

Adversarial Search

Lab 5

Optimal Decisions in Games

Game definition

- **Initial state** – includes board position and the player to move
- **Successor function** – returns a list of (move, state) pairs, where move is legal
- **Terminal test** – game over at some terminal state
- **Utility function** (objective or payoff function) – give numeric values of a terminal state

E.g., win: +1, loss: -1, draw: 0

MinMax Strategy

TicTacToe

Tic-Tac-Toe

Initial state and legal moves define game tree (see figure below for tic-tac-toe)

- Initial state: board empty, MAX turn
- Successor function: returns the list of 9 pairs (move, state)

E.g., from the initial state:

(0, [X, 1, 2, 3, 4, 5, 6, 7, 8])

(1, [0, X, 2, 3, 4, 5, 6, 7, 8])

(2, [0, 1, X, 3, 4, 5, 6, 7, 8])

(3, [0, 1, 2, X, 4, 5, 6, 7, 8])

(4, [0, 1, 2, 3, X, 5, 6, 7, 8])

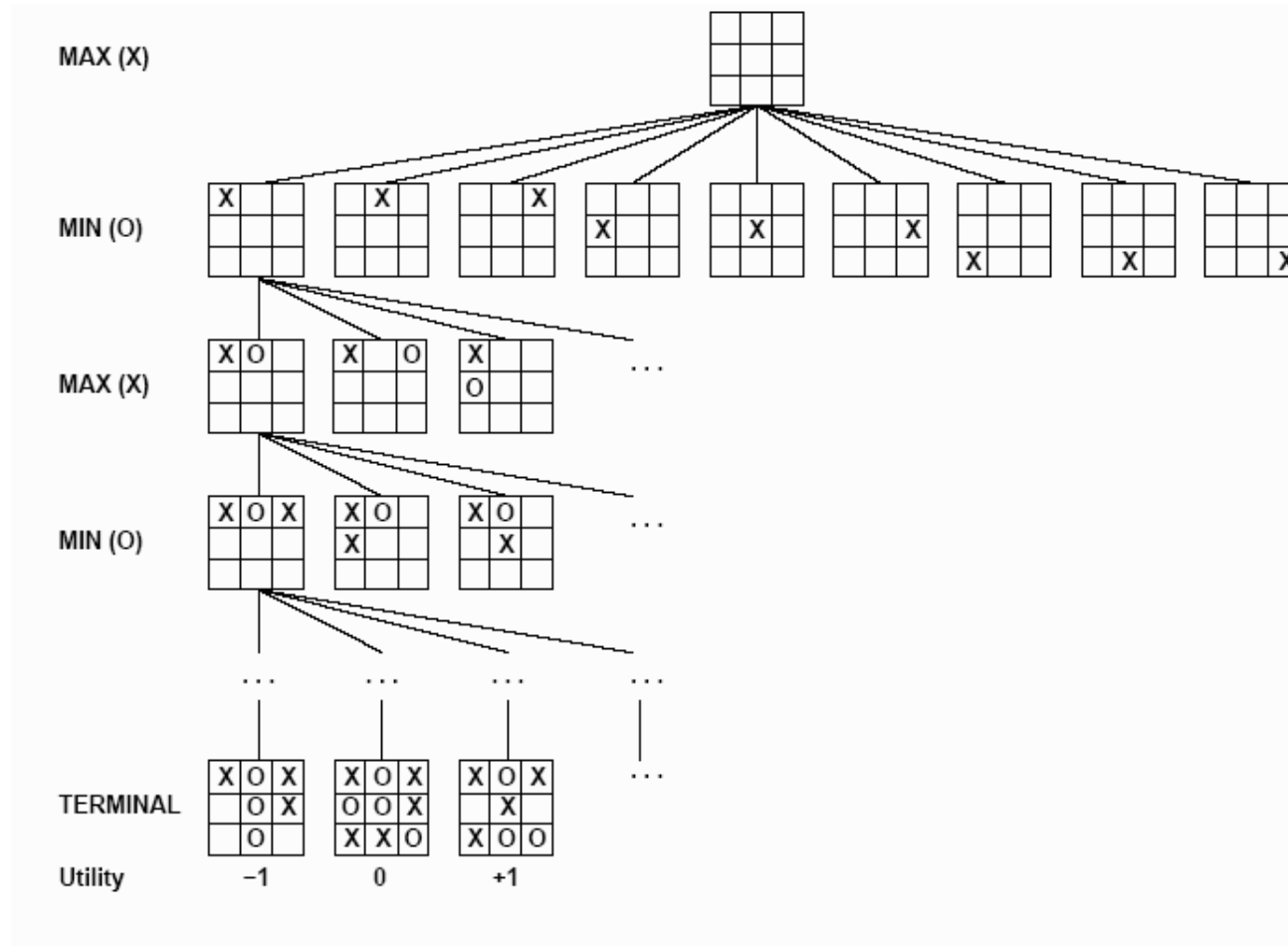
(5, [0, 1, 2, 3, 4, X, 6, 7, 8])

(6, [0, 1, 2, 3, 4, 5, X, 7, 8])

(7, [0, 1, 2, 3, 4, 5, 6, X, 8])

(8, [0, 1, 2, 3, 4, 5, 6, 7, X])

(Partial) Tic-Tac-Toe Search Tree



Tic-Tac-Toe

- The tic-tac-toe layout is

0	1	2
3	4	5
6	7	8

- The initial state of the board without moves is [0,1,2,3,4,5,6,7,8]
- After an X is placed on the central tile the state is [0,1,2,3,X,5,6,7,8]
- `utility_of(state)` returns +1 if winner is X (MAX player), -1 if winner is O (MIN player), or 0 otherwise
- `successors_of(state)` returns a list of pairs (move, state) as shown in the previous slides
- `is_terminal(state)` returns `True` if the state is either a win or a tie (board full)

Questions

1. What is the branching factor at depth 0? At depth 1?
2. What is the maximal depth?
3. Will a MIN move attempt to minimize or maximize the utility?
4. Are states after a terminal state explored?
5. Are all possible states explored to a terminal state?
6. Is this a depth-first or breadth-first search? How do you know? (see Python code)

Exercise Tic-Tac-Toe

Must be submitted

Run the MinMax Tic-Tac-Toe program by implementing these functions in `tictactoe_template.py`:

- `is_terminal(state)`
- `utility_of(state)`
- `is_terminal(state)`

Note: You will play as “O”, and the computer plays as “X”.

MinMax and Alpha-Beta

Nim

Homework Nim

Must be submitted

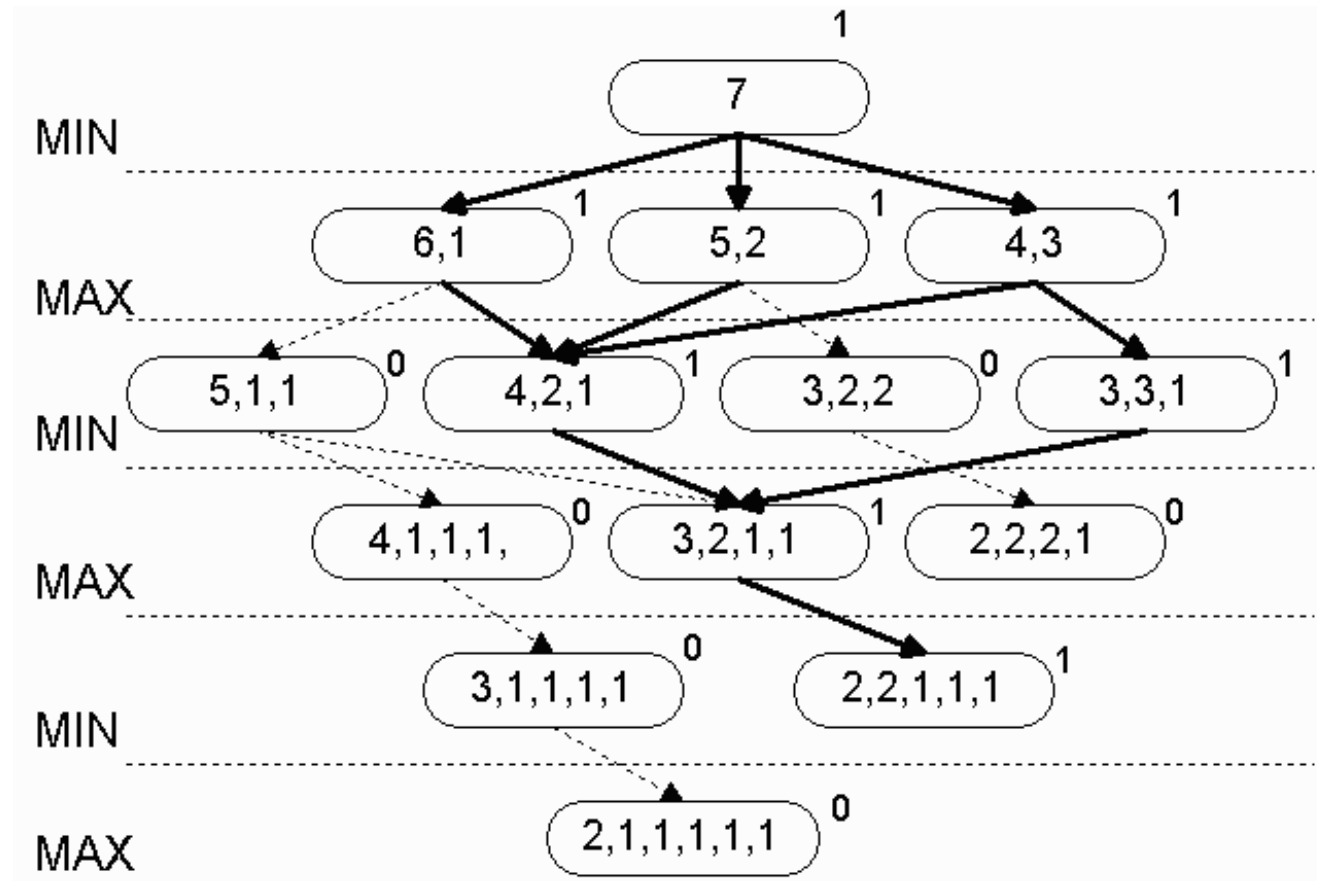
1. Write a nim game using `minmax_decision` from tic-tac-toe program. The state space of nim may be exhaustively searched.

To play nim, a number of tokens are placed on a pile between two opponents; at each move, the player must divide a pile into two nonempty piles of different sizes. Thus, a pile of six tokens [6] may be divided into two piles of [5,1] or [4,2] but not [3,3,]. The first player who can no longer move loses. The picture on the following slide illustrates a state space for a game with 7 tokens.

MIN should start the game and, with 7 tokens is certain to lose if MAX divides the piles following the **bold** arrows.

Test using 15 in the starting pile.

Nim Search Tree



Homework Nim

Must be submitted

2. Rewrite nim using `alpha_beta_decision` and compare with minmax over a pile of 20
3. Modify nim to play the MIN position, i.e., `utility_of` should return +1 for MIN and -1 for MAX

Note: MAX is stateless, no memory of previous pile is held. Any pile, e.g., [1,2,3,4], can be sent to MAX which will return its move.

Challenge

May optionally be submitted

Solve “Breakthrough” using alpha-beta search

- The game is played on a chessboard, every piece is a pawn.
- A piece may move one space straight or diagonally forward if the target square is empty.
- A piece may move into a square containing an opponent’s piece if and only if that square is one step diagonally forward. The opponent’s piece replaces it. Note that capturing is not compulsory, nor is it “chained” as in checkers.
- The first player to reach the opponent’s home row – the one farthest from the player – is the winner. If all pieces of a player are captured, that player loses.

Adapted from

[https://en.wikipedia.org/wiki/Breakthrough_\(board_game\)](https://en.wikipedia.org/wiki/Breakthrough_(board_game))