School of Engineering and Advanced Technology
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282758 Simulation, Modelling and Optimisation

Orbital Tower Notes

Tension

The tension for any point in the tower comes from the difference between the gravitational and centrifugal forces, f_c - f_g. This must be integrated from the base of the tower to the point in question.

$$T = \varrho A \int_{r_m}^{R} \left(\omega^2 \cdot R - \frac{GM}{R^2} \right) dR$$

where R is the distance from the centre of Mars, r_m is the radius of Mars, M is the mass of Mars, ω is the orbital frequency of Mars, G is the gravitational constant and ϱA is the mass of the volume element and is assumed to be constant.

This can be integrated to give

$$T = \varrho A \left(\frac{\omega^2}{2} \left(R^2 - r_m^2 \right) + GM \left(\frac{1}{R} - \frac{1}{r_m} \right) \right)$$

Plotting the tension with height will create a curve starting at zero, reaching a maximum at the areostationary point and declining to zero at the end of the tower.

Wave Equation

The tower can be approximated to a string that behaves according to the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial R^2}$$

where u is the sideways displacement of the tower and c^2 is the tension divided by the mass of the volume element.

Using the Crank-Nicholson method to convert this to a difference equation gives:

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} = c^2 \frac{1}{2} \left(\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} + \frac{u_{i+1,j+1} - 2u_{i+1,j} + u_{i+1,j-1}}{k^2} \right)$$

where $u_{i,j}$ is the displacement at the ith time step and the jth height step up the tower, h is the length of a time step and k is the length of a height step.

This can be rearranged to give the future displacements in terms of current and past displacements:

$$\begin{bmatrix} -\frac{r}{2} & 1+r & -\frac{r}{2} \end{bmatrix} \mathbf{u}_{i+1} = \begin{bmatrix} \frac{r}{2} & 2-r & \frac{r}{2} \end{bmatrix} \mathbf{u}_i - \mathbf{u}_{i-1}$$

where $r = c^2h^2/k^2$ and u_i is a column vector of all the displacements of the tower at the ith time step.