, x_2))\mathrm{d}t + \Xi_1(x_1, x_2)\mathrm{d}w, \\ \mathrm{d}x_2 = (\dfrac{1}{m}u_a - \dfrac{1}{m}\phi_1(x_1) - \dfrac{1}{m}\phi_2(x_2) + \wp_2(x_1, x_2))\mathrm{d}t + \Xi_2(x_1, x_2)\mathrm{d}w. \end{cases} \tag{70}$$

The simulation is run with the initial conditions $x_1(0) = 0.1$, $x_2(0) = 0.05$, $\hat{\xi}_1(0) = 0.2$, $\hat{\xi}_2(0) = 0.2$, $\hat{\theta}(0) = 0.5$, and $\zeta(0) = 0$. And the design parameters are chosen as $k_1 = k_2 = 10$, $c_1 = c_2 = 2.5$, and $\iota = 0.5$. To ensure a fair comparison, all the design parameters and initial conditions are set to be the same.

The comparison results between two methods are given in Figs. 2 and 3. Fig. 2 shows the output signal y under two control schemes. The comparison of control signals is shown in Fig. 3. Obviously, the normal control scheme in [24] needs a much larger control signal when the network attacks happened at 5s, which means more energy are consumed.

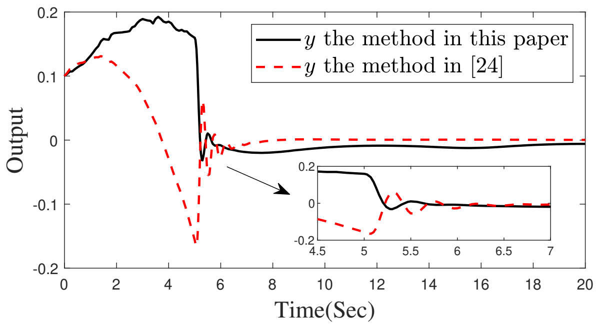

Fig. 2 Comparison of output y

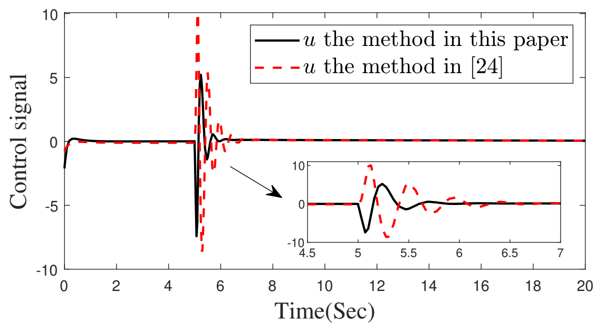

Fig. 3 Comparison of control signal u

Remark 6. *The system with strict-feedback form, which is a special case of non-strict feedback system, have relatively received an amount of research, so we choose to compare the proposed method with our previous work [24] in* ***Case 1****, which only considers* *the security control problem of strict-feedback stochastic systems. In the following simulations, we further choose a non-strict feedback stochastic system and different attack models.*

Case 2: In this case, we consider more complex system and attack models. Choose the following actuator attack

$$u_a = \begin{cases} u, & t \le 15s \\ 2u - 1.2\sin(3t), & t > 15s \end{cases} \tag{71}$$

and the sensor attack is specified as

$$y_s = \begin{cases} y, & t \le 10s \\ y(-2 + 0.8\cos(t)), & t > 10s \end{cases} \tag{72}$$

Furthermore, we select $\wp_1(x_1, x_2) = x_1 x_2 \sin(x_1)$, $\wp_2(x_1, x_2) = 0.2\sin(x_1 x_2)$, $\Xi_1(x_1, x_2) = x_1 \sin(x_2)$ and m, ϕ_1, ϕ_2, Ξ_2 remain unchanged in (70), thus (70) becomes a non-strict feedback system.

The initial conditions and the design parameters are same in **Case 1**. The simulation results are depicted in Figs 4-7. The trajectories of system states x_1 and x_2 are shown in Fig. 4. Fig. 5 describes the curves of the transformed states ξ_i and their estimations $\hat{\xi}_i$ for $i = 1, 2$. The trajectories of the input signal u and attacked signal u_a are depicted in Fig. 6. The trajectories of adaptive parameter $\hat{\theta}$ and Nussbaum variable ζ are shown in Fig. 7. From these simulation results, it is clearly seen that under the action of the proposed control laws, all the signals are bounded.

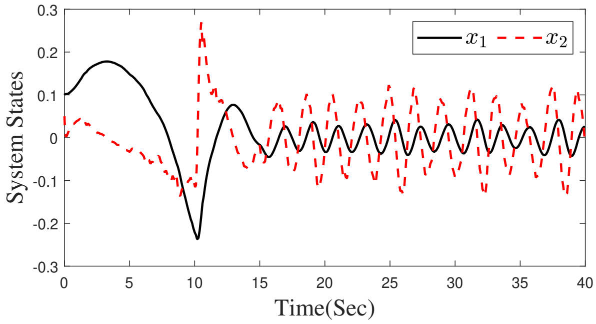

Fig. 4 System states x_1 and x_2 in Case 2

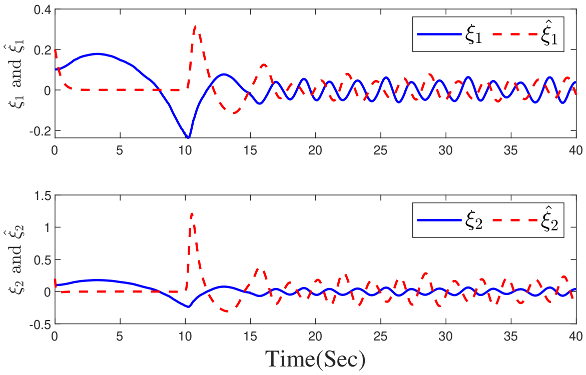

Fig. 5 The transformed variables ξ_i and their estimations $\hat{\xi}_i$ for $i = 1, 2$ in Case 2

Example 2: To further demonstrate the effectiveness of our method, we consider a one-link manipulator modeled as [5, 6]

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin(q) = \tau_t + \tau_d \\ M\dot{\tau} + H\tau = u - K_m\dot{q} \end{cases} \tag{73}$$

where q, $\dot{q}$, and $\ddot{q}$ are the link position, velocity, and acceleration, respectively; u denotes the control input; τ_t means the torque; and $\tau_d = \tau(q, \dot{q}, \tau_t)\dot{w}$, $\tau(q, \dot{q}, \tau_t) = q^2\cos(\dot{q}\tau_t)$ with w being the stochastic disturbances defined in (1). The rest of the parameters are chosen as $D = 1\text{kgm}^2$, $B = 1\text{Nms/rad}$, $N = 10\text{Nm}$, $M = 0.1\text{H}$, $H = 1\Omega$, and $K_m = 0.2\text{Nm/A}$, as the same in [6].

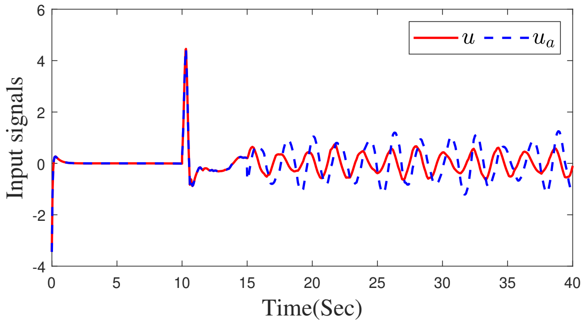

Fig. 6 The trajectories of u and u_a in Case 2

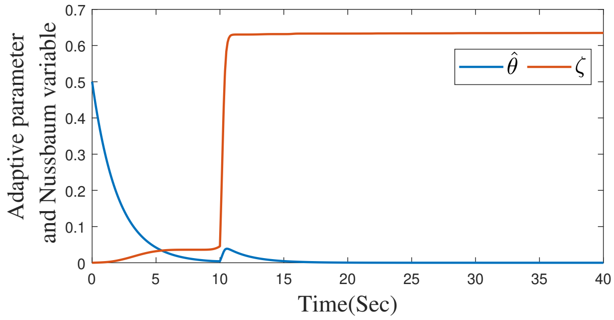

Fig. 7 The Nussbaum variable ζ and adaptive parameter $\hat{\theta}$ in Case 2

Let $y = x_1 = q$, $x_2 = \dot{q}$, and $x_3 = \tau$, and take the following actuator and sensor attacks into consideration

$$u_a = \begin{cases} u, & t \leq 5s \\ -1.5u - 0.5e^{\sin(2t)}, & t > 5s \end{cases} \tag{74}$$

$$y_s = \begin{cases} 0, & t \leq 5s \\ y(-1.5 + 0.5e^{\sin(2t)}), & t > 5s \end{cases} \tag{75}$$

then the attacked three-order system (73) can be remodelled as

$$\begin{cases} \mathrm{d}x_1 = x_2 \mathrm{d}t, \\ \mathrm{d}x_2 = (-x_2 + x_3 - 10\sin x_1)\mathrm{d}t + x_1^2 \cos(x_2 x_3)\mathrm{d}w, \\ \mathrm{d}x_3 = (10u_a - 2x_2 - 10x_3)\mathrm{d}t, \\ y = x_1. \end{cases} \tag{76}$$

The design parameters are designed as $k_1 = 6$, $k_2 = 4$, $k_3 = 3$, $c_1 = c_2 = c_3 = 2.5$, and $\iota = 1$. Moreover, the initial conditions are set as $[x_1(0), x_2(0), x_3(0)]^T = [0.2, 0.2, 0.2]^T$, $[\hat{\xi}_1(0), \hat{\xi}_2(0), \hat{\xi}_3(0)]^T = [0.2, 0.2, 0.2]^T$, and $[\zeta(0), \hat{\theta}(0)]^T = [0, 0.5]^T$. The simulation results are given in Figs. 8-9. Once again, the results show that the proposed control method can make the attacked system (76) secure.

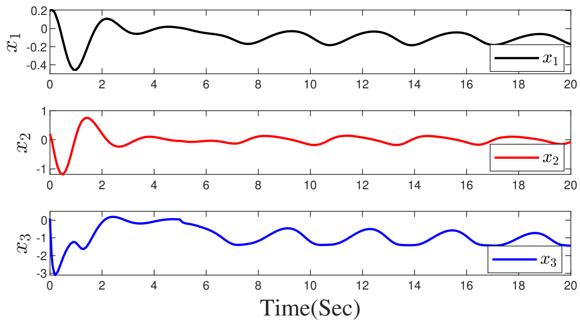

Fig. 8 System states in Example 2

Remark 7. *Due to the unknown attack signals in practice, it is necessary to consider multiple types of attack forms. The actuator attack (74) introduces different control direction compared to (71), which is a disaster for a control system. In Example 2, the additive attack signals take the form of* $e^{\sin(2t)}$*, which means that* $\psi_a(t)$ *does not satisfy* $\psi_a \to 0$ *as* t *approaches infinity. Instead, it transforms into a periodic attack and causes interference to the system continuously.*

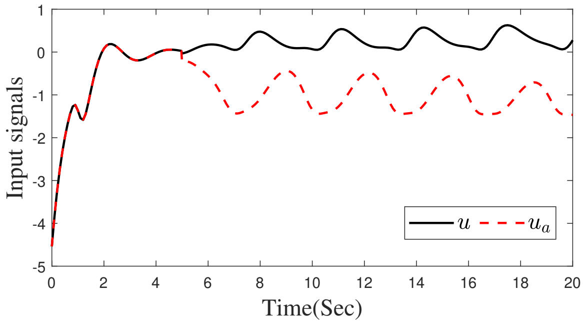

Fig. 9 The trajectories of u and u_a in Example 2

6 Conclusion

This paper has focused on the security control and inverse optimality control design problems for stochastic non-strict feedback nonlinear CPS under sensor and actuator attacks. The main difficulties in control design come from unknown network attacks, unknown control coefficients, and non-strict feedback forms. Firstly, a linear state transformation is introduced. Next, the Nussbaum gain function and FLS are banded to handle the above difficulties. Finally, the controller is proposed by combining output feedback security control and IOC. The effectiveness of our proposed control strategy against various network attacks is verified through two physical examples. How to solve the security consensus control problem of multi-agent systems under network attacks is worthy of future research.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under Grant 71871135, and Grant 72271155; and in part by the Fundamental Research Funds for the Central Universities under Grant 222201717006.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest

The authors declare that they have no conflict of interest.

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