Agent-Based Modelling of Emotion Contagion in Groups[1]

Tibor Bosse[a], Rob Duell[a], Zulfiqar A. Memon[a,b],
Jan Treur[a], C. Natalie van der Wal[a]

[a]Vrije Universiteit Amsterdam, Department of Artificial Intelligence
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands
[b]Sukkur Institute of Business Administration (Sukkur IBA),
Airport Road Sukkur, Sindh, Pakistan
{tbosse, rduell, treur, cwl210}@few.vu.nl, memon.zulfiqar@gmail.com

Abstract To avoid the development of negative emotion in their teams, team leaders may benefit from being aware of the emotional dynamics of the team members. To this end, the use of intelligent computer systems that analyze emotional processes within teams is a promising direction. As a first step towards the development of such systems, this paper uses an agent-based approach to formalize and simulate emotion contagion processes within groups, which may involve absorption or amplification of emotions of others. The obtained computational model is analysed both by explorative simulation and by mathematical analysis. In addition, to illustrate the applicability of the model, it is shown how the model can be integrated within a computational 'ambient agent model' that monitors and predicts group emotion levels over time and proposes group support actions based on that. Based on this description, a discussion is provided of the main contribution of the model, as well as the next steps needed to incorporate it into real world applications.

Keywords: multi-agent model, emotion contagion spirals, ambient agent model.

1 Introduction

The occurrence of emotion contagion in groups is a social phenomenon, where emotions of group members can be absorbed by other group members, but also can be amplified so that levels of emotion may occur that may substantially exceed the original emotion levels of group members. How to avoid such trends for negative emotions and how to stimulate them for positive emotions can be a real challenge for both group members and group leaders. This paper first presents an analysis and a computational model for the occurrence of emotion contagion in

[1] Parts of the work described here have been presented in a preliminary form as
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Bosse, T., Duell, R., Memon, Z.A., Treur, J., and Wal, C.N. van der, A Multi-Agent Model for Emotion Contagion Spirals Integrated within a Supporting Ambient Agent Model. In: Yang, J.-J.; Yokoo, M.; Ito, T.; Jin, Z.; Scerri, P. (eds.), *Proceedings of the 12th International Conference on Principles of Practice in Multi-Agent Systems, PRIMA'09*. Lecture Notes in Artificial Intelligence, vol. 5925. Springer Verlag, 2009, pp. 48–67.
Duell, R., Memon, Z.A., Treur, J., and Wal, C.N. van der, An Ambient Agent Model for Group Emotion Support. In: Cohn, J., Nijholt, A., Pantic, M.(eds.), *Proceedings of the Third International Conference on Affective Computing and Intelligent Interaction, ACII'09*. IEEE Computer Society Press, 2009, pp. 550-557.

groups. In addition, it is shown how this model can be integrated in a computational ambient agent model to support group leaders. The ambient agent can predict and analyze the team's emotional level for present and future time points. In case a team's emotional level is found (to become) deficient compared to a certain norm, the ambient agent proposes the team leader to take some measures.

Many definitions of emotions exist today. In this article emotions are defined as being intense and short-lived and focused on a specific target or cause [12]. Emotions can sometimes transfer into moods, which are global (positive) feelings which can last a few moments up to a few weeks. Hence, the difference between emotions and moods is mainly determined by two aspects, namely 1) their cause and 2) their duration. Regarding the cause, emotions are assumed to be triggered by a certain (internal or external) stimulus, whereas moods are assumed to be independent of a particular stimulus. Regarding the duration, emotions last only briefly, usually up to seconds after the occurrence of the stimulus that triggered them, whereas moods last much longer. Nevertheless, moods can be modified based on a (cumulative) effect of multiple emotions. Emotions, moods and other related concepts such as feeling traits and personal tendencies, are gathered under the general term affect. All terms that fall under the most general term affect are further explained in [3]. In the current work only emotions are into focus, with the possible extension into moods. Emotions allow humans to respond quickly and efficiently to events that affect their welfare [20]. In addition, they provide us with information about others' behavioral intentions, and script our social behavior. Research on the idea that emotion also has a strong social component, which can influence interactions, is found in, e.g. [15], [16]. The process of emotion contagion, in which a group member influences the emotions of another group member (and vice versa), through the conscious or unconscious induction of emotion states [27], is a primary mechanism through which individual emotions create a collective emotion. This process has been described as an inclination to mimic the gestural behavior of others, to "synchronize facial expression, utterances and attitudes" [16]. Emotion contagion has been shown to occur in many cases varying from emotions in small groups to panicking crowds; see [1], [2], [29], [21].

Emotion contagion has found a biological foundation in recent neurological findings on the *mirroring function* of certain neurons (e.g., [18], [19], [25]), in conjunction with the notion of internal simulation (by as-if body loops; cf. [37]; see also [38]). Mirror neurons are neurons which, in the context of the neural circuits in which they are embedded, show both a function to prepare for certain actions or bodily changes and a function to mirror states of other persons. They are active not only when a person intends to perform a specific action or body change, but also when the person observes somebody else intending or performing this action or body change. This includes expressing emotions in body states, such as facial expressions for emotions. For example, there is strong evidence that (already from an age of just 1 hour) sensing somebody else's face expression leads (within about 300 milliseconds) to preparing for and showing the same face expression ([13], p. 129-130). The idea is that these neurons and the neural circuits in which they are embedded play an important role in social functioning and in (empathic) understanding of others; (e.g., [18], [25]). The discovery of mirror neurons is often considered a crucial step for the further development of the discipline of social cognition, comparable to the role the discovery of DNA has played for biology, as it provides a biological basis for many social phenomena; cf. [18]. Indeed, when states of other persons are mirrored by some of the person's own states that at the same time are connected via neural circuits to states that are crucial for the own feelings and actions, then this provides an effective basic mechanism for how in a social context persons fundamentally affect each other's actions and feelings.

The positive effects of emotions have been investigated empirically in [10], where it is hypothesized that positive emotions trigger upward spirals toward enhanced emotional well-being. This prediction is based on Frederickson's broaden-and-build theory [11]. The broaden hypothesis states that positive emotions broaden people's momentary mind-sets: the scopes of attention, cognition, action and the array of percepts, thoughts, and actions presently in mind are widened. This in turn, serves to build their enduring personal resources. In this way the short-term effect of positive emotions can develop from a short-term effect into a longer lasting positive upward spiral. The complementary narrowing hypothesis predicts the reverse pattern: negative emotions shrink people's momentary thought-action repertoires. In turn, also in the reverse pattern the momentary or 'beginning' negative spiral can build into a longer lasting negative spiral. Support for the broaden and narrowing hypotheses can be found in [9]. The build hypothesis expresses that positive emotions encourage people to discover and explore new ways of thinking and action, by which they are building their personal resources such as socio-emotional and intellectual skills. The broaden hypothesis can predict upward trends in emotional well-being of a person, which the authors [9], [10], [11] and many other researchers in the field of positive psychology investigate. In [10], the authors demonstrated that initial experiences with positive affect can improve broad-minded-coping, which in turn can predict increases in positive affect over time, creating an upward trend towards improved emotional well-being.

This paper first introduces a multi-agent model that formalizes and simulates emotion contagion within groups, and can represent two different types of emotion contagion processes: emotion absorption and emotion amplification within groups. Next it is shown how this computational model can be used in applications within a teamwork context, supported by an intelligent ambient agent. Section 2 explains a formalized model of group emotion contagion processes. Next, in Section 3, simulation results for the model are presented and in Section 4, the model is analyzed mathematically. Section 5 addresses formal verification of the emotion contagion model and the simulation results. Section 6 describes how the model for emotion contagion has been integrated in an existing ambient agent model. In Section 7 some simulation results are discussed for the resulting ambient agent model. Section 8 is a discussion.

2 The Emotion Contagion Model

Modeling group emotion can be done at the level of the group or at the level of the individuals, which have been named the top-down and bottom-up approach respectively [2]. The bottom-up perspective sees group emotion as the sum of its parts, affected by the homogeneity or heterogeneity of the group and the mean emotions of the group members. Individual differences play an important role, such as specific personality traits and the underlying brain mechanisms. The top-down approach defines group emotion as being different from the sum of its individual parts. The authors describe this as that diverse emotional tendencies of individuals are submerged into a group emotion and the emotional character of the group can be more extreme than the individual tendencies. The model for emotion contagion introduced in this section subsumes different types of emotion contagion, varying from *emotion absorption* which occurs when group members adapt their emotion levels to each other by a kind of averaging process, to *emotion amplification*, in which case group members can use other group members' emotion as a trigger to generate higher or lower levels of emotions than available in the group. These two different types of models can be linked to a certain degree with the top-down and bottom-up approach defined in [2], *emotion absorption* being more a bottom-up approach and *emotion amplification* being more a top-down approach of modeling group emotion contagion. The model distinguishes multiple factors that influence emotion contagion. In [1] (following [21]) Barsade describes an informal model of emotion contagion in which the emotion being expressed and transferred among group members is characterized by the valence (positive or negative) and the energy level with which the emotion is expressed. Furthermore Barsade [1] suggests two categories of contagion mechanisms: automatic subconscious contagion through mimicry and feedback and conscious transfer through social comparison of moods and appropriate responses in groups, mediated by attention. Regardless of the mechanisms employed, it is claimed that the type of emotion and the degree of emotion contagion in groups, is influenced by the emotional valence and the emotional energy.

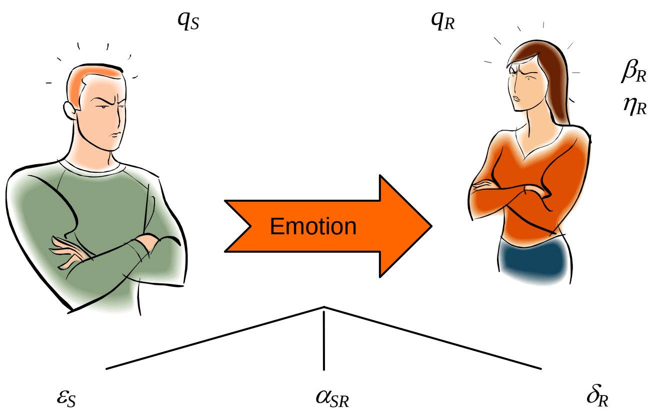

Figure 1: Aspects of Emotion Contagion

Inspired by these theories, in this section a computational model of emotion contagion is proposed. First a number of aspects are distinguished that play a role in the contagion, varying from aspects related to the sender, the channel between sender and receiver and the receiver of the transferred emotion. Accordingly, the model distinguishes three parts in the process of transfer of emotion and related parameters: an agent having the role of sender (denoted as *S*), an agent having the role of receiver (denoted by *R*), and the channel from *S* to *R* (see Fig. 1 and Table 1). Note that in a group in principle each agent A has both the role of sender and of receiver.

	Emotion state		Characteristics	
Sender	current level of the sender's emotion	q_S	extent to which the sender expresses the emotion	ε_S
Channel			the strength of the channel from sender to receiver	α_{SR}
Receiver	current level of the receiver's emotion	q_R	openness or sensitivity for received emotion	δ_R
			bias to adapt emotions upward or downward	β_R
			tendency to amplify emotions	η_R

Table 1: Parameters for aspects of emotion contagion

A basic assumption underlying the model is that each individual agent (no matter whether it acts as a sender or a receiver) experiences a particular emotion with a certain intensity in the domain [0,1], indicated by the variable q_S (for the level of emotion of the sender) or q_R (for the level of emotion of the receiver). The particular type of emotion to which these levels refer is not specified. In principle, this can be any type of emotion from a categorical model of emotion[2] (e.g., the classical model of Ekman [33], which distinguishes anger, disgust, fear, joy, sadness and surprise), as long as the type of emotion is the same for all agents included in a simulation.

The aspect ε_S depends on how introvert or extravert, expressive, active and energetic the person is. It represents the degree to which a person transforms internal emotion into external expression. In this sense, an introvert person will induce a weaker contagion of an emotion than an extravert person. The aspect α_{SR} depends on the type and intensity of the contact between the two persons (e.g., distance vs attachment). The aspect δ_R indicates the degree of susceptibility of the receiver: the extent to which the receiver allows the emotions received from others to affect his own emotion, and how flexible/persistent the receiver is emotionally. The aspect η_R describes the tendency to amplify emotions, when triggered by received emotions. When it is *0* the person does not amplify emotions, but only absorbs them; when it is *1* it does not absorb emotions but only amplifies them. The aspect β_R describes the bias when amplifying emotions (more positive or more negative), when triggered by received emotions.

As a first step, all aspects have been formalized numerically by numbers in the interval [*0, 1*]. In addition, the parameter γ_{SR} is used to represent the strength by which an emotion is transferred to *R* from sender *S*. It is assumed to depend on expressiveness ε_S, channel strength α_{SR}, and openness δ_R, as follows:

$$\gamma_{SR} = \varepsilon_S \, \alpha_{SR} \, \delta_R \qquad \textbf{(1)}$$

The stronger the channel, the higher α_{SR} and the more contagion takes place. The model works as follows: if gamma is set to *0* there is no contagion, if γ_{SR} is *1*, there is maximum

[2] In addition to *categorical* models, in principle it is also possible to integrate the model with so-called *dimensional* emotion categorization models (which represent emotions as coordinates in a multi-dimensional space, using e.g. dimensions like valence and arousal), or even more sophisticated *hybrid* models, such as the 'hourglass of emotions' [32]. One way to do this would be to unify the level of emotion used in this paper with one single dimension within a dimensional model. A more detailed investigation of the consequences of such an approach is left for future work.

strength of contagion. If γ_{SR} is not 0, there is contagion and the higher the value, the more contagion takes place. In this way, the parameter γ_{SR} can create the behavior as formulated by hypothesis (1) and (2) from [1]. In a way γ_{SR} expresses the energy by which an emotion is being expressed and transferred. Interestingly this γ_{SR} depends on situational factors (processes and influences) at both group and individual level. The overall strength by which emotions from all the other group members are received by *R* in a group *G*, indicated by γ_R, is defined as

$$\gamma_R = \Sigma_{S \in G\setminus\{R\}} \; \gamma_{SR} \tag{2}$$

The proposed model can simulate (starting from initial values set for the levels q_A for all agents *A*) upward and downward emotional spirals through mechanisms, with which not only an individual agent, but also the whole group of agents can get to a higher or lower level of emotion. Each agent *A* transfers an emotion value *q* between *0* and *1*. The model makes it possible for each agent *A* in certain situations to approximate values like *0* and *1*, or values in between. Each agent will reach its own emotional equilibrium within the group. Suppose *G* is a group of agents. The dynamics of an agent *A*'s emotion level is described as

$$dq_A/dt = \gamma_A \, [\, \eta_A \, (\beta_A \, PI \; + (1\text{-}\beta_A) \, NI) + (1\text{-}\eta_A) \, q_A{}^* \; - \; q_A] \tag{3}$$

Here

$$q_A{}^* = \Sigma_{S \in G\setminus\{R\}} \; w_{SA} \, q_S \tag{4}$$

is a weighted sum of the emotion levels of the other group members *S* with weights

$$w_{SA} = \varepsilon_S \, \alpha_{SA} \, / \, \Sigma_{C \in G\setminus\{A\}} \; \varepsilon_C \, \alpha_{CA} \tag{5}$$

The upward or downward direction of the change in an agent *A*'s emotional level over time depends on the bias parameter β_A, and the speed of ascend or descend on parameter γ_R. Furthermore, to determine amplification, *PI* and *NI* are the positive and negative impact of received emotion from the other group members respectively, which will be specified and explained in more detail below. The parameter β_A defines the overall impact as a weighted combination of the two contributions. By varying the values of the β_A's, upward as well as downward spirals can be simulated. If $\beta_A = 1$ then the receiver is only susceptible for positive impact. If $\beta_A = 0$, then the receiver is only susceptible to negative impact. Any number between *0* and *1* represents a person who is more or less susceptible to positive and negative impacts. E.g., if $\beta_A = 0.8$, the agent will be infected by *80%* with *PI* and by *20%* with *NI*.

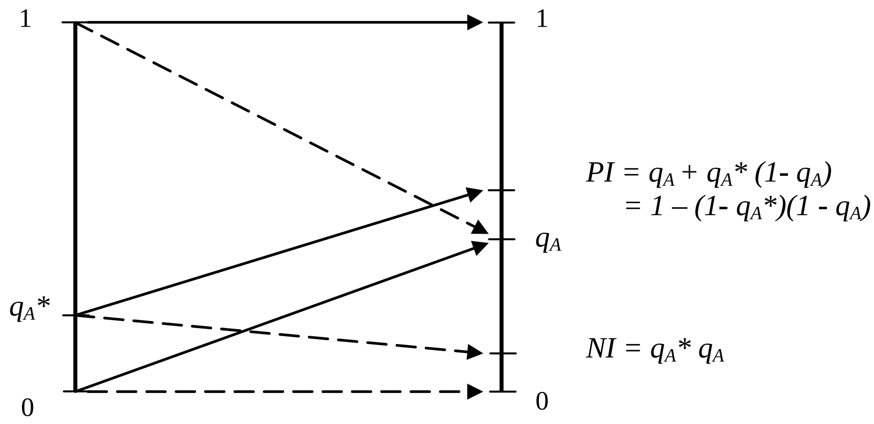

Figure 2: Positioning of *NI* and *PI* in the interval [0, 1]

In more detail the positive and negative impacts of the other group members are defined as:

$$PI = 1 – (1-q_A^*) (1-q_A) \quad \textbf{(6)}$$
$$NI = q_A^* q_A \quad \textbf{(7)}$$

These expressions can be explained using Fig. 1. For the upward, positive impact the argument is as follows. The agent A already has level q_A. In addition to that it receives extra intensity q_A^* from the other group members. This is in a sense added to q_A, but taking into account a normalisation to the interval ($1 - q_A$) which is the space left between q_A and 1. This results in a linear projection of the [0, 1] on the left hand side to the interval [q_A, 1] on the right hand side. This provides the mapping of q_A^* to $q_A + q_A^* (1- q_A)$, which can be rewritten as $PI = 1 – (1-q_A^*)(1 - q_A)$. At the same time a negative, downward impact can be determined by linear projection of the interval [0, 1] on the left hand side to the interval [0, q_A] on the right hand side, resulting in $NI = q_A^* q_A$. The overall resulting new level is determined as a weighted sum of *NI* and *PI*, where the weights $1-\beta_A$ and β_A directly relate to the bias parameter β_A.

By filling (6) and (7) in the equation (3), the detailed set of equations for group *G* is for all $A \in G$:

$$dq_A/dt = \gamma_A [\eta_A (\beta_A (1 – (1-q_A^*) (1-q_A)) + (1-\beta_A) q_A^* q_A) + (1-\eta_A) q_A^* - q_A] \quad \textbf{(8)}$$

Note that the model presented so far represents the emotional states of all agents within a group separately; the question of how these separate individual emotional states can be interpreted and aggregated, in order to assess the collective emotional state of a group, is addressed in Section 6.

3 Simulation Results for Emotion Contagion

In this section some simulation results of emotion contagion processes are discussed, first for the emotion absorption model, and next for the emotion amplification model that generates emotion contagion spirals. After that, a section shows simulation results for larger populations.The agents are deterministic in itself (every time a simulation with specific settings is run, the outcome will be the same). The simulation runs are based on partially stochastic settings. The simulations for small group sizes in Sections 3.1 and 3.2 do not have stochastic parameter settings. The simulations in Section 3.3 for large populations, include stochastically distributed parameters, such as a uniform distribution for the initial emotional value q of each group member. This allows an investigation of group patterns based on many different scenario's that approach reality. Furthermore, the agent characteristics of openness, expressiveness and channel strength are static in the simulations; they do not change. The emotional level of each agent and that of the group is dynamic. Therefore, the agent learns indirectly from its past. For every timestep, the emotional values are updated with the current percepts of that timepoint. Indirectly, the past is present in every point in time, because the update mechanism happens dynamically: the emotional value of the previous timestep is partially taken into account into the next point in time. This proces repeats every timestep, making the past percepts of the agent indirectly present in the current percept. See also the pseudo code below.

```
PSEUDO CODE EMOTION CONTAGION SIMULATIONS
Initialisation(time step == 1)
        For small group sizes (Sections 3.1 and 3.2)
                Manually set an emotional start value q for each agent between [0,1];
                Calculate contagion strengths for every agent, towards every other agents, based on
                the static values for agent characteristics: openness, channel strength and
                expressiveness;
        For larger group sizes (Section 3.3)
                Distribute an emotional start value q for each agent (0,1];
                Distribute contagion strengths uniformly (0,1];

Dynamic Updates(For every time step>1)
        Dynamically update emotion levels of every agent;
        Dynamically update emotion level of the group;
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3.1 Simulation Results for Emotion Absorption

A large number of simulations have been performed, using numerical simulation software, resulting in a variety of interesting patterns. These patterns are described in this section. The occurrences of a number of typical patterns were mathematically proven (under certain conditions) and are presented in Section 4. In this section some of the simulation results are discussed for the case of the absorption: $\eta_R = 0$ for all R. Simulation results for the case of amplification ($\eta_R = 1$ for all R) will be presented in Section 3.2.

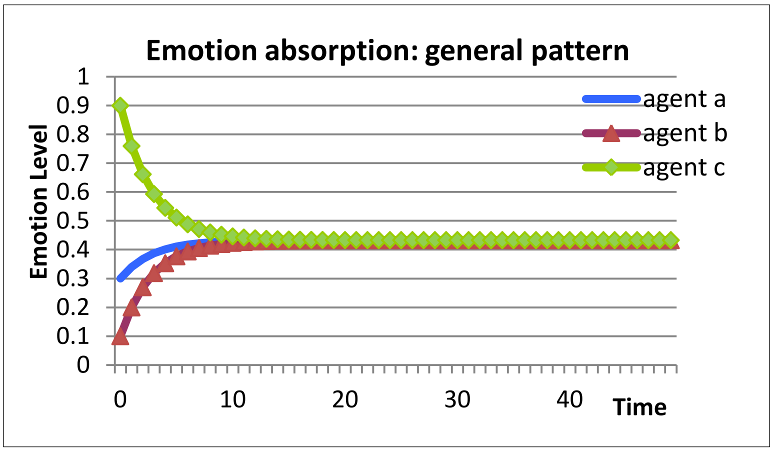

Figure 3: Simulation trace 1 for emotion absorption ($\eta_R = 0$ and $\gamma_R = 1$ for all R)

Simulations shown here are for a group of 3 agents *a*, *b*, and *c*. Time is at the horizontal axis, and the emotion level at the vertical axis. A first pattern found is that when the γ_R for all agents are not *0* (in this case they are all *1*), the emotion levels of all of them approximate the average of their initial emotion levels; hereby the speed depends on the δ_R (susceptibility) and $\varepsilon_S\alpha_{SR}$ (see Figure 3). The occurrence of this pattern has been confirmed mathematically; see Theorem 3 in the Section 4.

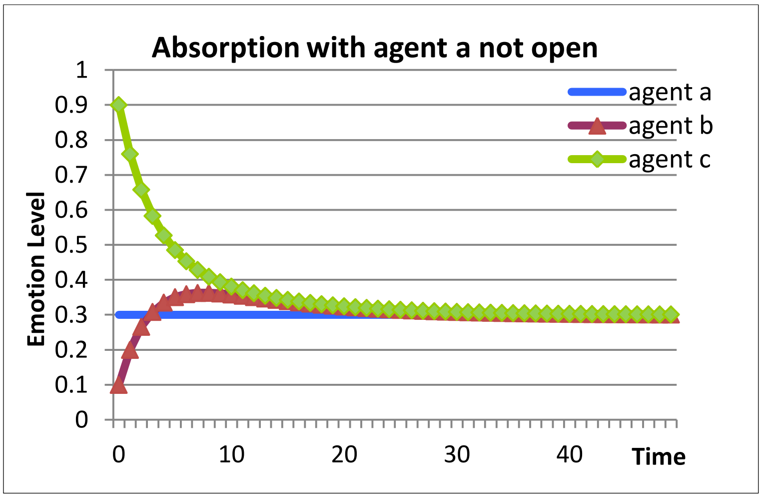

Figure 4: Simulation trace 2 for emotion absorption ($\eta_R = 0$ and $\delta_a = 0$)

A very specific pattern happens when all γ_R are *0*, in this case all agents will have equilibria that are equal to their individual initial emotional levels. In other words: the emotional levels of all agents will not change at all; see Theorem 1 in the next section. Another situation (see Figure 4) occurs when agent *a* has δ_a set to *0* and the other agents have this parameter $\neq 0$. This situation represents that agent *a* is not open to receive emotions, but can send emotions. As a result agent *a*'s initial emotion level will remain the same. Furthermore, the agents *b* and *c* will eventually reach the equilibrium of agent *a*, which is equal to its initial emotion level.

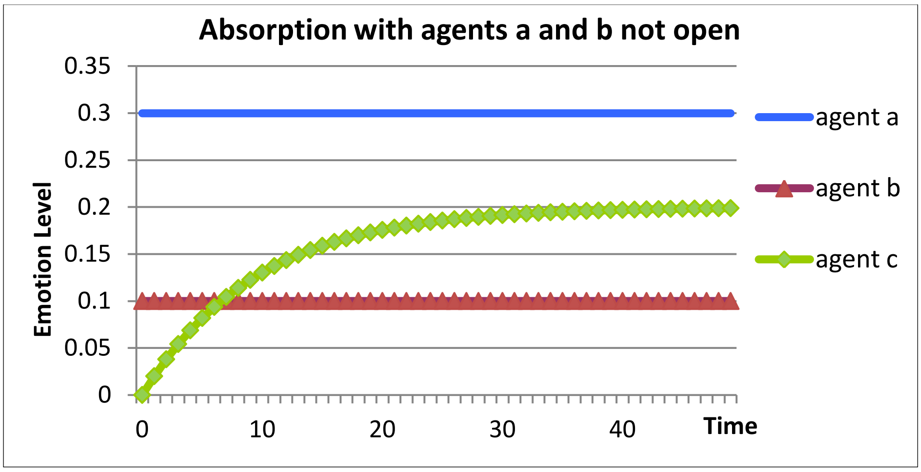

Figure 5: Simulation trace 3 for emotion absorption ($\eta_R = 0$ and δ_R $(a, b, c) = (0, 0, 0.5)$)

In Figure 5 it is shown that when agent *a* and *b* both have δ_R set to 0, agent *c* will reach a value in-between *a* and *b*'s initial emotion values. The actual value that agent *c* will reach, depends on the settings of the parameter settings for all agents. This situation represents a case, where two agents do not change their emotional level, because they are only open to sending emotions, but not to receiving emotions. As a result the third agent is forced to reach a value in between the emotional levels of the others. A next situation (see Figure 6) is one where δ_a *and* ε_a are set to 0. This represents agent *a* being bi-directionally excluded from emotion contagion: (s)he can not receive or send emotions. The agents *b* and *c* are forced to go to a certain average in between their initial emotion values. The exact value they will reach depends on the settings of their δ_R (susceptibility) and ε_S α_{SR}.

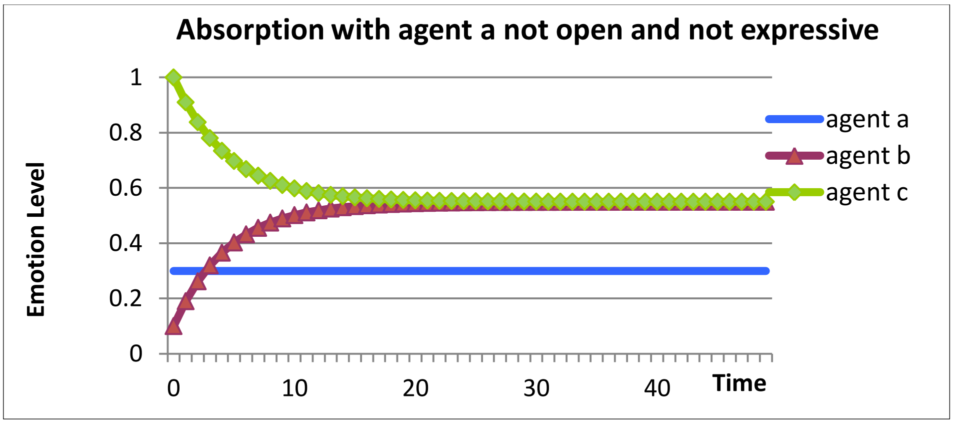

Figure 6. Simulation trace 4 for emotion absorption ($\eta_R = 0$ and $\delta_a = \varepsilon_a = 0$)

3.2 Simulation Results for Emotion Amplification

Inspired by the momentary emotion (contagion) effects that can turn into long lasting upward or downward emotional spirals in [11], for the case of amplification ($\eta_R = 1$ for all R) the proposed model can simulate both upward and downward emotional spirals. A large number of simulations have been performed, using numerical simulation software, resulting in a variety of interesting patterns. In this section some of the simulation results are discussed. The next section presents results of a mathematical analysis, in which for most patterns their occurrence was proven, under certain conditions.

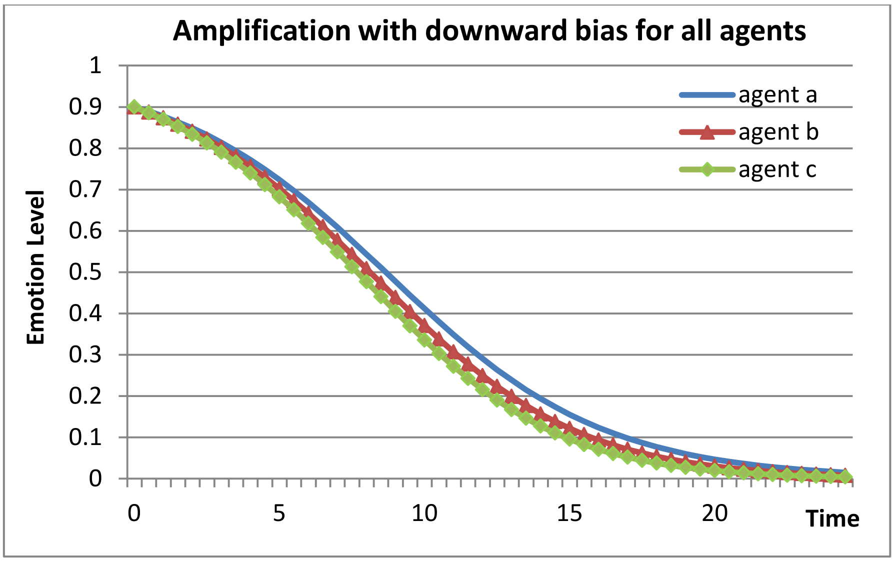

Figure 7: Simulation trace 1 (all $\beta=0$, all $\delta_a=0.6$, $\delta_b=0.7$, $\delta_c=0.8$, and all $w_{DC}=0.2$)

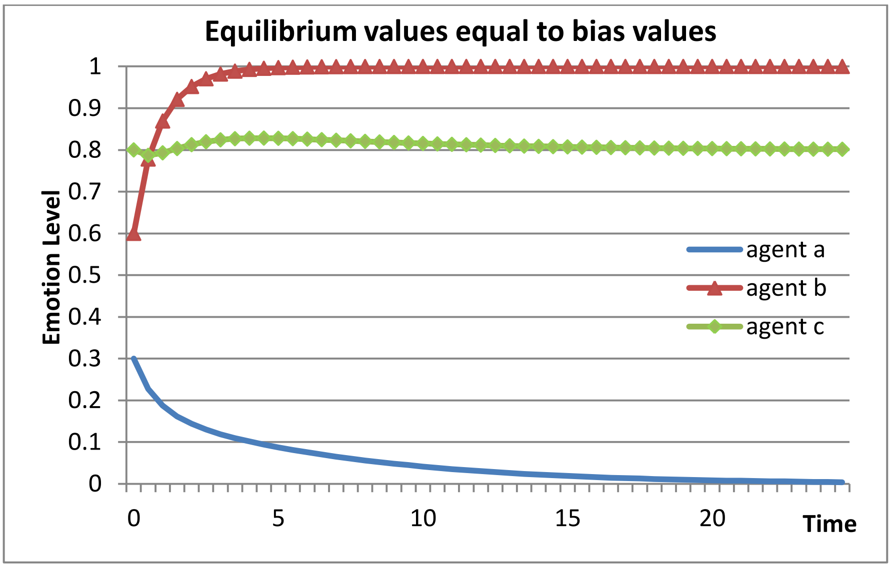

Figure 8: Simulation trace 2 ($\beta\,(a, b, c) = (0, 1, 0.8)$, all $\delta_R=0.9$, all $w_{DC}=0.9$)

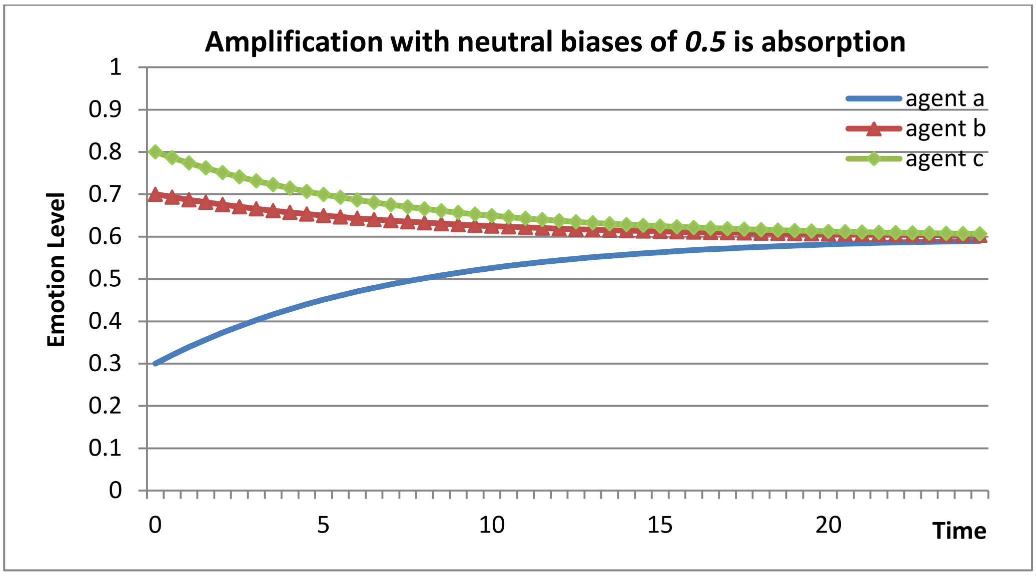

Figure 9: Simulation trace 3 (all β=*0.5*, all δ_R=*0.1*, all w_{DC}=*0.9*)

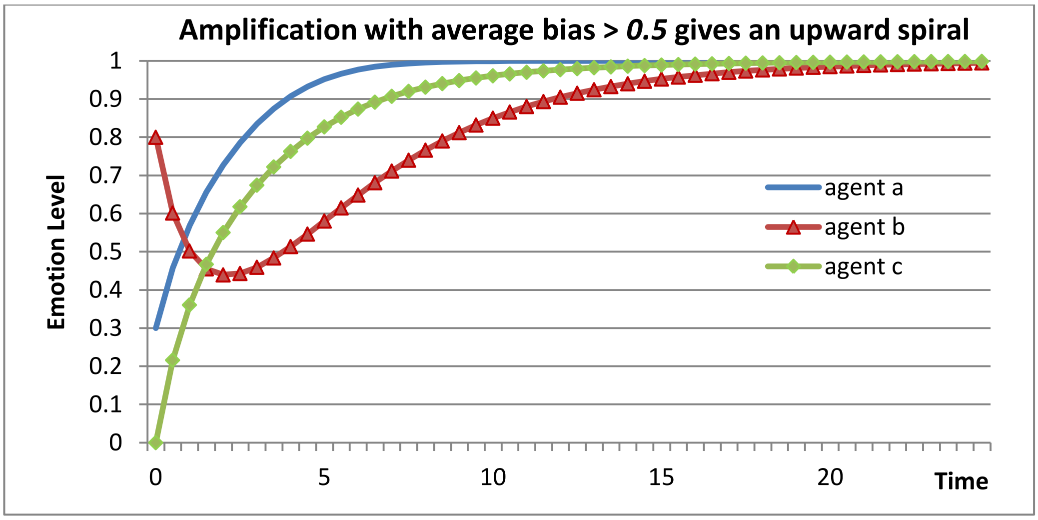

Figure 10: Simulation trace 4 (β *(a, b, c)= (1, 0.3, 0.8)*, all δ_R=*0.9*, w_{ba}=*0.625*, w_{ca}=*0.375*, w_{ab}=*0.64*, w_{cb}=*0.36*, w_{ac}=*0.4*, w_{bc}=*0.6*)

All simulations presented are for a group of 3 agents, infecting each other with the same emotion. A first pattern found is that when the bias parameters β of all three agents are set to *0* (strong downward bias), the emotion levels of all of them will approximate *0*, with speed depending on the δ_R (susceptibility) and ε_S α_{SR} (individual and group characteristics); see Figure 7 (with time on the x-axis and emotion strength on the y-axis). The reverse happens when all β's are set to *1*, then all agents will achieve an equilibrium of *1*. The occurrence of these patterns has been confirmed mathematically in Theorem 2 discussed in the next section.

Another situation occurs when the three agents have their β's set to: *0*, *1* and any other number. A situation was simulated in which agent *a* is susceptible only with negative impact (β = *0*), agent *b* is only susceptible with positive impact (β = *1*), and agent *c* is susceptible to more positive than negative impact ($\beta = 0.8$). In Figure 8, it is shown that in this case the equilibrium values match the agents' values of β. The speed of ascend or descend, depends on the susceptibility of the agent (setting of δ_R) and the situational factors at the individual and group level (represented by ε_S and α_{SR}). This illustrates the more general result expressed in Proposition 3, discussed in the next section. A next simulated situation (see Figure 9) is one where all three agents are equally susceptible to positive and negative impact, by setting every agent's β to *0.5*. In this situation all agents approximate an equilibrium value at 0.6; this equilibrium is the average of the initial emotional level; in this a case of neutral bias values in fact absorption takes place. This simulation illustrates Theorem 3, discussed in the next section. In the next situation presented the settings are: $\beta\ (a, b, c) = \beta\ (1, 0.3, 0.8)$, as shown in Figure 10. This represents a situation where agent *a* is only or fully susceptible to positive impact, agent *b* is susceptible more towards negative impact and agent *c* is more susceptible towards positive impact. Interestingly, agent *b* does not have an equilibrium of *0* or below *0.5*: all agents have an equilibrium of *1*. An indication for the height of the equilibrium could be the average β, which is 0.7 in this situation. This makes it possible to lift the emotional level of all group members to make the group-as-a-whole achieve an upward spiral [10]. In the mathematical analysis such behavior has been proved to occur (between two agents) in Theorem 4.

3.3 Larger Populations

In order to study the effect of an increase in population size on the results of the simulations, a number of experiments have been performed in which the model was run with a larger number of agents. To this end, for the case of the emotion absorption model[3] (i.e., $\eta_R = 0$ for all R) simulations have been run systematically with the population size increasing from 2 to 500 agents. For these simulations, the values of all q (i.e., the initial emotion levels of all agents) and γ_{SR} (i.e., the strength by which emotions are transferred) have been taken from a uniform distribution over the interval (0,1]. Note that values of 0 have been excluded to prevent agents from being completely 'excluded' from the contagion processes. Since the parameter settings were generated randomly, for each population size 100 simulations have been run, to be able to study differences between individual simulation runs.

Figure 11a-11c show three example simulation traces for the case of a population size of 10. As shown in the figures, the emotion levels of all agents converge to a value that approximates the average of the initial emotion levels q of the agents (assuming the same values of the emotion transfer variables γ_{SR}). In case the values for q and γ_{SR} are distributed more or less equally (as in Figure 11a), all emotion levels converge to a value of 0.5. The behaviour of this trace is similar to the pattern shown in Figure 2 for the case of 3 agents. In other cases, where the majority of the agents starts with an emotion level above or below 0.5 (see Figure 10b and 10c, respectively), an equilibrium value is reached that is higher or lower than 0.5.

[3] As the models for absorption and amplification only differ in using a different formula, there is no difference between them w.r.t. scalability. For this reason, only the results for absorption are shown.

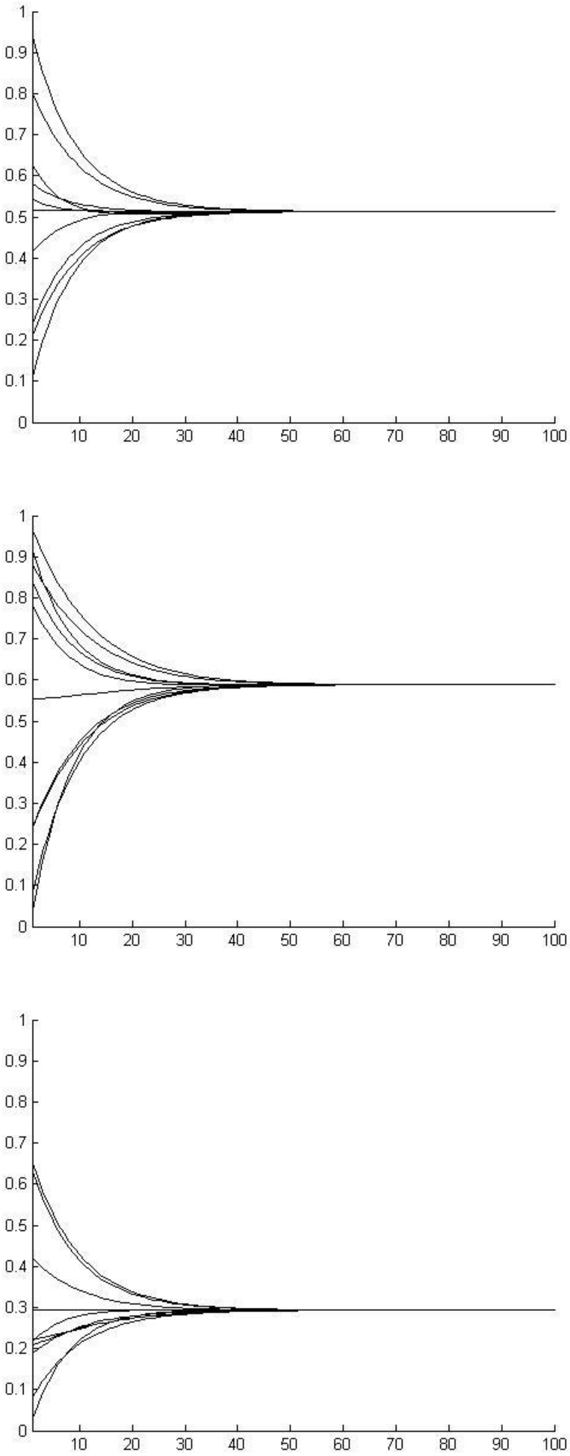

Figure 11(a-c): Three simulation traces for emotion absorption with a population of 10 agents

As the population size increases, the differences between individual runs of models with the same parameter settings become smaller, as the (randomly drawn) distribution of the values for q and γ_{SR} gradually becomes more uniform. This can be seen from Figure 12, 13, 14 and 15, which address arbitrary simulation runs of the model with 50, 100, 250 and 500 agents, respectively.

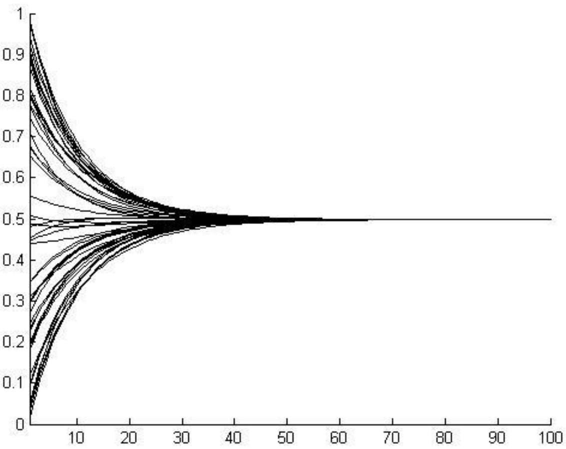

Figure 12: Simulation trace for emotion absorption with a population of 50 agents

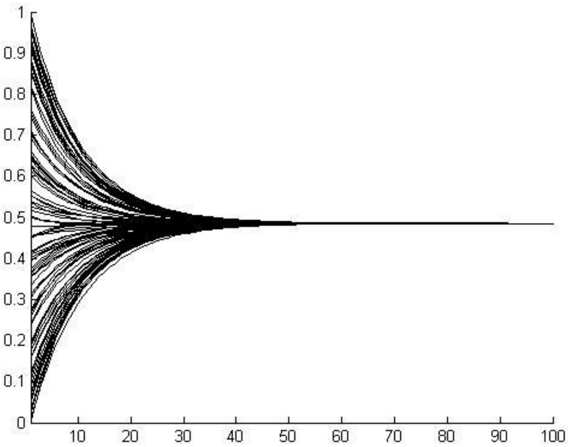

Figure 13: Simulation trace for emotion absorption with a population of 100 agents

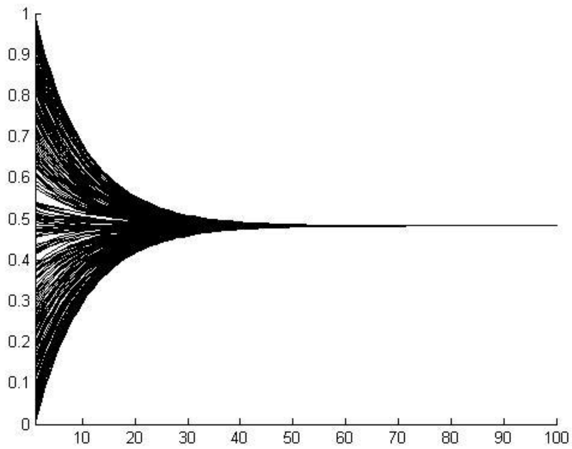

Figure 14: Simulation trace for emotion absorption with a population of 250 agents

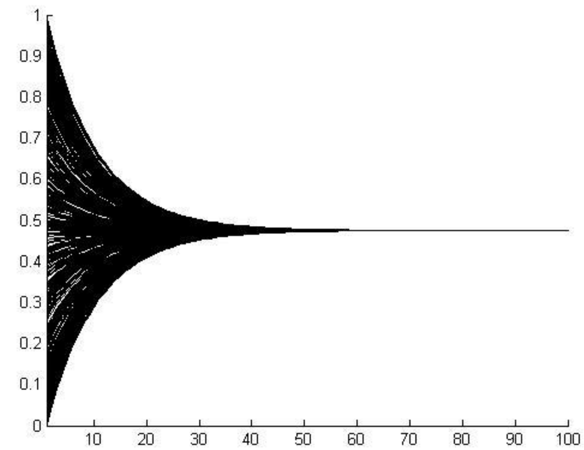

Figure 145: Simulation trace for emotion absorption with a population of 500 agents

For all population sizes considered, the average value of the equilibrium reached (over 100 simulations) approximated 0.5. Moreover, the time to convergence also turned out to be independent of the population size: in all cases, the equilibrium (which we defined as the case where for each pair of agents the difference in emotion level was smaller than 0.01) was reached after 68 time steps.

Finally, the effect of an increase in the number of agents on the actual simulation time was studied. This was done by measuring the CPU time of the simulations while executing them in Matlab on a regular notebook with 2200 MHz processor. The results are shown in Figure 15: as shown there, the simulation time (y-axis, in seconds) increases quadratically with an increase in population size (x-axis).

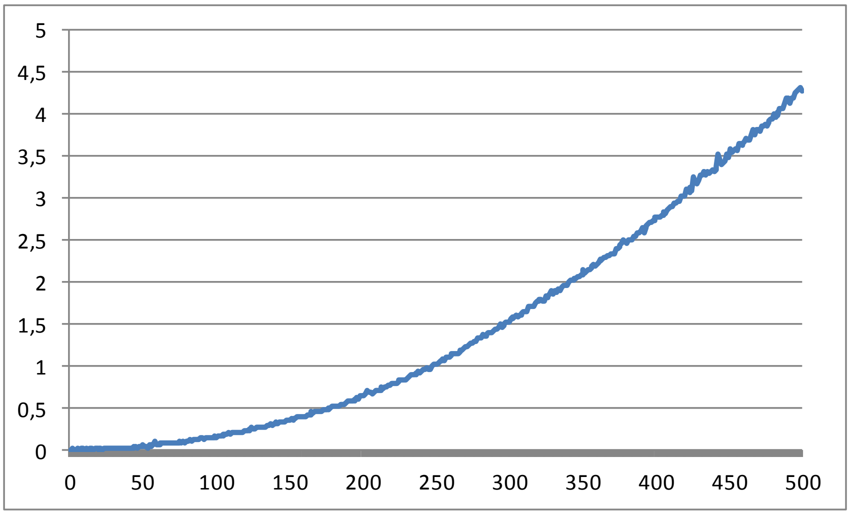

Figure 16: Dependence of Simulation Time on Population Size

4 Mathematical Analysis for the Emotion Contagion Model

In this section a mathematical analysis for the emotion contagion models is presented. First the emotion absorption model is addressed, and next the emotion amplification model.

4.1 Mathematical Analysis for the Emotion Absorption Model

This section presents some of the results of a mathematical analysis of the model. Note that $\gamma_A = 0$ iff $\Sigma_{B\in G\setminus\{A\}}\ \varepsilon_B\ \alpha_{BA}\ \delta_A = 0$ iff $\varepsilon_B\alpha_{BA}\delta_A = 0$ for all $B\neq A$. This means that $\gamma_A = 0$ can only occur when for each $B\neq A$ either $\varepsilon_B = 0$ or $\alpha_{BA} = 0$ or $\delta_A = 0$. This can be interpreted in the sense that A is isolated from emotional impact of all group members. In such a special case q_A will always be in an equilibrium state.

Theorem 1 (No change when $\gamma_A = 0$)
If $\gamma_A = 0$ then the emotion value for A will be in an equilibrium right from the start.

Next, conditions on monotonicity are addressed. Assuming $\gamma_A > 0$, from the equations it follows that $dq_A/dt \geq 0$ if and only if $q_A* \geq q_A$. In particular, for A with the lowest q_A it holds $q_B \geq q_A$ for all B ≠ A, and therefore via $q_A* = \Sigma_{B\in G\backslash\{A\}}\ w_{BA}\, q_B \geq \Sigma_{B\in G\backslash\{A\}}\ w_{BA}\, q_A = q_A$ it follows that q_A is monotonically increasing. Similarly the highest q_A is monotonically decreasing.

Theorem 2 (Monotonicity Conditions)

Suppose $\gamma_A > 0$. Then the following hold:

(a) q_A is monotonically increasing iff $q_A* \geq q_A$
   q_A is strictly monotonically increasing iff $q_A* > q_A$

(b) q_A is monotonically decreasing iff $q_A* \leq q_A$
   q_A is strictly monotonically decreasing iff $q_A* < q_A$

(c) If $q_B \geq q_A$ for all B ≠ A, then q_A is monotonically increasing.
   If in addition $q_B > q_A$ for at least one B ≠ A, then q_A is strictly increasing.

(d) If $q_B \leq q_A$ for all B ≠ A, then q_A is monotonically decreasing.
   If in addition $q_B < q_A$ for at least one B ≠ A, then q_A is strictly decreasing.

Next, equilibria are addressed for $\gamma_A > 0$. When at some point in time all q_A are the same, then from Theorem 2(c) and (d) it follows that they are both (non-strictly) monotonically increasing and decreasing, so they are in an equilibrium. Moreover, from Theorem 2(c) and (d) it follows that as long as the values of the q_A are different, then the lowest and highest values keep on changing (strictly increasing, resp. decreasing), so are not in an equilibrium. This implies the following identification of equilibria.

Theorem 3 (Equilibria when $\gamma_A > 0$ for all A)

Suppose $\gamma_A > 0$ for all A. Then the equilibria are the cases where all q_A are equal. Equilibria are reached between the lowest and highest initial value.

In some cases the equilibria are the average of the initial values, due to preservation of the (overall) sum of the emotion levels:

$\Sigma_{A\in G}\, q_A(t') = \Sigma_{A\in G}\, q_A(t)$ for all t and t' or $\Sigma_{A\in G}\, q_A(t+\Delta t) = \Sigma_{A\in G}\, q_A(t)$ for all t and Δt.

Taking the sum of the equations, the criterion for preservation is

$$\Sigma_{A\in G}\, \gamma_A\, (q_A* - q_A) = 0 \quad \text{or} \quad \Sigma_{A\in G}\, \gamma_A\, q_A* = \Sigma_{A\in G}\, \gamma_A\, q_A$$

Now

$$\gamma_A\, w_{BA} = \Sigma_{C\in G\backslash\{A\}}\ \gamma_{CA}\, w_{BA} = \Sigma_{C\in G\backslash\{A\}}\ \varepsilon_C\, \alpha_{CA}\, \delta_A\, w_{BA}$$
$$= (\Sigma_{C\in G\backslash\{A\}}\ \varepsilon_C\, \alpha_{CA})\, \delta_A\ \varepsilon_B\, \alpha_{BA} / \Sigma_{C\in G\backslash\{A\}}\ \varepsilon_C\, \alpha_{CA}$$
$$= \varepsilon_B\, \alpha_{BA}\, \delta_A = \gamma_{BA}$$

Therefore

$\gamma_A\, q_A^* = \Sigma_{B\in G\setminus\{A\}}\ \gamma_A\, w_{BA}\, q_B = \Sigma_{B\in G\setminus\{A\}}\ \gamma_{BA}\, q_B$

and taking the sum

$\Sigma_{A\in G}\ \gamma_A\, q_A^* = \Sigma_{A\in G}\Sigma_{B\in G\setminus\{A\}}\ \gamma_{BA}\, q_B = \Sigma_{B\in G}\Sigma_{A\in G\setminus\{B\}}\ \gamma_{BA}\, q_B = \Sigma_{B\in G}\ (\Sigma_{A\in G\setminus\{B\}}\ \gamma_{BA}\,)\, q_B$

It follows that the criterion for overall emotion preservation is equivalent to

$\Sigma_{A\in G\setminus\{B\}}\ \gamma_{BA} = \gamma_B = \Sigma_{A\in G\setminus\{B\}}\ \gamma_{AB}$ for all B

which in terms of the basic parameters is equivalent to

$\Sigma_{A\in G\setminus\{B\}}\ \varepsilon_B\, \alpha_{BA}\, \delta_A = \Sigma_{A\in G\setminus\{B\}}\ \varepsilon_A\, \alpha_{AB}\, \delta_B$ for all B.

Theorem 4 (Preservation of overall emotion)

The following are equivalent:

(i) The overall emotion in the group is preserved

(ii) $\Sigma_{A\in G\setminus\{B\}}\ \gamma_{BA} = \Sigma_{A\in G\setminus\{B\}}\ \gamma_{AB}$ for all B.

(iii) $\Sigma_{A\in G\setminus\{B\}}\ \varepsilon_B\, \alpha_{BA}\delta_A = \Sigma_{A\in G\setminus\{B\}}\ \varepsilon_A\, \alpha_{AB}\, \delta_B$ for all B.

When these conditions are satisfied, an equilibrium is reached where each emotion level is the average of the initial emotion levels. The conditions are satisfied in particular when all γ_{BA} are equal, or when, more specifically, all ε_A are equal, all α_{AB} are equal and all δ_B are equal. Finally, it is analyzed under which conditions the emotion values stay within the interval [$0, 1$] (*closure property*). It can easily be verified that the expression describing change reaches its maximum for $\varepsilon_A = \alpha_{AB} = \delta_B = q_S = 1$ and $q_R = 1$. Similarly, this function reaches its minimum for $\varepsilon_A = \alpha_{AB} = \delta_B = q_R = 1$ and $q_S = 0$. Using this, the following equations for upper and lower bounds are obtained:

$(1- (\#(G) -1))\Delta t = q_{min} \leq q_R(t+\Delta t) \leq q_{max} = (\#(G)-1)\Delta t$

In order to maintain the closure property for emotion contagion in the absorption model, q_{max} has to be constrained to 1 and q_{min} to 0. Therefore, respectively:

$(\#(G)-1).\Delta t \leq 1$ *and* $(1- (\#(G) -1))\Delta t \geq 0$

both lead to the same constraint $\Delta t \leq 1/(\#(G) -1)$. So, as long as this constraint is maintained, the closure property holds for the absorption model:

Theorem 5 (Closure property)

The emotion values generated remain in the interval [$0, 1$] if $\Delta t \leq 1/(\#(G) - 1)$.

4.2 Mathematical Analysis for the Emotion Amplification Model

This section presents some of the results of a mathematical analysis of the model. First, the following conditions on monotonicity have been found.

Proposition 1 (Monotonicity Conditions)

(a) If $\beta_A = 0$ then $q_A(t)$ is always monotonically decreasing;
it is strictly decreasing when $q_A^*(t) < 1$ and $q_A(t) > 0$.

(b) If $\beta_B = 1$ then $q_B(t)$ is always monotonically increasing;
it is strictly increasing when $q_B^*(t) > 0$ and $q_B(t) < 1$.

(c) If $\beta_A \leq 0.5$ and $q_A^*(t) \leq q_A(t)$ then $q_A(t)$ is monotonically decreasing;
it is strictly decreasing when $q_A^*(t) < q_A(t)$.

(d) If $\beta_B \geq 0.5$ and $q_B^*(t) \geq q_B(t)$ then $q_B(t)$ is monotonically increasing;
it is strictly increasing when $q_B^*(t) > q_B(t)$.

Next, equilibria have been investigated. First, conditions have been established for the case of an equilibrium with one of the emotion values 0 or 1.

Proposition 2

Suppose all w_{SR} are nonzero. Then for an equilibrium the following holds:

(a) If $q_A = 0$ then $\beta_A = 0$ or $q_C = 0$ for all C

(b) If $q_B = 1$ then $\beta_B = 1$ or $q_C = 1$ for all C

Based on this, the following theorem provides the possibilities for equilibria concerning those subgroups with β is 0 or 1.

Theorem 1 (Equilibria for members for which β is 0 or 1)

Suppose all w_{SR} are nonzero. Let the two subsets $S_0, S_1 \subseteq G$ be given by

$S_0 = \{ A \in G \mid \beta_A = 0 \}$ $\quad S_1 = \{ B \in G \mid \beta_B = 1 \}$

Then for an equilibrium the following holds:

(a) If $A \in S_0$ then $q_A = 0$ or $q_C = 1$ for all $C \neq A$. If $B \in S_1$ then $q_B = 1$ or $q_C = 0$ for all $C \neq B$.

(b) If $\#(S_0) \geq 2$, i.e., there are at least two members A_1 and A_2 with $\beta_{A1} = 0$ and $\beta_{A2} = 0$, then either $q_A = 0$ for all $A \in S_0$ or $q_C = 1$ for all $C \in G$.

(c) If $\#(S_1) \geq 2$, i.e., there are at least two members B_1 and B_2 with $\beta_{B1} = 1$ and $\beta_{B2} = 1$, then either $q_B = 1$ for all $B \in S_1$ or $q_C = 0$ for all $C \in G$.

(d) If $\#(S_0) \geq 2$ and $\#(S_1) \geq 2$, then there are three possibilities:

(i) $q_C = 0$ for all $C \in G$

(ii) $q_C = 1$ for all $C \in G$

(iii) $q_A = 0$ for all $A \in S_0$ and $q_B = 1$ for all $B \in S_1$

In the specific case, that for all group members β is 0 or 1, a complete classification of equilibria can be obtained; for an example, see Fig. 2.

Theorem 2 (Equilibria when all β's are equal to 0 or 1)

Suppose all w_{SR} are nonzero and for all C it holds $\beta_C = 0$ or $\beta_C = 1$, in other words, the whole group G is partitioned into the two subsets

$S_0 = \{ A \in G \mid \beta_A = 0 \}$ and $S_1 = \{ B \in G \mid \beta_B = 1 \}$.

Then for an equilibrium the following holds:

(a) If $S_0 = G$ and $S_1 = \varnothing$, i.e., $\beta_C = 0$ for all C, then
either $q_C = 0$ for all C (attracting) or $q_C = 1$ for all C (non-attracting).

(b) If $S_1 = G$ and $S_0 = \varnothing$, i.e., $\beta_C = 1$ for all C, then either $q_C = 0$ for all C (non-attracting) or $q_C = 1$ for all C (attracting).

(c) If $\#(S_0) = \#(S_1) = 1$, i.e., there is exactly one member A with $\beta_A = 0$, and exactly one member B with $\beta_B = 1$, then there are two possibilities:

(i) $q_A = 0$ for $A \in S_0$ and q_B has any value for $B \in S_1$

(ii) $q_B = 1$ for $B \in S_1$ and q_A has any value for $A \in S_0$

(d) If $\#(S_0) = 1$ and $\#(S_1) \geq 2$, i.e., there is exactly one member A with $\beta_A = 0$, and there are at least two members B_1 and B_2 with $\beta_{B1} = \beta_{B2} = 1$, then there are two possibilities:

(i) $q_C = 0$ for all $C \in G$

(ii) $q_B = 1$ for all $B \in S_1$ and q_A has any value for $A \in S_0$

(e) If $\#(S1) = 1$, and $\#(S0) \geq 2$, i.e., there is exactly one member B with $\beta_B = 1$, and there are at least two members A_1 and A_2 with $\beta_{A1} = \beta_{A2} = 0$, then there are two possibilities:

(i) $q_C = 1$ for all $C \in G$

(ii) $q_A = 0$ for all $A \in S_0$ and q_B has any value for $B \in S_1$

(f) If $\#(S_0) \geq 2$ and $\#(S_1) \geq 2$, i.e., there are at least two members A_1 and A_2 with $\beta_{A1} = \beta_{A2} = 0$ and also at least two members B_1 and B_2 with $\beta_{B1} = \beta_{B2} = 1$, then there are three possibilities:

(i) $q_C = 0$ for all $C \in G$

(ii) $q_C = 1$ for all $C \in G$

(iii) $q_A = 0$ for all $A \in S_0$ and $q_B = 1$ for all $B \in S_1$

For the specific case of three group members, where one member has β is 0, one has 1 and one has neither, the following holds; for an example, see Fig. 3.

Proposition 3 (A case for 3 members)

Consider a group G which consists of three members named by a, b, c with $\beta_a = 0$, $\beta_b = 1$, and $\beta_c = \beta$, where $0 < \beta < 1$ is assumed. Moreover, suppose all w_{SR} are nonzero. Then the following are the possibilities for equilibria:

(i) $q_a = q_b = q_c = 0$

(ii) $q_a = q_b = q_c = 1$

(iii) $q_a = 0$, $q_b = 1$ and $q_c = \beta w_{bc} / ((1-\beta)w_{ac} + \beta w_{bc})$

In particular, when $w_{ac} = w_{bc}$, then the value for q_c in (iii) is β.

The following proposition shows that only in trivial cases a group member with β not 0 or 1 can reach 0 or 1.

Proposition 4 ($q_A = 0$ with $\beta_A > 0$ or $q_B = 1$ with $\beta_B < 1$)

Suppose all w_{SR} are nonzero. Then for an equilibrium it holds

(i) If $q_A = 0$ for some A with $\beta_A > 0$ then $q_C = 0$ for all $C \in G$.

(ii) If $q_B = 1$ for some B with $\beta_B < 1$ then $q_C = 1$ for all $C \in G$.

The case that all group members converge to an equal equilibrium value, which is not 0 or 1, only occurs when all β's are 0.5; for an example, see Fig. 4.

Theorem 3 (Equal equilibrium values for all members)

Suppose all w_{SR} are nonzero, then for an equilibrium the following are equivalent:

(i) For some q with $0 < q < 1$ it holds $q_C = q$ for all C.

(ii) For all C it holds $\beta_C = 0.5$.

For the case of two persons, a complete classification can be found, as shown in the following theorem.

Theorem 4 (The case of two persons)

Suppose the group consists of two persons named by a and b. Then for an equilibrium, there are the following possibilities:

(i) When $\beta_a + \beta_b \neq 1$ the only two possibilities are:

$q_a = q_b = 0$ attracting when $\beta_a + \beta_b < 1$

$q_a = q_b = 1$ attracting when $\beta_a + \beta_b > 1$

(ii) When $\beta_a + \beta_b = 1$ attracting equilibria occur where q_a and q_b get values between 0 and 1.

5 Formal Verification of the Emotion Contagion Spiral Model

In this section, it is discussed how traces generated by the emotion contagion models have been formally verified. The temporal predicate logical language TTL [6] used to express properties to be verified supports formal specification and analysis of dynamic properties, covering both qualitative and quantitative aspects. TTL is built on atoms referring to states of the world, time points and traces, i.e., trajectories of states over time. In addition, dynamic properties are (sorted) temporal predicate logic statements that can be formulated with respect to traces based on the state ontology Ont in the following manner. Given a trace γ over state ontology Ont, the state in γ at time point t is denoted by state(γ, t). These states can be related to state properties via the formally defined satisfaction relation denoted by the infix predicate |=: state(γ, t) |= p denotes that state property p holds in trace γ at time t. Based on these statements, dynamic properties can be formulated in a formal manner in a sorted predicate logic, using quantifiers over time and traces and the usual logical connectives such as ¬, ∧, ∨, ⇒, ∀, ∃. A dedicated software environment has been developed for TTL, featuring both a Property Editor for building and editing TTL properties and a Checking Tool that enables automated formal verification of such properties against a set of (simulated or empirical) traces.

The purpose of the type of verification performed here is to check whether the model behaves as it should. A typical example of a property that may be checked is whether no unexpected situations occur, such as a variable running out of its bounds (e.g., $q_A(t) > 1$, for some t and A), or whether eventually, an equilibrium value is reached. Other more complex examples, can be found in the theorems presented in the previous section. For the emotion contagion model, a number of such dynamic properties have been formalized in TTL varying from properties addressing limit behavior (equilibria reached) to properties of the process from initial values to the equilibria. Below, a number of these properties are introduced, both in semi-formal and in informal notation (where state(γ, t) |= p denotes that p holds in trace γ at time t).

Note that the properties are all defined for a particular trace γ and sometimes for a particular time interval between tb and te.

P1a - Emotional Stability for Agent A

For all time points t1 and t2 between tb and te in trace γ, if at t1 the level of emotion of agent A is x1, then at t2 the level of emotion of agent A is between x1 - α and x1 + α.

P1a(γ:TRACE, tb, te:TIME, A:AGENT, α:REAL) ≡
∀t1,t2:TIME ∀x1,x2:REAL
state(γ, t1) |= emotion(agent(A), x1) &
state(γ, t2) |= emotion(agent(A), x2) &
tb ≤ t1 ≤ te & tb ≤ t2 ≤ te ⇒ x1-α ≤ x2 ≤ x1+α

This property can be used to verify in which situations a certain agent's level of emotion does not fluctuate much. It has been found, for example, that for the trace shown in Figure 2 and for α = 0.00001, the emotion of agent *a* remains stable between time point 28 and 50. In other words, checking P1a(traceFig2, 28, 50, a, 0.00001) was successful, where traceFig2 is the trace of Figure 2.

P1b - Emotional Stability for Agent A around Value x

For all time points t between tb and te in trace γ
the level of emotion of agent A is between x - α and x + α (where α is a constant).

P1b(γ:TRACE, tb, te:TIME, x:REAL, A:AGENT, α:REAL) ≡
∀t:TIME ∀y:REAL
    state(γ, t) |= emotion(agent(A), y) & tb ≤ t ≤ te ⇒ x-α ≤ y ≤ x+α

As a variant of P1a, property P1b can be used to check whether an agent's level of emotion stays around a certain (given) value. For example, for $\alpha = 0.0001$, property P1b(traceFig2, 25, 50, 0.4333, b, 0.0001) was true. One step further, P1a and P1b can be used as building blocks to check the propositions and theorems related to equilibria presented in the previous section against the generated traces. For example, property P1c checks whether Theorem 3 holds:

P1c - Equal Equilibria

If for all agents A and B, γ_{AB} is nonzero in trace γ then eventually the same equilibrium *q* (between *0* and *1*) will occur for all agents

P1c(γ:TRACE, α:REAL) ≡
[∀A,B:AGENT [A≠B ⇒ ∃g:REAL>0 [state(γ, 1) |= has_gamma_for(agent(A),agent(B),g)]]]
⇒ [∃q:REAL ∀C:AGENT P1b(γ,40,50,q,C,α)]

This property, which has been proven in the mathematical analysis, has been checked for α = 0.07 for all generated traces, and indeed was confirmed. In addition, similar properties have been formulated that make claims about the equilibria on the basis of the initial settings. Details of these properties are not shown here. However, some examples (in informal notation) are:

- In case $\gamma_{SR} = 0$ for all agents, then each agent ends up in an equilibrium that is equal to its initial emotion value.
- In case $\delta_R = 0$ for exactly 1 agent A (i.e., $\delta_A = 0$), and other δ_R are nonzero, and all α_{SR} and ε_S are nonzero for all agents, then each agent ends up in an equilibrium that is equal to the initial emotion value of agent A.

- In case $\delta_R = 0$ and $\varepsilon_S = 0$ for exactly 1 agent A (i.e., $\delta_A = \varepsilon_A = 0$), and other δ_R and ε_S are nonzero, and all α_{SR} are nonzero for all agents, then agent A ends up in an equilibrium that is equal to its initial emotion value, and all other agents end up in an equilibrium that is in between their initials emotion values.
- In case $\delta_R = 0$ for exactly 2 agents A and B (i.e., $\delta_A = \delta_B = 0$), and other δ_R are nonzero, and all α_{SR} and ε_S are nonzero for all agents, then agent A and B end up in an equilibrium that is equal to their initial emotion value, and all other agents end up in an equilibrium that is in between the initial emotion values of A and B.

P2a - Monotonic Increase of Emotion[4]

For all time points t1 and t2 between tb and te in trace γ, if at t1 the level of emotion of agent A is x1, and at t2 the level of emotion of agent A is x2 and t1 < t2, then x1 ≤ x2.

P2a(γ:TRACE, tb, te:TIME, A:AGENT) ≡
∀t1,t2:TIME ∀x1,x2:REAL
state(γ, t1) |= emotion(agent(A), x1) &
state(γ, t2) |= emotion(agent(A), x2) &
tb ≤ t1 ≤ te & tb ≤ t2 ≤ te & t1 < t2 ⇒ x1 ≤ x2

Property P2a and the variant P2b addressing monotonic decrease (by replacing ≤ in the consequent by ≥) can be used to check whether an agent's level of emotion increases or decreases monotonically over a certain interval. Such monotonicity, for example, occurs for agent *c* during the whole trace shown in Figure 2 (i.e., property P2b(traceFig2, 1, 50, c) succeeded). Furthermore, these properties can be used as building blocks to check the propositions and theorems related to monotonicity presented in the previous section against the generated traces. For example, property P2c checks whether part (c) and (d) of Proposition 1 hold:

P2c - Conditional Monotonicity

For all agents A, if $q_A* \geq q_A$ between tb and te in trace γ, then q_A is monotonically increasing during this interval, and if $q_A* \leq q_A$ between tb and te in trace γ, then q_A is monotonically decreasing during this interval.

P2c(γ:TRACE, tb, te:TIME) ≡
∀A:AGENT
[[□□∀t:TIME ∃A2,A3:AGENT ∃x1,x2,x3,w2,w3:REAL
state(γ, t) |= emotion(agent(A), x1) &
state(γ, t) |= emotion(agent(A2), x2) &
state(γ, t) |= emotion(agent(A3), x3) & A2≠A3 & tb ≤ t ≤ te &
state(γ, 1) |= has_w_for(agent(A2),agent(A),w2) &
state(γ, 1) |= has_w_for(agent(A3),agent(A),w3) &
w2*x2+w3*x3 ≥ w1] ⇒ p2a(γ, tb, te, A)] &
[[∀t:TIME ∃A2,A3:AGENT ∃ x1,x2,x3,w2,w3:REAL
state(γ, t) |= emotion(agent(A), x1) &
state(γ, t) |= emotion(agent(A2), x2) &
state(γ, t) |= emotion(agent(A3), x3) & A2≠A3 & tb ≤ t ≤ te &
state(γ, 1) |= has_w_for(agent(A2),agent(A),w2) &
state(γ, 1) |= has_w_for(agent(A3),agent(A),w3) &

[4] A strict variant of such properties can be created by replacing ≤ by <.

w2*x2+w3*x3 ≤ w1] ⇒ p2b(γ, tb, te, A)]

Here, q_A* is explained in the section after the introduction section. This property has been confirmed for all possible intervals in all generated traces.

P3 - Emotion between Boundaries

For all time points t between tb and te in trace γ
if at t the level of emotion of agent A is x, then min < x < max.
P3(γ:TRACE, tb, te:TIME, max, min:REAL, A:AGENT) ≡
∀t:TIME ∀x:REAL
state(γ, t) |= emotion(agent(A), x) & tb ≤ t ≤ te ⇒ min ≤ x ≤ max

This property can be used to check whether the emotion of an agent stays between certain boundaries. For example, no emotional value should ever become lower than 0 or higher than 1. This turned out to be the case for all generated traces where $\Delta t \leq 1/(\#(G) - 1)$. That is, property P3(trace, 1, 50, 0.0, 1.0, X) succeeded for all traces trace with these settings and agents X, which confirms Theorem 5 of the previous section. In addition, it was found that the property failed for some traces that do not have these settings. E.g., for a trace with $\Delta t = 0.7$, all $\gamma_{SR} = 1$, and initial values $q_a = 0.3$, $q_b = 0.1$, and $q_c = 0.9$, the emotion values eventually run out of their boundaries.

P4 - Emotion Agent A1 above Agent A2

For all time points t between tb and te in trace γ, if at t the level of emotion of agent A1 is x1
and the level of emotion of agent A2 is x2, then x1 ≥ x2.
P4(γ:TRACE, tb, te:TIME, A1, A2:AGENT) ≡
∀t:TIME ∀x1,x2:REAL
state(γ, t) |= emotion(agent(A1), x1) & state(γ, t) |= emotion(agent(A2), x2) & tb ≤ t ≤ te ⇒ x1 ≥ x2

Property P4 can be used to check whether an agent's emotion level stays above (or below) another agent's level during a specified interval. For example, in the trace of Figure 2, agent *c* always has a higher emotion than agent *a* (i.e., property P4(traceFig2, 1, 50, c, a) succeeded). However, in the end the difference becomes very small, and if the simulation were continued longer, eventually this property would fail.

P5 - Emotion Approaches Value x with Speed s

For all time points t1 and t2 between tb and te in trace γ, if at t1 the level of emotion of agent A is x1, and at t2 the level of emotion of agent A is x2, and t2 = t1+1, then s * |x-x1| ≥ |x-x2| (here s is a constant < 1).
P5(γ:TRACE, tb, te:TIME, x:REAL, A:AGENT) ≡
∀t1,t2:TIME ∀x1,x2:REAL
state(γ, t1) |= emotion(agent(A), x1) & state(γ, t2) |= emotion(agent(A), x2) &
tb ≤ t1 ≤ te & tb ≤ t2 ≤ te & t2 = t1+1 ⇒ |x-x1| * s ≥ |x-x2|

Property P5 can be used to check whether an agent's emotion level approaches a given value x, and to determine the speed *s* with which this happens (where $0 < s < 1$, and a high *s* denotes a slow speed). For example, for the trace shown in Figure 3, it turned out that agent *b* approaches emotion level *0.3* with a speed of approximately *0.9991*.

P6 - Higher Beta's lead to Higher Emotion Levels

If for all agents the initial level of emotion is higher (or equal) in trace γ1 than in γ2
and for all agents the beta is higher (or equal) in trace γ1 than in γ2

then for all agents the final level of emotion will be higher (or equal) in trace γ1 than in γ2.

P6(γ1, γ2:TRACE, tb, te:TIME) ≡
[∀A:AGENT ∃x1,x2:REAL
state(γ1, tb) |= emotion(agent(A),x1) & state(γ2, tb) |= emotion(agent(A),x2) & x1≥x2] & [∀A:AGENT ∃x1,x2:REAL
state(γ1, tb) |= has_beta(agent(A),x1) & state(γ2, tb) |= has_beta(agent(A),x2) & x1≥x2] ⇒
[∀A:AGENT ∃x1,x2:REAL state(γ1, te) |= emotion(agent(A),x1) & state(γ2, te) |= emotion(agent(A),x2) & x1≥x2]

This property can be used to compare traces with different parameter settings. It turned out to hold for all generated traces, as long as the initial values were not 0 or 1.

6 The Ambient Agent Model for Group Emotion Analysis

The emotion contagion model described in Section 2 above can be used by an ambient agent to analyse the past, present and future (expected) dynamics of a team's emotion contagion processes. The main goal of the designed ambient agent is to estimate and predict the level of a given type of emotion in the group at present and future points in time, and based on such an analysis propose actions whenever considered necessary. The emotion considered is assumed to be a positive emotion, so when the emotion level of the group is expected to become too low, this analysis process should detect this early enough to intervene.

Concepts needed in such a model for an ambient agent concern the ambient agent's estimations of the relevant human's states at different points in time; these estimations are described by the ambient agent's observations and beliefs; in addition an assessment of the (expected) group's emotion state is needed. An assessment is generated when the group emotion level at some (future) time point is expected to be too low, compared to a certain norm (*EN*). Moreover, to model direct observation of individual emotion levels, the concept *expressed emotion level* (ε_S q_A) is used, as the emotion level that can be observed from someone's face or speech expressions, for example, by use of methods discussed in [14], [31], [34], [35], [36]. This may differ from the emotion level in that the expressiveness factor also has effect on it.

To formalize the concepts introduced in this and the previous sections, a number of logical atoms are introduced that incorporate numerical representations; see Table 2. Note that in order to generate and analyze possible temporal patterns for the future, some of the atoms have an additional time variable *T*. This is used to make predictions about future emotion states, as part of the analysis.

The dynamic relationships of the model to reason about emotion contagion are described and formalised as follows. Note that the beliefs on emotion expressiveness, openness, and channel strengths are assumed to be initially given and to persist (until they are changed). Moreover, a scenario is considered where at some (initial) point in time the current emotion levels of the members are estimated or observed, and from that time point onwards, the beliefs on emotion levels for subsequent time points are determined, as a form of temporal projection (or prediction).

First the role of observed expressed emotions is formalised. The agent is assumed to possess observation equipment, for example, in the form of a face reader with software that detects emotion expressions from face images [31]. The face reader is a tool that is able to recognize six basic emotional expressions (happy, sad, scared, disgusted, surprised, angry) based on video images. This is done in three steps, namely: face finding, face modelling and face classification. For more details, see [31]. As an alternative also methods can be employed to recognize emotions from speech, for example, as discussed in [34], [35], [36].

The expressed emotion *EV* results from the emotion level *V* and the expressiveness *E* by which the emotion is displayed on the face. In the model it is assumed that the expressed emotion level is formalised as the product $V*E$. Note that this means that it is assumed that the expressiveness (being a number between *0* and *1*) always reduces the level of the emotion: $EV \leq E$.

concept	*formalisation*
observed that person *A* has expressed emotion level	observed(agent, has_expressed_emotion_level_at(A:AGENT, EV:REAL,T:REAL))
belief that person *A* has expressed emotion level *EV*	belief(agent, has_expressed_emotion_level_at(A:AGENT, EV:REAL,T:REAL))
belief that person *B* has expressiveness *E*	belief(agent, has_expressiveness(B:AGENT, E:REAL))
belief that person *A* has openness for received	belief(agent, has_openness(A:AGENT, D:REAL))
belief that the channel from *B* to *A* has strength *C*	belief(agent, has_channel_strength(B:AGENT, A:AGENT, C:REAL))
belief that the contagion strength from *B* to *A* is *CS*	belief(agent, has_contagion_strength(B:AGENT, A:AGENT, CS:REAL))
belief that the overall contagion strength to receiver *A*	belief(agent, has_overall_contagion_strength(A:AGENT, CS:REAL))
belief that step size is *DT*	belief(agent, stepsize(DT:REAL))
belief that person *A* has relevance *R*	belief(agent, has_relevance(A:AGENT, R:REAL))
belief that person *A* has emotion level *V* at time *T*	belief(agent, has_emotion_level_at(A:AGENT, V:REAL, T:REAL))
belief that the group emotion level at *T* is *GE*	belief(agent, group_emotion_level_at(GE:REAL, T:REAL))
belief that the group emotion norm is *EN*	belief(agent, group_emotion_norm(EN:REAL))
assessment that the deficient of the group emotion at	assessment(agent, group_emotion_deficient_at(ED:REAL, T:REAL))

Table 2: Concepts to reason about emotion contagion and their formalization

In other words, this assumption excludes the situation that an emotion level is expressed that is not there (no faking of emotions). Moreover, note that in ADR2 below it is assumed that the expressiveness factor *E* is nonzero. Then under the assumptions discussed above, from an expressed emotion level *EV* the emotion level *V* itself can be determined as $V = EV/E$.

ADR1 Observing group members' expressed emotion levels

If the agent observes an expressed emotion level
then the ambient agent will believe this.

observes(agent, has_expressed_emotion_level_at(A, V, T))
→ belief(agent, has_expressed_emotion_level_at(A, V, T))

ADR2 Generating a belief on an emotion level from a belief on an expressed emotion level

If the agent believes that a group member has expressed emotion level *EV*
and that this group member has expressiveness *E*
then it will generate a belief that this group member has emotion level *EV/E*

belief(agent, has_expressed_emotion_level_at(A, EV, T)) &
belief(agent, has_expressiveness(E))
→ belief(agent, has_emotion_level_at(A, EV/E, T))

ADR3 Generating beliefs on contagion strengths

If the ambient agent believes that *B* has expressiveness *E*

and the ambient agent believes that the channel from B to A has strength C
and the ambient agent believes that A has openness D
then the ambient agent will believe that the contagion strength from B to A will be $E*C*D$

belief(agent, has_expressiveness(B, E)) &
belief(agent, has_channel_strength(B, A, C)) &
belief(agent, has_openness(A, D))
→ belief(agent, has_contagion_strength(B, A, E*C*D))

ADR4 Updating beliefs on emotion levels

If $A \neq B$ and $B \neq C$ and $C \neq A$
and the ambient agent believes that B has emotion level $V2$ at time T
and the ambient agent believes that C has emotion level $V3$ at time T
and the ambient agent believes that the contagion strength from B to A is $CS2$
and the ambient agent believes that the contagion strength from C to A is $CS3$
and $Q1 = (CS2*V2+ CS3*V3)/(CS2+CS3)$
and the ambient agent believes that A has emotion level $V1$ at time T
and the ambient agent believes that A has beta $BE1$
and the ambient agent believes that A has eta $ETA1$
and the ambient agent believes that A has gamma $G1$
and the ambient agent believes that the step size is DT
then the ambient agent will believe that the emotion level of A will be
$G1* [ETA1* (BE1 (1 – (1-Q1) (1-V1)) + (1-BE1) Q1V1) +(1-ETA1) Q1 – V1)]*DT$ at time $T+DT$

A≠B & B≠C & C≠A &
belief(agent, has_emotion_level_at(B, V2, T)) &
belief(agent, has_emotion_level_at(C, V3, T)) &
belief(agent, has_contagion_strength(B, A, CS2)) &
belief(agent, has_contagion_strength(C, A, CS3)) &
Q1 = (CS2*V2+ CS3*V3)/(CS2+CS3)
belief(agent, has_emotion_level_at(A, V1, T)) &
belief(agent, has_beta(A, BE1)) &
belief(agent, has_eta(A, ETA1)) &
belief(agent, has_gamma(A, G1)) &
belief(agent, step_size(DT))
→ belief(agent, has_emotion_level_at(A,
V1+ G1* [ETA1* (BE1 (1 – (1-Q1) (1-V1)) + (1-BE1) Q1V1) +(1-ETA1) Q1 – V1)]*DT, T+DT))

An analysis also involves an assessment of the (expected) level of the group's emotion. To this end, first a belief on the group's emotion level is generated.

ADR5 Determining beliefs on the group's emotion level

If the ambient agent believes that the group members have emotion levels $V1, V2, V3$
and relevance $R1, R2, R3$ respectively
then it will believe that the group's emotion level is $R1*V1+ R2*V2+R3*V3$.

belief(agent, has_emotion_level_at(a1, V1, T)) &
belief(agent, has_emotion_level_at(a2, V2, T)) &
belief(agent, has_emotion_level_at(a3, V3, T)) &
belief(agent, has_relevance(a1, R1)) &
belief(agent, has_relevance(a2, R2)) &
belief(agent, has_relevance(a3, R3))
→ belief(agent, group_emotion_ level_at(R1*V1+ R2*V2+R3*V3, T))

An assessment is generated when the group emotion level at some (future) time point is expected to be too low, compared to a certain norm. In case of a negative outcome further action may be needed, to avoid this undesired situation. The assessment includes an estimation of how much the group emotion level is too low (the *group emotion deficient*):

ADR6 Assessment of the group's emotion level

If the ambient agent believes that the group emotion level *V* at time *T* is lower than the emotion norm *EN*,

then it will assess the situation as having a group emotion deficient *EN-V* at *T*.

```
belief(agent, group_emotion_ level_at( V, T)) &
belief(agent, group_emotion_norm(EN))  &  V<EN
→ assessment(agent, group_emotion_ deficient_at(EN-V, T))
```

7 The Agent Model for Group Emotion Support

In the previous sections, the emotion contagion model and the analysis process based on it have been discussed. In this section, the support model is introduced that uses these models to provide intelligent support to humans in cases where the group emotion level is expected to become below a certain norm. The support model introduced here, uses a heuristic approach. The idea is that an ambient agent will reason about the proper actions that should be undertaken by the team leader to keep the group emotion level optimal. For example, it uses knowledge expressing that in case the group emotion level (e.g., relaxedness or happiness) is lower than a certain norm, certain members are to be detected that play a crucial role in a negative or positive sense and give them either a pep talk to or to increase or decrease their impact on the other group members.

When a negative assessment of the (future) group emotion state is made, then the ambient agent is assumed to propose actions to the team leader, in order to avoid such states. Some examples of possible actions are:

- giving a group member that negatively affects the emotion in the team a less central role (decreasing the emotion contagion strengths from this person)
- ask a person with a positive emotion level (for example the team leader) either to not be too open for other members (decrease the person's openness; i.e., δ_R) or to be more expressive (increase the person's expressiveness; i.e., ε_S)

Two heuristics that are applied are the following:

- *let the group members with lower emotion levels get less impact on the other members, and get more impact from the other members*
- *let the group members with higher emotion levels get more impact on the other members, and get less impact from the other members*

Here 'higher' and 'lower' can be defined as the members with highest or lowest emotion level, but also as above or under the group's emotion level. In general, two (low and high) *emotion* *thresholds* are assumed for this, where a specific case is that these thresholds are both equal to the group's emotion level.

For a group member under the low threshold, his or her impact on the other members can be decreased by (encouragement for) decreasing the person's expressiveness, or by decreasing the channel strengths from this person to the other members. Moreover, the person's impact from other members can be increased by increasing the person's openness, and by increasing the channel strengths from the other members. For an overview of the action options based on the two heuristics, see Table 3.

	person under low threshold	*person above high threshold*
expressiveness	decrease	increase
openness	increase	decrease
channels to others	decrease	increase
channels from others	increase	decrease

Table 3: Overview of the action options

This approach does not give indications for the adjustment extent to which such an increase or decrease has to be applied. When such adjustment extents are chosen, the approach can also be combined with a feasibility ranking approach described next.

After the ambient agent generates the actions options, they have to be ranked on their (in)feasibility, expressing how difficult they are to achieve, because it may happen that some action options can be easily realized whereas others are difficult to realize. The action option with the lowest infeasibility will be chosen and proposed by the ambient agent to the team leader. A realistic ranking, from the most to the least infeasible parameter, could be:

1) openness (δ_A), because it seems that this personality characteristic is difficult to change over time;
2) expressiveness (ε_A) because this personality characteristic can be 'faked' (one can display emotions that are not experienced);
3) channel strength (α_{BA}) because it is easy to lower the channel strength's between a person and every other group member, by simply separating this individual from the group.

The formalization of the concepts is given in Table 4. The formalization of the dynamic relationships is as follows.

SDR1 Low emotion member identification

If the ambient agent believes that *A* has emotion level *V* at *T0*
and that the low threshold is *LT*
and $V \leq LT$
then the agent will believe that *A* is a low emotion member

```
belief(agent, has_emotion_level_at(A, V, T0))  &
belief(agent, low_threshold(LT))  &
```

V≤LT
→ belief(agent, low_emotion_member(A))

concept	*formalisation*
the agent believes that the low threshold for individual emotion levels is *LT*	belief(agent, low_threshold(LT:REAL))
the agent believes that the high threshold for individual emotion levels is *HT*	belief(agent, high_threshold(HT:REAL))
the agent believes that *A* is a low emotion member	belief(agent, low_emotion_member(A:AGENT))
the agent believes that *A* is a high emotion member	belief(agent, high_emotion_member(A:AGENT))
the agent believes that the adjustment extent is *AE*	belief(agent, adjustment_extent(AE:REAL))
the agent believes that an action option is to change the value *W1* for expressiveness of *A* to *W2*	belief(agent, action_option(adjust_to(expressiveness(A:AGENT), W1:REAL, W2:REAL)))
the agent believes that an action option is to change the value *W1* for channel strength from *A* to *B* to *W2*	belief(agent, action_option(adjust_to(channel_strength(A:AGENT, B:AGENT), W1:REAL, W2:REAL)))
the agent believes that an action option is to change the value *W1* for openness of *A* to *W2*	belief(agent, action_option(adjust_to(openness(A:AGENT), W1:REAL, W2:REAL)))
The agent believes that parameter *P* has adjustment infeasibility factor *IF*	belief(agent, has_infeasibility_factor(P: PARAMETER, IF:REAL))
The agent believes that the action option to adjust parameter *P* from *W1* to *W2* has infeasibility rank *R*	belief(agent, has_action_option_rank(adjust_to(P: PARAMETER, W1:REAL, W2:REAL), R:REAL)
The agent believes that the feasibility threshold is *FT*	belief(agent, feasiblity_threshold(FT:REAL))
The agent proposes the action to adjust parameter *P* from *W1* to *W2*	action_proposal(agent, adjust_to(P: PARAMETER, W1:REAL, W2:REAL))

Table 4: Formalization of concepts in the support model

SDR2 High emotion member identification

If the ambient agent believes that *A* has emotion level *V* at *T0*
and that the high threshold is *HT*
and *V≥HT*
then the agent will believe that *A* is a high emotion member

belief(agent, has_emotion_level_at(A, V, T0)) &
belief(agent, high_threshold(HT)) &
V≥HT
→ belief(agent, high_emotion_member(A))

SDR3 Heuristic generation of expressiveness action options for low emotion members

If the ambient agent believes that *A* is a low emotion member
and a group emotion deficient *ED* at *T* was identified
and it believes that the adjustment extent is *AE*
and the expressiveness of *A* is *E*
then the agent will believe that an action option is to change the value *E* for expressiveness to *E - AE*ED*E*

belief(agent, low_emotion_member(A)) &
assessment(agent, group_emotion_deficient_at(ED, T)) &
belief(agent, adjustment_extent(AE)) &

belief(agent, has_expressiveness(A, E))
→ belief(agent, action_option(adjust_to(expressiveness(A), E, E - AE*ED*E)))

SDR4 Heuristic generation of channel action options for low emotion members

If the ambient agent believes that *A* is a low emotion member
and that a group emotion deficient *ED* at *T* was identified
and that the adjustment extent is *AE*
and the channel from *A* to B has strength *C*
then the agent will believe that an action option is to change the value *C* for channel strength to *C - AE*ED*C*

belief(agent, low_emotion_member(A)) &
assessment(agent, group_emotion_deficient_at(ED, T)) &
belief(agent, adjustment_extent(AE)) &
belief(agent, has_channel_strength(A, B, C))
→ belief(agent, action_option(adjust_to(channel_strength(A, B), C, C - AE*ED*C)))

SDR5 Heuristic generation of openness action options for low emotion members

If the ambient agent believes that *A* is a low emotion member
and a group emotion deficient *ED* at *T* was identified
and it believes that the adjustment extent is *AE*
and the openness of *A* is *D*
then the agent will believe that an action option is to change the value *D* for openness to *D + AE*ED*(1-D)*

belief(agent, low_emotion_member(A)) &
assessment(agent, group_emotion_deficient_at(ED, T)) &
belief(agent, adjustment_extent(AE)) &
belief(agent, has_openness(A, D))
→ belief(agent, action_option(adjust_to(openness(A), D, D+ AE*ED*(1-D))))

SDR6 Heuristic generation of expressiveness action options for high emotion members

If the ambient agent believes that *A* is a high emotion member
and a group emotion deficient *ED* at *T* was identified
and it believes that the adjustment extent is *AE*
and that the expressiveness of *A* is *E*
then the agent will believe that an action option is to change the value *E* for expressiveness to *E + AE*ED*(1-E)*

belief(agent, high_emotion_member(A)) &
assessment(agent, group_emotion_deficient_at(ED, T)) &
belief(agent, adjustment_extent(AE)) &
belief(agent, has_expressiveness(A, E))
→ belief(agent, action_option(adjust_to(expressiveness(A), E, E + AE*ED*(1-E))))

SDR7 Heuristic generation of channel action options for high emotion members

If the ambient agent believes that *A* is a high emotion member
and that a group emotion deficient *ED* at *T* was identified
and that the adjustment extent is *AE*
and that the channel from *A* to B has strength *C*
then the agent will believe that an action option is to change the value *C* for channel strength to *C + AE*D*(1-C)*

belief(agent, high_emotion_member(A)) &
assessment(agent, group_emotion_deficient_at(ED, T)) &

belief(agent, adjustment_extent(AE)) &
belief(agent, has_channel_strength(A, B, C))
→ belief(agent, action_option(adjust_to(channel_strength(A, B), C, C + AE*ED*(1-C))))

SDR8 Heuristic generation of openness action options for high emotion members

If the ambient agent believes that *A* is a high emotion member
and a group emotion deficient *ED* at *T* was identified
and it believes that the adjustment extent is *AE*
and that the openness of *A* is *D*
then the agent will believe that an action option is to change the value *D* for openness to *D - AE*ED*D*

belief(agent, high_emotion_member(A)) &
assessment(agent, group_emotion_deficient_at(ED, T)) &
belief(agent, adjustment_extent(AE)) &
belief(agent, has_openness(A, D))
→ belief(agent, action_option(adjust_to(openness(A), D, D - AE*ED*D)))

SDR9 Ranking action options

If the ambient agent believes that an action option is to change the value *W1* for *P* to *W2*
and it believes that *P* has infeasibility factor *IF*
then the agent will believe that the action option has infeasibility rank *IF*(W2-W1)*

belief(agent, action_option(adjust_to(P, W1, W2)))
belief(agent, has_infeasibility_factor(P, IF))
→ belief(agent, has_action_option_rank(adjust_to(P, W1, W2), IF*(W2-W1)))

SDR10 Generation of action proposals

If the ambient agent believes that the action option has infeasibility rank *R*
and that the feasibility threshold is *FT*
and $R \leq FT$ and $R \geq -FT$
then it will generate the action option as an action proposal.

belief(agent, has_action_option_rank(adjust_to(P, W1, W2), R)) &
belief(agent, feasiblity_threshold(FT)) &
R≤FT & R≥-FT
→ action_proposal(agent, adjust_to(P, W1, W2))

8 Simulation Results of the Ambient Agent Model

To illustrate the group emotion support model described in previous sections, by a specific example, a specific scenario is addressed. The simulation for the analysis process for an absorption case is discussed in Section 8.1. Section 8.2 shows the simulation for the support mechanisms. Similarly simulation results for an amplification case are presented in Sections 8.3 and 8.4.

8.1 Simulation of analysis process of absorbed emotion contagion

In this section the simulation results of the analysis process are shown in an example scenario for absorption that represents a situation where the group emotion is happiness and is analyzed by the ambient agent. The LEADSTO software environment [8] has been used to perform a number of simulation experiments. In this example, the ambient agent generates beliefs on the individual emotion levels of three group members, named Arnie, Bernie and Charlie (see ADR2), and of the group emotion level at different points in time (see ADR5). The agent also assesses the (expected) group's emotion deficient at a future time point based on its belief of the group emotion level and the norm for the group emotion level. The norm of the group emotion can be set by the modeler and represents in this example an optimal level of happiness, at which the team can perform as optimal as possible. The norm was set to *0.62* in this example.

In this example scenario, Arnie is very happy (initial emotion level = *0.9*), he cannot receive other emotions (because his δ and receiving α's are zero), however, he is able to send emotions (his ε is not zero). Bernie is not happy (initial emotion level is *0.05*), he cannot receive emotions (his δ is zero), but he can send emotions (his ε is not zero). The contagion strengths toward Arnie and Bernie are zero. If these strengths stay zero, Arnie and Bernie will stay on the same emotion level. Finally, Charlie is also not happy (initial emotion level = *0.3*), but he can receive and send emotions quite strongly (because his δ is *0.9* and his ε is *1*). For an overview of the settings, see Table 5.

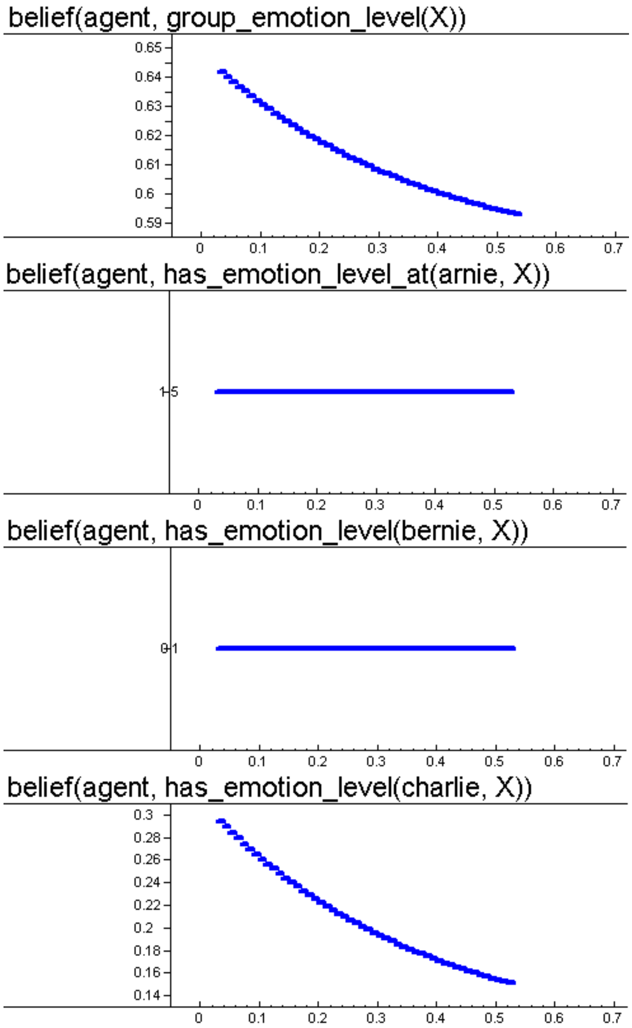

Figure 17: Simulation trace of the analysis process for emotion absorption

In Figure 17 a simulation trace is shown in which the horizontal axis represents time, and the vertical axis represents quantitative information about generation of ambient agent's beliefs on the individual and group emotion levels at different (future) time points. In this situation the total group emotion level goes from *0.64* to *0.59* in 500 time steps. This means that the group emotion level is above the norm of *0.62* at first, but will get below this norm later. The idea of the analysis model is that our ambient agent predicts this downward development early in time (long before it actually happens), so it can propose appropriate actions to the team leader early in time, to prevent this from happening. The simulations are based on step size $\Delta t = 0.1$.

	Arnie	*Bernie*	*Charlie*
Initial emotion level q	*0.9*	*0.05*	*0.3*
expressiveness ε	*0.6*	*0.5*	*1*
outgoing channel strengths α	*0*	*0.6*	*0.6*
openness δ	*0*	*0*	*0.9*

Table 5: Parameter Settings in Example Scenario

In Figure 16, on the x-axis time is represented and goes from *0* to *0.7*. This is the processing time of the ambient agent. The idea is that the agent reads the emotions of the persons at time point 0 and from that time point the ambient agent starts to generate beliefs on the development of the emotion levels of the group members and the group as a whole. This simulation can be found in all the graphs of the individual and group emotion. The developments of the emotion levels (simulated by the ambient agent from time point *0* to *0.5*) are estimated for the future time points *0* to *5*. Figure 18 shows the assessment of the expected emotion deficient by the ambient agent (see ADR6). Only the part where assessment is generated is shown. At time point *0.55* on the x-axis the ambient agent makes an assessment of the future group emotion level deficient for time point *5*. The ambient agent assesses that on future time point *5* indeed there is a group emotion deficient to be expected (of about *0.027*).

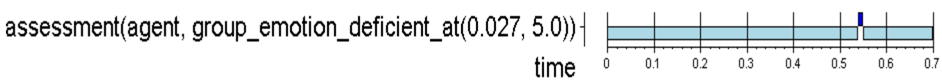

Figure 18: Simulation trace for assessment of emotion deficient

8.2 Simulation of the support process of absorbed emotion contagion

In this section the example scenario of the previous section is extended with the support of the ambient agent. The assumption is made that Arnie is working separately from Bernie and Charlie; i.e., he works in a different office than Bernie and Charlie. Therefore, Arnie's channels to Bernie and Charlie have strength *0*. Previously, the ambient agent assessed that there is a nonzero emotion deficient expected: the group emotion level slowly gets below the group emotion level norm of *0.62*. Therefore, based on its heuristics, the ambient agent detects which group members are high or low emotion members, and generates action options that decrease or increase parameters related to these members: expressiveness, openness or channel strength. After ranking these options, the agent proposes to the group leader those options that do not exceed a certain feasibility threshold. An example of (a part of) such a trace is shown in Figure 3. Here, time is on the horizontal axis, and state properties are on the vertical axis. A dark box indicates that a state property is true. Figure 3 shows that the ambient agent detects the high and/or low emotion members (Arnie is detected as a high emotion member and Bernie as a low emotion member; see SDR1 and SDR2), the action-options are ranked (see SDR9) and the ambient agent proposes the actions that do not exceed the feasibility threshold to the group leader (see SDR10).

8.3 Simulation of the assessment of amplified emotion contagion

In this section the simulation results of the analysis process are shown in an example scenario for an amplification case. The ambient agent generates beliefs on the individual emotion levels of three group members, named Arnie, Bernie and Charlie (see ADR2), and of the group emotion level at different points in time (see ADR5). The agent also assesses the (expected) group's emotion deficient at a future time point based on its belief of the group emotion level and the norm for the group emotion level. The norm of the group emotion can be set by the modeler and represents in this example an optimal level of happiness, at which the team can perform as optimal as possible. The norm was set to *0.60* in this example.

	Arnie	*Bernie*	*Charlie*
Initial emotion level q	*0.3*	*0.1*	*0.1*
Impact β	*0.3*	*1*	*0.6*
Contagion strength $\varepsilon_a*\alpha_{ab}$	*0.72*		
Contagion strength $\varepsilon_a*\alpha_{ac}$	*0.72*		
Contagion strength $\varepsilon_b*\alpha_{ba}$		*0.45*	
Contagion strength $\varepsilon_b*\alpha bc$		*0.45*	
Contagion strength $\varepsilon_c*\alpha_{ca}$			*0.09*
Contagion strength $\varepsilon_c*\alpha_{cb}$			*0.09*
Openness δ	*0.9*	*0.9*	*0.9*

Relevance ρ	*0.34*	*0.33*	*0.33*

In this example scenario Arnie, Bernie and Charlie are all not very happy (initial emotion levels are *0.3* and *0.1*). They are all very open to receive each other's happiness emotions; all have an openness δ of *0.9*.

Table 6: Overview of the parameter settings

Arnie can send his emotions most effectively to others, because his contagion strength, which is his channel α multiplied by his expressiveness ε, is *0.72* for both Bernie and Charlie. Bernie can send emotions less effectively, his contagion strength is *0.45*. Charlie can send his emotions with even less power: his contagion strength is *0.09*. For an overview of the settings, see Table 6.

In Figure 19 a simulation trace is shown in which the horizontal axis represents time, and the vertical axis represents quantitative information about generation of ambient agent's beliefs on the individual and group emotion levels at different (future) time points. In this situation, the total group emotion level goes from 0.49 downwards and through an upwards spiral mechanism to 0.58 in 500 time steps. This means that the group emotion level is always below the norm of 0.60. In this analysis model, our ambient agent predicts the future development of the group emotion level and this prediction shows that it will stay below the norm for all the future time steps. In this case it can propose appropriate actions to the team leader early in time, to help the group emotion level get above the norm faster. The simulations are based on step size $\Delta t = 0.1$.

On the x-axis in Figure 18, time goes from *0* to *1*. This time actually represents the processing time of the ambient agent. The idea is that the agent reads the emotions of the persons at time point *0* and from that time point the ambient agent starts to generate beliefs on the development of the emotion levels of the group members and the group as a whole. The developments of the emotion levels (simulated by the ambient agent from time point *0* to *0.5*) are estimated for real future time points *0* to *5*. At time point *0.5* on the x-axis, the agent makes the assessment of an expected emotion deficiency for real future time point *5*. The ambient agent assesses that on future time point 5, there is a group emotion deficiency to be expected (of about *0.04)*.

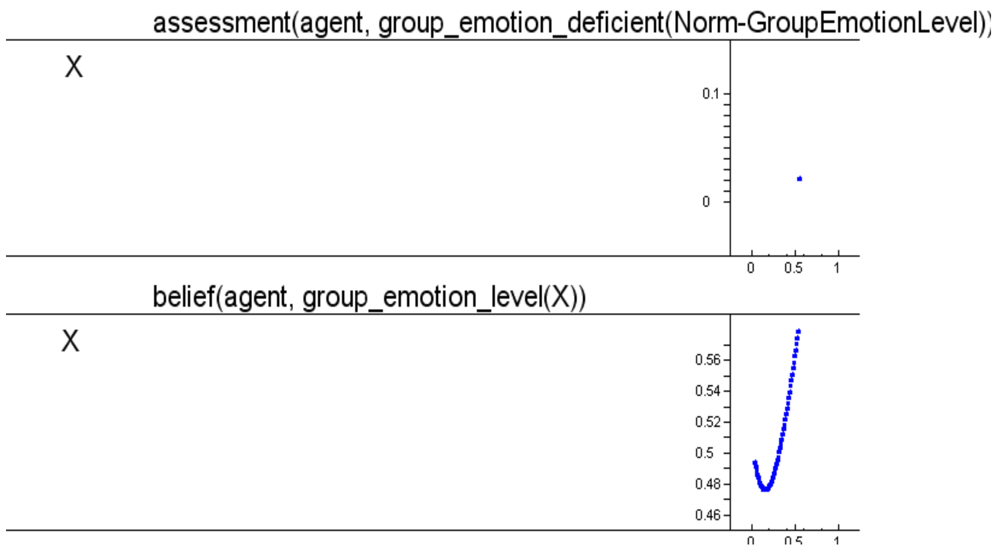

Figure 19: Simulation trace of the analysis process for emotion amplification

8.4 Simulation of the support process for amplified emotion contagion

In this section, the example amplification scenario of the previous section is extended with the support of the ambient agent. The assumption is made that Arnie, Bernie and Charlie are working on a task together that is perhaps stressful, since they are not very happy (initial emotion levels are *0.3* or *0.1*). Arnie is very charismatic and he works together a lot with Bernie and Charlie; this is represented in his high contagion strength. Charlie on the other hand is very introvert and therefore his contagion strength is weak. Bernie has a medium contagion strength. All three are open to receive happy emotions from others, since they all have a high level of openness. In the previous section it was shown that the ambient agent predicted the future development of the group emotion level, namely an upward spiral that still was below the norm at future time point *5*. Therefore, based on its heuristics, the ambient agent detects which group members are high or low emotion members, and generates action options that decrease or increase parameters related to these members: expressiveness or channel strength. After ranking these options, the agent proposes to the group leader those options that do not exceed a certain feasibility threshold. An example of (a part of) such a trace is shown in Figure 20. Here, time is on the horizontal axis, and state properties are on the vertical axis. A dark box indicates that a state property is true. Figure 19 shows that the ambient agent detects the high and/or low emotion members (Charlie is detected as a high emotion member and Bernie as a low emotion member; see SDR1 and SDR2), the action-options are ranked (see SDR9) and the ambient agent proposes the actions that do not exceed the feasibility threshold to the group leader (see SDR10).

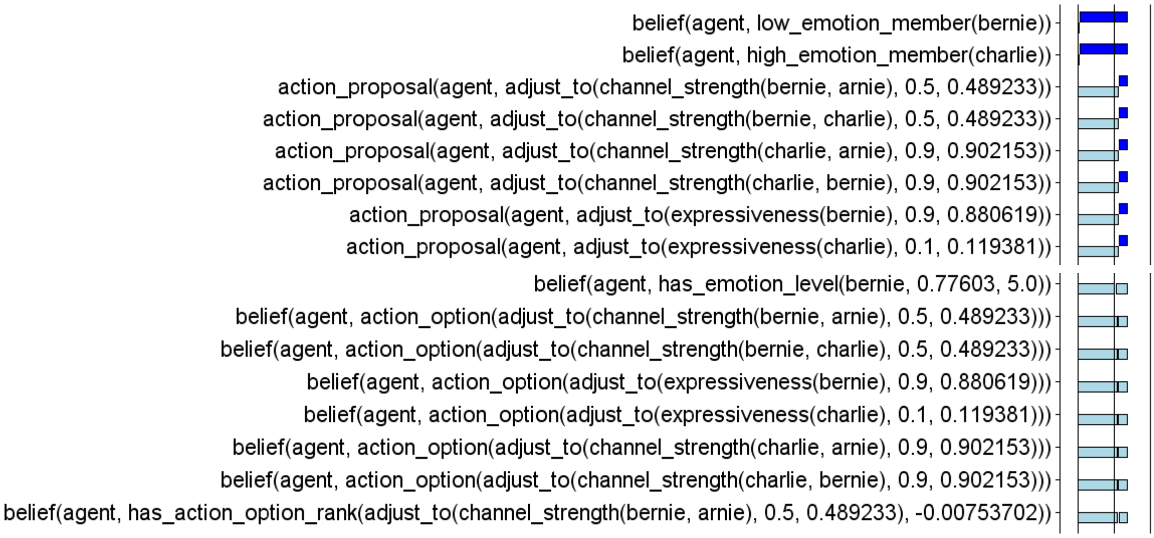

Figure 20: Simulation trace of a support process for amplified emotion contagion

9 Discussion

Within teams performing critical tasks, a team leader is responsible for a good spirit in the team. Due to high pressure, emotions within the team may easily take the form of a negative spiral. Therefore, it is challenging to regulate such patterns. Recent literature on emotion contagion spirals addresses how such spirals may occur. Most existing computational models of emotional processes represent emotion as a process or state that depends on observed stimuli by a single agent; e.g., [8], [22], [28]. These models of emotion differ from our proposed model, in that the focus in these models lies more on individual emotions, not on collective emotion. Recently researchers have started to investigate emotions in a social context more extensively. For the work reported in the current paper, more specific work on emotion contagion spirals was taken as a point of departure; cf. [9], [10], [11]. In the current paper, first a multi-agent-based model for emotion contagion spirals has been presented and analyzed. Although an extensive empirical validation is left for future work, it turned out that the model is able to produce various interesting emerging patterns as described (informally) in the psychological literature, including the upward and downward emotion spirals discussed in [10]. Although this is not an exhaustive proof, it is an important indication that the model behaves as expected. In contrast to most existing (symbolic) agent-based modeling approaches, the current approach represents a multi-agent system using numerical techniques.

Literature on computational models of emotion contagion is scarce. The only computational models that come close to the process modeling of this current work can be found in the area of social science, named social diffusion modeling. Examples of social diffusion models are: the diffusion of social movements like political interests and parties, see [17], and crowd behavior, as in emergency evacuation, see [23][5]. Most social diffusion models follow the diffusion of innovations model of Rogers, in which it is posed that the diffusion process of innovations proceeds in the form of an S-shaped curve: the contagion of an innovation starts slow, but then accelerates rapidly, followed by a rapid deceleration [26]. Even though social diffusion models can simulate the contagion of a certain innovation and use similar concepts as the current work does, such as a sender, receiver and communication channel, these computational models of social diffusion also differ from our model, in the way that they model the complex spread of innovations as diffusion that is asymmetric in time, irreversible, and nondeterministic. Our model of emotion contagion, models the continuous spread of emotions among the group members over time, which can have many patterns in it and is reversible in time.

The model for emotion contagion was taken as a point of departure for an ambient agent model that uses the computational model to assess the expected emotion levels at future time points, and to propose actions to the team leader to regulate these levels. The generic agent

[5] The question to what extent our model is able to simulate such completely different processes is beyond the scope of this paper. Although these processes share some characteristics with the process of emotion contagion, for other factors (e.g. openness, or the tendency to adapt emotions upward or downward) it is not trivial to find a counterpart.

model for human-aware ambient intelligence applications described in [5] was taken as a point of departure. One of the possible applications of the resulting ambient agent model could be analyzing and supporting group emotion in virtual meetings. For example, when two groups at two locations in the world are video-conferencing, a software agent could measure the group emotion of both groups and could show the emotion level of the other group to the group leaders. The ambient software agent could then, if necessary, provide support to the group leaders, e.g. when is the best time to let the other group make a decision, or how to calm the other group down after their anger level got too high during decision making.

Another important role of the ambient agent could be to control the impact of individual emotions on important collective decisions. According to Barsade [1,2], emotion contagion may have direct significance for social, political and economic behaviour. She states that "if people 'catch' each other's emotions, then this can influence their decisions accordingly. This can be problematic, however, if people are not aware that the mood they are in, or the subsequent actions, originated from someone else's emotions—not their own. For this reason, making people aware of the phenomena of emotional contagion is important. For example, emotional contagion could have an influence on how economic processes operate as the anxiety/worry—or exuberance—that originates with fewer people "ripples" out via emotional contagion to a larger group of people, influencing collective behavior" [30]. The process of 'catching' emotions of other agents is represented in our model by the combination of parameters shown in Table 1 (for instance, having a higher 'openness' δ_R enables the receiver to better 'catch' the emotion of the sender. If an ambient agent is able to analyse such mechanisms, it might also be able to make the people involved aware of them, thereby avoiding the pitfalls of collective decision making sketched above.

In follow-up research, more attention will be paid to the model's more detailed external validation of the model for emotion contagion spirals. The mathematical and automated analyses described above have been successfully performed to guarantee internal validity, and it fits to patterns described informally in (social) psychological literature. Nevertheless, this does not guarantee that the model is directly applicable to humans in a more detailed and more quantitative manner, and in particular it does not show which personality parameter values fit which person. Therefore, as a next step, a more detailed validation of the model in laboratory experiments is planned. The idea is to create a setting in which various humans interact in a room, while continuously being subject to (physiological) measurements (e.g., using emotion recognition approaches as discussed in [14], [34], [35], [36]) to assess their emotions. The obtained data can then be used in order to fine-tune the model using adaptive and machine learning techniques. This will not only provide a more detailed validation of the model, but also result in realistic parameter settings for different types of individuals.

Concerning further work, a number of factors can be refined or added to the model. For instance, recently a new perspective has emerged that describes leaders as the managers of group emotions [24]. According to this perspective, every group member can assume a leadership role by providing certainty and direction during times of ambiguity to create shared emotion within the group. The gender of this group leader also has an important impact on the emerging emotion contagion processes.

A final possibility to extend the model is to consider multiple emotions. Currently, the group contagion spirals of only one emotion can be modeled. It will be interesting, for example, to study the impact of simultaneous occurrences of happiness and anger within the same group, or the interaction between anger and fear within a group. For specific types of emotions, specific values may have to be estimated, e.g. α, δ, ρ. However, if also interaction between different emotions is to be addressed (for example, anger in one person affecting fear in another person), more specific work is needed, which is planned for the future.

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