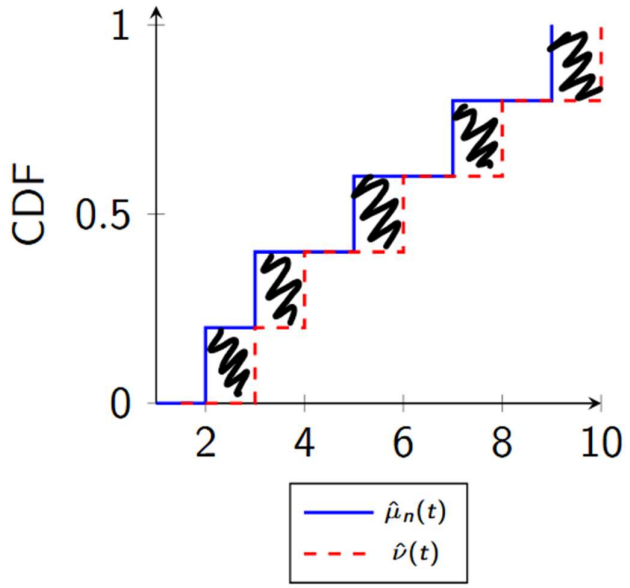
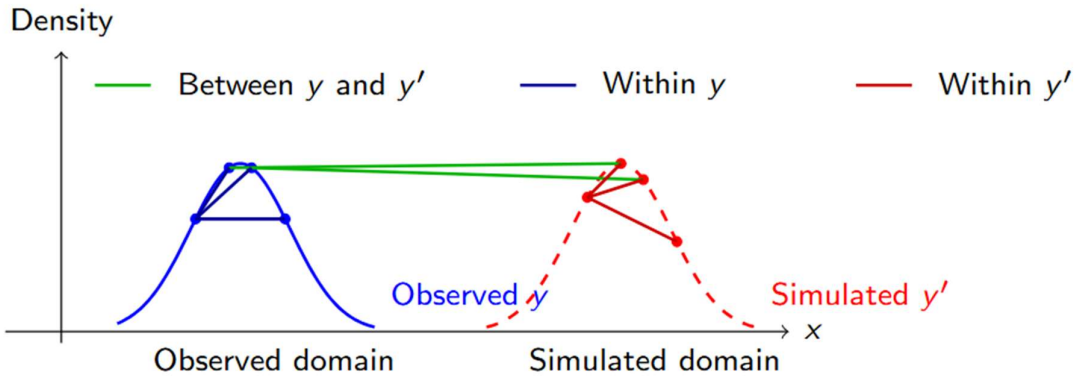
Distance Metric	Definition	Formula (Empirical Estimation)
Wasserstein Distance	**Minimal transport** from one distribution into another.	$\frac{1}{n}\sum_{i=1}^{n}\lvert y_{(i)} - y'_{(i)}\rvert$
Energy Distance	Difference between **pairwise distances**.	$\frac{2}{n^2}\sum_{i,j}\lVert y_i - y'_j\rVert_p - \frac{1}{n^2}\sum_{i,j}\lVert y_i - y_j\rVert_p - \frac{1}{n^2}\sum_{i,j}\lVert y'_i - y'_j\rVert_p$
Maximum Mean Discrepancy (MMD)	Generalisation of Energy Distance, with choice of kernel.	$\frac{1}{n(n-1)}\sum_{i\neq j}^{n}k(y_i, y'_j) + \frac{1}{n(n-1)}\sum_{i\neq j}^{n}k(y'_i, y'_j) - \frac{2}{n^2}\sum_{i\neq j}^{n}k(y_i, y'_j)$
Cramer-von Mises Distance (CvMD)	**Area** between the CDF of simulated and observed.	$\hat{C} = \frac{U}{2n^4} - \frac{4n^2-1}{12n}$ $\frac{U}{n} = \sum_{i=1}^{n}(r_{(i)} - i)^2 + \sum_{j=1}^{n}(s_{(j)} - j)^2$
Kullback-Leibler Divergence (KLD)	**Log difference** between densities.	$\frac{1}{n}\sum_{i=1}^{n}\ln\frac{\min_j \lVert y'_i - y_j\rVert}{\min_{j\neq i}\lVert y'_i - y'_j\rVert} + \ln\left(\frac{n}{n-1}\right)$


1
0.5
0
CDF
2
4
6
8
10
$\hat{\mu}_n(t)$
$\hat{\nu}(t)$


Density
Between y and y'
Within y
Within y'
Observed y
Simulated y'
x
Observed domain
Simulated domain