“Game Tree Searching by Min/Max Approximation” Article Summary.

The article “Game Tree Searching by Min/Max Approximation” by Ronald L. Rivest describes the application of Min and Max functions, approximated by generalized mean value operators, in searching minimax game trees. The author suggests that the combination of min/max approximated functions and penalty-based iterative search method may be used as an alternative to the minimax algorithm.

The introductory part of the paper describes the method of generalized mean values, which is, in a nutshell, a way to transform discrete min and max operators into functions with continuous partial derivatives for all arguments. The author suggests using partial derivatives of these functions to determine the leaves and branches with stronger impact to the root-node of the searching tree.

In order to set a background for the implementation of the suggested method, the author describes the game tree search methodology along with some heuristic functions for two-players, zero-sum, perfect information games. He introduces the penalty-based iterative search method which suggests assigning penalties (or weights) to edges in order to reflect the quality of the respective move and searching for the path with minimized sum of penalties. In the min/max approximation heuristics, the penalties are defined as partial derivatives of the approximating functions described above. The method suggests summing all penalties between the tip and the root node in order to estimate the quality of a search path. The nodes with least penalty will be chosen for the further discovery.

The disadvantage of the method described is a high computational cost because it involves powers and roots. The paper suggests using the reverse approximation to reduce the computational complexity. The author explains that, since, the generalized mean values are used for the approximation of the min and max values, using the approximate min and max values anyway is acceptable and may not introduce much error.

The paper uses the *Connect-Four* game to demonstrate performance of the method in comparison to the depth-first minimax search with alpha-beta pruning. The experiment revealed that the performance strongly depends of the type of resource bound. When the time per move is limited alpha-beta search performs better. In cases where number of moves is limited, the min/max approximation method demonstrates stronger results. Based on the experiment, the author concludes that the method he described may find more practical value in cases when a move operator has a high computational cost.

Link to the original article: https://people.csail.mit.edu/rivest/pubs/Riv87c.pdf